Privacy-Preserving Energy Flow Control in Smart Grids

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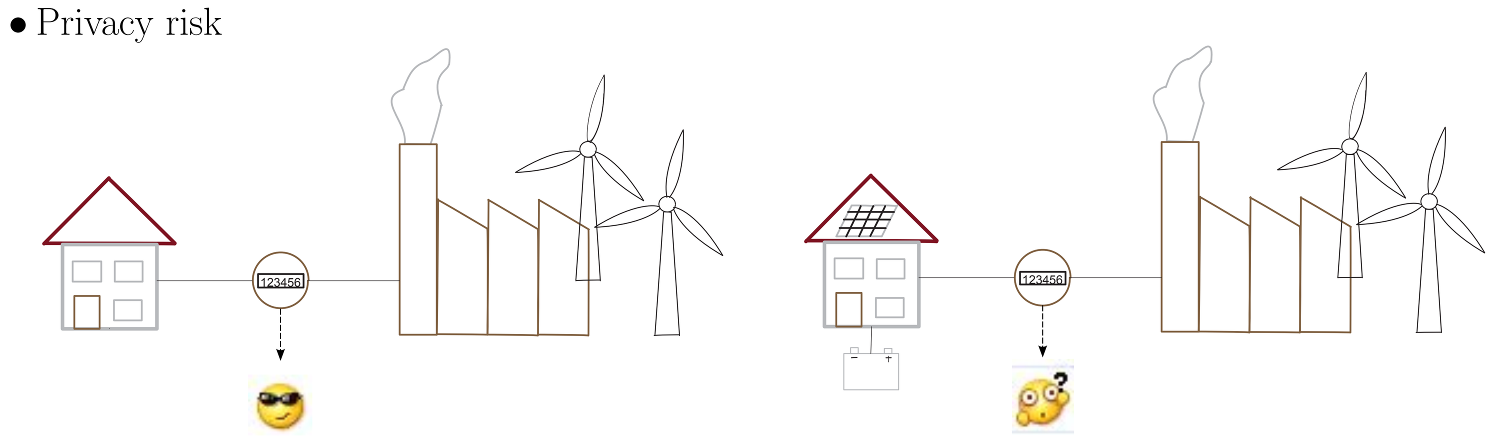
1 Motivation

Smart grid

- Monitor the grid more granularly.
- Predicate demand; detect failure; and adapt pricing.
- A more adaptive, reliable, and efficient grid

Smart meter

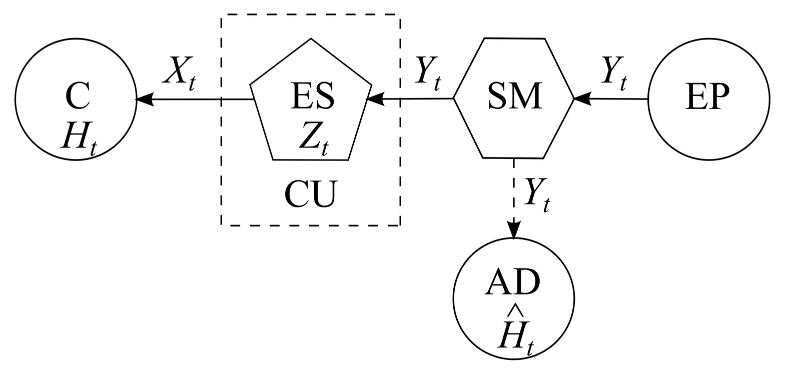
- Utility
- Privacy risk



State of arts

- Encryption
  - Do not work in the case of having inner threats.
- Distortion
  - Distort the energy supply from energy demand profile.
  - Use alternative energy sources or energy storage devices.
  - Information theoretic objective to maximize adversary uncertainty about the energy demand profile [1, 2, 3, 4, 6, 7]
  - Online algorithm to flatten smart meter readings [5]
  - Belief state MDP formulation [6, 7]
  - Detection theoretic objective [8]

2 Smart Grid Model



Settings

- H_t, $\hat{H}_t$, X_t, Y_t, and Z_t are defined on finite sets.
- Control strategy: $p_{Y_t|X_t,Z_t}$ under a constraint $z_t - z_{t+1} + y_t = x_t$
- Markov property:

$$P_{H_{t+1},X_{t+1},Z_{t+1},Y_{t+1}|H^t,X^t,Z^t,Y^t} = p_{Y_{t+1}|X_{t+1},Z_{t+1}} \cdot p_{X_{t+1}|H_{t+1},X_t} \cdot p_{Z_{t+1}|X_t,Z_t} \cdot p_{H_{t+1}|H_t}$$

3 Bayesian-Detection Operational Privacy Leakage

Assumptions

- **Informed** and **greedy** adversary
- Bayesian detection model of adversary behavior

Instantaneous privacy leakage

- **Minimal Bayesian risk** of the adversary to infer on the hypothesis H_t:

$$r_t = \sum_{y_t} \left\{ \min_{\hat{h}_t} \sum_{h_t,x_t,z_t} c(\hat{h}_t, h_t) \cdot p_{Y_t|X_t,Z_t}(y_t|x_t,z_t) \cdot p_{H_t,X_t,Z_t}(h_t,x_t,z_t) \right\}$$

4 Optimal Energy Flow Control

Optimal privacy-preserving design

$$\left\{p^*_{Y_t|X_t,Z_t}\right\}_{t=0}^{\infty} = \arg\max_{\{p_{Y_t|X_t,Z_t}\}_{t=0}^{\infty}} V$$

- Accumulated discounted minimal Bayesian risk: $V = \sum_{t=0}^{\infty} \beta^t \cdot r_t$ where $0 \le \beta < 1$
- Current control strategy affects the future as

$$p_{H_{t+1},X_{t+1},Z_{t+1}|H_t,X_t,Z_t} = p_{Z_{t+1}|X_t,Z_t} \cdot p_{X_{t+1}|H_{t+1},X_t} \cdot p_{H_{t+1}|H_t}.$$

How to solve it?

- View it as a **belief state Markov decision process**.
  - State: $s_t = (h_t, x_t, z_t) \in \mathcal{S}$
  - Belief state: $b_t = p_{H_t,X_t,Z_t} \in \mathcal{B}$
  - Action: $a_t = p_{Y_t|X_t,Z_t} \in \mathcal{A}$
  - Reward: $r_t(b_t, a_t)$
  - Policy: $\delta_t : \mathcal{B} \to \mathcal{A}$
  - Belief state update: $b_{t+1}(s_{t+1}) = \sum_{s_t \in \mathcal{S}} Pr(s_{t+1}|s_t) \cdot b_t(s_t)$

  Define $\Delta = \{\delta_0, \delta_1, \dots\}$. Then,

$$\Delta^* = \arg\max_{\Delta} V(\Delta, b_0), \text{ for all } b_0 \in \mathcal{B}.$$

- **Bellman's principle of optimality**

  For all $t \in \{0, 1, \dots\}$ and all $b \in \mathcal{B}$,

$$V(\Delta^*, b) = \max_{a \in \mathcal{A}} r_t(b, a) + \beta \cdot V(\Delta^*, b'(b, a)),$$
$$\delta^*_t(b) = \arg\max_{a \in \mathcal{A}} r_t(b, a) + \beta \cdot V(\Delta^*, b'(b, a)).$$

- Optimal privacy-preserving energy control strategies
  - Established algorithms to solve Δ^* and $V(\Delta^*, b_0)$
  - With b_0 and Δ^*, solve $\left\{p^*_{Y_t|X_t,Z_t}\right\}_{t=0}^{\infty}$ and $\{b_t\}_{t=1}^{\infty}$ successively.

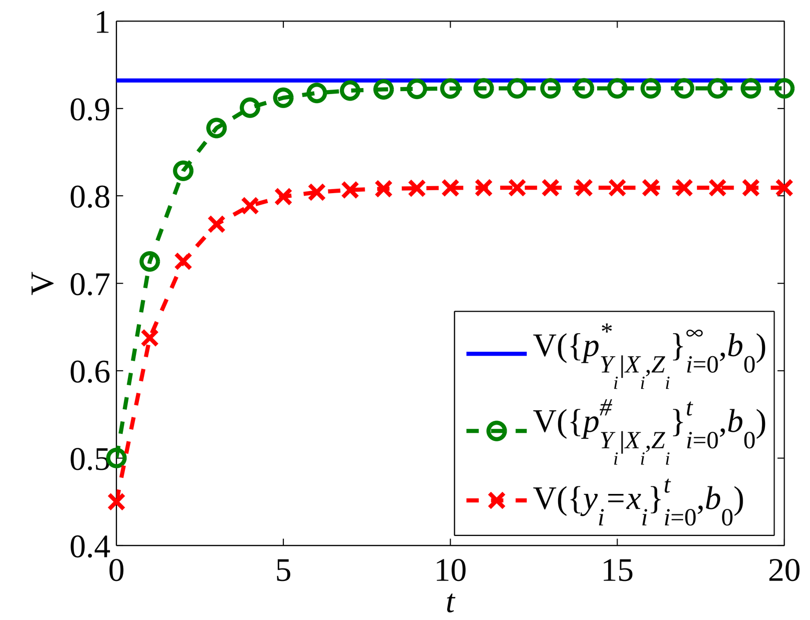
5 Instantaneous Energy Flow Control

Instantaneous privacy-preserving energy control strategy

$$p^{\#}_{Y_t|X_t,Z_t} = \arg\max_{p_{Y_t|X_t,Z_t} \in \mathcal{A}} r_t\left(b_t\left(b_0, \left\{p^{\#}_{Y_i|X_i,Z_i}\right\}_{i=0}^{t-1}\right), p_{Y_t|X_t,Z_t}\right)$$

- Ignore the impact of the current control strategy on the future.
- With b_0, solve $\left\{p^{\#}_{Y_t|X_t,Z_t}\right\}_{t=0}^{\infty}$ and $\{b_t\}_{t=1}^{\infty}$ successively.
- The optimization of instantaneous privacy-preserving energy control strategy $p^{\#}_{Y_t|X_t,Z_t}$ can be equivalently reformulated as a set of linear programmings.

6 Numerical Result



Settings

- Binary hypothesis H_t, e.g., "in" or "out"
- $x_t \in \{0, u\}$, $z_t \in \{0, u, 2u\}$, and $y_t \in \{0, u, 2u, 3u\}$
- $c(\hat{h}_t, h_t) = 0$ if $\hat{h}_t = h_t$; otherwise, $c(\hat{h}_t, h_t) = 1$. Then, r_t is the minimal probability of error of the adversary to infer on H_t.
- $\beta = 0.5$

An upper bound:

$$V\left(\left\{p^*_{Y_i|X_i,Z_i}\right\}_{i=0}^{\infty}, b_0\right) \le 1$$

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