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Quantum Gravity

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Abstract

The article analyzes the Einstein equations from a quantum theoretical point of view. The basic equation of the quantum gravity and a new uncertainty relation between the Schwarzschild radius and the radial coordinate are obtained. Using the new uncertainty relation, the equations of the general theory of relativity are estimated in relation to the Planck scale. It is established that the lower limit for the space-time scale is justified. The appearance of virtual black holes and wormholes (quantum foam) on the Planck scale is predicted.The three-dimensionality of the observed space is substantiated. A hypothesis has been put forward about the nature of the singularities of black holes and the Metagalaxy.

1. Introduction

The search for a consistent and testable theory of quantum gravity is one of the most important open problems in fundamental physics.The general theory of relativity is what is called classical, that is, non-quantum theory. The current theories for the other interactions are all quantum theories, or, moreover, it is these interactions that are described within the framework of quantum theory, which uses concepts radically different from classical physics.

Quantum theory is usually applied in the field of microphysics. This is the world of molecules, atoms, nuclei and elementary particles. Thus, quantum theory underlies not only physics, but also chemistry and biology. The smallest scales explored experimentally so far are those explored by particle accelerators such as the Large Hadron Collider. These smallest studied scales are on the order of 10^{-18} m.

All fields of the Standard Model carry energy and thus generate a gravitational field. Since these are quantum fields, they cannot be inserted directly into Einstein's classical field equations. Only a consistent unification of gravity with quantum theory can describe the interaction of all fields at a fundamental level.

We call quantum gravity any theory (or approach) that applies the principle of superposition to a gravitational field. Einstein's theory itself is incomplete. It is possible to prove singularity theorems, which state that, under certain assumptions, there are regions of spacetime where the theory fails. Specific examples include regions inside black holes and the origin of our universe. There is another type of singularity. Quantum field theories fail due to discrepancies that arise when studying space-time on arbitrarily small scales.

The physical scale where we definitely expect quantum effects of gravity to become relevant is the Planck level. The three constants G, h (and, accordingly, h/2) and c provide the corresponding scales of quantum gravity, since from them it is possible to construct (in addition to numerical coefficients) unique expressions for fundamental length, time and mass (or energy). Since Max Planck formulated them back in 1899, they are named Planck units in his honor.

To generate particles with masses on the order of the Planck mass and higher, it is necessary to build an accelerator of galactic dimensions. This is one of the most important problems in the search for quantum gravity: we cannot directly probe the Planck scale by experimental means.

Everything that has been said so far points to the need for a quantum theory of gravity. For more than a hundred years we have not had a complete quantum theory of gravity. But how can one construct such a theory? Let's consider the main approaches along this path.

First, the connection between quantum mechanics (quantum theory with a finite number of degrees of freedom) and gravity is studied using the Schrodinger (or Dirac) equation in a Newtonian gravitational field.

There are also two approaches to constructing quantum gravity: the covariant approach and the canonical approach. Both approaches are aimed at constructing a quantum version of general relativity. The covariant approach gets its name from the fact that the four-dimensional (covariant) formalism is used throughout. In most cases, this formalism uses path integrals (in which four-dimensional spacetimes are summed up according to the principle of superposition). Like the photon in quantum electrodynamics, the particle is identified as a mediator of the quantum gravitational field - the graviton. It is massless, but has spin 2 (whereas a photon has spin 1). The fact that it is truly massless is indirectly confirmed by the detection of gravitational waves - they move at the speed of light c.

It is believed that quantum general relativity is only an effective field theory, that is, this approach, using standard quantum field theory up to the Planck scale theory, is asymptotically safe. One promising approach is dynamic triangulation, so named because the spacetimes to be summed in the path integral are discretized into tetrahedra. One of the candidates for the creation of a final quantum field theory of gravity is supergravity. A candidate for the creation of a final theory of quantum gravity of a completely different nature is superstring theory (or M-theory) An alternative to covariant quantization is the canonical (or Hamiltonian) approach. The procedure here is similar to the procedure in quantum mechanics, where quantum operators are constructed for positions, momenta and other variables. This also includes a quantum version of energy called the Hamilton operator.

In quantum mechanics, the Hamilton operator generates evolution in time according to the formula of the Schrodinger equation. In quantum gravity the situation is different. Instead of the Schrodinger equation, there are restrictions - the Hamiltonian (and other functions) are forced to vanish. This is due to the disappearance of space-time at a fundamental level. This is due to the fact that classical theory no longer has a fixed background. Background independence is one of the main obstacles to quantum gravity. An alternative formulation uses variables that have some similarities to the gauge fields used in the Standard Model. This approach is known as Loop Quantum Gravity [38]. In addition to the approaches already mentioned, there are many others. This article proposes another approach to constructing quantum gravity. We call it the integral method.

2. General information

The Planck length (denoted ℓ_P) is a fundamental unit of length in Planck System of Units, equal in International System of Units (SI) approximately $1{:}6^{-35}$ meters. The Planck length is a natural unit of length because it only includes fundamental constants: speed of light, Planck's constant, and the gravitational constant. The Planck length is: $\ell_P = \sqrt{(G/c^3)\hbar}$ = $1.616229(38)^{-35}$m, where: $\hbar$ is Dirac constant $(\hbar = h/2\pi)$, where h is Planck constant; G - gravitational constant; c is the speed of light in a vacuum. Dimensional analysis shows that measuring the position of physical objects accurate to the Planck length is problematic.

In a thought experiment, to determine the position of an object, a stream of electromagnetic radiation, that is, photons, is sent to it. The higher the energy of the photons, the shorter their wavelength and the more accurate the measurement will be. It is assumed that if photons had enough energy to measure objects the size of the Planck length, then when interacting with the object they would collapse into a microscopic black hole and it would be impossible to measure. This imposes fundamental limitations on the accuracy of length measurements [1-3].

3. Qualitative substantiation of photon collapse on the Planck scale

According to general relativity, any form of energy, including photon energy, must generate a gravitational _eld. And the greater this energy, the more powerful the gravitational eld they generate [2]. Further: let's introduce the concept of \kinetic energy of photons", which is determined by the formula $E_{kin} = P\,c$, where P is the photon momen-tum, and c - their speed; this energy is a positive quantity and, with the free movement of photons, is not limited by anything; the total energy of a photon beam also includes the potential energy of interaction of photons with each other and this energy is a negative quantity [2].

3.1. Initial Reasoning

For two massive particles each with mass m, the potential energy of interaction depends only on the distance between them. Based on Newton's equation of gravity, the potential energy of interaction, when taking the state of innate removal of particles as zero, has the form [3].

$E_{pot} = -Gm^2/r$, where G is the gravitational constant; m is the mass of each particle; r is the distance between particles.

To find the total energy of a system of two bodies with mass m, you need to add up the kinetic energies of both bodies and add here their mutual gravitational potential energy, which together gives a constant:

$$\sum (1/2) m_i v_i^2 - Gm^2/r_{ij} = const; \quad i = 1, 2 \tag{3.1}$$

Based on the admissible analogy with the potential energy of massive particles, taking into account the fact that photons have no mass, we believe that it is permissible for two photons to substitute the value of the photon momentum divided by the speed of light into this equation instead of the mass m, then there is P/c [4, 5].

This allows us to introduce the concept of potential energy of interaction of photons with each other and define it as

$$E_{pot} = G\,P^2/c^2 r \tag{3.2}$$

Here r must be compared with the photon wavelength λ.

Then the total energy of interacting photons is equal to the sum of the kinetic (in order of magnitude) and potential energies and has the form

$$E = E_{kin} + E_{pot} \approx 1/2 \left(\frac{2P}{c}\right) c^2 - \frac{G\,P^2}{c^2\,\lambda} = P\,c\left(1 - \frac{G\,P}{c^3\,\lambda}\right) = P\,c\left(1 - \frac{\lambda_g}{\lambda}\right) \qquad (3.3)$$

(photon spin is not taken into account here, but for now this is not significant). The quantity $\lambda_g \approx (G/c^3)P$ for a system of gravitationally interacting photons is an analogue of the gravitational radius $r_g \approx (G/c^3)$mc for a massive particle. To use this equation in

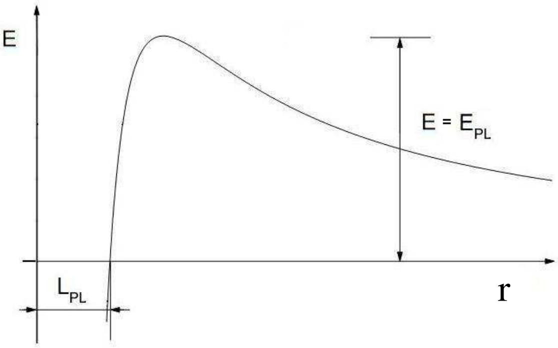

Figure 1: The graph of the function $E(\lambda)$

quantum theory, we consider these quantities P and λ using the Heisenberg uncertainty relation as the momentum and position uncertainties. By allowing one to obtain important estimates in a fairly simple way, uncertainty relations turn out to be a useful "working tool" of quantum theory. According to the uncertainty relation, these quantities are related to each other.

Assuming that $P \approx \hbar/\lambda$, where $\hbar$ is the Dirac constant and using this relation (by substituting $P \approx \hbar/\lambda$), we find the function $E(\lambda)$ from the last equation

$$E(\lambda) = \frac{\hbar c}{\lambda}\left(1 - \frac{\ell_P^2}{\lambda^2}\right) \qquad (3.4)$$

where $\ell_P = \sqrt{\hbar G/c^3}$ is the fundamental Planck length, which appears automatically.
The graph of the function $E(\lambda)$ constructed on the basis of this equation (Fig.1) shows that as the wavelength of photons λ decreases, their total energy increases, since the second term in the last equation at low photon momentum is practically zero. In this case, the maximum total energy $E(\lambda)$ turns out to be approximately equal to the Planck energy E_P, and the photon wavelength λ is almost comparable to the Planck length .

However, if the momentum of photons continues to increase, the total energy of the system of photons will begin to decrease due to an increase in the negative gravitational component of the total energy, which until this moment did not play a significant role. When the photon wavelength λ is equal to the Planck length $\ell_P \approx 10^{-35}$m, the total energy of interaction of photons with each other becomes equal to zero, the photons collapse and turn into microscopic Planck black hole.

Thus, when electromagnetic radiation acquires Planck energy (that is, its wavelength λ becomes equal to the Planck length ℓ_P), the electromagnetic radiation collapses. Therefore, it is no longer possible to use it as a tool for "probing" ultra-small distances. We have discovered the limit, the frontier of scientific research. A system of two or more gravitationally interacting photons is called a geon [6].

3.2. More Rigorous Reasoning

If we think more strictly, then we need to proceed from the Hamilton-Jacobi equation [6]

$$g^{ik}\partial^2 S/\partial x^i \partial x^k = (m')^2\, c^2 \qquad (3.5)$$

with metric coefficients g^{ik}, taken from the Schwarzschild solution, where S - action, m' is the mass of the particle (we denote the mass of the central body here as m).This equation is a generalization of the equation between relativistic energy and momentum of a particle in special relativity

$$E^2 - \mathbf{p}^2 c^2 = (m')^2 c^4 \qquad (3.6)$$

The generalized equation is covariant (the physical content of the equation does not depend on the choice of coordinate system). In expanded form, the indicated Hamilton-Jacobi equation has the form

$$\left(1-\frac{r_g}{r}\right)^{-1}\left(\frac{\partial S}{c\,\partial t}\right)^2-\left(1-\frac{r_g}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2-\frac{1}{r^2}\left(\frac{\partial S}{\partial\varphi}\right)^2-(m')^2c^2=0 \qquad (3.7)$$

It can be rewritten as follows

$$E^2=\left(1-\frac{r_g}{r}\right)^2P^2c^2+\left(1-\frac{r_g}{r}\right)\frac{N^2c^2}{r^2}+\left(1-\frac{r_g}{r}\right)(m')^2c^4 \qquad (3.8)$$

where N - angular momentum of the particle; r_g is the gravitational radius of the central attracting body with mass m.
For the above approximation, it is necessary to put in this equation the mass of particles (photons) m' equal to zero, neglect the angular momentum (spin) of photons N and use the Heisenberg uncertainty relations $P\,r\ \approx\hbar$. Then we obtain an approximate equation for the total energy

$$E\approx\left(1-\frac{r_g}{r}\right)P\,c=\left(1-\frac{2G\,m}{c^2r}\right)P\,c\approx\left(1-\frac{2\ell_P^2}{\lambda^2}\right)\frac{\hbar c}{\lambda} \qquad (3.9)$$

where $r=\lambda$ - photon wavelength; $r_g=2Gm/c^2$ is the gravitational radius of the central body.
In this expression, the gravitational mass m must be replaced by P/c, where P is the momentum of the photons; $P\approx\hbar/\lambda$. The resulting equation, up to a coefficient 2, coincides with the equation established above for the total energy of the photon system.

To take into account the angular momentum of photons in the specified equation, you need to perform the substitution $N^2=h^2\,l(l+1)$ where l is the quantum number of the total angular momentum of photons. Taking into account the angular momentum of photons leads to the appearance of a second, internal event horizon in the resulting Planck black hole (point 2 on the graph, Fig.2).

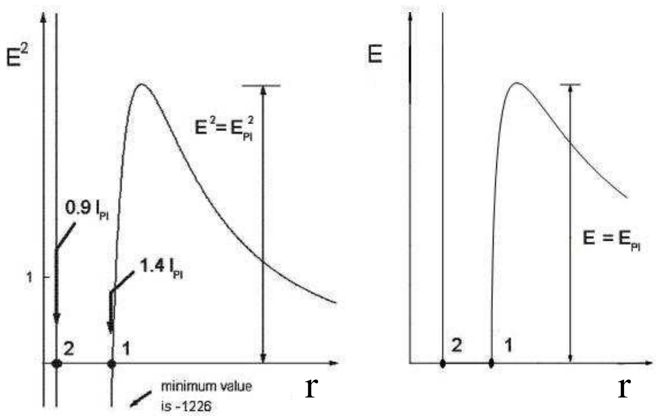

Figure 2: The graph of the function $E(\lambda)$ with allowance for the angular momentum $l=1$

For a charged black hole, the metric coefficient g_{00}, according to the Reissner-Nordstrom solution, has the form [7]

$$g_{00}=1-\frac{r_g}{r}+\frac{GQ^2}{c^4r^2} \qquad (3.10)$$

where Q is the total charge of the black hole.
Considering that the Planck charge is $Q=\sqrt{\hbar c}$ then at the Planck level

$$\frac{GQ^2}{c^4r^2}=\frac{G\hbar}{c^3\lambda^2}=\frac{\ell_P^2}{\lambda^2} \qquad (3.11)$$

Therefore, the metric coefficient g_{00} takes the form

$$g_{00}=1-\frac{r_g}{r}+\frac{GQ^2}{c^4r^2}=1-\frac{2\ell_P^2}{\lambda^2}+\frac{\ell_P^2}{\lambda^2}=1-\frac{\ell_P^2}{\lambda^2} \qquad (3.12)$$

That is, the charge has virtually no effect on the overall functional dependence $E(\lambda)$. The general rule is that the metric coefficient gik cannot be greater than 1 [7, 8].

This thought experiment uses both general relativity and the uncertainty principle of quantum mechanics. Both theories predict that it is impossible to measure with precision that exceeds the Planck length [7]. In any theory of quantum gravity that combines general relativity and quantum mechanics, the traditional concept of space and time does not apply at distances smaller than the Planck length or for periods of time shorter than the Planck time. It follows that at the Planck level all particles are massless and move at the speed of light. This conclusion follows from the very course of reasoning in this article, since the Planck length naturally appears as a result of the interaction of only massless energy quanta [9].

3.3. Planck Length and Dimension of Space

Now, according to the general belief of experts, "true" physics is formed under the Planck parameters $l \sim \ell P$, $t \sim t_p$, $M \sim M_p$. Understanding the ongoing processes in this area will lead to the construction of a unified field theory, a quantum theory of gravity, the creation of a theory of the origin of the Metagalaxy and a quantitative representation of physical geometry [10]. The same applies to the dimension of space.

Analysis of the Hamilton-Jacobi equation for photons in spaces of different dimensions n indicates the preference (energy advantage) of three-dimensional space for the emergence of Planck black holes, real or virtual (quantum foam). When considering this issue, we will use the results obtained at one time by P. Ehrenfest [11,12].

Ehrenfest considers "physics" in n-dimensional space U^n. In this case, he derives the law of interaction with a point center (similarly to the three-dimensional case) from the Poisson differential equation in U^n for the potential that determines this interaction. Fundamental physical laws of interactions are given in variational form. The Lagrangian for the simplest case of a scalar massless field $\varphi(t, x^1, x^2, \cdots, x^n)$ has the form

$$L = \left(\frac{\partial \varphi}{\partial t}\right)^2 + \sum_{k=1}^{n} \left(\frac{\partial \varphi}{\partial x^k}\right)^2$$

This Lagrangian leads to the Poisson equation and hence to the point center field $\varphi \sim r^{n-2}$($\varphi \sim \ln r$ for $n = 2$). The dimension of space is taken into account here only as a condition on the set of values that the index k can take. In the (3 + 1)-dimensional case $k = 1,2,3$. Thus, this Lagrangian allows us to obtain the corresponding part of physics in a space of any dimension. The Poisson equation is just mathematically equivalent to the indicated Lagrangian (with a natural generalization to other fields).

In the spherically symmetric case in U^n, from the Poisson equation or from Gauss's law for the field strength, expressions for the potential energy follow

$$E_{pot}^{(n\geq 3)} \approx -\frac{k\,m^2}{(n-2)r^{n-2}}; \quad n \geq 3 \tag{3.13}$$

$$E_{pot}^{(2)} \approx k\,m^2 \ln r; \quad n = 2 \tag{3.14}$$

$$E_{pot}^{(1)} \approx k\,m^2\, r; \quad n = 1 \tag{3.15}$$

where k is the interaction constant in n-dimensional space. With the usual Newton's constant, it is found through the matching of potentials for 3-dimensional space and the corresponding n-dimensional space.

For the potential energy of interacting photons, these equations take the form (taking into account that $m \to P/c\,; P \approx \hbar/\lambda;$ $r = \lambda$)

$$E_{pot}^{(n\geq 3)} \approx -\frac{k\,(P/c)^2}{(n-2)r^{n-2}} = -\frac{k\,(\hbar/\lambda\,c)^2}{(n-2)\lambda^{n-2}}; \quad n \geq 3 \tag{3.16}$$

$$E_{pot}^{(2)} \approx k\,(P/c)^2 \ln r = k\,(\hbar/\lambda\,c)^2 \ln \lambda; \quad n = 2 \tag{3.17}$$

$$E_{pot}^{(1)} \approx k\,(P/c)^2\, r = k\,(\hbar/\lambda\,c)^2\, \lambda; \quad n = 1 \tag{3.18}$$

Then the total energy of interacting photons in spaces of dierent dimensions is approxi-mately equal to

$$E^{(n)}(\lambda) \approx E_{kin} + E_{pot}^{(n)} \tag{3.19}$$

where the kinetic energy $E_{kin} = Pc = \hbar c/\lambda$ does not depend on the dimension of space. Equations for the total energy $E^{(n)}(\lambda) \approx E_{kin} + E_{pot}^{(n)}$ in spaces U^n will have the form

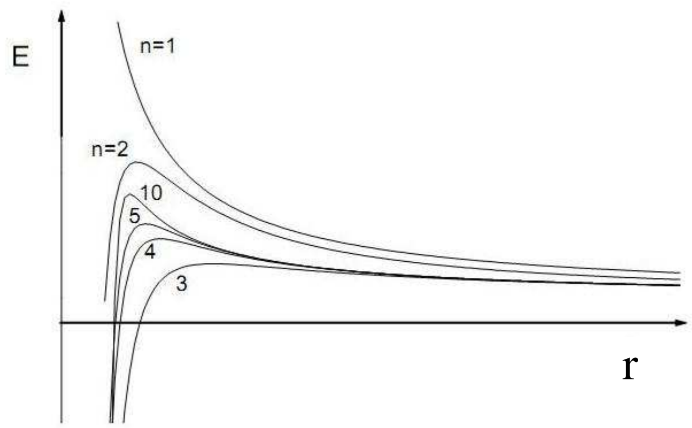

Figure 3: The graphs of the functions $E(\lambda)$ in n -dimensional spaces

(taking into account that $k = c = \hbar = 1$)

$$E^{(n\geq 3)}(\lambda) \approx E_{kin} + E_{pot}^{(n\geq 3)} = \frac{Pc}{2} - \frac{kP^2}{c^2(n-2)\lambda^{n-2}} = \left(1 - \frac{2}{(n-2)\lambda^{n-1}}\right)\frac{1}{2\lambda}; \quad n \geq 3 \tag{3.20}$$

$$E^{(2)}(\lambda) \approx E_{kin} + E_{pot}^{(2)} = \frac{Pc}{2} + \frac{k}{c^2}P^2 ln\,\lambda = \left(1 + \frac{2ln\,\lambda}{\lambda}\right)\frac{1}{2\lambda}; \quad n = 2 \tag{3.21}$$

$$E^{(1)}(\lambda) \approx E_{kin} + E_{pot}^{(1)} = \frac{Pc}{2} + \frac{k}{c^2}P^2\lambda = \frac{1,5}{\lambda}; \quad n = 1 \tag{3.22}$$

Graphs of the functions $E^{(n)}(\lambda)$ are shown in the Fig.3 and indicate that the formation of Planck black holes (real or virtual) is energetically most favorable in 3-dimensional space [13]. If we assume that on the Planck scale virtual black holes form the so-called space-time quantum foam, which is the basis of the "fabric" of the Universe, then the energetic advantage during the formation of Planck black holes most likely predetermined the 3dimensionality of the observable space [14]. It is not space that exists and imprints its form on things (in the form of a box filled with material objects according to Newton), but things and the physical laws governing them that define space. This point of view reaches its maximum validity in Einstein's general theory of relativity [15].

3.4. Philosophy of Space Dimension

The concept of the dimension of space is associated with a specific physical law and is involved in one of the ideological confrontations in the history of physics - the confrontation between the concepts of absoluteness and relativity of space.
The first concept assumes that space is something absolute, given, something like a ready-made stage on which physical phenomena are played out, but which does not depend on these phenomena.

The second concept of the relativity of space means that spatial relations are some relationships between physical bodies.If space can be likened to a stage, then this scene is created during the performance itself, created by physical phenomena, interactions between bodies. And this scene cannot even be imagined to exist independently of interactions.The concept of absolute space prevailed in Newtonian mechanics.

The general theory of relativity was won by the concept of the relativity of space, of which Leibniz was a staunch supporter. Kant was also influenced by Leibniz's views. At age 23, he wrote: "Three-dimensionality appears to result from the fact that substances in the existing world act on each other in such a way that the force of action is inversely proportional to the square of the distance... It is easy to prove that there would be no space and no extension if substances would not have any power to act externally. Without this force there is no connection - no order, without order there is no space [16]." That is, space is order in the totality of bodies, space is the relationship of bodies. These relationships are manifested in the forces acting between bodies [17].

Kant talks about a force inversely proportional to the square of the distance, which simply physically substantiates the three-dimensionality of the observed space. We are considering general patterns in multidimensional spaces, once established by Ehrenfest, but in relation to massless energy quanta, the existence of which is characteristic of the Planck scale. Here it is natural to assume that interactions between massless energy quanta create a system of relations that is energetically the most favorable. On the Planck scale, interactions between massless energy quanta (photons, gravitons, etc.), as a result of which Planck black holes, real and virtual, are formed (quantum "foam", the basis of the fabric of the Universe), are energetically most favorable in the system of relations that form space of dimension three.

We come to the conclusion that the three-dimensionality of space is associated with the fundamental properties of the material world at the Planck level.

4. Towards Quantum Gravity

4.1. Uncertainty Relations on The Planck Scale

A particle of mass m has a reduced Compton wavelength

$$\bar{\lambda}_C = \frac{\lambda_C}{2\pi} = \frac{\hbar}{mc} \tag{4.1}$$

On the other hand, the Schwarzschild radius of the same particle is equal to

$$r_g = \frac{2Gm}{c^2} = 2\,\frac{G}{c^3}\,mc \tag{4.2}$$

The product of these quantities is always constant and equal

$$r_g \bar{\lambda}_C = 2\,\frac{G}{c^3}\,\hbar = 2\ell_P^2 \tag{4.3}$$

Accordingly, the uncertainty relation between the Schwarzschild radius of the particle and the Compton wavelength of the particle will have the form

$$\Delta r_g \Delta\bar{\lambda}_C \geq \frac{G}{c^3}\,\hbar = \ell_P^2 \tag{4.4}$$

which is another form of the Heisenberg uncertainty relation on the Planck scale. Indeed, substituting here the expression for the Schwarzschild radius, we obtain

$$\Delta\left(2\,\frac{G}{c^3}\,mc\right)\Delta\bar{\lambda}_C \geq \frac{G}{c^3}\,\hbar \tag{4.5}$$

By canceling identical constants, we arrive at the Heisenberg uncertainty relation [18]

$$\Delta\left(mc\right)\Delta\bar{\lambda}_C \geq \frac{\hbar}{2} \tag{4.6}$$

4.2. Uncertainty Relations and Einstein's Equation

The uncertainty relation between the gravitational radius and the Compton wavelength of a particle is a special case of the general Heisenberg uncertainty relation on the Planck scale

$$\Delta R_\mu \Delta x_\mu \geq \ell_P^2 \tag{4.7}$$

where R_μ is a component of the radius of curvature of a small region of spacetime; x_μ is the conjugate coordinate of the small region. In fact, the indicated uncertainty relations can be obtained based on Einstein's equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{4.8}$$

where is the Einstein tensor, which combines the Ricci tensor, scalar curvature and metric tensor, $R_{\mu\nu}$ - Ricci tensor, obtained from the spacetime curvature tensor R_{abcd} by convolving it over a pair of indices, R is the scalar curvature, that is, the convoluted Ricci tensor, $g_{\mu\nu}$ is the metric tensor, Λ is the cosmological constant, and $T_{\mu\nu}$ is the energy-momentum tensor of matter, π is pi, c is speed of light in vacuum, G is Newton's gravitational constant.

In this form, the essence of the right side of Einstein's equations (4.8) is greatly obscured. It is advisable to rewrite these equations by grouping the constants into separate factors that have a specific meaning

$$\left(\frac{1}{4\,\pi}\right)(G_{\mu\nu}+\Lambda g_{\mu\nu}) = 2\left(\frac{G}{c^3}\right)\left(\frac{1}{c}\,T_{\mu\nu}\right) \tag{4.9}$$

A simple rearrangement of the factors allows us to gain deeper insight into the physical nature of the phenomenon [19]. It is known that the factor $(1/c)T_{\mu\nu}$ is associated with the density and flow of energy-momentum of matter and with the help of the factor (G/c^3) you can make the transition to the Planck scale, since the same factor is present in the expression for the Planck length $\ell_P = \sqrt{(G/c^3)\,\hbar}$.

When deriving his equations, Einstein assumed that physical space-time is Riemannian, that is, curved. A small region of Riemannian space is close to flat space. Example: if you cut out a small enough area from a sphere, the geometry will be imitated by Euclidean geometry. A similar technique—isolating the simplest from a more complex geometry (in this case, Euclidean geometry) by isolating a small part of the total space (here a sphere)—is a very common technique. Using the example of a sphere, it becomes clear that with a decrease in curvature or an increase in size, the surface locally approaches Euclidean space. Locally - in the small - the sphere can be approximated by part of the plane; globally - as a whole - impossible. This approximation is also realized in a more general case, when all curvature components decrease [10].

For any tensor field $N_{\mu\nu}$... the value $N_{\mu\nu}$... $\sqrt{-g}$ can be called the tensor density, where g is the determinant of the metric tensor $g_{\mu\nu}$ When the region of integration is small, $\int N_{\mu\nu\ldots}\sqrt{-g}\,d^4x$ is a tensor. If the region of integration is not small, then this integral will not be a tensor, since it represents the sum of tensors given at dierent points and, there-fore, is not transformed according to any simple law when transforming coordinates [20]. Only small areas are considered here. The above is also true when integrating over the three-dimensional hypersurface S^ν.

Thus, Einstein's equations for a small region of pseudo-Riemannian spacetime can be integrated over the three-dimensional hypersurface S^ν.

$$\frac{1}{4\pi}\int(G_{\mu\nu}+\Lambda g_{\mu\nu})\,\sqrt{-g}\,dS^\nu = 2\left(\frac{G}{c^3}\right)\frac{1}{c}\int T_{\mu\nu}\sqrt{-g}\,dS^\nu \tag{4.10}$$

Since the integrable region of spacetime is small, that is, it is practically at, from (4:10) we obtain the tensor equation

$$R_\mu = \frac{2G}{c^3}P_\mu \tag{4.11}$$

where $P_\mu = \frac{1}{c}\int T_{\mu\nu}\sqrt{-g}\,dS^\nu$ is the 4-pulse component matter; $R_\mu = \frac{1}{4\pi}\int(G_{\mu\nu}+\Lambda g_{\mu\nu})\,\sqrt{-g}\,dS^\nu$ is a component of the radius of curvature of a small region of spacetime.

The resulting tensor equation (4.11) can be rewritten in another form. Since $P_\mu = mcU_\mu$ then

$$R_\mu = \frac{2G}{c^3}\,P_\mu = \frac{2G}{c^3}mc\,U_\mu = r_g\,U_\mu \tag{4.12}$$

where r_g is the Schwarzschild radius (invariant of the radius of curvature), U_μ is the 4-speed, m is the gravitational mass. This entry reveals the physical meaning of the quantity R_μ, as the μ-component of the Schwarzschild radius. Note that here (compare, for example, with $dx_\mu dx^\mu = dS^2$).

Here the expression for the gravitational radius $r_g = 2(G/c^3)$mc is a more convenient form of notation than the form $r_g = 2(G/c^2)$m. In this case, continuity is visible between the resulting tensor equation (4.11) and the expression for the gravitational radius of a massive body or a similar expression for interacting massless photons $\lambda_g = 2(G/c^3)P$ and their connection with Planck length. This happens due to the presence of the (G/c^3) multiplier.

For a static spherically symmetric field and a static matter distribution we have $U_0 = 1, U_i = 0 (i = 1,2,3)$. In this case we get

$$R_0 = \frac{2G}{c^3}mc\,U_0 = \frac{2G}{c^3}\,mc = r_g \tag{4.13}$$

In a small region, space-time is practically at and the tensor equation (4:11) can be written in operator form

$$\hat{R}_\mu = \frac{2G}{c^3}\hat{P}_\mu = \frac{2G}{c^3}(-i\hbar)\frac{\partial}{\partial\,x^\mu} = -2i\,\ell_P^2\frac{\partial}{\partial\,x^\mu} \tag{4.14}$$

where $\hbar$ is the Dirac constant. Then the commutator of the operators $\hat{R}_\mu$ and $\hat{x}_\mu$ is equal to

$$[\hat{R}_\mu, \hat{x}_\mu] = -2i\ell_P^2 \tag{4.15}$$

This implies the above uncertainty relations

$$\Delta R_\mu \Delta x_\mu \geq \ell_P^2 \tag{4.16}$$

Substituting into (4:16) the values $R_\mu = \frac{2G}{c^3}P_\mu$ and $\ell_P^2 = \frac{\hbar G}{c^3}$ and canceling the same constants on the right and left, we arrive at the Heisenberg uncertainty relations.

$$\Delta P_\mu \Delta x_\mu \geq \frac{\hbar}{2} \tag{4.17}$$

Note that now, according to the equation $R_\mu = (2G/c^3)P_\mu$, along with the expressions for energy-momentum quanta P_{\mu}= h k_{\mu}. the expressions for the quantity $P_\mu = hK_\mu$ are valid (but not spacetime quanta), where k_μ is the wave 4-vector. That is, the quantity $R_\mu = 2\ell_P^2 k_\mu$ (component of the Schwarzschild radius) is quantized, but the quantization step is extremely small. This could serve as the basis for constructing a quantum theory of gravity.

In the static case, the relation must be valid

$$R_0^{(n)} = r_g^{(n)} = 2\ell_P^2 k_0(n + 1/2); \quad n = 0, 1, 2, ... \tag{4.18}$$

that is, at the Planck level, the gravitational radius of black holes is quantized. Such Planck black holes can be called space quanta, if a space quantum is dened as a minimal volume that is further indivisible. In vacuum ($n = 0$) the gravitational radius of virtual Planck black holes will be $R_0^{(0)} = r_g^{(0)} = \ell_P^2 k_0$.

For a static spherically symmetric eld and a static distribution of matter, the found uncertainty relation takes the form

$$\Delta R_0 \Delta x_0 = \Delta r_g \Delta r \geq \ell_P^2 \tag{4.19}$$

where r_g is the Schwarzschild radius, r is the radial coordinate. Here $R_0 = r_g$, and $x_0 = ct = r$, since at the Planck level matter moves at the speed of light.
For vacuum at the Planck level, the last uncertainty relation $\Delta r_g \Delta r \geq \ell_P^2$ will be characteristic, since a state of motion or a velocity vector cannot be assigned to vacuum. In Minkowski space, due to its high symmetry, vacuum is the same state for all inertial frames of reference; in any frame of reference it will appear to be at rest (static). Therefore, the Planck vacuum, according to the specified uncertainty relation, will generate wormholes and tiny virtual black holes (quantum foam).

4.3. Basic Equation of Quantum Gravity

From equations (4.11) and (4.14) it is clear that the basic equation of the quantum theory of gravity (Klimets equation)[21] should have the following form (similar to the Schrodinger equation) [22].

$$-2i\ell_P^2 \frac{\partial}{\partial x^\mu}|\Psi(x_\mu)\rangle = \hat{R}_\mu|\Psi(x_\mu)\rangle \tag{4.20}$$

In equation (4.20), spatial and temporal coordinates are equal. The R_μ operator acts as a generator of infinitesimal displacements of quantum states. Its form depends on the specific situation.

4.4. Estimation of The Equations of General Relativity at The Planck Level

The last uncertainty relation (4.19) allows us to perform some estimates of the equations of general relativity in relation to the Planck scale. For example, the expression for the invariant interval dS in the Schwarzschild solution has the form

$$dS^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - r_g/r} - r^2(d\Omega^2 + \sin^2 \Omega d\varphi^2) \tag{4.21}$$

Substituting here, according to the uncertainty relations, instead of r_g the value $r_g \approx \ell_P^2/r$ we get

$$dS^2 \approx \left(1 - \frac{\ell_P^2}{r^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \ell_P^2/r^2} - r^2(d\Omega^2 + \sin^2 \Omega d\varphi^2) \tag{4.22}$$

It can be seen that at the Planck level $r = \ell_P$ the invariant interval dS is limited from below by the Planck length; at this scale, division by zero appears, which means the formation of real and virtual Planck black holes.

Similar estimates can be made for other general relativity equations. In macroscopic physics, when encountering a heavy body, we must first of all estimate the ratio of the gravitational radius to the distance to the center of gravity $\zeta = r_g/r$ and we will already know a lot about the magnitude of the effects associated with general theory of relativity. For example, the ζ parameter determines the scale of change in the clock rate. For the Sun, the ζ parameter is approximately $4•10^{-6}$ or 1.76 arcsec, that is, a ray of light passing near the edge of the solar disk, it will deviate by an amount of the order of $4•10^{-6}$ radians. For Mercury, this parameter will be 10^{-7}, which in one hundred Earth years gives the displacement of Mercury's perihelion 43 arcsec. The ζ parameter is included in all other estimates. But, as we found out above, the parameter $\zeta = r_g/r$ at the Planck level has the form $\sim \ell_P^2/r^2$, so in order to, in order to estimate any relation obtained within the framework of the general theory of relativity in relation to the Planck scale, it is necessary to replace the relation rg/r with the expression $\zeta \sim \ell_P^2/r^2$. Indeed, we saw above that the parameter ζ determines at the Planck level the collapse of photons, the dimension of space, the non-Euclidean nature of spacetime, and fluctuations of the spacetime metric.

4.5. Shimmering Spacetime Geometry and Virtual Black Holes

The gravitational field makes zero oscillations, and the geometry associated with it also oscillates. The ratio of the circumference to the radius fluctuates around the Euclidean value: the smaller the scale, the greater the deviations from Euclidean geometry become.

Let us estimate the order of the wavelength of zero gravitational oscillations, at which the geometry becomes completely different from Euclidean [23]. The degree of deviation of the ζ geometry from the Euclidean one in the gravitational field is determined by the ratio of the gravitational potential φ and the square of the speed of light $c : \zeta = \varphi/c^2$. When $\zeta << 1$, the geometry is close to Euclidean; at $\zeta \sim 1$ all similarity disappears. The oscillation energy of the scale L is equal to $E = \hbar v \sim \hbar c/L$ (c/L is the order of the oscillation frequency). The gravitational potential created by the mass m at such a length is $\varphi = Gm/L$, where G is the constant of universal gravity. Instead of m you should substitute the mass, which, according to Einstein's formula, corresponds to the energy E ($m = E/c^2$). We get $\varphi = GE/Lc^2 = G\hbar/L^2c$. Dividing this expression by c^2, we obtain the deviation value $\zeta = G\hbar/c^3L^2 = \ell_P^2/L^2$. Equating $\zeta = 1$, we find the length at which the Euclidean geometry is completely distorted. It is equal to the Planck length $\ell_P = \sqrt{G\hbar/c3} \approx 10^{-35}$ m. This is where quantum foam comes in.

The spacetime metric $g_{00} \approx 1-\Delta_g = 1-\ell_P^2/(\Delta r)^2$ fluctuates, generating the so-called spacetime quantum foam, consisting of virtual Planck black holes and wormholes [14]. But these fluctuations $\Delta_g = 1-\ell_P^2/(\Delta r)^2$ in the macroworld and in the world of atoms are very small compared to 1 and become noticeable only on the Planck scale. Fluctuations Δ_g must be taken into account when using the special relativity metric (+1,−1,−1,−1) in very small regions of space and at large momenta. Therefore, the expression for the invariant interval dS in spherical coordinates must always be written in the form

$$dS^2 = \left(1 - \frac{\ell_P^2}{(\Delta r)^2}\right) c^2 dt^2 - \frac{dr^2}{1 - \ell_P^2/(\Delta r)^2} - r^2(d\Omega^2 + \sin^2 \Omega d\varphi^2) \qquad (4.23)$$

However, due to the smallness of $\ell_P^2/(\Delta r)^2$, the expression for the invariant interval is usually written in Galilean form (+1,−1,−1,−1), which is incorrect. The correct expression must take into account fluctuations of the spacetime metric and the gravitational collapse of matter at the Planck distance scale. It can be seen that on the Planck scale Lorentz invariance is violated.

In physical work, a certain small parameter is usually determined, which can be neglected under clearly dened conditions. As a rule, the approximation is expressed in the form of an inequality when the dimensionless quantity dening the approximation becomes small compared to unity. For example, classical Newtonian mechanics is true if two conditions are met: $v/c << 1$; $\hbar / S << 1$ (c is the speed of light, v is the speed of the body, $\hbar$ is Planck's constant, S is an action).[10] In our case, special and general relativity are true when $\ell_P^2/ << 1$ (ℓ_p is the Planck length, L is the macroscopic length). When $\ell_P^2/L^2 \sim 1$ the laws of quantum gravity apply. Approximations reign in physics.

It is known that the coordinate speed of light c_k in some place with gravitational potential $\varphi = -Gm/r$ is equal to $c_k = c\,(1+2\,\varphi/c^2) = c\,(1-r_g/r)$, where c is the physical speed of light. Then on the Planck scale, due to quantum uctuations of the potential, the expression for the coordinate speed of light will take the form $c_k = c\,(1 - \ell_P^2/(\Delta\lambda)^2)$.

Here λ is the wavelength of light emitted by the source. The greater the distance from the source the light travels and the shorter its wavelength, the more noticeable the dispersion of the rays will be due to accumulated distortions. In this case, the photon velocity inho-mogeneities $\Delta c = \ell_P^2/(\Delta\lambda)^2$ are determined not by the Planck length, but by its square, so that these inhomogeneities are immeasurably small (of the order of $10^{-56}\,c$ for $\lambda = 10^{-5}$ cm) and images of distant sources will be sharp even at metagalactic distances [24].

As noted in for a region of spacetime with dimensions L, the uncertainty of the Christoel symbols should be of order ℓ_P^2/L^3, and the uncertainty of the metric tensor should be of order ℓ_P^2/L^2. If L is a macroscopic length, then quantum limitations are fantastically small and can be neglected even at atomic scales. If the value of L is comparable to ℓ_P, then maintaining the previous (ordinary) concept of space becomes more and more dicult and the in uence of microcurvature becomes obvious [25].

The expression for metric uctuations is consistent with the Bohr-Rosenfeld uncer-tainty relation $\Delta g\,(\Delta L)2 \geq \ell_P^2$ [8]. From this point of view, other expressions for uctuations of the metric tensor, namely $\Delta g \sim \ell_P/L$ and its rst derivatives (Christoel symbols) $\Delta r \sim \ell_P^2/L^2$, set to 1 by analogy with electrodynamics do not correspond to reality, since gravity (ge ometrodynamics) is fundamentally dierent from electrodynamics [26, 27]. Observations of the degree of blurring of distant stellar objects did not conrm these expressions. The correct expression is $\Delta g \sim \ell_P^2/L^2$ [28].

As emphasized in these small-scale uctuations indicate that everywhere in space something similar to gravitational collapse is happening all the time, that gravitational collapse is essentially constantly occurring, but the reverse process is also constantly occurring, that in addition to the gravitational collapse of the Universe and stars, it is also necessary to consider a third and the most important level of gravitational collapse at the Planck distance scale.

The uncertainty relations written above are valid for any gravitational elds, since in a suciently small 4-region of any gravitational eld space-time is practically at. Note that according to Markov M.A. real Planck black holes with a mass of 10^{-5} g may not \ evaporate", but be stable formations [29]. The fact is that the entire mass of a black hole can \evaporate", with the exception of that part of it that is associated with the energy of zero-point, quantum oscillations of the black hole's matter. Such vibrations do not increase the temperature of the object and their energy cannot be radiated. On the other hand, the quantum laws of conservation of baryon and lepton charges should also prevent the complete evaporation" of a black hole. The residual mass is 10^{-5} g. Planck black holes have an extremely small interaction cross section 10^{-66}cm^2. This leads to the fact that stars and planets are almost completely transparent to them - the mean free path of a Planck black hole in matter of nuclear density is comparable to the radius of the visible part of the Universe, and therefore they are very dicult to detect. Therefore, Planck black holes, which arose as a result of the collapse of radiation in the rst fractions of a second of the Big Bang (for example, during the collision of energetic photons), could hypothetically serve as a source of mysterious dark matter. As is known, dark matter does not manifest itself in any way, except for the gravitational eect on the visible part of matter [14].

On the other hand, the uncertainty relation rgr $\Delta r_g\, \Delta r \geq \ell_P^2$ indicates that on the Planck scale there is a vacuum consisting of virtual Planck black holes. The energy density of such a vacuum does not change as the Universe expands, which creates negative vacuum pressure. This vacuum can serve as a source of dark energy. From the uncertainty relation $\Delta r_g\, \Delta r \geq \ell_P^2$ it follows that a decrease in Δr will lead to an increase in Δr_g and vice versa. When $r << \ell_P$ the Schwarzschild radius r_g exceeds both r and the Planck length ℓ_P Therefore, any attempt to probe length scales $r << \ell_P$ will require localizing the energy within a radius that is much smaller than the corresponding Schwarzschild radius, $r_g >> \ell_P$. Thus, the corresponding act of measurement will result in the formation of a macroscopic classical black hole long before we have a chance to measure the distance $r << \ell_P$ [1].

It can be seen that the Planck length is the limit of distance, less than which the very concepts of space and length cease to exist. Any attempt to explore the existence of shorter distances (less than $1{:}66^{-35}$m) by carrying out collisions at higher energies would inevitably end in the birth of a black hole. Collisions at high energies, instead of breaking matter into smaller pieces, will lead to the birth of black holes of ever larger sizes [30]. Decreasing the Compton wavelength of the particle will lead to an in- crease in the Schwarschild radius of the black hole. The uncertainty relation between the Schwarzschild radius and the Compton wavelength gives rise to virtual black holes on the Planck scale [31].

Virtual Planck black holes are also important for the theory of elementary parti- cles. The fact is that when carrying out calculations in modern quantum theory and, in particular, when calculating the intrinsic energy of particles, the contribution of in-termediate states with arbitrarily high energy is usually taken into account, which leads to the appearance of known divergences. Taking into account the gravitational interac-tion of the corresponding virtual particles and the possibility of the emergence of virtual (short-lived) black holes in the intermediate state should lead to the elimination of these divergences [14].

It can be seen that at the Planck level, matter is in a black hole state, and Planck black holes are characterized by dierent quantum numbers. It is assumed that the basis (nuclei) of quarks and leptons are Planck black holes and this may be an alternative to string theory [32]. Signicant matter can be built from Planck black holes. In a free state, Planck black holes, as noted above, can act as so-called dark matter.

The problem of singularities in Planck black holes is resolved if we assume that the singularities are multidimensional and therefore have unlimited capacity and nite density of matter [2]. It is assumed that the additional dimensions of space in the singularity are compactied (folded into rings). Thus, the three-dimensionality of the external, observ-able space is due to the energetic advantage in the formation of virtual Planck black holes, and the multidimensional nature of the singularities hidden under the event horizon in black holes solves the problem of the innite density of collapsing matter.

4.6 Space quantization and Planck length

In the 1960s, the hypothesis of the quantization of spacetime along the path of unifying quantum mechanics and general relativity led to the assumption that there are cells of spacetime with the minimum possible length equal to the fundamental length. According to this hypothesis, the degree of in uence of space quantization on transmitted light depends on the size of the cell. Research requires intense radiation that travels as far as possible. From the picture of space-time foam presented by Wheeler it follows that for photons with a wavelength λ propagating in the foam, the travel time T from the source to the detector must be indenite in accordance with the law , which can only depend on the distance traveled x, the wavelength of the particle λ and the Planck scale ℓ_P with a shape of type $\delta T \sim x^n \ell_p^{1+m-n} / \lambda^m$, where m and n are model-dependent powers, and $1 + m - n > 0$. The phenomenology of quantum gravity currently focuses mainly on eects suppressed at the rst power of the Planck scale, since stronger suppression leads to even weaker eects [33-36]. Therefore, the picture that experimenters are now focusing on corresponds to the following choice: $n = m = 1$, that is, $\delta T \sim x\, \ell_p/\lambda$.

Currently, a group of scientists has used data from the gamma-ray burst GRB 041219A, taken from the European space telescope Integral. The gamma-ray burst GRB 041219A was included in the one percent of the brightest gamma-ray bursts over the entire observation period, and its source is at least 300 million light years away. The Integral observation made it possible to estimate the cell size several orders of magnitude more accurately than all previous experiments of this kind.

Analysis of the data showed that if the granularity of space exists at all, then it should be at a level of 10^{-48} meters or less [24]. The theory of spacetime quantization is discredited by this. There are two options available to explain this fact. The rst option assumes that at the micro level|on the Planck scale|space and time vary simultaneously with each other, so that the speed of photon propagation does not change. The second explanation assumes that photon velocity inhomogeneities are determined not by the Planck length, but by its square (of the order of 10^{-66}cm^2), so that these inhomogeneities become immeasurably small. Indeed, in a gravitational eld, the coordinate speed of light changes, as a result of which light rays are bent. If we denote by c the physical speed of light at the origin, then the coordinate speed of light c_k at some place with a gravitational potential φ will be equal to $c_k \approx c(1 + \varphi/c^2)$. But then, as was shown above, on the Planck scale $c_k \approx c(1- \ell_p^2/l^2)$. That is, uctuations in the speed of light $\Delta c \approx c\ell_p^2/l^2$ are determined not by the Planck length, but by the square of the Planck length and therefore are immeasurably small. In fact, if the wavelength of visible light is $\lambda \approx 10^{-5}$ cm, then in this case the ratio $\ell_p^2/\lambda^2 = 10^{-66} / 10^{-10} = 10^{-56}$ is less than the ratio $\ell_p/\lambda = 10^{-3}3/ 10^{-5} = 10^{-28}$ by 28 orders of magnitude [33]. From a modern point of view, the hypothesis of the quantization of spacetime is unsatisfactory. In fact, from Einstein's equations, as has been shown, the quantization of the curvature of spacetime (quantization of the Schwarzschild radius) follows. In accor-dance with this, the dispersion of light rays from distant galaxies is determined not by the Planck length, but by its square, $n = 1$; $m = 2$ and $\delta T \sim x\, \ell_p^2/ \lambda^2$, therefore, uctuations in the speed of light will be immeasurably small and images of distant sources will be sharp even at metagalactic distances [37-40].

5. On the Problem of Singularities

5.1. Introductory Statements

One of the difficulties of the general theory of relativity is the problem of singularities, which actually arose from the moment Friedman obtained non-stationary cosmological solutions to the equations of the general theory of relativity and became even more acute in connection with the problem of gravitational collapse. Singularity denotes a state of infinite density of matter, which indicates the insufficiency of the general theory of relativity. Multidimensionality solves these problems.

5.2. How to Place the Universe at A "Point"

The Universe at a "point" is the author's asserted possibility of placing spaces of any extent in a multidimensional "point" with a given size (that is, in a small region of multidimensional space), including the free placement of our entire Universe in a multidimensional "point" with a diameter of 10^{-33}cm.For a book, as an example of a 3-dimensional object, the amount of information in the form of letters takes up V volume in the book.

If the same amount of information is placed in 2-dimensional space, that is, on a plane, then in the form of lines the information will occupy an area S with a square side a(2), and a(2) > a(3), where a(3) is the side of a 3-dimensional cube representing a book.The same amount of information, placed in a one-dimensional space, in the form of a string will stretch in length by the value a(1), and

$$a(1) > a(2) > a(3) \tag{5.1}$$

Accordingly, as the number of dimensions of space increases, to accommodate the same amount of information (in the form of letters), we will need an n-dimensional cube with an ever smaller side $a(n)$ of the corresponding n-dimensional cube, that is

$$a(1) > a(2) > \cdots > a(k) > \cdots > a(n) \tag{5.2}$$

It is easy to show that $a(n)$ and $a(k)$ are related by the following relation

$$a(n) = a(k)^{k/n} \tag{5.3}$$

Indeed, (5.3) follows from the equality of volumes of information (or matter) in one or another n-dimensional space

$$V(1) = V(2) = \cdots = V(k) = \cdots = V(n) \tag{5.4}$$

where $V(n)$ are "volumes" of n-dimensional spaces containing the same (equal) number of units of information (or units of matter - atoms), located at the nodes of n-dimensional cubic lattices with a step of d in one or another n-dimensional space. One can imagine that the distance d between particles (atoms) becomes smaller and smaller. Chains of particles in the direction of each coordinate axis transform into what we call continuum. And our rows of atoms turn into solid lines $V(1)$, planes $V(2)$, volumes $V(3)$, etc. to

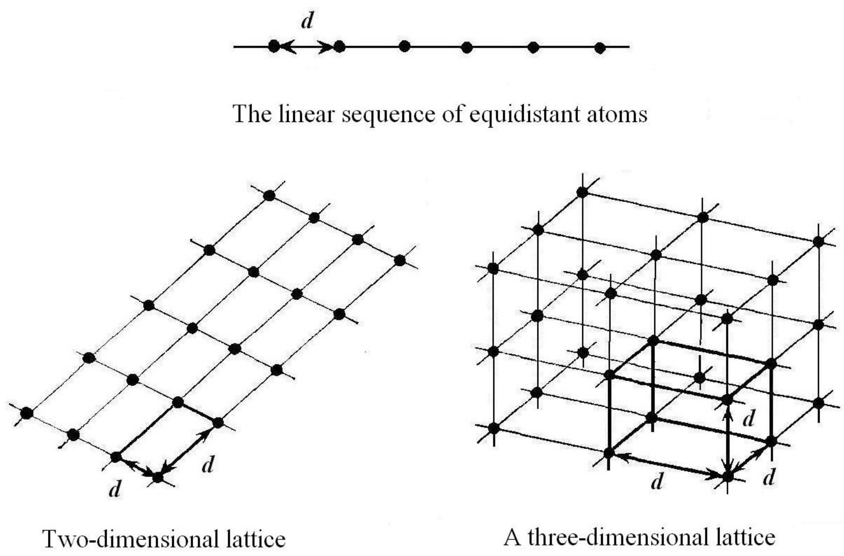

Figure 4:

$$V(1) = a(1)^1; V(2) = a(2)^2; \cdots ; V(k) = a(k)^k; \cdots ; V(n) = a(n)^n \tag{5.5}$$

then (5.3) follows from here. Here, for example, $a(1) = d \cdot t$, where t is the number of lattice steps.
For a 3-dimensional space from (5.3) we obtain the following relation

$$a(n) = a(3)^{3/n} \tag{5.6}$$

An interesting conclusion follows from the relation (5.6). Suppose we need to place the entire observable Universe together with matter in an elementary n-dimensional "cube" with side $a(n)$ equal to 10 10^{-33}cm = 10. ℓ_p (then there are ten units of Planck length), where $\ell_p = 10^{-33} cm$ s one unit of Planck length. How many dimensions of space do we need for this?
The size of the observable Universe is 10^{28} cm or, in Planck length units, 10^{61} ℓ_p Planck length units. From the relation (5:6) we have

$$10^1 \ell_P = (10^{61} \ell_P)^{3/n} \tag{5.7}$$

Hence $n = 183$. From (5.7) it is clear that already with 183 dimensions of space, the entire observable Universe can be placed in a 183-dimensional \cube" with a side $10\ell_P$, that is, in fact, in a "point" (183-dimensional).

The density of matter in such a "cube" remains equal to the density of matter located in the 3-dimensional space of the observable Universe. Indeed, the density of matter in n-dimensional space is determined as follows: $\rho(n) = M / V(n)$, where M is the mass of matter of the observable Universe, $V(n)$ is the volume n-dimensional space, $\rho(n)$ is the density of matter in n-dimensional space. And since, by condition, $V(3) = V(183)$, then $\rho(3) = \rho(183)$.

An illustrative example: folding a one-dimensional thread of length r1 into a flat twodimensional "mat" in the form of a spiral with a diameter of r_2 or into a three-dimensional ball with a diameter of r_3. It is clear that $r_1 > r_2 > r_3$, that is, the compactness of the placement of the thread increases with increasing dimension of space, but the density of placement of the substance of the thread remains the same (the atoms of the substance of the thread will still be located at a distance of d from each other in the direction of

each n-th coordinate axis. (Fig.4)
Based on the above, we claim that any finite-dimensional space can be placed in an infinite-dimensional "point".

It can be assumed that the singular "point" (that is, a very small region of space), from which, according to the general theory of relativity, our Universe arose, was multidimensional.
It can also be assumed that during the collapse of black holes, when the matter of the black hole reaches a certain (for example, Planck?) density, the collapsing matter in the center of the black hole (in the singularity) is squeezed out into other dimensions of space, which can be folded (compactified) into rings with a diameter on the order of the Planck density length.

6. Conclusion

The approach to quantum gravity outlined in the article is based on the assumption that in a small region Riemannian space-time is practically flat. This allows us to reduce the Einstein equation to a tensor equation of the first rank and study it from a quantum mechanical point of view. This brings us to the Planck limit. In this case, three-dimensionality arises as the most energetically favorable state of quantum foam, the basis of the fabric of the Universe. Taking into account metric fluctuations in special relativity (at the Planck level) should lead to the elimination of divergences in quantum field theory. The hypothesis that the singularity of black holes is multidimensional eliminates the question of its infinite density. In constructing the theory, philosophical considerations played a certain role.

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