

Voting Theory

In many decision making situations, it is necessary to gather the group consensus. This happens when a group of friends decides which movie to watch, when a company decides which design to produce, and when a democratic country elects its leaders.

While the basic idea of voting is fairly universal, the method by which those votes are used to determine a winner can vary. Amongst a group of friends, you may decide upon a movie by voting for all the movies you're willing to watch, with the winner being the one with the greatest approval. A company might eliminate unpopular designs then revote on the remaining. A country might look for the candidate with the most votes.

In deciding upon a winner, there is always one main goal: to reflect the preferences of the people in the most fair way possible.

Preference Schedules

To begin, we're going to need more information than a traditional ballot normally provides. A traditional ballot usually asks you to pick your favorite from a list of choices. This ballot fails to provide any information on how a voter would rank the alternatives if their first choice was unsuccessful.

A **preference ballot** is a ballot in which the voter ranks the choices in order of preference.

Example: A vacation club is trying to decide which destination to visit this year: Hawaii (H), Orlando (O), or Anaheim (A). Their votes are shown below:

	Bob	Ann	Marv	Alice	Eve	Omar	Lupe	Dave	Tish	Jim
1 st choice	A	A	O	H	A	O	H	O	H	A
2 nd choice	O	H	H	A	H	H	A	H	A	H
3 rd choice	H	O	A	O	O	A	O	A	O	O

These individual ballots are typically combined into one **preference schedule**, which shows the number of voters in the top row that voted for each option:

	1	3	3	3
1 st choice	A	A	O	H
2 nd choice	O	H	H	A
3 rd choice	H	O	A	O

Notice that by totaling the vote counts across the top of the preference schedule we can recover the total number of votes cast.

Plurality

The voting method we're most familiar with in the United States is the **plurality method**. In this method, the choice with the most first-preference votes is declared the winner. Ties are possible, and would have to be settled through some sort of run-off vote.

This method is sometimes mistakenly called the majority method, but it is not necessary for a choice to have gained a majority of votes to win. A majority is over 50%; it is possible for a winner to have a plurality without having a majority.

Example:

In our election from above, we had the preference table:

	1	3	3	3
1 st choice	A	A	O	H
2 nd choice	O	H	H	A
3 rd choice	H	O	A	O

For the plurality method, we only care about the first choice options. Totaling them up:

Anaheim: 4 votes

Orlando: 3 votes

Hawaii: 3 votes

Anaheim is the winner.

What's wrong with plurality?

The election above may seem totally clean, but there is a problem lurking that arises whenever there are three or more choices. Looking back at our preference table, how would our members vote if they only had two choices?

Anaheim vs Orlando: 7 out of the 10 would prefer Anaheim

	1	3	3	3
1 st choice	A	A	O	H
2 nd choice	O	H	H	A
3 rd choice	H	O	A	O

Anaheim vs Hawaii: 6 out of 10 would prefer Hawaii

	1	3	3	3
1 st choice	A	A	O	H
2 nd choice	O	H	H	A
3 rd choice	H	O	A	O

This doesn't seem right, does it? Anaheim just won the election, yet 6 out of 10 voters would prefer Hawaii! That hardly seems fair. Condorcet noticed how this could happen and for him we name our first **fairness criterion**. The fairness criteria are statements that seem like they should be true in a fair election.

Condorcet Criterion: If there is a choice that is preferred in every one-to-one comparison with the other choices, that choice should be the winner. We call this winner the Condorcet Winner.

In the example above, Hawaii is the Condorcet Winner. (Check for yourself that Hawaii is preferred over Orlando)

Example: Consider a city council election in a district that is 60% democratic voters and 40% republican voters. Even though city council is technically a nonpartisan office, people generally know the affiliations of the candidates. In this election there are three candidates: Don and Key, both democrats, and Elle, a republican. A preference schedule for the votes looks as follows:

	342	214	298
1 st choice	Elle	Don	Key
2 nd choice	Don	Key	Don
3 rd choice	Key	Elle	Elle

We can see a total of $342+214+298=854$ voters participated in this election. Computing percentage of first place votes:

Don: $214/854 = 25.1\%$

Key: $298/854 = 34.9\%$

Elle: $342/854 = 40.0\%$

So in this election, the democratic voters split their vote over the two democratic candidates, allowing the republican candidate Elle to win under the plurality method with 40% of the vote.

Analyzing this election closer, we see that it violates the Condorcet Criterion. Analyzing the one-to-one comparisons:

Elle vs Don: 342 prefer Elle; 512 prefer Don

Elle vs Key: 342 prefer Elle; 512 prefer Key

Don vs Key: 556 prefer Don; 298 prefer Key

So even though Don had the smallest number of first-place votes in the election, he is the Condorcet Winner, being preferred in every one-to-one comparison with the other candidates.

Insincere Voting

Situations like the one above, when there are more than one candidate that share somewhat similar points of view, can lead to **insincere voting**. Insincere voting is when a person casts a ballot counter to their actual preference for strategic purposes. In the case above, the democratic leadership might realize that Don and Key will split the vote, and encourage voters to vote for Key by officially endorsing him. Not wanting to see their party lose the election, as happened in the scenario above, Don's supporters might insincerely vote for Key, effectively voting against Elle.

Instant Runoff Voting

Instant Runoff Voting (IRV), also called Plurality with Elimination, is a modification of the plurality method that attempts to address the issue of insincere voting. In IRV, voting is done with preference ballots, and a preference schedule is generated. The choice with the *least* first-place votes is then eliminated from the election, and any votes for that candidate are redistributed to the voters' next choice. This continues until a choice has a majority (over 50%).

This is similar to the idea of holding runoff elections, but since every voter's order of preference is recorded on the ballot, the runoff can be computed without requiring a second costly election.

This voting method is used in several political elections around the world, including election of members of the Australian House of Representatives, and for county positions in Pierce County, Washington (until it was eliminated by voters in 2009). A version of it is used by the International Olympic Committee to select host nations.

Example: Consider the preference schedule below, in which a company's advertising team is voting on five different advertising slogans, called A, B, C, D, and E here for simplicity.

Round 1: Initial votes

	3	4	4	6	2	1
1 st choice	B	C	B	D	B	E
2 nd choice	C	A	D	C	E	A
3 rd choice	A	D	C	A	A	D
4 th choice	D	B	A	E	C	B
5 th choice	E	E	E	B	D	C

If this was a plurality election, note that B would be the winner with 9 votes, compared to 6 for D, 4 for C, and 1 for E.

There are total of $3+4+4+6+2+1 = 20$ votes. A majority would be 11 votes. No one yet has a majority, so we proceed to elimination rounds.

Round 2: We make our first elimination. Choice A has the fewest first-place votes, so we remove that choice, shifting everyone's options to fill the gaps.

	3	4	4	6	2	1
1 st choice	B	C	B	D	B	E
2 nd choice	C	D	D	C	E	D
3 rd choice	D	B	C	E	C	B
4 th choice	E	E	E	B	D	C

Still no choice has a majority, so we eliminate again.

Round 2: We make our second elimination. Choice E has the fewest first-place votes, so we remove that choice, shifting everyone's options to fill the gaps.

	3	4	4	6	2	1
1 st choice	B	C	B	D	B	D
2 nd choice	C	D	D	C	C	B
3 rd choice	D	B	C	B	D	C

Notice that the first and fifth columns have the same preferences now, we can condense those down to one column.

	5	4	4	6	1
1 st choice	B	C	B	D	D
2 nd choice	C	D	D	C	B
3 rd choice	D	B	C	B	C

Now B has 9 votes, C has 4 votes, and D has 7 votes. Still no majority, so we eliminate again.

Round 3: We make our third elimination. C has the fewest votes.

	5	4	4	6	1
1 st choice	B	D	B	D	D
2 nd choice	D	B	D	B	B

Condensing this down:

	9	11
1 st choice	B	D
2 nd choice	D	B

D has now gained a majority, and is declared the winner under IRV.

What's Wrong with IRV?

Example: Let's return to our City Council Election

	342	214	298
1 st choice	Elle	Don	Key
2 nd choice	Don	Key	Don
3 rd choice	Key	Elle	Elle

In this election, Don has the smallest number of first place votes, so Don is eliminated in the first round. The 214 people who voted for Don have their votes transferred to their second choice, Key.

	342	512
1 st choice	Elle	Key
2 nd choice	Key	Elle

So Key is the winner under the IRV method.

We can immediately notice that in this election, IRV violates the Condorcet Criterion. On the other hand, the temptation has been removed for Don's supporters to vote for Key; they now know their vote will be transferred to Key, not simply discarded.

Example: Consider the voting system below

	37	22	12	29
1 st choice	Adams	Brown	Brown	Carter
2 nd choice	Brown	Carter	Adams	Adams
3 rd choice	Carter	Adams	Carter	Brown

In this election, Carter would be eliminated in the first round, and Adams would be the winner with 66 votes to 34 for Brown.

Now suppose that the results were announced, but election official accidentally destroyed the ballots before they could be certified, and the votes had to be recast. Wanting to “jump on the bandwagon”, 10 of the voters who had originally voted in the order Brown, Adams, Carter change their vote to favor the presumed winner, changing those votes to Adams, Brown, Carter.

	47	22	2	29
1 st choice	Adams	Brown	Brown	Carter
2 nd choice	Brown	Carter	Adams	Adams
3 rd choice	Carter	Adams	Carter	Brown

In this re-vote, Brown will be eliminated in the first round, having the fewest first-place votes. After transferring votes, we find that Carter will win this election with 51 votes to Adams' 49 votes! Even though the only vote changes made *avored* Adams, the change ended up costing Adams the election. This doesn't seem right, and introduces our second fairness criterion:

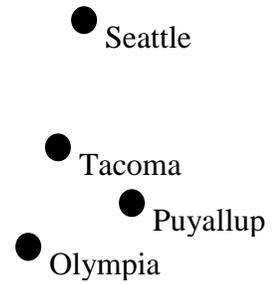
Monotonicity Criterion: If voters change their votes to increase the preference for a candidate, it should not harm that candidate's chances of winning.

This criterion is violated by this election. Note that even though the criterion is violated in this particular election, it does not mean that IRV always violates the criterion; just that IRV has the potential to violate the criterion in certain elections.

Borda Count

Borda Count is another voting method, named for Jean-Charles de Borda, who developed the system in 1770. In this method, points are assigned to candidates based on their ranking; 1 point for last choice, 2 points for second-to-last choice, and so on. The point values for all ballots are totaled, and the candidate with the largest point total is the winner.

Example: A group of mathematicians are getting together for a conference. The members are coming from four cities: Seattle, Tacoma, Puyallup, and Olympia. Their approximate relationship on a map is shown to the right. The votes for where to hold the conference were:



	51	25	10	14
1 st choice	Seattle	Tacoma	Puyallup	Olympia
2 nd choice	Tacoma	Puyallup	Tacoma	Tacoma
3 rd choice	Olympia	Olympia	Olympia	Puyallup
4 th choice	Puyallup	Seattle	Seattle	Seattle

In each of the 51 ballots ranking Seattle first, Puyallup will be given 1 point, Olympia 2 points, Tacoma 3 points, and Seattle 4 points. Multiplying the points per vote times the number of votes allows us to calculate points awarded:

	51	25	10	14
1 st choice	Seattle	Tacoma	Puyallup	Olympia
4 points	204	100	40	56
2 nd choice	Tacoma	Puyallup	Tacoma	Tacoma
3 points	153	75	30	42
3 rd choice	Olympia	Olympia	Olympia	Puyallup
2 points	102	50	20	28
4 th choice	Puyallup	Seattle	Seattle	Seattle
1 point	51	25	10	14

Adding up the points:

- Seattle: $204+25+10+14 = 253$ points
- Tacoma: $153+100+30+42 = 325$ points
- Puyallup: $51+75+40+28 = 194$ points
- Olympia: $102+50+20+56 = 228$ points

Under the Borda Count method, Tacoma is the winner of this election.

What's Wrong with Borda Count?

You might have already noticed one potential flaw of the Borda Count from the previous example. In that example, Seattle had a majority of first-choice votes, yet lost the election! This seems odd, and prompts our next fairness criterion:

Majority Criterion: If a choice has a majority of first-place votes, that choice should be the winner.

This example under the Borda Count violates the Majority Criterion. Notice also that this automatically means that the Condorcet Criterion will also be violated, as Seattle would have been preferred by 51% of voters in any head-to-head comparison.

Borda count is sometimes described as a consensus-based voting system, since it can sometimes choose a more broadly acceptable option over the one with majority support. In the example above, Tacoma is probably the best compromise location.

Because of this consensus behavior, Borda Count (or some variation) is commonly used in awarding sports awards. It is used to determine the Most Valuable Player in baseball, to rank teams in NCAA sports, and to award the Heisman trophy.

Copeland's Method (Pairwise Comparisons)

So far none of our voting methods have satisfied the Condorcet Criterion. The Copeland Method specifically attempts to satisfy the Condorcet Criterion by looking at pairwise (one-to-one) comparisons. In this method, each pair of candidates is compared, using all preferences to determine which of the two is more preferred. The more preferred candidate is awarded 1 point. If there is a tie, each candidate is awarded $\frac{1}{2}$ point. After all pairwise comparisons are made, the candidate with the most points, and hence the most pairwise wins, is declared the winner.

Variations of Copeland's Method are used in many professional organizations, including election of the Board of Trustees for the Wikimedia Foundation that runs Wikipedia.

Example: Consider our vacation group example from the beginning of the chapter.

	1	3	3	3
1 st choice	A	A	O	H
2 nd choice	O	H	H	A
3 rd choice	H	O	A	O

Comparing Anaheim to Orlando, the 1 voter in the first column clearly prefers Anaheim, as do the 3 voters in the second column. The 3 voters in the third column clearly prefer Orlando. The 3 voters in the last column prefer Hawaii as their first choice, but if they had to choose between Anaheim and Orlando, they'd choose Anaheim, their second choice overall. So, altogether $1+3+3=7$ voters prefer Anaheim over Orlando, and 3 prefer Orlando over Anaheim. So, comparing Anaheim vs Orlando: 7 votes to 3 votes: Anaheim gets 1 point

Anaheim vs Hawaii: 4 votes to 6 votes: Hawaii gets 1 point

Hawaii vs Orlando: 6 votes to 4 votes: Hawaii gets 1 point

Hawaii is the winner under Copeland's Method, having earned the most points.

Notice this process is consistent with our determination of a Condorcet Winner.

Example: Consider the advertising group's vote we explored earlier:

	3	4	4	6	2	1
1 st choice	B	C	B	D	B	E
2 nd choice	C	A	D	C	E	A
3 rd choice	A	D	C	A	A	D
4 th choice	D	B	A	E	C	B
5 th choice	E	E	E	B	D	C

With 5 candidates, there are 10 comparisons to make:

A vs B: 11 votes to 9 votes A gets 1 point
 A vs C: 3 votes to 17 votes C gets 1 point
 A vs D: 10 votes to 10 votes A gets $\frac{1}{2}$ point, D gets $\frac{1}{2}$ point
 A vs E: 17 votes to 3 votes A gets 1 point
 B vs C: 10 votes to 10 votes B gets $\frac{1}{2}$ point, C gets $\frac{1}{2}$ point
 B vs D: 9 votes to 11 votes D gets 1 point
 B vs E: 13 votes to 7 votes B gets 1 point
 C vs D: 9 votes to 11 votes D gets 1 point
 C vs E: 17 votes to 3 votes C gets 1 point
 D vs E: 17 votes to 3 votes D gets 1 point

Totaling these up:

A gets $2\frac{1}{2}$ points
 B gets $1\frac{1}{2}$ points
 C gets $2\frac{1}{2}$ points
 D gets $3\frac{1}{2}$ points
 E gets 0 points

So Copeland's Method would choose D as the winner. Notice that in this case, D is not a Condorcet Candidate, but Copeland's Method still found a winner.

What's Wrong with Copeland's Method

As already noted, Copeland's Method does satisfy the Condorcet Criterion. It also satisfies the Majority Criterion and the Monotonicity Criterion. So is this the perfect method? Well, no.

Example: A committee is trying to award a scholarship to one of four students, Anna (A), Brian (B), Carlos (C), and Dimitry (D). The votes are shown below:

	5	5	6	4
1 st choice	D	A	C	B
2 nd choice	A	C	B	D
3 rd choice	C	B	D	A
4 th choice	B	D	A	C

Making the comparisons:

A vs B: 10 votes to 10 votes A gets $\frac{1}{2}$ point, B gets $\frac{1}{2}$ point
 A vs C: 14 votes to 6 votes: A gets 1 point
 A vs D: 5 votes to 15 votes: D gets 1 point
 B vs C: 4 votes to 16 votes: C gets 1 point
 B vs D: 15 votes to 5 votes: B gets 1 point
 C vs D: 11 votes to 9 votes: C gets 1 point

Totaling:

A has $1\frac{1}{2}$ points

B has $1\frac{1}{2}$ points

C has 2 points

D has 1 point

So Carlos is awarded the scholarship. However, the committee then discovers that Dimitry was not eligible for the scholarship (he failed his last math class). Even though this seems like it shouldn't affect the outcome, the committee decides to recount the vote, removing Dimitry from consideration:

	5	5	6	4
1 st choice	A	A	C	B
2 nd choice	C	C	B	A
3 rd choice	B	B	A	C

A vs B: 10 votes to 10 votes A gets $\frac{1}{2}$ point, B gets $\frac{1}{2}$ point

A vs C: 14 votes to 6 votes A gets 1 point

B vs C: 4 votes to 16 votes C gets 1 point

Totaling:

A has $1\frac{1}{2}$ points

B has $\frac{1}{2}$ point

C has 1 point

Suddenly Anna is the winner! This leads us to another fairness criterion.

The Independence of Irrelevant Alternatives (IIA) Criterion: If a non-winning choice is removed from the ballot, it should not change the winner of the election.

Equivalently, if choice A is preferred over choice B, introducing or removing a choice C should not make B preferred over A.

In this election, the IIA Criterion was violated.

This anecdote illustrating the IIA issue is attributed to Sidney Morgenbesser:

After finishing dinner, Sidney Morgenbesser decides to order dessert. The waitress tells him he has two choices: apple pie and blueberry pie. Sidney orders the apple pie. After a few minutes the waitress returns and says that they also have cherry pie at which point Morgenbesser says "In that case I'll have the blueberry pie."

Another disadvantage of Copeland's Method is that it is fairly easy for the election to end in a tie. There are a number of alternative methods based on satisfying the Condorcet Criterion that use more sophisticated methods for determining the winner when there is not a Condorcet Candidate.

So Where’s the Fair Method?

At this point, you’re probably asking why we keep looking at method after method just to point out that they are not fully fair. We must be holding out on the perfect method, right?

Unfortunately, no. A mathematical economist, Kenneth Arrow, was able to prove in 1949 that there is no voting method that will satisfy all the fairness criteria we have discussed. This is dubbed **Arrow’s Impossibility Theorem**.

To see a very simple example of how difficult voting can be, consider the election below:

	5	5	5
1 st choice	A	C	B
2 nd choice	B	A	C
3 rd choice	C	B	A

Notice that in this election:

- 10 people prefer A to B
- 10 people prefer B to C
- 10 people prefer C to A

No matter whom we choose as the winner, 2/3 of voters would prefer someone else! This scenario is dubbed Condorcet’s Voting Paradox, and demonstrates how voting preferences are not transitive (just because A is preferred over B, and B over C, does not mean A is preferred over C). In this election, there is no fair resolution.

It is because of this impossibility of a totally fair method that Plurality, IRV, Borda Count, and Copeland’s Method are all still used. Usually the decision of which method to use is based on what seems most fair for the situation.

Approval Voting

Up until now, we’ve been considering voting methods that require ranking of candidates on a preference ballot. There is another method of voting that can be more appropriate in some decision making scenarios. With Approval Voting, the ballot asks you to mark all choices that you find acceptable. The results are tallied, and the option with the most approval is the winner.

Example: A group of friends is trying to decide upon a movie to watch. Three choices are provided, and each person is asked to mark with an “X” which movies they are willing to watch. The results are:

	Bob	Ann	Marv	Alice	Eve	Omar	Lupe	Dave	Tish	Jim
Titanic		X	X			X		X		X
Scream	X		X	X		X	X		X	
The Matrix	X	X	X	X	X		X			X

Totaling the results, we find
 Titanic received 5 approvals
 Scream received 6 approvals
 The Matrix received 7 approvals.

In this vote, The Matrix would be the winner.

What's Wrong with Approval Voting?

Approval voting can very easily violate the Majority Criterion. Consider the voting schedule:

	80	15	5
1 st choice	A	B	C
2 nd choice	B	C	B
3 rd choice	C	A	A

Clearly A is the majority winner. Now suppose that this election was held using Approval Voting, and every voter marked approval of their top two candidates.

A would receive approval from 80 voters
 B would receive approval from 100 voters
 C would receive approval from 20 voters

B would be the winner. Some argue that Approval Voting tends to vote the least disliked choice, rather than the most liked candidate.

Additionally, Approval Voting is susceptible to strategic insincere voting, in which a voter does not vote their true preference to try to increase the chances of their choice winning. For example, in the movie example above, suppose Bob and Alice would much rather watch Scream. They remove The Matrix from their approval list, resulting in a different result.

	Bob	Ann	Marv	Alice	Eve	Omar	Lupe	Dave	Tish	Jim
Titanic		X	X			X		X		X
Scream	X		X	X		X	X		X	
The Matrix		X	X		X		X			X

Totaling the results, we find Titanic received 5 approvals, Scream received 6 approvals, and The Matrix received 5 approvals. By voting insincerely, Bob and Alice were able to sway the result in favor of their preference.

Exercises

Skills

- To decide on a new website design, the designer asks people to rank three designs that have been created (labeled A, B, and C). The individual ballots are shown below. Create a preference table.

ABC, ABC, ACB, BAC, BCA, BCA, ACB, CAB, CAB, BCA, ACB, ABC

- To decide on a movie to watch, a group of friends all vote for one of the choices (labeled A, B, and C). The individual ballots are shown below. Create a preference table.

CAB, CBA, BAC, BCA, CBA, ABC, ABC, CBA, BCA, CAB, CAB, BAC

- The planning committee for a renewable energy trade show is trying to decide what city to hold their next show in. The votes are shown below.

Number of voters	9	19	11	8
1st choice	Buffalo	Atlanta	Chicago	Buffalo
2nd choice	Atlanta	Buffalo	Buffalo	Chicago
3rd choice	Chicago	Chicago	Atlanta	Atlanta

- How many voters voted in this election?
 - How many votes are needed for a majority? A plurality?
 - Find the winner under the plurality method.
 - Find the winner under the Borda Count Method.
 - Find the winner under the Instant Runoff Voting method.
 - Find the winner under Copeland's method.
- A non-profit agency is electing a new chair of the board. The votes are shown below.

Number of voters	11	5	10	3
1st choice	Atkins	Cortez	Burke	Atkins
2nd choice	Cortez	Burke	Cortez	Burke
3rd choice	Burke	Atkins	Atkins	Cortez

- How many voters voted in this election?
- How many votes are needed for a majority? A plurality?
- Find the winner under the plurality method.
- Find the winner under the Borda Count Method.
- Find the winner under the Instant Runoff Voting method.
- Find the winner under Copeland's method.

5. The student government is holding elections for president. There are four candidates (labeled A, B, C, and D for convenience). The preference schedule for the election is:

Number of voters	120	50	40	90	60	100
1st choice	C	B	D	A	A	D
2nd choice	D	C	A	C	D	B
3rd choice	B	A	B	B	C	A
4th choice	A	D	C	D	B	C

- How many voters voted in this election?
 - How many votes are needed for a majority? A plurality?
 - Find the winner under the plurality method.
 - Find the winner under the Borda Count Method.
 - Find the winner under the Instant Runoff Voting method.
 - Find the winner under Copeland's method.
6. The homeowners association is deciding a new set of neighborhood standards for architecture, yard maintenance, etc. Four options have been proposed. The votes are:

Number of voters	8	9	11	7	7	5
1st choice	B	A	D	A	B	C
2nd choice	C	D	B	B	A	D
3rd choice	A	C	C	D	C	A
4th choice	D	B	A	C	D	B

- How many voters voted in this election?
 - How many votes are needed for a majority? A plurality?
 - Find the winner under the plurality method.
 - Find the winner under the Borda Count Method.
 - Find the winner under the Instant Runoff Voting method.
 - Find the winner under Copeland's method.
7. Consider an election with 129 votes.
- If there are 4 candidates, what is the smallest number of votes that a plurality candidate could have?
 - If there are 8 candidates, what is the smallest number of votes that a plurality candidate could have?
8. Consider an election with 953 votes.
- If there are 7 candidates, what is the smallest number of votes that a plurality candidate could have?
 - If there are 8 candidates, what is the smallest number of votes that a plurality candidate could have?

9. Does this voting system having a Condorcet Candidate? If so, find it.

Number of voters	14	15	2
1st choice	A	C	B
2nd choice	B	B	C
3rd choice	C	A	A

10. Does this voting system having a Condorcet Candidate? If so, find it.

Number of voters	8	7	6
1st choice	A	C	B
2nd choice	B	B	C
3rd choice	C	A	A

11. The marketing committee at a company decides to vote on a new company logo. They decide to use approval voting. Their results are tallied below. Each column shows the number of voters with the particular approval vote. Which logo wins under approval voting?

Number of voters	8	7	6	3
A	X	X		
B	X		X	X
C		X	X	X

12. The downtown business association is electing a new chairperson, and decides to use approval voting. The tally is below, where each column shows the number of voters with the particular approval vote. Which candidate wins under approval voting?

Number of voters	8	7	6	3	4	2	5
A	X	X			X		X
B	X		X	X			X
C		X	X	X		X	
D	X		X		X	X	

Concepts

13. An election resulted in Candidate A winning, with Candidate B coming in a close second, and candidate C being a distant third. If for some reason the election had to be held again and C decided to drop out of the election, which caused B to become the winner, which is the primary fairness criterion violated in this election?
14. An election resulted in Candidate A winning, with Candidate B coming in a close second, and candidate C being a distant third. If for some reason the election had to be held again and many people who had voted for C switched their preferences to favor A, which caused B to become the winner, which is the primary fairness criterion violated in this election?

15. An election resulted in Candidate A winning, with Candidate B coming in a close second, and candidate C being a distant third. If in a head-to-head comparison a majority of people prefer B to A or C, which is the primary fairness criterion violated in this election?
16. An election resulted in Candidate A winning, with Candidate B coming in a close second, and candidate C being a distant third. If B had received a majority of first place votes, which is the primary fairness criterion violated in this election?

Exploration

17. In the election shown below under the Plurality method, explain why voters in the third column might be inclined to vote insincerely. How could it affect the outcome of the election?

Number of voters	96	90	10
1st choice	A	B	C
2nd choice	B	A	B
3rd choice	C	C	A

18. In the election shown below under the Borda Count method, explain why voters in the second column might be inclined to vote insincerely. How could it affect the outcome of the election?

Number of voters	20	18
1st choice	A	B
2nd choice	B	A
3rd choice	C	C

19. Compare and contrast the motives of the insincere voters in the two questions above.
20. Consider a two party election with preferences shown below. Suppose a third candidate, C, entered the race, and a segment of voters sincerely voted for that third candidate, producing the preference schedule from #17 above. Explain how other voters might perceive candidate C.

Number of voters	96	100
1st choice	A	B
2nd choice	B	A

21. In question 18, we showed that the outcome of Borda Count can be manipulated if a group of individuals change their vote.
- Show that it is possible for a single voter to change the outcome under Borda Count if there are four candidates.
 - Show that it is not possible for a single voter to change the outcome under Borda Count if there are three candidates.

22. Show that when there is a Condorcet Candidate, it is impossible to manipulate the vote under Pairwise Comparisons and help your preferred candidate. (possibly show that it would be possible to change the winner, just not to your preferred candidate)
23. The Pareto criterion is another fairness criterion that states: *If every voter prefers choice A to choice B, then B should not be the winner.* Explain why plurality, instant runoff, Borda count, and Copeland’s method all satisfy the Pareto condition.
24. Sequential Pairwise voting is a method not commonly used for political elections, but sometimes used for shopping and games of pool. In this method, the choices are assigned an order of comparison, called an agenda. The first two choices are compared. The winner of is then compared to the next choice on the agenda, and this continues until all choices have been compared against the winner of the previous comparison.

- a. Using the preference schedule below, apply Sequential Pairwise voting to determine the winner, using the agenda: A, B, C, D.

Number of voters	10	15	12
1st choice	C	A	B
2nd choice	A	B	D
3rd choice	B	D	C
4th choice	D	C	A

- b. Show that Sequential Pairwise voting can violate the Pareto criterion.
- c. Show that Sequential Pairwise voting can violate the Majority criterion.
25. The Coombs method is a variation of instant runoff voting. In Coombs method, the choice with the most last place votes is eliminated. Apply Coombs method to the preference schedules from questions 5 and 6.
26. Copeland’s Method is designed to identify a Condorcet Candidate if there is one, and is considered a Condorcet Method. There are many Condorcet Methods, which vary primarily in how they deal with ties, which are very common when a Condorcet winner does not exist. Copeland’s method does not have a tie-breaking procedure built-in. Research the Schulze method, another Condorcet method that is used by the Wikimedia foundation that runs Wikipedia, and give some examples of how it works.
27. The plurality method is used in most U.S. elections. Some people feel that Ross Perot in 1992 and Ralph Nader in 2000 changed what the outcome of the election would have been if they had not run. Research the outcomes of these elections and explain how each candidate could have affected the outcome of the elections (for the 2000 election, you may wish to focus on the count in Florida). Describe how an alternative voting method could have avoided this issue.

