

Title: Proposal to Represent the Slashed Zero Variant of Empty Set
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Abstract

As of Unicode Version 8.0, the symbol for empty set is encoded as the character U+2205 EMPTY SET, with no standardized variation sequences. In scientific publications, the symbol is typeset in one of three variant forms, chosen by notational style: a slashed circle, \emptyset , a slashed wide oval in the shape of a letter, \varnothing , or a slashed narrow oval in the shape of a digit zero, $\mathbb{0}$. The slashed circle and the slashed zero forms are the most widely used, and correspond to the LaTeX commands `\varnothing` and `\emptyset`, respectively.

Having one Unicode representation to map to, fonts that provide glyphs for more than one form of the symbol typically implement a stylistic variant or a mapping to a PUA code point. However, the widespread use of the slashed zero variant and its mapping to the main LaTeX command for the symbol, `\emptyset`, make it a candidate for a dedicated means to distinctly represent it in Unicode.

This document evaluates three approaches for representing in Unicode the slashed zero variant of the empty set symbol and proposes a solution based on a variation sequence. Additional related characters resulting from the investigation and needed to complete the solution are also proposed for encoding.

1. The empty set symbol

1.1. Historical references

The introduction of the modern symbol for the empty set is attributed to André Weil of the Nicholas Bourbaki group. According to his autobiography, Weil used the capital letter \emptyset of the Norwegian alphabet to create the symbol, as shown in Figure 1 [Weil]. The symbol appeared for the first time in Bourbaki's book *Théorie des ensembles*, published in 1939 [Bourbaki39].

reçus, ont été assez généralement approuvées. Bien plus tard la part que j'avais prise à ces débats me valut le respect de ma fille Nicolette quand je lui dis que j'étais personnellement responsable de l'adoption du symbole \emptyset pour l'ensemble vide, symbole dont elle venait d'apprendre l'usage à l'école. Le \emptyset appartenait à l'alphabet norvégien, et j'étais seul dans Bourbaki à le connaître.

Figure 1: Excerpt from Weil's autobiography mentioning the creation of the symbol for the empty set, in its slashed letter form [Weil, p. 119].

As the symbol gained adoption, its shape seems to have morphed at some point into that of a slashed circle: already by 1970 that form was used in a list of mathematical characters published by [Monotype]. The circular shape appears to be the most widely used today. As an example, a reproduction from an early edition of Bourbaki's *Théorie des ensembles* in a modern book shows the symbol in its original letter form, Ø, while the body text of the same book uses the circular form, ∅ [Mashaal, p. 52, 56].

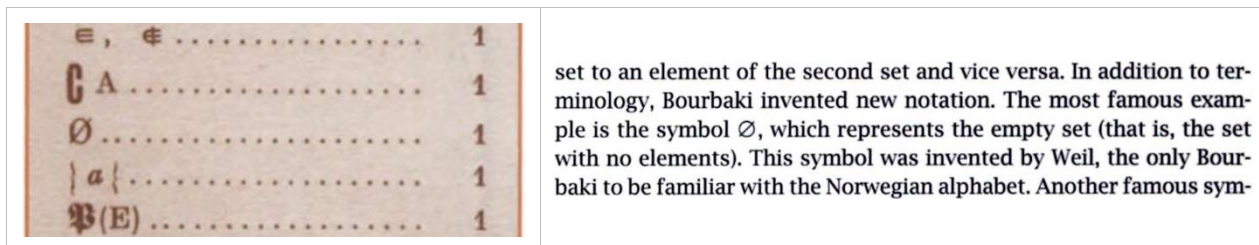


Figure 2: The original shape of the symbol for the empty set from an early edition of Bourbaki group's *Théorie des ensembles* reproduced in [Mashaal, p. 52], in contrast with the circular shape used in the body text of the same book (p. 56).

Starting in the late 1970s, Donald Knuth popularized the slashed zero form of the empty set symbol through TeX. Knuth chose the slashed zero glyph for the TeX command with that very name, `\emptyset`, published in one of the early TeX manuals in 1979 [Knuth]. Later, the American Mathematical Society added the slashed circle glyph, with the TeX command `\varnothing`, in its first provision of additional symbols, documented in 1985 [Beeton]. By default, i.e., without explicitly redefining them, those commands produce the same glyphs in today's LaTeX implementations [Pakin, Oetiker].

<p>math symbols:</p> <p>∞ <code>\infty</code></p> <p>∅ <code>\emptyset</code></p> <p># <code>\#</code></p>	<p>em (em unit), 40.</p> <p>Em-dash, 5–6, 27, 29, 80.</p> <p><code>\emptyset</code> (∅), 179.</p> <p>En-dash, 5–6, 17, 35, 148,</p> <p><code>\end</code> (terminate TeX), 23,</p>
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Figure 3: The original version of the slashed zero form, used for `\emptyset` in TeX [Knuth, p. 179, 189].

<i>Miscellaneous symbols of type Ord.</i>	
2 "7E	\hbar <code>\hbar</code>
2 "7D	\hslash <code>\hslash</code>
2 "3F	\varnothing <code>\varnothing</code>
1 "73	\textcircled{S} <code>\circledS</code>
2 "40	\nexists <code>\nexists</code>

Figure 4: The first mention of the TeX command `\varnothing`, using the circular shape [Beeton, p. 62].

Although publicized through TeX, the slashed zero form seems to have been in existence before its introduction in TeX. Figure 5 shows the symbol in a 1979 German handbook of mathematics (based on a book of Russian origin) whose overall appearance is that of metal type rather than TeX [Bronstein].

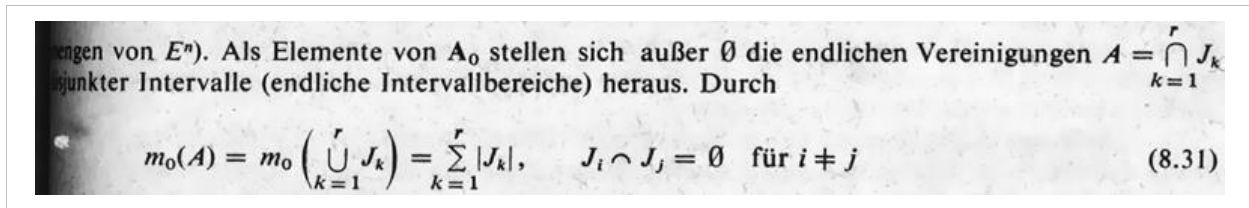


Figure 5: The slashed zero form, in a 1979 German handbook not typeset in TeX [Bronstein, p. 63].

1.2. Modern practice

The symbol U+2205 EMPTY SET is widely used in mathematical notation, to denote the concept of the set with no elements, or sometimes the empty word, and is employed in other fields, e.g., in linguistics to denote a morphological or phonological zero. In mathematical notation, the glyph of the symbol can vary from a perfectly circular shape, to a wide ellipse similar to the capital letter O, to a narrow ellipse shaped like the digit zero, crossed in all cases by a diagonal bar that protrudes from the ellipse: \emptyset , \emptyset , \emptyset . Evidence shows that all three variant forms of the empty set symbol are in use. Of the three, the slashed circle shape, \emptyset , and the slashed zero shape, \emptyset , are dominant in modern publications.

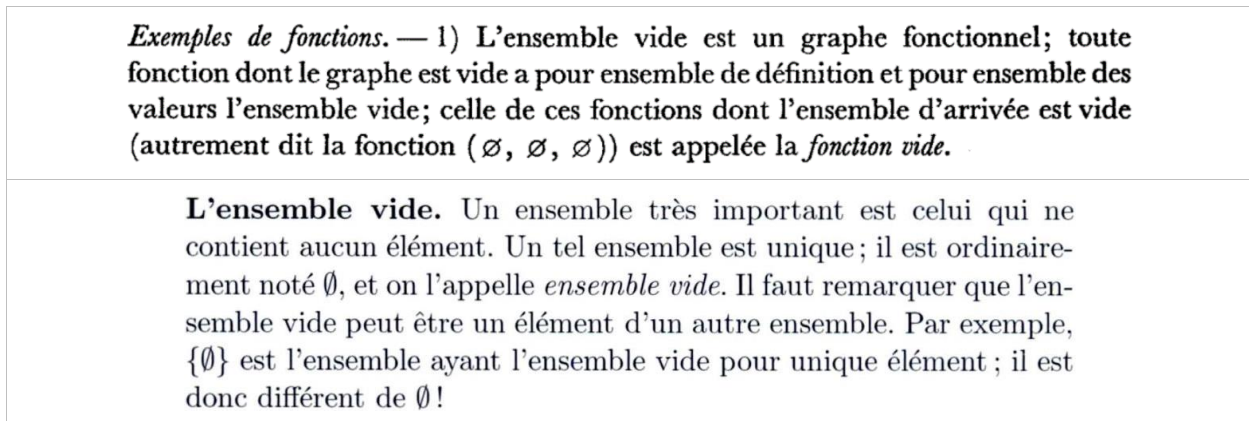


Figure 6: Widely used forms: slashed circle [Bourbaki07, p. E II.14] and slashed zero [Matoušek, p. 15].

The slashed letter form, \emptyset , while being the original shape of the symbol, is encountered to a lesser extent in more modern publications, as illustrated in Figure 7 in a scan from a Romanian translation of a compendium of mathematics, published in 1980. The English edition of that book, published in 1975, also uses the slashed letter form [Gellert].

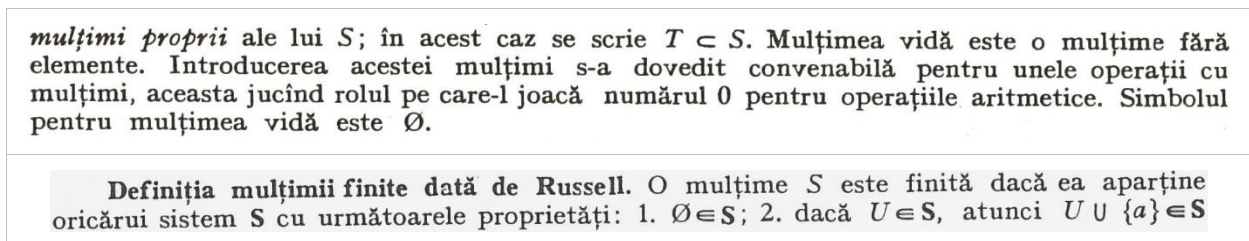


Figure 7: Slashed letter variant of the symbol, in context with digit 0 and letter O [Gellert, p. 397, 405].

A notable implementation of the wide oval form is the glyph for U+2205, \emptyset , in the Windows Cambria Math font, which is the default font for mathematical symbols in Microsoft Office applications. While not identical in shape to the capital letter \emptyset , the two are barely distinguishable.

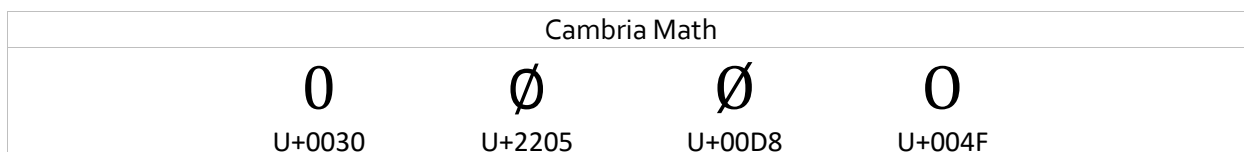


Figure 8: Wide oval form of U+2205 EMPTY SET in the Cambria Math font, in comparison with U+0030 DIGIT ZERO, U+00D8 LATIN CAPITAL LETTER O WITH STROKE, and U+004F LATIN CAPITAL LETTER O.

While semantically there is no difference between the various forms of the empty set symbol in a given domain, a preference for one variant or another can be seen in practice, driven by notational (rather than stylistic) considerations, as the evidence in Section 4 will illustrate. Some fonts provide glyphs for more than one variant. For instance, the STIX fonts use the circular form for the default glyph mapped to U+2205, following the style in the Unicode code charts, and relegate the slashed zero form to a stylistic set variant glyph with no Unicode mapping [STIX]. The lack of a mapping for the slashed zero form is due to the absence from Unicode of a supporting code point or variation sequence. Consequently, the specific shape of the symbol is not preserved when textual data is interchanged.

1.3. Current encoding

As of Unicode Version 8.0, a single character, U+2205 EMPTY SET, and no variation sequences with that character, are encoded for representing the empty set symbol in all of its forms [TUS80]. The representative glyph for U+2205 in the Unicode code charts uses the circular form and has been stable since the beginning:

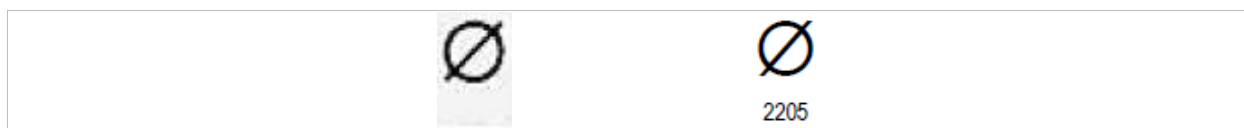


Figure 9: Representative glyphs for U+2205 in the code charts of Unicode Versions 1.0 and 8.0.

The UTC has previously considered the slashed zero variant for the empty set and has dismissed the prospect of representing it by a separate character or a variation sequence on the basis that it can be created with a combining character sequence of the form $\langle U+0030$ DIGIT ZERO, $U+0338$ COMBINING LONG SOLIDUS OVERLAY \rangle . As a result of that decision, that combining character sequence is documented in the subsection “Miscellaneous Symbols” in Section 2.7 of [UTR25] as a means to create the slashed zero variant.

However, combining overlay diacritics are not used productively: U+0338 can be used in combination with a base character to indicate negation of the concept denoted by that base character, but should not be used as a mechanism to create the visual of an atomic symbol which has a slash as an integral part of its shape. The slashed zero form of the empty set symbol is a preexisting symbol, with domain-specific semantics attached to it, such as the notion of empty set in mathematics. It is not a negated zero, which would be meaningless, any more than the slashed letter \emptyset is a negated letter O.

1.4. Other mappings

In Versions 2.0 and 3.0 of the *Mathematical Markup Language (MathML)* specification, as well as the *XML Entity Definitions for Characters* specification, both the slashed circle and the slashed zero forms are mapped to U+2205 [MathML2, MathML3, XMLEnt]. However, in MathML Version 2.0, the two forms were distinguished for a period of time, in three drafts between two stages of that specification: the W3C Candidate Recommendation of 2000-11-13 and the W3C Recommendation of 2001-02-21. In those drafts, the slashed zero form was mapped to a nonstandard variation sequence <U+2205, U+FE00>:

Name	Unicode	Glyph	Unicode Name	Description	Aliases
empty	02205-0FE00	∅	EMPTY SET	/emptyset - zero, slash	emptyset
emptyv	02205	∅	EMPTY SET	/varnothing - circle, slash	varnothing

Figure 10: Mappings of XML entities *empty* and *emptyv* in intermediate stages of MathML Version 2.0.

Both images used for the glyphs looked like slashed zeros, but the descriptions clearly distinguished the two forms. By the W3C Working Draft of 2002-12-19 stage of MathML Version 2.0, the two entities, *empty* and *emptyv*, were both mapped to U+2205 with no variation sequences.

MathML Version 3.0 refers to [XMLEnt] for the two entities, which switched from slashed zero glyphs to slashed circle glyphs between its 2010-02-11 W3C Proposed Recommendation and 2010-04-01 W3C Recommendation stages. The slashed circle glyphs are also used as of the latest version of [XMLEnt]:

Name	Unicode	Glyph	Unicode Name	Description	Aliases
empty	U+2205	∅	EMPTY SET	/emptyset - zero, slash	emptyset, emptyv, varnothing
emptyv	U+2205	∅	EMPTY SET	/varnothing - circle, slash	empty, emptyset, varnothing

Figure 11: Mappings of XML entities *empty* and *emptyv* in MathML Version 3.0.

2. Other slashed-oval symbols

The symbol for the empty set is one of several symbols with similar shape, all having different semantics. Some are distinctly encoded as of Unicode 8.0, others reuse the code point U+2205 (where it is possible to determine the underlying coded representation), and in other cases the mapping cannot be inferred from the rendered text.

2.1. Symbols with circular shape

Symbols with the shape of a slashed circle can be encountered in a variety of semantic domains. The common features of the symbols analyzed here are the circular shape and the trait that the diagonal

stroke protrudes from the circle. That trait distinguishes these symbols from other symbols where the stroke does not protrude, such as U+2298 CIRCLED DIVISION SLASH, \oslash , which are thus excluded from this analysis. Built-up symbols such as U+29B3 EMPTY SET WITH RIGHT ARROW ABOVE, $\emptyset\rightarrow$, although related to the symbols of interest, are also not relevant for this discussion.

Signs other than the empty set symbol but with the same visual characteristics are used to denote concepts including the following: diameter, average value, empty word, morphological or phonological zero, and slashed degree in musical notation. The first two are listed in a typeface catalog of mathematical symbols published by Monotype in 1970, sharing a common glyph with the symbol for the empty set.

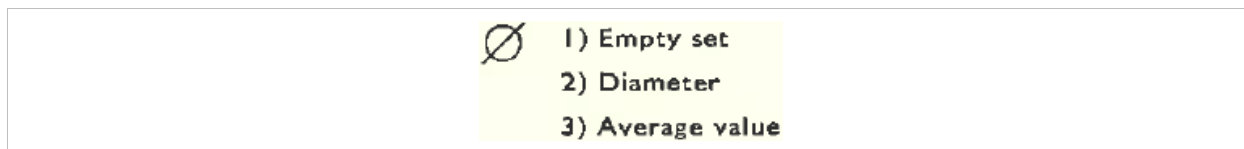


Figure 12: Slashed circle with three meanings, in a catalog of mathematical characters [Monotype].

The interpretation of *average value* is documented in [Pakin, p. 23]: “the average value of a variable x is written [...] by some people as “ $\emptyset x$ ” or “ $\oslash x$ ” (\diameter x or \varnothing x).” This notation can also be found on some Volkswagen car dashboards for average fuel consumption or speed [Photobucket]. It is interesting to note that the German for “average” is “Durchschnitt,” directly translated as “cut-through.”

As of Unicode 8.0, there are three separately encoded characters for the concepts mentioned earlier: U+2205 EMPTY SET, U+2300 DIAMETER SIGN, and U+1D1A9 MUSICAL SYMBOL DEGREE SLASH. In the code charts, U+2205 is annotated with “used in linguistics to indicate a null morpheme or phonological “zero””.



Figure 13: Representative glyphs for U+2205, U+2300, and U+1D1A9 in the Unicode 8.0 code charts.

2.2. The barred digit zero

The narrow oval form of the empty set symbol, while modeled as a slashed zero, must not be conflated with the variant of the digit zero with diagonal bar. The barred form of digit zero is sometimes used for contrasting the digit zero and the letter O, e.g., in programming environments [Bigelow]. That form is still a variant of U+0030 DIGIT ZERO and not a variant of the symbol U+2205 EMPTY SET. The glyphs can vary: they may have a bar with positive or negative slope, or have a dot instead of a bar. The common characteristics of barred zero glyphs are that all are mapped to U+0030 and are easily distinguished from the letter O.

Figure 14 compares a few barred glyphs for U+0030 DIGIT ZERO from the Windows Consolas font and a few monospaced fonts used with LaTeX [Robson, FontCat]. In Consolas, the barred form is the default glyph, and the hollow and dotted forms are mapped from the default via GSUB stylistic set substitutions ss07 and ss08. In the TTX font, the glyph has a bar of negative slope.

Consolas			Bera Mono	DejaVu Sans Mono	TXTT
Ø	0	⊘	⊘	⊘	⊘
Default	ss07	ss08			

Figure 14: Barred form of U+0030 DIGIT ZERO in Consolas (with glyph variants) and a few LaTeX fonts.

3. Representing the slashed zero variant of empty set and its implications

There are three conceivable approaches for distinctly representing the slashed zero variant of the empty set symbol in Unicode, corresponding to three models: a shape-based model, involving separate code points for different shapes; a semantic model, involving variation sequences of a single character that encodes the symbol for empty set semantically; and a hybrid model, which is a combination of the first two. The three approaches, with their pros and cons, are outlined below.

3.1. Shape-based model

This is the model of hyphen-minus or decimal point, in which dedicated characters are separately encoded for each shape and in which multiple characters can have the same semantics. There can be at least one character with explicitly ambiguous shape and semantics, such as U+002D HYPHEN-MINUS, and separate characters for the specific shapes and roles, such as U+2010 HYPHEN and U+2212 MINUS SIGN. Also, multiple characters can be used for the same semantics, such as U+002E FULL STOP and U+002C COMMA to represent a decimal point.

This model lets convention select a specific shape and deal with duplicate semantics between different characters, such as the decimal point vs. comma. As distinct code points are used, no variation sequences are needed. However, a disadvantage is that input and search are complicated by the additional choices in selecting or matching characters.

Applied to the symbol for the empty set, the shaped-based model would encode two separate characters for the variants of the symbol:

- A character with explicitly ambiguous shape – the existing U+2205 EMPTY SET
- A new character specifically for the slashed zero shape

An additional new character could also be encoded specifically for the slashed circle shape, but that is not strictly required by precedent. The existing, explicitly ambiguous character can be used for the dominant shape (slashed circle form) and the new, unambiguous character for the other main, though non-dominant, shape (slashed zero form). The existing character formally remains ambiguous, so glyph variation is expected, but font designers wishing to make a distinction would be advised to use the dominant shape, as they would, for instance, use mathematical style for Greek letters.

As characters are encoded by shape, in this approach the new character for the slashed zero shape could be used for other purposes than the narrow oval variant of empty set. The slashed letter form (the form

between slashed circle and slashed zero) would not be encoded as a new character, as that shape would be represented by the existing U+00D8 LATIN CAPITAL LETTER O WITH STROKE.

3.2. Semantic model

The semantic model requires one code point per semantic domain. In this model, there is a single character, U+2205 EMPTY SET, with well-defined semantics, viz. the semantics of empty set used in mathematical notation. The typographic style used in rendering the symbol varies by convention, but all typographic variants share the same semantics. As the identity of the character is defined by its precise semantics, symbols of similar shape but different meaning are separately encoded, one code point per semantic domain. The separate encoding of U+2205 EMPTY SET, U+2300 DIAMETER SIGN, and U+1D1A9 MUSICAL SYMBOL DEGREE SLASH aligns with this logic, even though there are also differences in glyph size and position, as seen in Figure 13.

To represent the various shapes established by the major notational conventions in current use, standardized variation sequences are defined, such as a variation sequence for the slashed zero variant of empty set. An older convention like the slashed letter variant, \emptyset , does not need to be given its own variation sequence, because its use in modern publications appears to be limited and there is indication that the actual letter was originally used [Bourbaki39]. Instead, the slashed zero shape \emptyset which is the barred digit zero rather than the empty set symbol would be separately represented by a new character or variation sequence, for its distinct semantics as well as to avoid any abuse of the variation sequence of slashed zero for anything other than the empty set.

Specifically, the semantic model would define the following entities:

- A character with precise semantics and ambiguous shape – the existing U+2205 EMPTY SET
- A new standardized variation sequence, $\langle U+2205, U+FE00 \rangle$, for the slashed zero form of the empty set symbol
- New characters encoded for the specific symbols for average value, empty word, and morphological or phonological zero

The main advantage of this model is that input and search are simplified and continue to work with the existing U+2205 EMPTY SET character. The semantic model is supported by the fact that the MathML entities *empty* and *emptyv* both used either the slashed circle or the slashed zero as representative shapes, which indicates that it is the semantics that defines their identity. Also in support is the fact that, for a period of time, MathML Version 2.0 used a nonstandard variation sequence, as shown earlier in Section 1.4. These observations indicate that the semantic solution fits the needs of the mathematical user community.

The disadvantage of the semantic model is that multiple characters need to be encoded, for all of the semantic domains in which slashed circle symbols are used. Furthermore, a means to represent the barred form of digit zero also needs to be defined. All this ensures that the semantics of U+2205 are guaranteed to be mathematical and that the proposed variation sequence $\langle U+2205, U+FE00 \rangle$ does not get abused for mimicking the barred digit zero.

3.3. Hybrid model

A compromise model is to encode both a dedicated character for the slashed zero *shape* and a variation sequence attached to U+2205 in its mathematical *semantics* of empty set.

In this model, U+2205 is shared for the symbols for empty set (with ambiguous shape), average value, empty word, and morphological or phonological zero. The approach is viable because evidence indicates that the last three symbols share the basic, rather than the varied, shape of the empty set symbol: LaTeX documentation [Pakin] and electronic display [Photobucket] for average value; use of LaTeX `\varnothing` for empty word, illustrated in Figure 18 [Jaerisch]; Unicode code chart annotation for morphological or phonological zero [TUS80]. If the shared use had instead been for the narrow variant, then that would have invalidated the approach because it would have made it susceptible to falling back to an inadequate shape when the variation selector is legitimately ignored.

Precisely, the hybrid model would define the following entities:


- A character with explicitly ambiguous shape – the existing U+2205 EMPTY SET
- A new character encoded specifically for the slashed zero shape
- A new standardized variation sequence, `<U+2205, U+FE00>`, for the slashed zero form of the empty set symbol

This approach offers the input and search advantages of the semantic model, via the variation sequence. It fits the needs of the mathematical user community, while providing a more robust solution to the broader user community, by using a new character with a determined shape, which does not risk falling back deficiently. The solution aligns with existing practice and also borrows from the practicality of the shaped-based model, reducing the number of characters to encode.

3.4. Proposed approach

Comparing the pros and cons of the three alternative models outlined above, and considering the precedent of U+2205 EMPTY SET and U+2300 DIAMETER SIGN being separately encoded, the semantic and hybrid models, and particularly the latter, seem better suited for solving the problem of distinctly representing the slashed zero variant of the empty set symbol. The corresponding proposal statement consists of a few parts, given below.

1. Define a standardized variation sequence for the slashed zero variant of the empty set symbol, as follows:

	<code><U+2205, U+FE00></code> EMPTY SET digit zero form
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The glyph of the variant is that of a digit zero with the diagonal bar protruding from the ellipse.

2. Replace the text in UTR #25 which suggests approximating visually the variant shape of empty set using the combining character sequence `<U+0030, U+0338>` with a recommendation to use the proposed standardized variation sequence `<U+2205, U+FE00>`.


3. Encode a new character specifically for the slashed zero shape. If the semantic model is deemed superior, then encode new characters for each semantic domain in which slashed circle symbols are used, as mentioned in Section 3.2.

Note: Fully qualified character names and code points, and a filled out proposal summary form, will be provided at a later date, following the consideration of this proposal by the UTC and a preliminary decision about the characters that qualify for encoding.

3.5. Auxiliary proposal

While orthogonal to the issue of the empty set, a consequence of the proposed solution is that a means to represent the barred form of digit zero is needed. This ensures that the proposed variation sequence <U+2205, U+FE00> is used only for its intended role.

The barred form of digit zero can be created either by encoding a new character:

	U+218C DIGIT ZERO WITH NONEMPTY COUNTER
---	---

or by defining a new standardized variation sequence:

	<U+0030, U+FE00> DIGIT ZERO with nonempty counter
---	---

This character or variation sequence would be limited to cases where the numbers do not need to be parsed. The glyph is somewhat generic, as no distinction is made between the variants of the digit zero with dot or bar slanted backwards.

Note: A proposal summary form will be provided later, if the UTC favors the encoding of a new character to the introduction of a standardized variation sequence for the barred digit zero.

3.6. Security considerations

The proposed variation sequence <U+2205, U+FE00> would explicitly result in a shape that is confusable with the barred glyph for the digit zero discussed in Section 2.2. The two shapes can already be confused, but only with fonts that implement the slashed zero form of empty set and the barred form of digit zero: cf. \emptyset and \emptyset . While the proposed variation sequence would add to the confusability of the two shapes, the security aspects need to be evaluated in context.

First, the character U+2205 is not commonly supported in identifiers, nor is a variation sequence used in comparing the internal forms of identifiers. This makes it unlikely that the variation sequence could be used as direct spoof (different but lookalike identifier). Second, there is already variety in the glyphs for digit zero across fonts, as sampled in Figure 14 and examined by other authors [Bigelow, Robson]. These observations mitigate the security concerns associated with the proposed introduction of a variation sequence for the empty set symbol.

4. Supporting evidence

4.1. Slashed zero variant of empty set

The slashed zero variant of the empty set symbol is widely used in current publications, as illustrated earlier in Figures 5 and 6. Additional evidence is provided in the following reproductions, which show the slashed zero variant in contexts where the choice of the variant form appears to be influenced by other notation in the same document, such as the digit zero.

$\{x_0, x_1, \dots, x_n\}$ — the set containing x_0, x_1, \dots, x_n and no other elements; \emptyset — the empty set; \in — the membership relation;
<p>There is one exception to the claim made in Exercise 19(b). Note that ${}^0\emptyset = \{\emptyset\}$, since the empty set is a function with empty domain and empty range. Therefore, in cardinal arithmetic, $0^0 = {}^0\emptyset = 1$. Recall that in calculus the number 0^0 is not defined; it is treated as an indeterminate form.</p>
<p>For example, if $\mathfrak{M} \models \text{ZFC}$, and $\psi(x)$ is the formula “$\forall y (y \notin x)$”, then $\psi^{\mathfrak{M}}$ is \mathfrak{M}'s version of the empty set. We shall write $\emptyset^{\mathfrak{M}}$ rather than $\psi^{\mathfrak{M}}$. Moreover, as we explained in Chapter 3, \emptyset is the set-theoretic version of zero. Therefore, we shall sometimes write $0^{\mathfrak{M}}$ instead of $\emptyset^{\mathfrak{M}}$. Other natural numbers have similar definitions, and we shall in general write $n^{\mathfrak{M}}$ for \mathfrak{M}'s version of the natural number n.</p>

Figure 15: Excerpts from [Just, p. xvii, 37, 102]. The choice of the slashed zero form seems to have been deliberate, as the slashed circle form would have been jarring in the context of the last two paragraphs.

<p>Lemma 2.12 ([11]). <i>For $M \in \mathcal{C}(R)$ where $M \neq 0$, $\text{Ass}(M) \neq \emptyset$; moreover, every associated prime is homogeneous and contains $\text{Ann}(M)$. In fact, if $\mathfrak{p} \in \text{Ass}(M)$, then $\mathfrak{p} = \text{ann}(x)$ for some homogeneous element $x \in M$.</i></p>

Figure 16: Excerpt from [Lynn, p. 312].

<p>Similarly, the commutative square on the left below is a system of categories indexed by the poset of all the subsets of $\{0, 1\}$. Its core consists of cores $F_\emptyset \subset \mathcal{F}_\emptyset$, $F_0 \subset \mathcal{F}_0$, $F_1 \subset \mathcal{F}_1$, and $F_{0,1} \subset \mathcal{F}_{0,1}$, making the right cube commutative:</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> </div> <div style="text-align: center;"> </div> </div>
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Figure 17: Excerpt from [Blomgren, p. 528].

4.2. Slashed circle as variant of empty set and symbol for empty word

The excerpt in Figure 18 illustrates the usage of the slashed circle form, by means of the LaTeX command `\varnothing`, for both empty set and empty word, which is relevant for the discussion in Section 3.3.

for each $i \in I$ there exists $j \in I$ such that $a(i, j) = 1$. The set of *A-admissible words* of length $n \in \mathbb{N}$ is given by

$$\Sigma^n := \{(\omega_1, \dots, \omega_n) \in I^n : a(\omega_i, \omega_{i+1}) = 1 \text{ for all } i \in \{1, \dots, n-1\}\},$$

and we set $\Sigma^0 := \{\emptyset\}$, where \emptyset denotes the empty word. Note that \emptyset will also be used to denote the empty set. The set of all finite *A-admissible words* is denoted by

$$\Sigma^* := \bigcup_{n \in \mathbb{N}} \Sigma^n.$$

Let us also define the *word length function* $|\cdot| : \Sigma^* \cup \Sigma \cup \{\emptyset\} \rightarrow \mathbb{N}_0 \cup \{\infty\}$, where for $\omega \in \Sigma^*$ we set $|\omega|$ to be the unique $n \in \mathbb{N}$ such that $\omega \in \Sigma^n$, for $\omega \in \Sigma$ we set $|\omega| := \infty$ and \emptyset is the unique word of length zero. For each $\omega \in \Sigma^* \cup \Sigma \cup \{\emptyset\}$ and

Figure 18: Excerpt from [Jaerisch, p. 5456].

References

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