A Moment Magnitude Scale

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The nearly coincident forms of the relations between seismic moment M_0 and the magnitudes M_L, M_s, and M_w imply a moment magnitude scale $\mathbf{M} = \frac{2}{3}\log M_0 - 10.7$ which is uniformly valid for $3 \lesssim M_L \lesssim 7$, $5 \lesssim M_s \lesssim 7\frac{1}{2}$, and $M_w \gtrsim 7\frac{1}{2}$.

It is well known that the most widely used earthquake magnitude scales, M_L (local magnitude), M_s (surface wave magnitude), and m_b (body wave magnitude), are, in principle, unbounded from above. It is equally well known that, in fact, they are so bounded, and the reasons for this are understood in terms of the operation of finite bandwidth instrumentation on the magnitude-dependent frequency characteristics of the elastic radiation excited by earthquake sources. Using just these ideas, *Hanks* [1979] demonstrated how the maximum reported $m_b \simeq 7$ and maximum reported $M_s \simeq 8.3$ can be rationalized rather precisely. (The upper limit to m_b at $\simeq 7$ occurs when m_b is determined from body wave amplitudes at periods near 1 s, for example, when m_b is determined from World-Wide Standard Seismograph Network (WWSSN) data. Prior to the establishment of the WWSSN in the early 1960's, many m_b values were determined from longer-period amplitude measurements, and this is especially so in the case of the larger earthquakes [*Geller and Kanamori,* 1977]. Many of these older, longer-period values are considerably greater than 7; this period dependence of m_b is also understood in terms of frequency-dependent source excitation.) Similarly, the maximum M_L reported in southern California after more than 40 years of magnitude determination is 6.8 [*Hileman et al.*, 1973], the values of 7.1 for the earthquake of December 31, 1934, and 7.7 for the Kern County earthquake of July 21, 1952, listed there being M_s. Although *Kanamori and Jennings* [1978] have obtained M_L slightly larger than 6.8 by synthesizing Wood-Anderson seismograms from available strong-motion accelerograms and *Bolt* [1978] has recently determined $M_L = 7.2$ for the Kern County earthquake from distant stations, it seems likely that the upper limit to M_L is also near 7, principally because m_b and M_L are both obtained from amplitudes at ~1-s period and because of the form of the correlation between m_b and M_L [*Gutenberg and Richter,* 1956].

Thus the magnitude scales M_L, M_s, and m_b are said to saturate at large magnitude. Just as in the case of peak acceleration data at a fixed distance R [*Hanks and Johnson,* 1976; *Hanks,* 1979], M_L, M_s, and m_b for crustal earthquakes saturate for the same physical reason: for large enough earthquakes, all of these narrow-band time domain amplitude measurements no longer measure gross faulting characteristics but only limiting conditions on localized failure along crustal fault zones. Peak acceleration data at $R \simeq 10$ km no more measure gross source properties for an earthquake with seismic moment $M_0 \gtrsim 10^{25}$ dyn cm than does m_b or M_L for an earthquake of $M_0 \gtrsim 10^{27}$ dyn cm or M_s for an earthquake of $M_0 \gtrsim 10^{29}$ dyn cm.

Hanks and Thatcher [1972] pointed out that a magnitude scale based directly on an estimate of the radiated energy, rather than the converse, would not only circumvent the difficulties associated with characterizing earthquake source strength with narrow-band time domain amplitude measurements, specifically magnitude saturation, but had become practical with the increased understanding of the gross spectral characteristics of earthquake sources that developed in the early 1970's. *Kanamori* [1977] realized this possibility by independently estimating the radiated energy E_s with the relation

$$E_s = \frac{\Delta\sigma}{2\mu} M_0 \tag{1}$$

where $\Delta\sigma$ is the earthquake stress drop and μ is the shear modulus, reducing (1) to

$$E_s = \frac{1}{2 \times 10^4} M_0 \tag{2}$$

by taking advantage of the constancy of earthquake stress drops for shallow earthquakes [*Aki,* 1972; *Thatcher and Hanks,* 1973; *Kanamori and Anderson,* 1975; *Hanks,* 1977], and using (2) in the Gutenberg-Richter relation between E_s and M_s

$$\log E_s = 1.5 M_s + 11.8 \tag{3}$$

where E_s is in ergs. The idea is that if M_s is bounded, so too is E_s as obtained from (3), but if E_s is known independently from (2), it may be used on the left-hand side of (3) to determine a magnitude M_w that will not saturate. A significant feature of *Kanamori*'s [1977] definition of M_w by (3) through use of (2) is that he found that M_w so defined is quite similar to M_s for a number of earthquakes with $M_s \lesssim 8$, that is, well below the saturation level of M_s. This agreement attests to the general validity of both the Gutenberg-Richter E_s-M_s relation (3) for $M_s \lesssim 8$ and the use of (2) to estimate E_s independently.

A second important feature of M_w as defined by *Kanamori* [1977] is that it is intrinsically a moment magnitude scale. This moment magnitude relation is, upon substituting (2) on the left-hand side of (3) and M_w for M_s on the right-hand side of (3),

$$\log M_0 = 1.5 M_w + 16.1 \tag{4}$$

which is remarkably coincident with the M_0-M_s relationship empirically defined by *Purcaru and Berckhemer* [1978] for $5 \lesssim M_s \lesssim 7\frac{1}{2}$:

$$\log M_0 = 1.5 M_s + (16.1 \pm 0.1) \tag{5}$$

and the M_0-M_L relationship empirically defined by *Thatcher and Hanks* [1973] for southern California earthquakes ($3 \lesssim$

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$M_L \lesssim 7$):

$$\log M_0 = 1.5M_L + 16.0 \quad (6)$$

Thus a single moment magnitude **M** may be written from (4), (5), and (6):

$$\mathbf{M} = \tfrac{2}{3} \log M_0 - 10.7 \quad (7)$$

Apart from the scatter of the observations about the empirically defined relations (3), (5), and (6), **M** as defined by (7) is uniformly valid with respect to $3 \lesssim M_L \lesssim 7$, $5 \lesssim M_s \lesssim 7\frac{1}{2}$, and M_w at larger magnitude. To the extent that coda duration magnitudes used extensively for $M_L \lesssim 3$ earthquakes are tied to M_L [e.g., *Lee et al.*, 1972], **M** as given by (6) should apply to them as well, although it would be desirable to verify this with M_0-coda duration data.

Table 1 presents M_0, M_L, M_s, and **M** calculated from (7) for a number of significant southern California earthquakes between 1918 and 1973, and Table 2 presents these parameters, as available, for five large California earthquakes between 1857 and 1906. For the earthquakes between 1918 and 1973, there is, on balance, good agreement between M_L, M_s, and **M**. In the case of the Imperial Valley (1940) earthquake, M_L is 0.6 units less than **M**, revealing anomalously low 1-s ground motion amplitudes for what is otherwise a fairly large earthquake by California standards. In the case of the Kern County (1952) main shock, M_L is 0.5 units less than M_s and 0.3 units less than **M**, almost certainly reflecting the saturation level of M_L around 7. In the case of the Point Mugu (1973) earthquake the large M_L relative to M_s and **M** reflects anomalously large 1-s excitation for an earthquake with such a small M_0, in turn suggesting a relatively high stress drop [*Ellsworth et al.*, 1973]. Similarly, M_L for the Desert Hot Springs (1948) earthquake is 0.5 units larger than **M**, which may again reflect a high stress drop [*Thatcher and Hanks*, 1973], but M_s, although it is apparently less well determined, it also larger than **M** by the same amount. As is the case for the observational scatter of the moment magnitude pairs from the appropriate empirical relation (i.e., equations (5) or (6)), deviations of **M** from M_s or M_L can in most cases be attributed to variable stress drop or saturation of M_L or M_s.

TABLE 1. Seismic Moments and Magnitudes for Southern California Earthquakes (1918–1973)

Date	M_0, $\times 10^{25}$ dyn cm	M_L	M_s	M
April 21, 1918	15		6.8	6.8
July 23, 1923	1		6¼	6.0
June 29, 1925	20		6¼	6.8
Nov. 4, 1927	100, 65*		7.3	7.3, 7.2
March 11, 1933	2	6.3	6¼	6.2
May 19, 1940	30	6.4†	6.7	7.0
July 1, 1941	0.9	5.9	5.9	6.0
Oct. 21, 1942	9	6.5	6½	6.6
March 15, 1946	1	6.3	6¼	6.0
April 10, 1947	7	6.2	6.4	6.5
Dec. 4, 1948	1	6.5	6.5±	6.0
July 21, 1952 (main shock)	200	7.2‡	7.7	7.5
July 21, 1952 (aftershock)	3	6.4		6.3
July 29, 1952 (aftershock)	3	6.1		6.3
March 19, 1954	4	6.2		6.4
April 9, 1968	6	6.4	6.7	6.5
Feb. 9, 1971	10	6.4	6.6	6.6
Feb. 21, 1973	0.1	5.9	5.2	5.3

Unless otherwise specified, M_0 entries are from *Hanks et al.* [1975], and M_L entries from *Hileman et al.* [1973]. All but the last three M_s entries are from *Gutenberg and Richter* [1954], taken as M_s according to *Geller and Kanamori* [1977]. M_s for April 9, 1968, and February 9, 1971, are from *Kanamori and Anderson* [1975], and M_s for February 21, 1973, is from the National Earthquake Information Service, U.S. Geological Survey.

*From *Yeh* [1975].

†From *Trifunac and Brune* [1970].

‡From *Bolt* [1978].

TABLE 2. Seismic Moments and Magnitudes for Some Large California Earthquakes (1857–1906)

Date	M_0, $\times 10^{25}$ dyn cm	M_s	M
Jan. 9, 1857	900, 530–870*		7.9, 7.8–7.9
March 26, 1872	500		7.8
Feb. 9, 1890	15		6.8
Dec. 25, 1899	15		6.8
April 18, 1906	400,† 350–430*	8¼‡	7.7, 7.7

Unless otherwise specified, M_0 entries are from *Hanks et al.* [1975].

*From *Sieh* [1977].

†From *Thatcher* [1975].

‡From *Gutenberg and Richter* [1954].

There are several interesting features of Table 2, all associated with the three 'great' California earthquakes of the historical record, the Fort Tejon (1857), Owens Valley (1872), and San Francisco (1906) earthquakes. First, none of the earthquakes can be classified as a great earthquake on the **M** scale. *Kanamori* [1977] lists three dozen earthquakes between 1904 and 1969 larger than the San Francisco earthquake, and in fact there are more than this, since the M_0 given here for this earthquake is approximately 2½ times smaller than that given by *Kanamori* [1977]. Second, these three earthquakes all have very similar M_0 and **M**, suggesting that an upper limit of $M_0 \simeq 10^{28}$ dyn cm and $\mathbf{M} \simeq 8.0$ may exist for California earthquakes. This upper limit is physically reasonable if, for California earthquakes, the seismogenic depth of faulting does not exceed 15–20 km and if fault lengths do not exceed several hundred kilometers. Finally, these three earthquakes are only somewhat larger than the Kern County (1952) earthquake, for which close-in instrumental ground motion records are available. While the coverage is far from ideal, the importance of these records for aseismic design practices and the mitigation of other earthquake hazards is obvious enough, if indeed the Kern County earthquake is near the upper limit in source strength for California earthquakes.

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