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Closed-Form Solutions for Minimizing Sum MSE in Multiuser Relay Networks

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***Abstract*—A scenario consisting of single antenna source-destination node pairs communicating using multiple single antenna relays is considered. A transmission scheme utilizing two subsequent time-slots assuming a time invariant channel throughout the transmission period is applied to this scenario. In the first time-slot, the source nodes transmit to both the relays and the destination nodes. Both the source nodes and the relays retransmit to the destination nodes in the second time-slot. As the relays can not decode the received signals, amplify and forward relaying is used. By exploiting the direct links between the source and the destination nodes, each destination node observes a two dimensional signal space. In the present paper, the sum mean square error can be written as a convex quadratic function of the relays' gain and part of the temporal filters at the source and the destination nodes. Moreover, closed form solutions for the minimum sum mean square error with interference zero forcing constraints and a total energy constraint are obtained. The results show that the proposed schemes outperform the interference alignment scheme at low and moderate SNRs.**

I. Introduction

Due to the increasing demands of high aggregate throughput at user equipments (UE), upcoming multiuser wireless systems have to smartly utilize their limited resources to serve many UEs with a high QoS. Interference alignment (IA) is a promising transmission technique as it maximizes the degrees of freedom (DoF) [1]. By definition, the relationship between the DoF and the sum rate of such an interference network is linear only at infinite SNRs [2]. As a result, IA may only achieve asymptotically the sum capacity at high SNR. In practice, only low to moderate SNRs are achievable at the destination nodes in most of the current communications systems. Therefore, finding a transmission technique which maximizes the performance at the low and moderate SNR regime is of interest.

In the low and moderate SNR regime, the received noise has to be tackled together with the interference as it significantly affects the performance. As a consequence, minimizing the sum mean square error (sum MSE) is considered in this paper. By minimizing the sum MSE, both the received noise and the interference are compensated keeping only the desired signal.

Many previous contributions tackle the problem of minimizing the sum MSE. For MIMO interference channels, the authors of [3] and [4] propose an iterative method which minimizes the sum MSE. First, the transmit filters are optimized in a closed form with a priori known receive filters of the previous iteration. Then the optimum transmit filters are applied to optimize the receive filters and the process iteratively starts again. Because the optimization problem is non-convex and this method does not jointly optimize both the transmit and the receive filters, this method may lead to a local optimum.

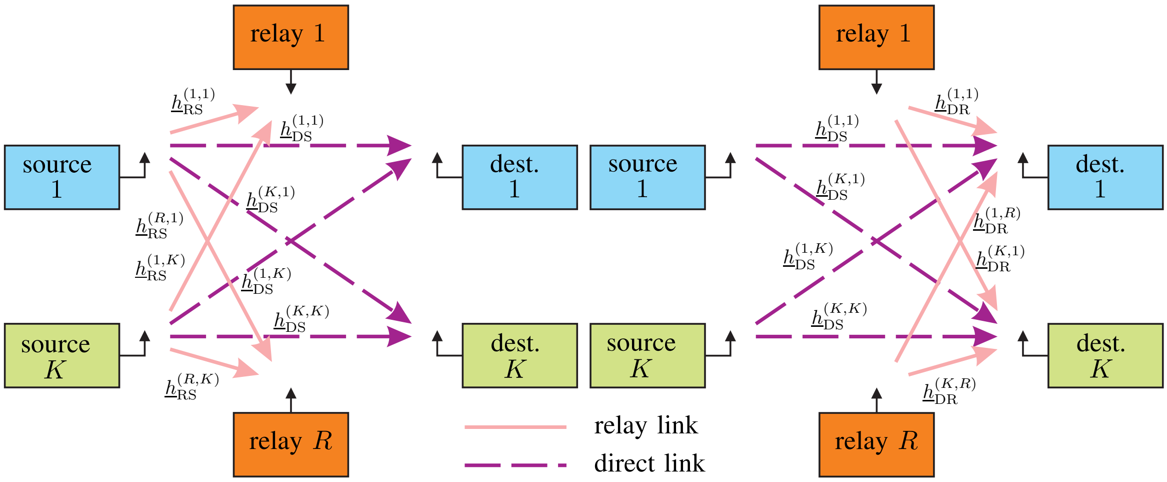
In [5], a scenario consisting of a single antenna node pair communicating through several single antenna relays is considered. The authors optimize the relays' gains aiming at minimizing the MSE at the destination node. Extending this scenario to multiple node pairs, a closed form solution which minimizes the sum MSE is derived in [6]. However, IA is not feasible in the scenario of [6] as the dimension of the receive signal space of every destination node is one, i.e., each destination node is equipped with a single antenna and it receives once. By exploiting the direct links among the node pairs in this scenario, IA achievability is shown in [7].

In the present paper, a scenario of several node pairs communicating through a number of relays is considered. Each of the nodes and the relays is equipped with a single antenna. By exploiting the direct links among the node pairs, fully adapting the relays and partially adapting the transmit and the receive filters, the minimum sum MSE is found under an IA constraint and a total energy constraint.

The rest of the paper is organized as follows. The system model is discussed in the next section. In Section III, the sum MSE of the considered scenario is derived. Closed form solutions for the minimum MSE are obtained in Section IV. Section V shows and discusses some numerical results. Finally, the conclusions are drawn in Section VI.

II. System Model and Transmission Scheme

A scenario consisting of K source-destination node pairs and R one-way relays is considered. Every node and relay is equipped with a single antenna so that it is impossible for a relay to separate the received signals. Consequently, the amplify and forward relaying strategy is used. Moreover, full CSI is assumed to be available at the nodes and the relays. A two time-slot transmission scheme is considered where τ denotes the index of the time-slot. At the first time-slot $\tau = 1$, the source nodes transmit to both the relays and the destination nodes as shown in Fig. 1a. Both the source nodes and the relays retransmit to the destination nodes at the second time-slot $\tau = 2$ as illustrated in Fig. 1b. The channel coefficients between the l-th source node and k-th destination



(a) (b)

Fig. 1: Two time-slot scenario: (a) the source nodes transmit to both the relays and the destination nodes, (b) both the source nodes and the relays retransmit to the destination nodes.

node, the l-th source node and r-th relay and the r-th relay and the k-th destination node are denoted by $\underline{h}_{\mathrm{DS}}^{(k,l)}$, $\underline{h}_{\mathrm{RS}}^{(r,l)}$ and $\underline{h}_{\mathrm{DR}}^{(k,r)}$, respectively. All channels are considered to be constant throughout the transmission duration. Also, it is assumed that the noise signals $\underline{n}_{\mathrm{D}}^{(k,\tau)}$, $k = 1,\ldots,K$, $\tau = 1,2$, and $\underline{n}_{\mathrm{R}}^{(r)}$, $r = 1,\ldots,R$, at the destination nodes and at the relays, respectively are additive white Gaussian noise (AWGN) with zero mean and the same variance σ^2.

Let $\underline{d}^{(l)}$ be the transmitted data symbol of the l-th source node which is scaled by $\underline{v}_{\tau}^{(l)}$ in the τ-th time-slot. It is assumed that the data symbols are uncorrelated and that they have equal average energies $E_{\mathrm{d}} = \mathrm{E}\left\{\left|\underline{d}^{(k)}\right|^2\right\}$, $\forall k$. Additionally, let $\underline{e}_{\tau}^{(k)}$, $\tau = 1,2$, and $\underline{e}_{\mathrm{R}}^{(r)}$ be the received signals at the k-th destination node in the τ-th time-slot and the r-th relay, respectively. At the first time-slot, the received signals at the k-th destination node and the r-th relay are

$$\underline{e}_1^{(k)} = \sum_{l=1}^{K} \underline{h}_{\mathrm{DS}}^{(k,l)} \underline{v}_1^{(l)} \underline{d}^{(l)} + \underline{n}_{\mathrm{D}}^{(k,1)}, \tag{1}$$

and

$$\underline{e}_{\mathrm{R}}^{(r)} = \sum_{l=1}^{K} \underline{h}_{\mathrm{RS}}^{(r,l)} \underline{v}_1^{(l)} \underline{d}^{(l)} + \underline{n}_{\mathrm{R}}^{(r)}, \tag{2}$$

respectively. At the second time-slot, the received signal at the k-th destination node reads

$$\underline{e}_2^{(k)} = \sum_{l=1}^{K} \underline{h}_{\mathrm{DS}}^{(k,l)} \underline{v}_2^{(l)} \underline{d}^{(l)} + \sum_{r=1}^{R} \underline{h}_{\mathrm{DR}}^{(k,r)} \underline{g}^{(r)} \underline{e}_{\mathrm{R}}^{(r)} + \underline{n}_{\mathrm{D}}^{(k)}, \tag{3}$$

where $\underline{g}^{(r)}$ is a complex scaling factor of the r-th relay. As each destination node receives twice, it combines the received signals with the weights $\underline{u}_{\tau}^{(k)*}$, $\forall k, \tau = 1,2$. The received data symbol at the k-th destination node reads

$$\hat{\underline{d}}^{(k)} = \underline{u}_1^{(k)*} \underline{e}_1^{(k)} + \underline{u}_2^{(k)*} \underline{e}_2^{(k)}. \tag{4}$$

If $\underline{v}_1^{(l)}$ and $\underline{u}_2^{(k)}$ $\forall k, l$ are fixed, the received data symbol becomes a linear function of $\underline{g}^{(r)}$, $\underline{v}_2^{(l)}$, and $\underline{u}_1^{(k)}$, $\forall r, l, k$. Accordingly, the received data symbol can be written in a vector form as

$$\hat{\underline{d}}^{(k)} = \sum_{l=1}^{K} \underline{\mathbf{g}}^{*\mathrm{T}}\, \underline{\mathbf{q}}^{(k,l)} \underline{d}^{(l)} + \underline{\mathbf{g}}^{*\mathrm{T}}\, \underline{\mathbf{z}}^{(k)} + \underline{u}_2^{(k)*} \underline{n}_{\mathrm{D}}^{(k,2)} \tag{5}$$

where

$$\underline{\mathbf{g}} = \left(\underline{g}^{(1)*}, \cdots, \underline{g}^{(R)*} \middle| \underline{v}_2^{(1)*}, \cdots, \underline{v}_2^{(K)*} \middle| \underline{u}_1^{(1)}, \cdots, \underline{u}_1^{(K)}\right)^{\mathrm{T}}, \tag{6}$$

$$\begin{aligned}\underline{\mathbf{q}}^{(k,l)} = \big(&\underline{u}_2^{(k)*} \underline{v}_1^{(l)} \underline{h}_{\mathrm{DR}}^{(k,1)} \underline{h}_{\mathrm{RS}}^{(1,l)}, \ldots, \underline{u}_2^{(k)*} \underline{v}_1^{(l)} \underline{h}_{\mathrm{DR}}^{(k,R)} \underline{h}_{\mathrm{RS}}^{(R,l)} \big| \\ &\underbrace{0,\ldots,0}_{l-1}, \underline{u}_2^{(k)*} \underline{h}_{\mathrm{DS}}^{(k,l)}, \underbrace{0,\ldots,0}_{K-l} \big| \underbrace{0,\ldots,0}_{k-1}, \underline{v}_1^{(l)} \underline{h}_{\mathrm{DS}}^{(k,l)}, \underbrace{0,\ldots,0}_{K-k}\big)^{\mathrm{T}},\end{aligned} \tag{7}$$

and

$$\begin{aligned}\underline{\mathbf{z}}^{(k)} = \big(&\underline{u}_2^{(k)*} \underline{h}_{\mathrm{DR}}^{(k,1)} \underline{n}_{\mathrm{R}}^{(1)}, \ldots, \underline{u}_2^{(k)*} \underline{h}_{\mathrm{DR}}^{(k,R)} \underline{n}_{\mathrm{R}}^{(R)} \big| \\ &\underbrace{0,\ldots,0}_{K} \big| \underbrace{0,\ldots,0}_{k-1}, \underline{n}_{\mathrm{D}}^{(k,1)}, \underbrace{0,\ldots,0}_{K-k}\big)^{\mathrm{T}}.\end{aligned} \tag{8}$$

III. Sum Mean Square Error (Sum MSE)

In this section, the sum MSE function is derived and formulated as a convex quadratic function. The MSE of the k-th user is calculated as

$$\begin{aligned}\delta^{(k)} &= \mathrm{E}\left\{\left|\hat{\underline{d}}^{(k)} - \underline{d}^{(k)}\right|^2\right\} \\ &= E_{\mathrm{d}}\Big(\sum_{l=1}^{K} \underline{\mathbf{g}}^{*\mathrm{T}} \underline{\mathbf{q}}^{(k,l)}\, \underline{\mathbf{q}}^{(k,l)*\mathrm{T}} \underline{\mathbf{g}} - \underline{\mathbf{q}}^{(k,k)*\mathrm{T}}\, \underline{\mathbf{g}} \\ &\quad - \underline{\mathbf{g}}^{*\mathrm{T}}\, \underline{\mathbf{q}}^{(k,k)} + 1\Big) + \sigma^2\Big(\underline{\mathbf{g}}^{*\mathrm{T}} \mathbf{N}^{(k)} \underline{\mathbf{g}} + \left|\underline{u}_2^{(k)}\right|^2\Big),\end{aligned} \tag{9}$$

where

$$\mathbf{N}^{(k)} = \mathrm{diag}\Big(\left|\underline{h}_{\mathrm{DR}}^{(k,1)}\right|^2 \left|\underline{u}_2^{(k)}\right|^2, \cdots, \left|\underline{h}_{\mathrm{DR}}^{(k,R)}\right|^2 \left|\underline{u}_2^{(k)}\right|^2 \Big| \underbrace{0,\cdots,0}_{K} \Big| \underbrace{0,\cdots,0}_{k-1}, 1, \underbrace{0,\ldots,0}_{K-k} \Big). \quad (10)$$

Because there are multiple destination nodes in our considered scenario, the objective should be to minimize the individual MSE at each destination node. Unfortunately, this results in a multi-objective optimization problem. Instead, it is reasonable to consider the sum MSE as an objective function. The sum MSE can be written as

$$\delta = \sum_{k=1}^{K} \delta^{(k)} = \underline{\mathbf{g}}^{*\mathrm{T}} \underline{\mathbf{A}}\, \underline{\mathbf{g}} - \underline{\mathbf{b}}^{*\mathrm{T}} \underline{\mathbf{g}} - \underline{\mathbf{g}}^{*\mathrm{T}} \underline{\mathbf{b}} + K E_{\mathrm{d}} + \sigma^2 \left|\underline{u}_2^{(k)}\right|^2, \quad (11)$$

where

$$\underline{\mathbf{A}} = \sum_{k=1}^{K}\sum_{l=1}^{K} E_{\mathrm{d}} \underline{\mathbf{q}}^{(k,l)} \underline{\mathbf{q}}^{(k,l)*\mathrm{T}} + \sum_{k=1}^{K} \sigma^2 \mathbf{N}^{(k)}, \quad (12)$$

and

$$\underline{\mathbf{b}} = \sum_{k=1}^{K} E_{\mathrm{d}} \underline{\mathbf{q}}^{(k,k)}. \quad (13)$$

It can be observed from (11) and (12) that the sum MSE is essentially affected by the noise especially at low and moderate SNRs. However, at high SNRs, the noise term in (12) can be neglected.

IV. Minimizing Sum MSE

A. Interference alignment constrained minimum sum MSE

In this section, the sum MSE is minimized with the constraint of IA. To achieve IA, two conditions have to be satisfied [7]

$$\underline{\mathbf{g}}^{*\mathrm{T}}\ \underline{\mathbf{q}}^{(k,l)} = 0, \quad \forall k,\ l \neq k, \quad (14)$$

and

$$\underline{\mathbf{g}}^{*\mathrm{T}}\ \underline{\mathbf{q}}^{(k,k)} \neq 0, \quad \forall k. \quad (15)$$

Accordingly, the optimization problem is formulated as

$$\underline{\mathbf{g}}_{\mathrm{IAC}} = \underset{\underline{\mathbf{g}}}{\mathrm{argmin}} \left\{ \underline{\mathbf{g}}^{*\mathrm{T}} \underline{\mathbf{A}}\, \underline{\mathbf{g}} - \underline{\mathbf{b}}^{*\mathrm{T}} \underline{\mathbf{g}} - \underline{\mathbf{g}}^{*\mathrm{T}} \underline{\mathbf{b}} \right\} \quad (16)$$

subject to

$$\underline{\mathbf{H}}\ \underline{\mathbf{g}} = \mathbf{0}, \quad (17)$$

where the rows of $\underline{\mathbf{H}}$ correspond to $\underline{\mathbf{q}}^{(k,l)*\mathrm{T}}$, $\forall k,\ l \neq k$, for all interference links. The linear system of equations of (17) is homogenous and it has nontrivial valid solutions if $R \geq K^2 - 3K + 2$ [7]. The optimization problem is convex as the objective function is convex and the constraints form an affine set. The Lagrangian is written as

$$\mathrm{L}\left(\underline{\mathbf{g}}, \underline{\lambda}\right) = \underline{\mathbf{g}}^{*\mathrm{T}} \underline{\mathbf{A}}\, \underline{\mathbf{g}} - \underline{\mathbf{b}}^{*\mathrm{T}} \underline{\mathbf{g}} - \underline{\mathbf{g}}^{*\mathrm{T}} \underline{\mathbf{b}} + \underline{\lambda}\ \underline{\mathbf{H}}\ \underline{\mathbf{g}}, \quad (18)$$

where $\underline{\lambda}$ is a vector of Lagrangian multipliers each of which corresponds to a constraint of (17). Taking the generalized derivatives of (18) with respect to $\underline{\mathbf{g}}$ and $\underline{\lambda}$ yields

$$\frac{\partial \mathrm{L}}{\partial \underline{\mathbf{g}}} = \underline{\mathbf{A}}^{\mathrm{T}} \underline{\mathbf{g}}^{*} - \underline{\mathbf{b}}^{*} + \underline{\mathbf{H}}^{\mathrm{T}} \underline{\lambda}^{\mathrm{T}} \stackrel{!}{=} 0 \quad (19)$$

and

$$\frac{\partial \mathrm{L}}{\partial \underline{\lambda}} = \underline{\mathbf{H}}\ \underline{\mathbf{g}} \stackrel{!}{=} \mathbf{0}, \quad (20)$$

respectively. Solving (19) for $\underline{\mathbf{g}}$ and substituting the result in (20) yields

$$\underline{\mathbf{H}} \left(\underline{\mathbf{A}}^{*\mathrm{T}}\right)^{-1} \left(\underline{\mathbf{b}} - \underline{\mathbf{H}}^{*\mathrm{T}} \underline{\lambda}^{*\mathrm{T}}\right) = \mathbf{0}. \quad (21)$$

Solving (21) for $\underline{\lambda}$ yields

$$\underline{\lambda} = \underline{\mathbf{b}}^{*\mathrm{T}} \underline{\mathbf{A}}^{-1} \underline{\mathbf{H}}^{*\mathrm{T}} \left(\underline{\mathbf{H}}\ \underline{\mathbf{A}}^{-1} \underline{\mathbf{H}}^{*\mathrm{T}}\right)^{-1}. \quad (22)$$

By substituting (22) in (19), the optimum $\underline{\mathbf{g}}_{\mathrm{IAC}}$ is obtained as

$$\underline{\mathbf{g}}_{\mathrm{IAC}} = \left(\underline{\mathbf{A}}^{*\mathrm{T}}\right)^{-1} \underline{\mathbf{b}} - \left(\underline{\mathbf{A}}^{*\mathrm{T}}\right)^{-1} \underline{\mathbf{H}}^{*\mathrm{T}} \left(\underline{\mathbf{H}} \left(\underline{\mathbf{A}}^{*\mathrm{T}}\right)^{-1} \underline{\mathbf{H}}^{*\mathrm{T}}\right)^{-1} \underline{\mathbf{H}} \left(\underline{\mathbf{A}}^{*\mathrm{T}}\right)^{-1} \underline{\mathbf{b}}. \quad (23)$$

B. Total energy constrained minimum sum MSE

By partially adapting the filters, the transmitted energy in the first time-slot is fixed. In this section, the energy transmitted in the second time-slot $\tau = 2$ of both the source nodes and the relays is considered as a total energy constraint E_{tot}. Accordingly, the optimization problem is formulated as

$$\underline{\mathbf{g}}_{\mathrm{EC}} = \underset{\underline{\mathbf{g}}}{\mathrm{argmin}} \left\{ \underline{\mathbf{g}}^{*\mathrm{T}} \underline{\mathbf{A}}\, \underline{\mathbf{g}} - \underline{\mathbf{b}}^{*\mathrm{T}} \underline{\mathbf{g}} - \underline{\mathbf{g}}^{*\mathrm{T}} \underline{\mathbf{b}} \right\} \quad (24)$$

subject to

$$\underline{\mathbf{g}}^{*\mathrm{T}} \mathbf{C}\ \underline{\mathbf{g}} = E_{\mathrm{tot}}, \quad (25)$$

where $\mathbf{C}$ is a diagonal matrix with the relays' received energies at the first R diagonal elements, E_{d} at the next K diagonal elements and zeros at the last K diagonal elements. Because of the structure of $\mathbf{C}$, just the first $R + K$ elements of $\underline{\mathbf{g}}$ corresponding to both the relays' scaling factors and the transmit filters are constrained by (25). Accordingly, let $\underline{\mathbf{g}}$ and $\underline{\mathbf{b}}$ be split into $R + K$ and K dimensional vectors as

$$\underline{\mathbf{g}} = \begin{pmatrix} \underline{\mathbf{g}}_1 \\ \underline{\mathbf{g}}_2 \end{pmatrix}, \ \underline{\mathbf{b}} = \begin{pmatrix} \underline{\mathbf{b}}_1 \\ \underline{\mathbf{b}}_2 \end{pmatrix}. \quad (26)$$

Moreover, the matrix $\underline{\mathbf{A}}$ is split into $(R+K) \times (R+K)$, $(R+K) \times K$, $K \times (R+K)$ and $K \times K$ blocks denoted by $\underline{\mathbf{A}}_{11}$, $\underline{\mathbf{A}}_{12}$, $\underline{\mathbf{A}}_{21}$ and $\underline{\mathbf{A}}_{22}$, respectively. Finally, the block of $\mathbf{C}$ which covers the non-zero diagonal elements, i.e., the first $R + K$ diagonal elements, is denoted as $\mathbf{C}_1$. Then the constraint of (25) can be rewritten as

$$\underline{\mathbf{g}}_1^{*\mathrm{T}} \mathbf{C}_1\ \underline{\mathbf{g}}_1 = E_{\mathrm{tot}}. \quad (27)$$

This constraint forms a $R + K$ dimensional ellipsoid. To simplify the optimization problem of (24)-(25), the constraint can be reformulated as a $R + K$ dimensional sphere by decomposing the matrix $\mathbf{C}_1$. Because each relay is equipped with a single antenna, $\mathbf{C}_1$ is a diagonal matrix so $\mathbf{T}_1 = \mathbf{C}_1^{-1/2}$. Then (27) can be rewritten as

$$\underline{\mathbf{y}}_1^{*\mathrm{T}}\underline{\mathbf{y}}_1 = E_{\mathrm{tot}}, \tag{28}$$

where $\underline{\mathbf{g}}_1 = \mathbf{T}_1\underline{\mathbf{y}}_1$. By substituting $\underline{\mathbf{y}}_1$ into the objective function (24), the Lagrangian reads

$$\begin{aligned}
&\mathrm{L}\left(\begin{pmatrix}\underline{\mathbf{y}}_1\\ \underline{\mathbf{g}}_2\end{pmatrix},\lambda\right) = \begin{pmatrix}\underline{\mathbf{y}}_1^{*\mathrm{T}} & \underline{\mathbf{g}}_2^{*\mathrm{T}}\end{pmatrix}\begin{pmatrix}\underline{\tilde{\mathbf{A}}}_{11} & \underline{\tilde{\mathbf{A}}}_{12}\\ \underline{\tilde{\mathbf{A}}}_{21} & \underline{\mathbf{A}}_{22}\end{pmatrix}\begin{pmatrix}\underline{\mathbf{y}}_1\\ \underline{\mathbf{g}}_2\end{pmatrix}\\
&-\begin{pmatrix}\underline{\tilde{\mathbf{b}}}_1^{*\mathrm{T}} & \underline{\mathbf{b}}_2^{*\mathrm{T}}\end{pmatrix}\begin{pmatrix}\underline{\mathbf{y}}_1\\ \underline{\mathbf{g}}_2\end{pmatrix} - \begin{pmatrix}\underline{\mathbf{y}}_1^{*\mathrm{T}} & \underline{\mathbf{g}}_2^{*\mathrm{T}}\end{pmatrix}\begin{pmatrix}\underline{\tilde{\mathbf{b}}}_1\\ \underline{\mathbf{b}}_2\end{pmatrix}\\
&+\lambda\left(\underline{\mathbf{y}}_1^{*\mathrm{T}}\underline{\mathbf{y}}_1 - E_{\mathrm{tot}}\right),
\end{aligned} \tag{29}$$

where λ is the Lagrangian multiplier, $\underline{\tilde{\mathbf{b}}}_1 = \mathbf{T}_1\underline{\mathbf{b}}_1$, $\underline{\tilde{\mathbf{A}}}_{11} = \mathbf{T}_1\underline{\mathbf{A}}_{11}\mathbf{T}_1$, $\underline{\tilde{\mathbf{A}}}_{12} = \mathbf{T}_1\underline{\mathbf{A}}_{12}$ and $\underline{\tilde{\mathbf{A}}}_{21} = \underline{\mathbf{A}}_{21}\mathbf{T}_1$. Taking the derivatives of (29) with respect to $\begin{pmatrix}\underline{\mathbf{y}}_1\\ \underline{\mathbf{g}}_2\end{pmatrix}$ yields

$$\begin{aligned}
\frac{\partial \mathrm{L}}{\partial\begin{pmatrix}\underline{\mathbf{y}}_1\\ \underline{\mathbf{g}}_2\end{pmatrix}} &= \begin{pmatrix}\underline{\tilde{\mathbf{A}}}_{11}^{\mathrm{T}} & \underline{\tilde{\mathbf{A}}}_{21}^{\mathrm{T}}\\ \underline{\tilde{\mathbf{A}}}_{12}^{\mathrm{T}} & \underline{\mathbf{A}}_{22}^{\mathrm{T}}\end{pmatrix}\begin{pmatrix}\underline{\mathbf{y}}_1^*\\ \underline{\mathbf{g}}_2^*\end{pmatrix}\\
&-\begin{pmatrix}\underline{\tilde{\mathbf{b}}}_1^*\\ \underline{\mathbf{b}}_2^*\end{pmatrix} + \lambda\begin{pmatrix}\underline{\mathbf{y}}_1^*\\ \mathbf{0}\end{pmatrix} \overset{!}{=} \mathbf{0}.
\end{aligned} \tag{30}$$

From (30), the optimum solution is calculated as

$$\begin{pmatrix}\underline{\mathbf{y}}_1\\ \underline{\mathbf{g}}_2\end{pmatrix}_{\mathrm{EC}} = \begin{pmatrix}\underline{\tilde{\mathbf{A}}}_{11}^{*\mathrm{T}} + \lambda\mathbf{I}_{R+K} & \underline{\tilde{\mathbf{A}}}_{21}^{*\mathrm{T}}\\ \underline{\tilde{\mathbf{A}}}_{12}^{*\mathrm{T}} & \underline{\mathbf{A}}_{22}^{*\mathrm{T}}\end{pmatrix}^{-1}\begin{pmatrix}\underline{\tilde{\mathbf{b}}}_1\\ \underline{\mathbf{b}}_2\end{pmatrix}, \tag{31}$$

where λ has to be adapted to satisfy the total energy constraint of (28). In (30), two equalities are represented which are

$$\underline{\tilde{\mathbf{A}}}_{11}^{*\mathrm{T}}\underline{\mathbf{y}}_1 + \underline{\tilde{\mathbf{A}}}_{21}^{*\mathrm{T}}\underline{\mathbf{g}}_2 - \underline{\tilde{\mathbf{b}}}_1 + \lambda\underline{\mathbf{y}}_1 = \mathbf{0} \tag{32}$$

and

$$\underline{\tilde{\mathbf{A}}}_{12}^{*\mathrm{T}}\underline{\mathbf{y}}_1 + \underline{\mathbf{A}}_{22}^{*\mathrm{T}}\underline{\mathbf{g}}_2 - \underline{\mathbf{b}}_2 = \mathbf{0}. \tag{33}$$

Solving (33) for $\underline{\mathbf{g}}_2$ and substituting the result in (32) yields

$$\left(\underline{\hat{\mathbf{A}}} + \lambda\mathbf{I}_{R+K}\right)\underline{\mathbf{y}}_1 - \underline{\hat{\mathbf{b}}} = 0, \tag{34}$$

where

$$\underline{\hat{\mathbf{A}}} = \underline{\tilde{\mathbf{A}}}_{11}^{*\mathrm{T}} - \underline{\tilde{\mathbf{A}}}_{21}^{*\mathrm{T}}\left(\underline{\mathbf{A}}_{22}^{*\mathrm{T}}\right)^{-1}\underline{\tilde{\mathbf{A}}}_{12}^{*\mathrm{T}} \tag{35}$$

and

$$\underline{\hat{\mathbf{b}}} = \underline{\tilde{\mathbf{b}}}_1 - \underline{\tilde{\mathbf{A}}}_{21}^{*\mathrm{T}}\left(\underline{\mathbf{A}}_{22}^{*\mathrm{T}}\right)^{-1}\underline{\mathbf{b}}_2. \tag{36}$$

Using the Eigenvalue decomposition, the matrix $\underline{\hat{\mathbf{A}}}$ is decomposed as $\underline{\hat{\mathbf{A}}} = \underline{\mathbf{Q}}\boldsymbol{\Lambda}\underline{\mathbf{Q}}^{-1}$, where $\boldsymbol{\Lambda}$ is a $(R+K)\times(R+K)$ diagonal matrix with the eigenvalues $\rho_1,\ldots,\rho_{R+K}$ of $\underline{\hat{\mathbf{A}}}$ at the diagonal. Moreover, $\underline{\mathbf{Q}}$ is a $(R+K)\times(R+K)$ matrix where the i-th column is the eigenvector of $\underline{\hat{\mathbf{A}}}$ corresponding to the i-th eigenvalue ρ_i. Then

$$\left(\underline{\hat{\mathbf{A}}} + \lambda\mathbf{I}_{R+K}\right)^{-1} = \left(\underline{\mathbf{Q}}\boldsymbol{\Lambda}\underline{\mathbf{Q}}^{-1} + \lambda\mathbf{I}_{R+K}\right)^{-1} \tag{37}$$

$$= \frac{1}{\lambda}\underline{\mathbf{Q}}\left(\mathbf{I}_{R+K} - \left(\lambda\boldsymbol{\Lambda}^{-1} + \mathbf{I}_{R+K}\right)^{-1}\right)\underline{\mathbf{Q}}^{-1}, \tag{38}$$

where (38) follows from (37) by applying the Woodbury matrix inversion lema. Substituting (38) in (34) yields

$$\underline{\mathbf{y}}_1 = \frac{1}{\lambda}\underline{\mathbf{Q}}\left(\mathbf{I}_{R+K} - \left(\lambda\boldsymbol{\Lambda}^{-1} + \mathbf{I}_{R+K}\right)^{-1}\right)\underline{\mathbf{Q}}^{-1}\underline{\hat{\mathbf{b}}}. \tag{39}$$

To find λ which satisfies the total energy constraint, equation (39) is substituted in (28) as

$$\frac{1}{\lambda^2}\underline{\mathbf{p}}^{*\mathrm{T}}\left(\mathbf{I}_{R+K} - \left(\lambda\boldsymbol{\Lambda}^{-1} + \mathbf{I}_{R+K}\right)^{-1}\right)^2\underline{\mathbf{p}} = E_{\mathrm{tot}} \tag{40}$$

where

$$\underline{\mathbf{p}} = \underline{\mathbf{Q}}^{-1}\underline{\hat{\mathbf{b}}} = \left(\underline{p}^{(1)},\ldots,\underline{p}^{(R+K)}\right)^{\mathrm{T}}. \tag{41}$$

Equation (40) can be simplified as

$$\sum_{i=1}^{R+K}\frac{\left|\underline{p}^{(i)}\right|^2}{\left(\lambda+\rho_i\right)^2} = E_{\mathrm{tot}}\ . \tag{42}$$

As $\underline{\mathbf{A}}$ is a positive semidefinite matrix, λ should be adapted in such a way that the Hessian of the Lagrangian is kept positive semidefinite. Moreover, $\underline{\mathbf{y}}_1$ corresponds to the total energy constraint and thus corresponds to λ. Instead of taking the second derivative of (29), (34) is a function of only $\underline{\mathbf{y}}_1$ where $\underline{\mathbf{g}}_2$ satisfies the first order optimality condition of (30) and hence (34) is equivalent to (30). The second order optimality condition in this case is obtained by taking the derivative of (34) with respect to $\underline{\mathbf{y}}_1$. Accordingly,

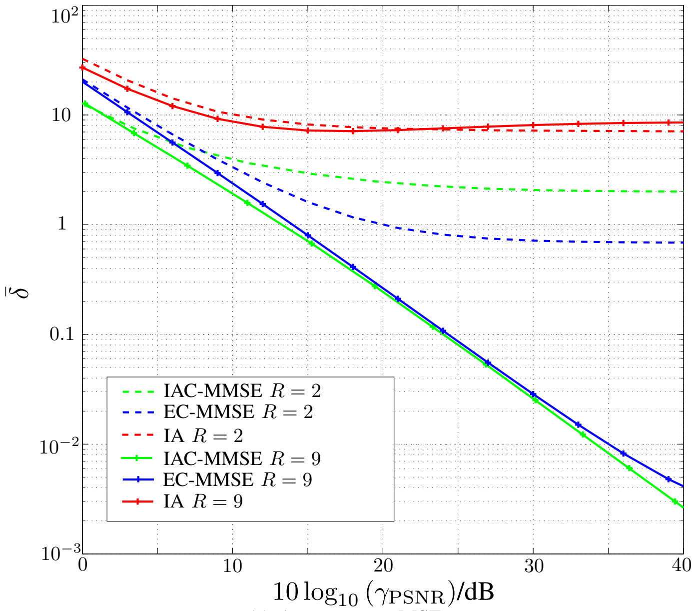
$$\underline{\mathbf{x}}^{*\mathrm{T}}\left(\underline{\hat{\mathbf{A}}} + \lambda\mathbf{I}_{R+K}\right)\underline{\mathbf{x}} \geq 0 \tag{43}$$

should hold for any vector $\underline{\mathbf{x}}$. This condition implies that an optimum solution exists if the matrix $\left(\underline{\hat{\mathbf{A}}} + \lambda\mathbf{I}_{R+K}\right)$ is positive semidefinite. As a result, the eigenvalues of this matrix are lower bounded by zero. Because $\underline{\hat{\mathbf{A}}}$ is a positive semidefinite matrix, $\lambda \geq -\rho_{\min}$ where $\rho_{\min}$ is the minimum eigenvalue of $\underline{\hat{\mathbf{A}}}$. Moreover, if $\lambda > -\rho_{\min}$, the constraint of (42) is a monotonically decreasing function of λ in the interval $]-\rho_{\min},\infty[$ [8]. Accordingly, there is only one zero of this function in the interval $]-\rho_{\min},\infty[$ and hence, a unique optimum solution for λ_{opt} exists. With an initial value $\lambda_{\mathrm{ini}} = -\rho_{\min} + \epsilon$ where ϵ is an arbitrary small value, a numerical equation solver can be used to find the optimum λ_{opt} which can be substituted in (31) to find the optimum solution for $\underline{\mathbf{y}}_1$ and $\underline{\mathbf{g}}_2$.

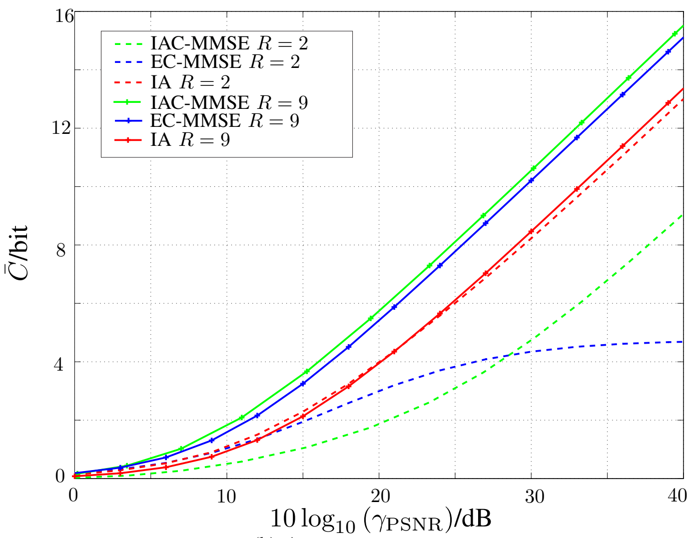
V. Numerical Results

In this section, both the sum rate C and the sum MSE δ are considered as performance measures. The performance of the proposed scheme is measured as a function of the pseudo SNR γ_{PSNR} which can be defined as the whole energy transmitted in both time-slots to the noise variance where one half of the total energy is transmitted in the first time-slot. The sum-rate per time-slot is calculated as

$$C = \frac{1}{2}\sum_{k=1}^{K}\log_2\left(1+\gamma_k\right), \tag{44}$$



(a) Average sum MSE



(b) Average sum rate

Fig. 2: The performance measures as a function of the PSNR for $K = 3$ node pairs.

where

$$\gamma_k = \frac{E_{\mathrm{d}}}{\sigma^2} \frac{\left|\underline{\mathbf{g}}^{*\mathrm{T}}\ \underline{\mathbf{q}}^{(k,k)}\right|^2}{\left|\underline{u}_1^{(k)}\right|^2 + \left|\underline{u}_2^{(k)}\right|^2 \left(\sum_{r=1}^{R} \left|\underline{h}_{\mathrm{DR}}^{(k,r)} \underline{g}^{(r)}\right|^2 + 1\right) + I_k}, \tag{45}$$

and

$$I_k = \sum_{l \neq k} \frac{E_{\mathrm{d}}}{\sigma^2} \left|\underline{\mathbf{g}}^{*\mathrm{T}}\ \underline{\mathbf{q}}^{(k,l)}\right|^2, \tag{46}$$

are the received SINR and the unaligned interference signals normalized to the noise variance at the k-th destination node, respectively. A Rayleigh fading channel is employed with a unit average channel gain. In the following, a scenario consisting of $K = 3$ node pairs and R relays is considered. The reference scheme is the IA scheme proposed in [7]. For the IA scheme, simply an arbitrary IA solution is picked and scaled to satisfy the total energy constraint of (25).

Fig. 2a and Fig. 2b show the sum MSE and the sum rate as a function of PSNR, respectively. For $R = 2$, the energy constraint minimum sum MSE (EC-MMSE) achieves a significantly smaller sum MSE than the other two schemes. In terms of capacity, the EC-MMSE outperforms the IAC-MMSE scheme at low and moderate PSNR but it saturates at a certain sum rate at high PSNRs. If the number of relays is relatively large, e.g., $R = 9$, lower sum MSE and thus higher sum rates are achieved by the proposed schemes as compared to the IA scheme. Furthermore, the EC-MMSE implicitly performs IA at high PSNR.

VI. Conclusion

In this paper, a scenario consisting of single antenna node pairs communicating through single antenna relays is considered. When fully adapting the relays and partially adapting the transmit and receive filters, closed form solutions for minimizing the sum MSE under IA constraint and energy constraint are proposed. The results show that the proposed schemes outperform the conventional IA scheme. Moreover, at high SNR, IA is implicitly achieved by minimizing sum MSE if the number of relays is sufficient.

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References

[1] V. Cadambe and S. Jafar, "Interference alignment and degrees of freedom of the K-user interference channel," in *IEEE Transactions on Information Theory*, vol. 54, no. 8, August 2008, pp. 3425–3441.

[2] S. Jafar and S. Shamai, "Degrees of freedom region of the MIMO X channel," *IEEE Transactions on Information Theory*, vol. 54, no. 1, pp. 151–170, January 2008.

[3] D. Schmidt, C. Shi, R. Berry, M. Honig, and W. Utschick, "Minimum mean squared error interference alignment," in *Proc. the 43rd Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, USA, November 2009, pp. 1106–1110.

[4] H. Shen, B. Li, M. Tao, and X. Wang, "MSE-based transceiver designs for the MIMO interference channel," *IEEE Transactions on Wireless Communications*, vol. 9, no. 11, pp. 3480–3489, November 2010.

[5] N. Khajehnouri and A. Sayed, "Distributed MMSE relay strategies for wireless sensor networks," *IEEE Transactions on Signal Processing*, vol. 55, no. 7, pp. 3336–3348, July 2007.

[6] S. Berger and A. Wittneben, "Cooperative distributed multiuser MMSE relaying in wireless Ad-Hoc networks," in *Proc. 39th Asilomar conference on Signals, Systems and Computers*, November 2005, pp. 1072–1076.

[7] H. Al-Shatri and T. Weber, "Interference alignment aided by non-regenerative relays for multiuser wireless networks," in *Proc. 8th International Symposium on Wireless Communication Systems*, Aachen, Germany, November 2011, pp. 271–275.

[8] H. Tuy, *Convex Analysis and Global Optimization*, ser. Nonconvex Optimization and Its Applications. Kluwer Academic Publishers, 1998, vol. 22.