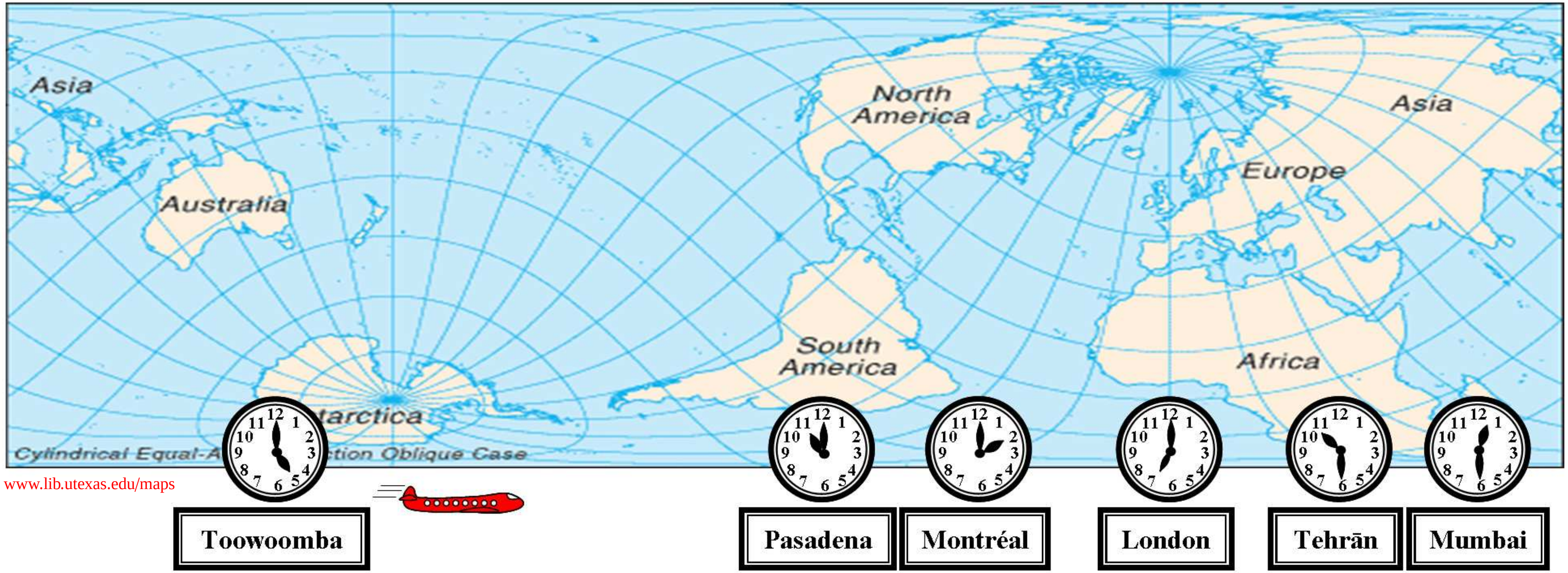
# THEOREM OF THE DAY

**The Sims Conjecture** *There is a function $f$ which, for any finite permutation group $G$ acting primitively on a set $\Omega$, bounds the order of a point stabilizer $G_\alpha$, $\alpha \in \Omega$, as $|G_\alpha| \leq f(k)$, $k$ being the size of a non-trivial orbit of $G_\alpha$ acting on $\Omega$.*




www.lib.utexas.edu/maps

**Imprimitive (aviation...)** Local time in Toowoomba may be thought of as an independent clock face: the permutation $(1_T\ 2_T\ \ldots\ 11_T\ 12_T)$. Circling the globe permutes local time hours: $(1_T\ 1_P\ 1_{Mo}\ 1_L\ 1_{Te}\ 1_{Mu})(2_T\ 2_P\ 2_{Mo}\ 2_L\ 2_{Te}\ 2_{Mu})\ldots(12_T\ 12_P\ 12_{Mo}\ 12_L\ 12_{Te}\ 12_{Mu})$, the actual rates of change depending on your trajectory. For $n$ cities, these two permutations generate a *wreath product* group $\mathbb{Z}_{12}$ wr $\mathbb{Z}_n$ acting on $12 \times n$ clock points. The action is *imprimitive*: clocks are always mapped to each other in entirety, never piecemeal; and the theorem does not apply: $G_{1_T}$, fixing time in Toowoomba, acts, regardless of $n$, with orbits of size $k = 12$, i.e. the other $n - 1$ clocks. But $|G_{T_1}| = 12^{n-1}$, since these clocks rotate independently.

**Primitive (... vs teleportation)** Now we equip our traveller with a separate cycle $(1_T\ 1_P\ 1_{Mo}\ 1_L\ 1_{Te}\ 1_{Mu})$, breaking out of the clock face to give a primitive group: the symmetric or, for even $n$, the alternating group. (By the way, Cameron, Neumann and Teague have shown that, for almost all $n$, no other primitive groups exist.) Fixing $1_T$ now leaves all other clock points free to move, so $|G_{1_T}| = (12n - 1)!$ or, for even $n$, $(12n - 1)!/2$; and orbits have size $k = 12n - 1$. So $f(k) = k!$ works here, but to assert the existence of an $f$ which works for *all* primitive groups is another matter!

A deep and difficult conjecture (1968) of Charles Sims, proved in 1983 by Peter Cameron, Cheryl Praeger, Jan Saxl and Gary Seitz using the then brand new Classification of the Finite Simple Groups. Thirty years on this remains the only proof, carrying therefore the official caveat ‘CFSG’.

**Web link:** www.asiapacific-mathnews.com/toc/0103.html: Cheryl Praeger: “Using the finite simple groups”.

**Further reading:** *Permutation Groups* by P.J. Cameron, Cambridge University Press, 1999.