PERGAMON

Annual Reviews in Control 23 (1999) 91-108

Annual Reviews in Control

FROM SIOUX CITY TO THE X-33 [1]

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Abstract: The history of the "Propulsion Controlled Aircraft (PCA)" problem is reviewed. While there had been repeated warnings that life-threatening hydraulic failure in a modern airliner can occur despite an estimated probability of 10^{-9}, only after the Sioux City accident was the possibility of using some automatic fly-by-throttle back-up control system for crippled airplane seriously considered. Several different schemes to help pilots fly hydraulically depleted aircraft by using collective and differential thrust of the engines will be reviewed. Special attention will be devoted to the system theoretic "model matching" methodology, in which the propulsion controlled aircraft is compensated so as to respond as if it were under normal aerodynamic control. Applications of these concepts to Unhabited Air Vehicles (UAV's) will also be considered. Finally, it will be shown how the propulsion control concepts can be extrapolated to the X-33, the reduced scale demonstration vehicle of the new space shuttle, the engines of which cannot be gimballed so that differential thrust instead of thrust vectoring has to be used. The latter application is more in the spirit of "reconfigurable control" and is based on the novel methodology of Linear Set Valued Dynamically Varying (LSVDV) control.

Keywords: Robust Control, Model Reference Control, Failure Isolation, Aircraft Control, Propulsion Control.

1. INTRODUCTION

In the aftermath of the Sioux City accident—in which a DC-10 lost all of its hydraulics and the crew managed to crash land the aircraft using engine thrust only—the National Transportation Safety Board (NTSB) recommended that the possibility of controlling aircraft by throttle only be explored and that some sort of emergency propulsion control system to help pilots fly-by-throttle aircraft with totally or partially disabled hydraulics be developed. NASA Dryden Flight Research Center took up NTSB recommendation and developed some conventional propulsion control schemes for the F-15 and the MD-11 (Burcham and Fullerton 1991, Burcham *et al.* 1993, Burcham *et al.* 1995). The University of Southern California, on the other hand, focused on the modern H^∞ approach to the propulsion control problem, and used the Lockheed L-1011 "Tristar" as testbed aircraft for its design.

[1] This research was supported by NASA Marshall Grant NAS8-97-292 and NSF Grant ECS-98-02594.

PII: S1367-5788(99)00011-5

In a parallel development, propulsion control has also been considered for some Single Stage To Orbit (SSTO) Reusable Launch Vehicle (RLV) concepts. Cheaper access to space indeed requires such higher efficiency engines as the new "aerospike" rocket engine technology, where, instead of ejecting the gas through a gimballed engine bell, the gas are ejected between a "plug" profile and empty space. In this newer concept, gimballing is impossible, so that differential thrust has to be used instead. In the X-33 demonstration vehicle (Wilson 1999), this aerospike engine concept is merged with the lifting body concept, so that this vehicle behaves somewhat like a hypersonic aircraft partially controlled by differential thrusting. Another SSTO concept uses the higher efficiency afforded by air-breathing scramjet engine. Again, these engines are fixed relative to the vehicle structure so that differential thrust has to be used instead of thrust vectoring, in addition to some aerodynamic control. The latter concept is implemented in the so-called "Winged Cone."

More recently, propulsion control has also been used in Uninhabited Air Vehicles (UAV's). Indeed, the extreme sizes of these flying machines— ranging from a palm top design to the 80 feet wingspan Centurion— compounded with their very low airspeed, call for new control concepts, like attitude control by propulsion.

In this paper, we develop two approaches to the propulsion control problem. The first one is the system theoretic "model matching" approach, where an aerospace vehicle with differential thrust actuators only is compensated to respond as if it were controlled by conventional control surfaces. The second approach is in the spirit of "reconfigurable" control. Basically, we look at the X-33 that has a variety of conventional (elevon, rudder, aileron,...) and unconventional (differential thrust) actuators and we imagine scenarios in which some actuators are lost and stability should be preserved by reconfiguring the controller to make heavier use of the remaining actuators.

2. PROPULSION CONTROL BACKGROUND—THE SIOUX CITY ACCIDENT

We have entitled our first approach "propulsion control by H^∞ model matching." Indeed, this approach is to duplicate, mathematically, some sort of model matching which the Sioux City crew improvised to manage the emergency.

Recall the sequence of events (Haynes 1991): Soon after leveling off at 37,000 feet, the tail (# 2) engine of the DC-10 sustained a "catastrophic uncontained" failure that created a hail of shrapnel that severed the hydraulic lines of all three systems, disabling the hydraulically-actuated control system of the aircraft. In the cockpit, the crew, aware of #2 engine failure but unaware of the damage to the hydraulics, implemented the standard emergency drill— the Captain, Al Haynes, went through the engine failure emergency checklist while the First Officer (FO) took the controls and flew the plane. Fairly quickly, the airplane slipped into a right down turn, to which the FO responded with left yoke, back pressure, and left rudder, which did not seem to help and soon thereafter the aircraft was into a dangerous, amplifying, right down turn, despite the first officer's attempt to correct the situation with full left yoke, the control column all the way back, and full left rudder. It is when the FO, stunned, called his captain repeatedly stating that he could no longer control the plane that the captain's attention was diverted from the emergency checklist to the odd situation of the aircraft in a right down turn while the controls were calling for left up turn! At that precise instant, the captain came to grips with the horrifying reality that he had lost just about all aerodynamic control. To salvage a situation that was becoming nearly desperate— the aircraft sinking more and more in a tighter-and-tighter down right turn—the captain boldly shut off the left engine and "firewalled" the right engine, and slowly the aircraft pulled itself out of the dangerous situation. For the next few minutes, the captain struggled to control the phugoid and Dutch roll oscillations by collective and differential thrust, respectively, when a DC-10 flight instructor, who was sitting in the passenger cabin when the engine exploded, came in the cockpit and volunteered his help. The captain complained that he had difficulties controlling the Dutch roll and that it was awkward to lean toward the center of the cockpit and maneuver the #1 and #3 throttle levers with the #2 throttle lever jammed in the cruise position while at the same time monitoring the jerky motion of the nose of the aircraft over the horizon. To get a better organization of the cockpit, it was decided that the captain would keep on maneuvering the yoke and the rudder pedals, as if nothing had happened, and the flight instructor would sit on the flight engineer seat between the captain and the first officer, holding the throttle, looking at the way the yoke was going and attempting to find a throttle signal that was going to reproduce the response desired by the captain. *In other words, the flight instructor was acting as a model matching controller, attempting to reproduce the normal response of the aircraft to the yoke position with the throttles.* (On top of this, the crew used a quick communication language: "up," "down," "right," "left.") The rationale for this organization was the following: The captain, from his fairly high position on the left pilot's seat, could concentrate on monitoring the motion of the nose of the aircraft above the horizon and simulate the timely corrective action required to fight the Dutch roll and the phugoid; on the other hand, the flight instructor, by observing the yoke position, could see the captain's intentions and find a matching throttle signal. In addition to ergonomics, another reason for this organization is that the crew did not want to totally give up on the hydraulics, because indeed, they did not know exactly what had happened, they were puzzled by some faulty readings of the hydraulic gauges on the flight engineer console, and they still had the hope that there would still be some response of the hydraulic system or that some response would be restored.

Gleaning on the above, we would perceive the propulsion control system as follows: During the emergency with the hydraulics of the airplane partially or totally disabled, the pilot would keep on maneuvering the stick (or yoke) and rudder pedals as if nothing had happened, the position of the stick (yoke) and rudder pedals would be picked up by sensors, the sensor output would be forwarded to a model matching compensator which would attempt to reproduce the normal response of the aircraft as if no failure had occurred by using engine thrust only.

There are several schools of thought as to how pilot's commands to a propulsion control system should be entered. The approach proposed here is to enter the commands through the normal input channels (the yoke and rudder pedals). In some other approaches, the commands to the propulsion system are entered through a different channel than the normal aerodynamic control input channel. A side stick as in the Airbus A-320 and A-340, is another possibility. NASA, on the other hand, elected to have the inputs to the propulsion control system of the MD-11 entered through a pair of thumb wheels on the autopilot panel— one for the pitch control and the other for the roll/yaw control. The reason for this choice is that the flight test of the PCA system would be done with the yoke "hot," that is, connected to a fully operational hydraulic system, so that the test pilot could grasp it and go back to normal aerodynamic control in case of problem with the propulsion system; besides, thumb wheels are slow to move and as such give the pilot the clear message that the aircraft would not respond quickly; finally, another reason is that thumb wheels tend to prevent pilot's induced oscillations (PIO's). However, it can be claimed that, in a real emergency, especially with a progressively degrading hydraulics, when the pilot has to supplement inadequate aerodynamic control with some propulsion control, it would be hard for the pilot to maneuver the yoke and the thumb wheels and ensure the proper combination of propulsion and aerodynamic control. Under those circumstances, it appears better to enter pilot's instructions through the yoke, leaving to the pilot the possibility to alter the relative weightings of normal aerodynamic control and propulsion control.

The concepts developed above can not only be applied to pure "propulsion" control (that is, total hydraulic failure and engine-only control), but can also be applied to "reconfigurable" control by considering situations where, say, the elevator is 50% effective because of partial failure and attempting to compensate for the partial elevator failure by using engine thrust. Another case study of reconfigurable/redundant control has been a thrust vectoring aircraft with failed elevator and the issue was to reproduce the elevator response by vectoring the thrust. Simulation studies have demonstrated that this works quite well.

Gleaning on the above broader interpretation, we would define propulsion control to be the utilization of secondary effects of engine thrust— that is, coupling between, on the one hand, thrust and, on the other hand, lateral and longitudinal dynamics— to achieve the following objectives:

- to compensate for total or partial hydraulic failure,
- to reconfigure the control system following substantial failure or combat damage,
- to compensate for the deliberate removal of control surfaces for stealth reason (e.g., the vertical stabilizer as in the Joint Strike Force (JSF) aircraft) or for thermal control reasons (e.g., the "lifting body" concept of the X-33),
- to supplement normal aerodynamic control with propulsion control for accrued maneuverability, unusual maneuvers (e.g., Herbs maneuver), or smoother ride.

3. EARLY WARNINGS

3.1 *hydraulic failure over North Vietnam*

One of the very first incidents of depleted hydraulic pressure is the story of Colonel Jack Broughton in his crippled F-105, "Thunderchief," over North Vietnam. In his book (Broughton 1969, pp. 113-115), Col. Broughton explains how, after being hit by ground gunners over North Vietman, the two primary hydraulic systems #1 and #2 of his F-105 sustained complete failure, while the pressure gauge of the 3rd emergency back up system was oscillating with a period of about 90 seconds between 0 psi and 3,000 psi. All the way from North Vietnam to the alternate airbase in Thailand, Col. Broughton had to plan to make his maneuvers when the hydraulic pressure was up.

3.2 *Preparing for the worst*

The first propulsion control story is that of Captain Bryce McCormick (Stewart 1991). When, in 1972, he did his DC-10 instruction, Cap. McCormick felt quite uncomfortable with the radical changes in the control system brought about by the "jumbo" generation of aircraft— no more direct cable connection between the cockpit controls and the control surfaces; instead, the control cables were just running from the cockpit to the hydraulic actuators themselves moving the control surfaces. Despite the re-assurance of the aircraft manufacturer that a hydraulic failure was "impossible" or had a probability of 10^{-9}, Cap. McCormick could not resist asking himself the tough question, "What if?" During a training session in a DC-10 flight simulator, he asked his flight instructor to shut off the hydraulics so that he could figure out what, if anything, could be done. To his great relief, he realized that, because of the exceptionally well placed engines in the DC-10, he could control the aircraft by propulsion only all the way from climb to approach. No later than a few months after this self-imposed training exercise, the aft cargo door of Cap. McCormick's aircraft failed, and the tremendous pressure across the floor caused it to collapse, taking with it the cables going from the cockpit to the tail of the aircraft. The aircraft was left with almost no elevator, no stabilizer trim, the rudder jammed in a shallow right turn position, and the tail engine throttle jammed at idle. The damage caused the plane to go in a descent that could not be kept under control by the grossly ineffective and sluggish elevator, so that the captain had to resort to increasing the wing engine thrust to level off. For the rest of the flight, a substantial amount of propulsion control had to be used to compensate for the deficient aerodynamic control, all the way to the landing roll where the crew had to steer the aircraft on the runway by differential reverse thrusting propulsion control—a maneuver never heard of before!

3.3 *disaster avoided by propulsion control*

Probably the first widely publicized example of "propulsion control" is the April 1977 L-1011 Delta Flight 1080 emergency (McMahan 1978). When the aircraft was taking off from San Diego airport, its left elevator got jammed in the nose up position. Soon after lift off, the nose started pitching dangerously up to 22° and the airspeed started dropping dangerously low to 140 KIAS, despite the crew attempt to correct the situation with full forward yoke and full nose down electric trim, while to make things worse a left roll tendency started manifesting itself. Stall was imminent when the captain idled off #1,#3 engines, firewalled engine #2, observed the pitch down response, and then advanced #1,#3 throttles to keep a safe airspeed in a safe, but still nose up, position. Another differential throttling maneuver was executed at 12,000 feet to level off. All the way from San Diego to Los Angeles, the aircraft flew with its pitch controlled by differential thrust between tail and wing engines, while the left roll tendency was compensated by wing differential thrust, and made a successful emergency landing in Los Angeles.

3.4 *asymmetric damage compensated by propulsion*

Another story is the April 1979 B-727-100 TWA incident (Stewart 1991). To avoid strong headwind, the captain, Harvey Gibson, after consulting with ATC and his flight engineer, took his aircraft to 39,000 ft, close to the limit of the flight envelope. All of a sudden, for a still unclear reason, the #7 right outboard leading edge slat deployed, and because of this asymmetric condition the airplane started rolling and yawing right to which the captain responded by applying full left aileron and rudder that didn't help much and the airplane went through a complete 180° roll from which it went into a spiraling dive. The aircraft quickly broke through the sound barrier and to reduce this excessive speed, which was freezing all controls, the captain boldly dropped the landing gear. The deployment of the undercarriage at such excessive speed created substantial damage to the right landing gear, to the flaps, and a hydraulic line was ruptured creating failure of system #A; however, the resulting airspeed drop allowed the captain to recover some control, first stopping the rolling motion, then recovering from the dive. Because of the asymmetric damage, below 200 knots, the aircraft had a strong left yaw tendency that required full right yoke and full right rudder to be compensated. Things got worse in the traffic pattern, as the aircraft veered out of control in a left turn despite all controls fully to the right. The captain idled #2 and #3 engines and "firewalled" engine #1 and gradually regained control of the aircraft and brought it to a landing with all controls to the right.

3.5 *total hydraulic failure*

This is the case of a Japan Air Lines B-747SR (anonymous 1985*a*, anonymous 1985*c*, anonymous 1985*b*). A few months before the fatal accident, the airplane had made a hard landing, its tail scraping the runway, seriously damaging the aft bulkhead that seals the pressurized passenger cabin. During the repair, the new bulkhead was improperly bolted on the pressurized cabin flange, leaving the whole assembly seriously weakened. When the ill-fated flight took off, as the improperly repaired bulkhead had to stand more and more pressure difference between the pressurized cabin and the rarefied air, all of a sudden the bulkhead disintegrated. The pressurized air from the cabin gushed through the tail cone to the vertical stabilizer and blew off one half of the vertical stabilizer; more dramatically, a piece of shrapnel from the disintegrating bulkhead sliced the hydraulic lines at the precise place where all four lines of all four independent hydraulic systems were running next to each other. Consequently, the aircraft sustained total hydraulic failure and, to make things worse, the grossly reduced vertical stabilizer area made the aircraft extremely susceptible to Dutch roll. The pilot attempted to make a turn by differential thrust, and immediately the aircraft slipped into Dutch roll oscillations, compounded by phugoid oscillations. It appears that the pilot could not find the corrective action to the Dutch roll that amplified to a 50° rolling motion, from where the situation became virtually unsalvageable. Eventually, the aircraft stalled, fell from the sky, killing 520 people.

4. THE DUTCH ROLL PROBLEM

It transpires from the simulation studies, the Sioux City accident, and especially the JAL B-747 air disaster, that a big problem in attempting to make turns by differential thrust is the Dutch roll. This is especially true for *pure jet power swept back wing aircraft* with engines away from the center of gravity, like the B-707, B-747, L-1011, DC-10 configurations. Indeed, the response to differential wing engine thrust with long arm length has *too much yaw* which does not help much to make a coordinated turn that requires the correct combination of yaw and roll. On the contrary, the excessive yaw input induces a side skid which, via the dihedral effect, brings the airplane in Dutch roll.

This Dutch roll problem is not as critical with airplane with engines close to the center of mass, like the B-727, F-15. Also, with a propeller aircraft, even though the engine are far from the C.G., this problem is not that critical, because indeed, the wash of the propeller of the engine with added manifold pressure increases the airspeed on that wing which increases the lift of the wing which in turn rolls the aircraft in the turn.

Dutch roll recovery is a bit of a tricky maneuver. The standard recovery maneuver calls for the pilot not to rush the corrective action, but instead monitor the motion of the aircraft, get the mental pattern, wait for the wings to be leveled, and *at that precise time* give a stroke of yoke toward the uprising wing (Davies 1973). This will not totally eradicate the Dutch roll, but will reduce it, so that the maneuver might have to be repeated. There is also a recovery maneuver using the rudder, which is not recommended, because the timing is critical and if not done properly the recovery attempt by rudder might make things worse. Now, if we observe that differential thrust is more like a rudder input than an aileron input, *we understand the difficulty of controlling the Dutch roll by propulsion.*

5. H^∞ MODEL MATCHING

The longitudinal or lateral motion of an aircraft, linearized around some point of the flight envelope, can be written

$$x(s) = (sI - A)^{-1} (B_c \;\; B_n) \begin{pmatrix} u \\ w \end{pmatrix}$$

where $x(s)$ is the attitude, u is the incremental throttle and w is the position of the control surfaces. Under normal conditions, the attitude can be controlled by the control surfaces w without throttle input and the motion of the aircraft is given by

$$x_n(s) = \underbrace{(sI - A)^{-1} B_n}_{G_n(s)} w(s) \qquad (1)$$

If the control surfaces fail ($w = 0$), the only hope for controlling the aircraft is the throttle u and under those circumstances the motion of the crippled aircraft becomes

$$x_c(s) = \underbrace{(sI - A)^{-1} B_c}_{G_c(s)} u(s) \qquad (2)$$

Instead of the pilot manually operating the throttles u, the throttle signal is generated by a stability augmentation scheme

$$u(s) = K_y(s)y(s) + K_w(s)w(s) \qquad (3)$$

where the hypothetical control surface position $w(s)$ acts as command signal, y is a sensor output that could be either the matching error $x_n - x_c = e$ or the state of the crippled aircraft $-x_c$, and the gains are designed so as to reproduce the normal response to w. As such the pilot would feel a normal aircraft.

5.1 *error feedback*

The objective of model matching by error $e = x_n - x_c$ feedback is to wrap a propulsion controller

$$u = K_e e + K_w w \qquad (4)$$

around the crippled aircraft in such a way that the resulting closed-loop transfer matrix, obtained by eliminating u between Equation 2 and Equation 4,

$$x_c = \underbrace{(I + G_c K_e)^{-1} G_c (K_e G_n + K_w)}_{\approx G_n} w,$$

matches as closely as possible the normal aircraft transfer matrix, without excessive thrust requirement. This part of the design is quite classical: Define the frequency-weighted state error $\tilde{e} = W_1 e$, the frequency-weighted control effort $\tilde{u} = W_2 u$, and put everything together in the open-loop transfer matrix

$$\begin{pmatrix} \tilde{e} \\ \tilde{u} \\ e \\ w \end{pmatrix} = \underbrace{\begin{pmatrix} W_1 G_n & -W_1 G_c \\ 0 & W_2 \\ G_n & -G_c \\ I & 0 \end{pmatrix}}_{G} \begin{pmatrix} w \\ u \end{pmatrix}$$

The key step is to find the stabilizing controller $K = \begin{pmatrix} K_e & K_w \end{pmatrix}$ that minimizes the closed-loop transfer matrix T_{zw} from w to the controlled variable;

$$z = \begin{pmatrix} \tilde{e} \\ \tilde{u} \end{pmatrix}$$

More specifically, the resulting H^∞ design procedure is

$$\inf_K \left\| \underbrace{\begin{pmatrix} W_1 (I + G_c K_e)^{-1} (G_n - G_c K_w) \\ W_2 (I + K_e G_c)^{-1} (K_e G_n + K_w) \end{pmatrix}}_{T_{zw}(G,K)} \right\|_\infty$$

Observe that in this problem setup, both the input and the output sensitivity functions appear quite explicitly in T_{zw}.

5.2 *crippled state feedback*

In this case, the propulsion controller has the form

$$u = -K_x x_c + K_w w \qquad (5)$$

and should be designed so that the resulting closed-loop transfer matrix

$$x_c = \underbrace{(I + G_c K_x)^{-1} G_c K_w}_{\approx G_n} w,$$

matches as closely as possible the nominal aircraft transfer matrix without excessive thrust requirement. Define the open-loop transfer matrix

$$\begin{pmatrix} \tilde{e} \\ \tilde{u} \\ x_c \\ w \end{pmatrix} = \underbrace{\begin{pmatrix} W_1 G_n & -W_1 G_c \\ 0 & W_2 \\ 0 & G_c \\ I & 0 \end{pmatrix}}_{G} \begin{pmatrix} w \\ u \end{pmatrix}$$

The key step is to find the stabilizing controller $K = \begin{pmatrix} K_x & K_w \end{pmatrix}$ that minimizes the closed-loop transfer matrix T_{zw} from w to the controlled variable z,

$$\inf_K \left\| \underbrace{\begin{pmatrix} W_1 (G_n - G_c (I + K_x G_c)^{-1} K_w) \\ W_2 (I + K_x G_c)^{-1} K_w \end{pmatrix}}_{T_{zw}(G,K)} \right\|_\infty$$

Observe that in this case, T_{zw} does not have as clear a robustness interpretation as in the error feedback case.

5.3 *findings*

These are the aircraft models used in the (Jonckheere and Yu 1999) study:

- the Lockheed L-1011-500 "Tristar," the data of which was obtained by courtesy of Lockheed Martin,
- a statically unstable thrust vectoring aircraft model designed by E. Shapiro, where the nominal model to be matched is the aircraft with an inner loop to prestabilize the unstable short periodic oscillations of the pitch dynamics,
- the Fokker F-27, a high-wing, turboprop propelled, European city hopper.

In the (Jonckheere and Yu 1998) study, the following was considered:

- "modified" L-1011 models, where the height of the tail engine above the C.G. was deliberately increased to understand the effect of such height on pitch control by propulsion.

Most of the research effort has focused on the L-1011, in both longitudinal and lateral control, because it is an interesting aircraft in the sense that the tail engine allows for some pitch control.

Simulation results have shown that, with infinite bandwidth actuators without saturation, very accurate longitudinal and lateral propulsion control by H^∞ model matching can be achieved for low-wing aircraft, especially trijet configuration like the L-1011. Longitudinal propulsion control of such a high-wing aircraft as the F-27 is a little more difficult to achieve because, among other things, of the very shallow pitch response to engine thrust. With real actuators modeled with delays, simulation results seem to indicate that the real limitation to achievable performance in PCA is not as much engine bandwidth as available thrust.

In lateral control, the eigenstructure analysis of the L-1011 has shown that the angle between the differential thrust vector and the Dutch roll left eigenspace is only a few degrees, which explains the substantial Dutch roll response to differential thrust. Furthermore, simulation studies of a coordinated turn by propulsion only have shown a substantial amount of engine activity, which can be traced to the matching controller attempting—successfully—to fight the Dutch roll. It follows that making a coordinated turn by propulsion is not a matter of advancing one throttle and retarding the other—this would send the aircraft in Dutch roll. Quite to the contrary, a complicated engine signal is necessary to put the aircraft in a coordinated turn by propulsion; this signal is complicated enough that test pilots have admitted that this kind of engine input could not be figured out from pilot's intuition. This is probably the most important motivation for having a "stability augmentation" system to help pilots fly aircraft by throttle only.

The propulsion control study of the L-1011 was first based on a linearized model of the airplane at Mach 0.84 and 31,000 ft. Subsequently, several points of the flight envelope were considered, a linear compensator was designed for every chosen point of the envelope, and then a neural network was trained for gain scheduling of the compensator as the aircraft moves across its flight envelope (Chu *et al.* 1996),(Jonckheere and Yu 1997). This proved to work quite well on the L-1011 example.

6. MORE INSIGHT INTO MODEL MATCHING

The model matching compensator configuration is inspired from algebraic system theory, the geometric theory of linear systems, and feedback linearization.

6.1 *structure algorithm*

In the algebraic system theory setup (Moore and Silverman 1972), assume $CG_c(s)$ has state space realization

$$\dot{x}_c = Ax_c + B_c u$$
$$y_c = Cx_c$$

Here, y_c is a vector of selected components of x_c to be matched. Differentiating y_c yields

$$\dot{y}_c = CAx_c + CB_c u$$

Assume the matrix CB_c is (right) invertible (if not, keep on differentiating y_c as dictated by Silverman's structure algorithm, and, if the geometric condition 13 is satisfied, this will eventually yield a nonsingular coefficient matrix of selected components of u, $\dot{u}$, $\ddot{u}$, ...). Assuming that $sCG_n(s)$ is proper, it is easily seen that the control

$$u = (CB_c)^{-1}(-CAx_c + sCG_n w) \qquad (6)$$

yields

$$y_c(s) = CG_n(s)w(s)$$

In other words, we have achieved *exact* matching along the C-direction. If the lack of properness of $sCG(s)$ is of concern, the control is modified to

$$u = (CB_c)^{-1}(-CAx_c - k_{den}y_c + k_{num}CG_n w)$$

which yields

$$y_c = \frac{k_{num}}{s + k_{den}} CG_n w$$

Clearly, the exact model matching controller dictated by algebraic system theory has the same structure as Equation 5. Furthermore, some elementary manipulation on the matching controller (6) yields

$$u = (CB_c)^{-1}(CAe + CB_n w)$$

which has basically the same structure as the error feedback matching controller (4).

Observe that this exact matching relies on such dangerous procedures as cancellation between unstable poles and nonminimum phase zeros, much too dangerous to be implemented in practice. Indeed, the controller 6 inverts the transfer function $CG_c(s)$ from the tail throttle to the pitch, which happens to be nonminimum phase in a trijet configuration with sufficiently big arm for the tail engine. Clearly, the unstable poles of $(CG_c(s))^{-1}$ based on a model are not going to be canceled exactly by the zeros of the real world $CG_c(s)$. This problem is avoided in the approximate H^∞ matching procedure.

6.2 *geometric theory*

The model matching problem can also be approached from a geometric point of view (Morse 1973),(Domenica and Isidori 1986). Geometric model matching relies on a generic problem—the disturbance decoupling with disturbance measurement (DDDM) problem, which is first reviewed. Then this fundamental result is specialized to error feedback and crippled state feedback, respectively.

6.2.1. *disturbance decoupling with disturbance measurement* If B is a matrix consisting several $n-D$ vectors, $\mathcal{B} = Im(B)$ denotes the subspace of

$\mathbb{R}^n$ spanned by the columns of B. The reachable subspace of a pair (A, B), written $< A|\mathcal{B} >$, is $\sum_{i=0}^{n-1} A^i \mathcal{B}$. A reachability subspace $\mathcal{R}$ of the pair (A, B) is a subspace of the form $< A + BF|Im(BG) >$ for some F, G of compatible dimension. The supremal reachability subspace of the triple (A, B, C), written $\mathcal{R}^*$, is the subspace such that $C\mathcal{R}^* = 0$ and, for any reachability subspace $\mathcal{R}$ in the kernel of C, we have $\mathcal{R} \subseteq \mathcal{R}^*$. The model matching problem relies on the following:

Theorem 1. ($DDDM$ problem) Consider the system

$$\dot{x} = Ax + B_1 u_1 + B_2 u_2 \qquad (7)$$
$$y = Cx$$

where u_1 is the control and u_2 is a disturbance or command. There exists a (possibly dynamic) feedback

$$u_1 = Mx + Nu_2 \qquad (8)$$

such that

$$C(sI - A - B_1 M)^{-1}(B_1 N + B_2) = 0 \qquad (9)$$

iff

$$\mathcal{B}_2 \subseteq \mathcal{R}^* + \mathcal{B}_1 \qquad (10)$$

where $\mathcal{R}^*$ is the supremal reachability subspace of $(A, \begin{pmatrix} B_1 & B_2 \end{pmatrix}, C)$. Furthermore if the $DDDM$ problem has a solution, it has a static solution.

Proof: Assume Eq. (10) holds and let us prove that the disturbance decoupling problem with disturbance measurement has a solution. From (10), clearly, there exists a matrix N such that $Im(B_1 N + B_2) \subseteq \mathcal{R}^*$. Next, let $\begin{pmatrix} M_1 & M_2 \end{pmatrix}$ be a feedback that makes $\mathcal{R}^*$ invariant, viz., $(A + B_1 M_1 + B_2 M_2)\mathcal{R}^* \subseteq \mathcal{R}^*$. The preceding, together with Eq. (10), yields a feedback M such that $(A + B_1 M)\mathcal{R}^* \subseteq \mathcal{R}^*$. Clearly,

$$< A + B_1 M|Im(B_1 N + B_2) > \subseteq \mathcal{R}^*$$

The above clearly implies that wrapping the feedback Eq. (8) around Eq. (7) yields Eq. (9). Conversely, assume that the decoupling problem has some solution for some dynamic feedback,

$$u_1 = \left[M_{11} + M_{12}(sI - M_{22})^{-1} M_{21}\right] x + \left[N_1 + M_{12}(sI - M_{22})^{-1} N_2\right] u_2$$

It is easily seen that this dynamic feedback around (7) is the static feedback

$$\begin{pmatrix} u_1 \\ v \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x \\ k \end{pmatrix} + \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} u_2$$

wrapped around the dynamically augmented plant $(\bar{A}, \begin{pmatrix} \bar{B}_1 & \bar{B}_2 \end{pmatrix}, \bar{C})$,

$$\begin{pmatrix} \dot{x} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ k \end{pmatrix} + \begin{pmatrix} B_1 & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} u_1 \\ v \end{pmatrix} + \begin{pmatrix} B_2 \\ 0 \end{pmatrix} u_2$$
$$y = \begin{pmatrix} C & 0 \end{pmatrix} \begin{pmatrix} x \\ k \end{pmatrix}$$

Define the subspace

$$\bar{\mathcal{R}} =< \bar{A} + \bar{B}_1 M|\bar{B}_1 N + \bar{B}_2 >$$
$$\bar{C}\bar{\mathcal{R}} = 0$$

and let $\mathcal{R}$ be its first component. Clearly, $\mathcal{R}$ is a reachability subspace of $(A, \begin{pmatrix} B_1 & B_2 \end{pmatrix})$ contained in the kernel of C and furthermore, $\mathrm{Im}(B_1 N_1 + B_2) \subseteq \mathcal{R}$. Let $\mathcal{R}^*$ be the supremal reachability subspace of $(A, \begin{pmatrix} B_1 & B_2 \end{pmatrix}, C)$. Clearly, $\mathrm{Im}(B_1 N_1 + B_2) \subseteq \mathcal{R} \subseteq \mathcal{R}^*$. From here on a little algebra yields $\mathcal{B}_2 \subseteq \mathcal{B}_1 + \mathcal{R}^*$. *Q.E.D.*

6.2.2. *error feedback* The error equations are

$$\dot{e} = Ae - B_c u + B_n w$$

Consider the static feedback

$$u = K_e e + K_w w$$

which yields

$$\dot{e} = (A - B_c K_e)e + (B_n - B_c K_w)w$$

The model matching problem is to find the feedback such that

$$C(sI - A + B_c K_e)^{-1}(B_n - B_c K_w) = 0 \qquad (11)$$

Theorem 2. There exists a static model matching feedback if there exists a reachability subspace $\mathcal{R}$ of the pair $(A, \begin{pmatrix} B_c & B_n \end{pmatrix})$ such that

$$\mathcal{B}_n \subseteq \mathcal{B}_c + \mathcal{R}$$
$$C\mathcal{R} = 0 \qquad (12)$$

Equivalently iff

$$\mathcal{B}_n \subseteq \mathcal{B}_c + \mathcal{R}^*$$

where $\mathcal{R}^*$ is the supremal reachability subspace of $(A, \begin{pmatrix} B_c & B_n \end{pmatrix})$ contained in kernel of C.

Proof: This is just a particular case of the disturbance decoupling problem with disturbance measurement ($DDDM$). *Q.E.D.*

Observe that if we attempt a matching of all state components ($C = I$), the matching condition reduces to $\mathcal{B}_n \subseteq \mathcal{B}_c$, which is very unlikely to occur because the throttle actuators space would have to cover the control surface actuator space.

6.2.3. *state feedback* First, observe that if an error feedback matching controller exists, it can be implemented as a crippled aircraft state feedback, viz.,

$$\begin{aligned} u &= K_e e + K_w w \\ &= K_e(G_n w - x_c) + K_w w \\ &= -K_e x_c + (K_e G_n + K_w) w \end{aligned}$$

Next, if the problem is formulated from the beginning on as a crippled aircraft state feedback the design has more degrees of freedom. To this end, consider the augmented plant $(\hat{A}, (\hat{B}_c \ \hat{B}_n), \hat{C})$,

$$\begin{pmatrix} \dot{x}_n \\ \dot{x}_c \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix} \begin{pmatrix} x_n \\ x_c \end{pmatrix} + \begin{pmatrix} 0 \\ B_c \end{pmatrix} u + \begin{pmatrix} B_n \\ 0 \end{pmatrix} w$$

$$e = \begin{pmatrix} C & -C \end{pmatrix} \begin{pmatrix} x_n \\ x_c \end{pmatrix}$$

Theorem 3. There exists a (possibly dynamic) crippled state feedback model matching compensator iff

$$\begin{pmatrix} \mathcal{B}_n \\ 0 \end{pmatrix} \subseteq \begin{pmatrix} 0 \\ \mathcal{B}_c \end{pmatrix} + \widehat{\mathcal{R}}^* \tag{13}$$

where $\widehat{\mathcal{R}}^*$ is the supremal reachability subspace of the triple

$$\left(\begin{pmatrix} A & 0 \\ 0 & A \end{pmatrix}, \begin{pmatrix} 0 & B_n \\ B_c & 0 \end{pmatrix}, \begin{pmatrix} C & -C \end{pmatrix} \right)$$

Proof: If the geometric condition (13) holds, the DDDM problem for the plant

$$(\hat{A}, (\hat{B}_c \ \hat{B}_n), \hat{C})$$

yields a matching controller in terms of a static feedback of x_n, x_c, w. In view of $x_n = G_n w$, this yields a dynamic feedback of x_c, w. Conversely, assume there exists some dynamic matching controller. As in Theorem 1, the dynamic controller is viewed as a static controller around some dynamic augmentation of $(\hat{A}, (\hat{B}_c \ \hat{B}_n), \hat{C})$. Following the same argument as in Theorem 1, one gets (13). *Q.E.D.*

6.3 *feedback linearization*

It should be clear that the structure algorithm approach can be extended to the nonlinear case using the differential-geometric theory (Nijmeijer and van der Schaft 1990). To be more specific, consider a Single-Input-Single-Output case of one failed control surface w being compensated for by one throttle signal u in such a way as to match a single component $h(x)$ of the state,

$$\begin{aligned} \dot{x}_n &= f(x_n) + g_n(x_n) w \\ y_n &= h(x_n) \end{aligned}$$

$$\begin{aligned} \dot{x}_c &= f(x_c) + g_c(x_c) u \\ y_c &= h(x_c) \end{aligned}$$

Differentiating y_c relative to time and substituting $f(x_c) + g_c(x)u$ for $\dot{x}_c$ yields

$$\dot{y}_c = L_f h + (L_{g_c} h) u$$

where

$$L_f h = \sum_i \frac{\partial h}{\partial x_i} f_i$$

denotes the Lie derivative of h along the flow of f and

$$L_{g_c} h = \sum_i \frac{\partial h}{\partial x_{c_i}} g_{c_i}$$

denotes the Lie derivative of h along the flow of g_c. The *crucial point* in feedback linearizability is an argument of the following form: If the following *constancy of structure* condition

$$L_{g_c} h(x_c) \neq 0, \forall x_c$$

holds, then the feedback linearization exists *globally* (that is, over the domain of validity of the chart x if x_i are local coordinates), because indeed, choosing the feedback control

$$u = \frac{1}{L_{g_c} h}(-L_f h - k_{den} y_c + k_{num} y_n) \tag{14}$$

achieves the desired relationship

$$y_c = \frac{k_{num}}{s + k_{den}} y_n$$

In essence, the specifications are achieved by manipulating the gains k_{num}, k_{den}.

Under the constancy of structure condition, the above scheme is very simple; however, the difficulty starts in a situation where

$$L_{g_c} h(x^1) = 0$$

for some $x^1 \in X$. Differentiating the output y_c twice yields

$$\frac{d^2y_c}{dt^2} = L_f^2h + (L_fL_{g_c}h)u + (L_{g_c}L_fh)u + (L_{g_c}^2h)u^2 + (L_{g_c}h)\dot{u}$$

The difficulty to achieve linearization by feedback is easily seen. Assume the equation

$$L_f^2h + (L_fL_{g_c}h + L_{g_c}L_fh)u + (L_{g_c}^2h)u^2 = -k_1\dot{y}_c - k_2y_c + k_{num}y_n$$

can be solved for u to yield

$$u = K(x_c, y_n, y_c, \dot{y}_c) \tag{15}$$

then the closed-loop system becomes

$$\frac{d^2y_c}{dt^2} + k_1\dot{y}_c + k_2y_c = k_{num}y_n + (L_{g_c}h)\dot{u}$$

Since $L_{g_c}h(x^1) = 0$, it can be assumed that $L_{g_c}h(x)$ is small in a neighborhood of x^1 so that (if we can keep $\dot{u}$ bounded) the approximate relationship becomes

$$y_c = \frac{k_{num}}{s^2 + k_1s + k_2}y_n$$

in some neighborhood O_{x^1} of the bad point x^1. Again, the specifications are achieved by manipulating the gains k_{num}, k_1, k_2.

Consider now a situation where the compensator 14 is valid in $X \setminus \{x^1\}$ and the compensator 15 is valid in some neighborhood O_{x^1}. The whole difficulty is to "glue" the compensator 14 and the compensator 15 within the intersection.

$$X \setminus \{x^1\} \cap O_{x^1}$$

This "gluing" can be approached using a partition of unity construction.

The above gluing problem will very specifically occur in an aircraft like the F-16 that has its short periodic oscillations crossing the imaginary axis. This means that, for some point x^1 of the envelope, the linearized open-loop transfer function has poles on the imaginary axis. The reader can easily verify for himself that the case $L_{g_c}h(x^1) = 0$ is equivalent to a pole at 0 for the transfer function of the system linearized around x^1. Clearly, the above gluing will be necessary to design a compensator able to keep control of the aircraft when its short periodic oscillations go from stable to unstable.

We also note that there is currently a substantial amount of work dealing with the linearization method subject to uncertain parameters using some adaptive scheme. However, it appears that these adaptive schemes rely heavily on strong assumptions on the way the uncertainty enters f and g. Among other assumptions is the constancy of structure; here, we have shown that some topological tools allow the lifting of this condition.

7. WINGED CONE BENCHMARK EXAMPLE

The so-called "Winged Cone," a Reusable Launch Vehicle (RLV) concept still being considered by NASA (Buschek 1995), is chosen as benchmark example for the geometric theory of model matching. The Winged-Cone accelerator air vehicle consists of an axis-symmetric conical forebody, a cylindrical engine nacelle section with engine modules all around the body, a pair of delta wings, and a cone frustrum engine nozzle section. The linearized longitudinal model represents a flight condition for trimmed accelerated flight at Mach 8 and 86,000 ft. The system is described by the linear time-invariant matrix differential equation where the coefficient matrices are given by

$$A = \begin{bmatrix} 3.65\cdot10^{-3} & -9.66\cdot10^{-1} & 0.00 & -5.56\cdot10^{-1} & -1.43\cdot10^{-3} \\ -3.91\cdot10^{-5} & -8.16\cdot10^{-2} & 1.00 & -8.44\cdot10^{-5} & 9.25\cdot10^{-6} \\ 2.01\cdot10^{-3} & 3.03 & -9.52\cdot10^{-2} & 1.55\cdot10^{-5} & -1.07\cdot10^{-5} \\ 2.72\cdot10^{-6} & 7.76\cdot10^{-6} & 1.00 & -7.76\cdot10^{-6} & -1.01\cdot10^{-9} \\ 2.07\cdot10^{-2} & -1.37\cdot10^{2} & 0.00 & 1.37\cdot10^{2} & 0.00 \end{bmatrix} \tag{16}$$

$$B_n = \begin{bmatrix} 9.69\cdot10^{-2} & 7.59 \\ 3.34\cdot10^{-2} & -2.09\cdot10^{-3} \\ 1.08 & 0.00 \\ 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} = \begin{bmatrix} b_e & b_\eta \end{bmatrix} \tag{17}$$

$$B_c = \begin{bmatrix} 0.00 & 7.60 \\ 0.00 & -2.09\cdot10^{-3} \\ 7.87 & 0.00 \\ 0.00 & 0.00 \\ 0.00 & 0.00 \end{bmatrix} = \begin{bmatrix} b_d & b_\eta \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The state vector and the control effort vector are:

$$x = \begin{bmatrix} V \\ \alpha \\ q \\ \theta \\ h \end{bmatrix} \tag{18}$$

$$= \begin{bmatrix} \text{incremental velocity (ft/sec)} \\ \text{incremental angle of attack (deg)} \\ \text{pitch rate (deg/sec)} \\ \text{incremental pitch attitude (deg)} \\ \text{incremental altitude (ft)} \end{bmatrix}$$

$$u_n = \begin{bmatrix} \delta e \\ \delta \eta \end{bmatrix} \tag{19}$$

$$= \begin{bmatrix} \text{symmetric elevon deflection (deg)} \\ \text{fuel equivalent ratio (-)} \end{bmatrix}$$

$$u_c = \begin{bmatrix} \delta d \\ \delta \eta \end{bmatrix} = \begin{bmatrix} \text{differential thrust (-)} \\ \text{fuel equivalent ratio (-)} \end{bmatrix}$$

The purpose of this example is to check whether there exist model matching feedback matrices K_e and K_w such that the differential thrust emulates the elevon for pitch attitude control, i.e.,

$$C\,(sI - A + B_c K_e)^{-1}\,(B_n - B_c K_w) = 0$$

and the closed-loop poles of this system are chosen consistently with MILSPEC,

$$\Lambda = \{-0.1750 \pm i0.1785, -7 \pm i7.1414, -0.1\}.$$

By the geometric theory, it is sufficient to check

$$\mathcal{B}_n \subseteq \mathcal{B}_c + \mathcal{R}^*$$

Let R^* be a matrix such that $\mathrm{Im}\,(R^*) = \mathcal{R}^*$. It is clear that the geometric matching condition is equivalent to

$$rank\,([\,R^* \mid B_c\,]) = rank\,([\,R^* \mid B_c \mid B_n\,])$$

where the rank is numerically evaluated using the singular value decomposition. The computation of such a matrix R^* is based on the following theorem.

Theorem 4. Let $\Lambda = \{\lambda_i\}$ be a self conjugate (i.e., invariant under reflection across the real axis) set of stable closed-loop poles, none of which is a transmission zero of $C\,(sI - A)^{-1}\,B$. For every i compute X_i, U_i such that

$$\begin{bmatrix} \lambda_i I - A & -B_c & -B_n \\ C & 0 & 0 \end{bmatrix} \begin{bmatrix} X_i \\ Uc_i \\ Un_i \end{bmatrix} = 0$$

Then

$$\mathcal{R}^* = \mathrm{Im}\,(X_1, X_2, \ldots)$$

Straightforward computation reveals a 5×15 R^* matrix of rank 3. Further, numerical evaluation of ranks using singular value decomposition yields $rank\,([\,R^* \mid B_c\,]) = 4$ and

$$rank\,([\,R^* \mid B_c \mid B_n\,]) = 4.$$

Therefore the matching can be achieved.

8. GAIN SCHEDULING

Since linear H^∞ model matching compensation is based on a linearized model G_p of the aircraft around some point p of the flight envelope, some gain scheduling is needed to cover the whole flight envelope P. The gain scheduling approach consists of linearizing the system about every point p, computing the ball of compensators achieving the matching specifications in a neighborhood of the operating point, and then "piecing together" all of the compensators into a continuously p-dependent compensator K_p, so that identification of the parameter vector together with a slow adaptation law would smoothly update the compensator and ensure good performance all over the envelope.

To get to the deeper topological aspects, observe that H^∞ design is a map

$$f : M_d(2n_x + n_u + n_w, n_w + n_u) \to M_d(n_u, n_x + n_w)$$

$$G = \begin{pmatrix} W_1 G_n & -W_1 G_c \\ 0 & W_2 \\ 0 & G_c \\ I & 0 \end{pmatrix} \mapsto (\,K_x \; K_w\,)$$

In the above $M_d(m,n)$ = set of $m \times n$ transfer matrices of fixed McMillan degree d, $n_u = \dim u$, etc.

It should be stressed that M_d has a very complicated topological structure. In the SISO case and in some multivariable symmetric cases (unlikely to occur in aircraft problems but quite likely to occur in large space structures with colocated actuators and sensors), the space M_d breaks into several connected components, each uniquely specified by the Cauchy index. It appears from some early work that the map f is "cellular" in the sense that it maps the cell of plants of a given Cauchy index into the cell of compensators of a specific Cauchy index.

One might think that a more pragmatic approach to the gain scheduling problem would be to parameterize M_d and rewrite the map f as a map between subsets of Euclidean spaces. This is certainly correct locally, but the global parameterization of the space M_d is a bit of a tricky problem.

8.1 *neural network gain scheduling map*

Since the gain scheduling map could be complicated, an idea is to take a few sample points of the flight envelope, compute the corresponding H^∞ compensators, and then use this data to train a neural network that would approximate the gain scheduling map. This approach has been used to schedule the gain of a propulsion controller for the L-1011 from cruise to approach to landing. This simple approach has worked on the L-1011 aircraft, because it is a well-designed aircraft, that only exhibits smooth variations of its stability derivatives across the flight envelope.

8.2 *simplicial approximation gain scheduling map*

Neural networks are used in a variety of problems because they provide a "universal approximator" for any map $\mathbb{R}^m \to \mathbb{R}^n$. It is in this spirit that neural networks have been used to approximate the gain scheduling map of the L-1011. However, in the wake of successful applications of neural networks, another "universal approximator" that has been known by topologists ever since the beginning of this century—the simplicial approximation theorem—has remained grossly overlooked. In a few words, the simplicial approximation theorem says that any continuous map can be approximated by a piecewise linear map, homotopic and "star related" to the original map. (The reason why the homotopic property is useful is that the spaces of plants and compensators are "full of holes," have nontrivial homology, and choosing an approximation homotopic to the original map would ensure that the approximate compensator does not "fall in a hole.") Probably the reason why the simplicial approximation theorem has remained somewhat overlooked is that its implementation requires modern tools from computational geometry.

Here is a very crude implementation of the simplicial approximation ideas to the gain scheduling problem: Take a few sample points $a^0, a^1, ..., a^N$ in the flight envelope. For each such point a^i, let $K^i = \left(K^i_x \; K^i_w\right)$ be the H^∞ compensator. Do a Delaunay triangulation of the set of vertices $\{a^i\}$; in other words, the convex hull of $\{a^i\}$ is decomposed into simplexes having the a^i's as vertices. Now, take a new point p in the flight envelope. The point location problem of computational geometry will identify the unique simplex $a^{i_0}...a^{i_n}$ of the triangulation such that

$$p \in a^{i_0}...a^{i_n}$$

More precisely, in barycentric coordinates,

$$p = \sum_{j=0}^{n} \lambda_j a^{i_j}, \;\; \lambda_j > 0, \;\; \sum_{j=0}^{n} \lambda_j = 1$$

Then, the scheduled controller would be

$$f(G_p) = \sum_{j=0}^{n} \lambda_j K^{i_j}(s)$$

We hasten to say that the above is merely a "piecewise-linear extension," not a simplicial map. Constructing a simplicial map would require some way to triangulate the space of compensators (a first crack would be to triangulate the space of local parameters as a subset of the Euclidean space) and from there follow the lines of our SimplicialVIEW project.

8.3 *gain scheduling map as cross section through bundle of compensators*

To be somewhat specific, consider a nominal and a crippled nonlinear p-dependent dynamics. Define G_{p^0} to be the open-loop G matrix corresponding to the nominal and crippled systems linearized about p^0. Define

$$\gamma^*(G_{p^0}) = \inf_{K stabilizing} ||T_{zw}(G_{p^0}, K)||_\infty$$

It can be shown that $\gamma^*(G)$ is lower semicontinuous provided G keeps the same number of RHP poles. Therefore, $\forall \epsilon > 0$, there exists a ball of plants $\mathcal{G}_{p^0}$ and a ball of compensators $\mathcal{K}_{p^0}$ such that

$$||T_{zw}(T, K)||_\infty \leq \gamma^*(G_{p^0}) + \epsilon$$
$$\forall G \in \mathcal{G}_{p^0}, \forall K \in \mathcal{K}_{p^0}$$

The set of compensators is parameterized by the unit ball of H^∞, BH^∞. One such compensator is the two Riccati equation solution compensator K_{p^0}. The problem is that it is hard to use $p^0 \mapsto K_{p^0}$ as gain scheduling map because the two Riccati equation solutions might be a complicated functions of the parameters and exhibit bifurcations, etc. Here the approach is to construct an easier map by carefully choosing a compensator in each $\mathcal{K}_p$ and making sure that they are properly "pieced together." This selection problem is called cross sectioning.

Consider the disjoint union

$$\mathcal{K} = \sqcup_{p^0 \in P} \mathcal{K}_{p^0}$$

topologized as a subspace of

$$P \times M_d(n_u, n_x + n_w)$$

where $M_d(n_u, n_x + n_w)$ is the set of $n_u \times (n_x + n_w)$ transfer matrices of fixed McMillan degree d and n_u, n_x, n_w denote the sizes of the vectors u, x, w, respectively. Define the mapping

$$\pi : \mathcal{K} \to P$$
$$(p, K) \mapsto p$$

Under some conditions, yet to be clarified, ($\mathcal{K} \xrightarrow{\pi} P$) is a fiber bundle over the space P with fiber

BH^∞. Clearly, a cross section through that bundle, that is, a mapping

$$c : P \to \mathcal{K}, \ \pi \circ c = 1$$

provides a compensator continuously depending on the initial condition and the parameter vector.

Topology provides us with some answers as to when a cross section exists and provides us with some high-level construction of the cross section. It is basically a trial-and-error procedure. We first construct a cross section defined over some low order skeleton of P and then figure out the primary and secondary obstructions to extending the cross section to higher skeleta. The simplicial ideas can be implemented at this stage: define the cross section over some vertices, check the obstruction, then define it over 2-simplexes by piecewise linear extension, until a piecewise linear cross section is constructed. The obstruction test somehow allows us to navigate between the holes in the plant and compensator spaces and make sure that no compensator falls in a hole. In our book, we have developed a variety of tools—ranging from combinatorial, simplicial, piecewise-linear techniques to obstruction computation by integration of invariant differential forms on Lie groups of matrices—to attack these kinds of problems.

A result of topology is that if the base space P is contractible, a cross section exists. Clearly, for an aircraft, the flight envelope P is a contractible space, so that a cross section is guaranteed to exist.

9. LINEAR SET VALUED DYNAMICALLY VARYING SYSTEMS AND RECONFIGURABLE CONTROLLERS

We briefly discuss a new method that is applicable to stabilizing time-varying systems subject to possible, but known, failures. This method is well suited for "reconfigurable control." This method is novel and is an outgrowth of recently completed work concerning Linear Dynamically Varying (LDV) systems. As an example of this method, a controller for the X-33 subject to possible failures is presented.

9.1 *Introduction*

There are a large number of control methodologies for linear time-invariant systems. However, time-varying systems present a challenge. If the system is running over a finite horizon, then a finite horizon controller can be found. Unfortunately, to design an optimal finite horizon controller, all the system parameters must be known a priori. For many systems, this information is not known and can never be known. For example, if a failure occurs, then the system parameters may be altered in a nondeterministic fashion. It is assumed here that all types of failures are known and hence the set of all possible system parameters is known. If a large number of flight conditions and many types of failures are possible, then the set of possible system parameters is large. However, the system parameters do not vary in an arbitrary fashion. At a particular flight condition, the set of possible system parameters at the next time step is a small subset of all possible parameters. Therefore, the entire set of parameters is known and the way in which the parameters vary is partially known. The objective of the controllers developed here is to use all of the known information. These controllers will be optimal in some LQ sense and the stabilizability of the system can be ascertained. That is, if these controllers don't stabilize the linear system, no controller will. (We only consider linear systems here; non-linearities and robustness are separate issues that can be addressed once linear stabilizability is guaranteed.)

This section proceeds as follows: First a brief discussion of two approaches to controlling time-varying systems is presented. The inadequacies of these methods motivate a new method which is the subject of this section. The next subsection deals with how one assesses the stabilizability of a finite horizon time-varying linear system. Unfortunately, eigenvalues and the like are only valid in the infinite horizon time-invariant case. In subsection 9.4 an algorithm that generates stabilizing time-varying controllers is developed. Then in subsection 9.5 this algorithm is modified to accommodate the uncertainties presented by the failures. Finally, an example of controlling the X-33 is presented.

First some terminology is defined. During the launch of the X-33, the vehicle will travel along a specific path. That is, its desired altitude, velocity, etc., will vary in a specific way. We call this a *flight path*. Depending on whether failures occur, and if emergency action is necessary, there are many possible flight paths. At each point along a flight path, a linear model for the system can be found. As the flight point changes along the flight path, the system parameters of the linear model change. We call this path through the space of system parameters a *parameter trajectory*. If no failures could ever occur, there is a one to one mapping between the flight paths and parameter trajectories. If failures are allowed, then the mapping from flight paths to parameter trajectories is one to many. Furthermore, if a failure occurs, then at the moment of failure the system parameters may shift whereas the flight point does not. Thus, flight paths are smooth, whereas parameter trajectories are not.

9.2 *LPV versus LDV*

Consider the following linear system with varying parameters:

$$\begin{aligned} x_{k+1} &= A_{\theta_k} x_k + B_{1_{\theta_k}} u_k + B_{2_{\theta_k}} w_k \qquad (20) \\ z_k &= \begin{bmatrix} C_{\theta_k} x_k \\ D_{\theta_k} u_k \end{bmatrix} \\ \theta_{k+1} &= f(\theta_k) \end{aligned}$$

Depending on f and our a priori knowledge of f, system 20 defines a wide variety of systems. For example, if $f : \mathbb{Z} \to \mathbb{Z}$, $f(l) = l+1$, then system 20 is a general time-varying system. If f is a Markov chain, i.e.,

$$f(\theta) = \begin{cases} \varphi_1 \text{ with probability } p_1(\theta) \\ \varphi_2 \text{ with probability } p_2(\theta) \\ \vdots \end{cases}$$

then system 20 is a *Jump Linear (JL)* system(Ji and Chizeck 1990). If $f : \Theta \to \Theta$ where Θ is compact, and f, A, B, C, D are known continuous functions, then system 20 is a continuous *Linear Dynamically Varying (LDV)* system(Bohacek and Jonckheere n.d.). If $f : \Theta \to \Theta$ where Θ is compact, but f is unknown, then system 20 is a *Linear Parametrically Varying (LPV)* system(Becker and Packard 1995). In the case of LDV systems, the function f is completely known, and in the case of LPV systems, f is completely unknown. LDV and LPV systems are the opposite ends of the spectrum and typically neither occurs. Usually, and in the case of reconfigurable controllers, the designer has a rough idea of the function f but there is some uncertainty. To account for this information, we introduce a new type of linear systems in which f is a set valued function, i.e., $f(\theta)$ is a set. These linear systems are referred to as *Linear Set Valued Dynamically Varying (LSVDV)* systems.

The X-33 is an example of a LSVDV system. During launch, the ideal flight path is a perfect launch to sub-orbit, in which case f is known. However, failures cause the system parameters to change drastically. Thus at each point θ, there are many values of $f(\theta)$. One value, φ_{nf}, corresponds to the case of no failures and the other values, φ_{f_i}, correspond to cases of failures. Thus $f(\theta) \in \{\varphi_{nf}, \varphi_{f_1}, \varphi_{f_2}, \cdots\}$. A stabilizing controller must be stable for all possible parameter trajectories.

Controllers will be designed by minimizing a linear objective function subject to LMI constraints. Before developing a controller, stability of time-varying systems must be formalized.

9.3 *Stability of Time-varying Systems*

For time-invariant systems, stabilizability is equivalent to the existence of an LQ controller F and a solution $Y \geq 0$ to a Lyapunov inequality. That is, there must exist $\infty > Y \geq 0$ and F such that

$$\begin{aligned} Y - (A+BF)' Y (A+BF) & \qquad (21) \\ -F'D'DF - C'C \geq 0 & \end{aligned}$$

where, for simplicity, we assume that $C'C > 0$. The optimal LQ controller is given by minimizing Y over all F and $Y \geq 0$ that satisfy 21. At optimality, the matrix inequality becomes an equality, $F = -(D'D + B'YB)^{-1}B'YA$, and the matrix equality becomes the usual algebraic Riccati equation

$$\begin{aligned} Y + A'YB(D'D + B'YB)^{-1}B'YA & \\ -A'YA - C'C = 0 & \end{aligned}$$

The stability of time-invariant linear systems is characterized by the eigenvalues of $A + BF$ and, if $C'C > 0$, then the rate of decay is a simple function of $\left\|(C'C)^{-1}\right\|$ and $\|Y\|$. Similarly, for time-varying systems, LQ controllers are characterized by a set of positive semi-definite matrices $\{Y_k \geq 0 : k \geq 0\}$ and a set of feedback matrices $\{F_k : k \geq 0\}$ such that

$$\begin{aligned} Y_k - (A_k + B_k F_k)' Y_{k+1} (A_k + B_k F_k) & \qquad (22) \\ -F_k' D_k' D_k F_k - C_k' C_k \geq 0 & \end{aligned}$$

However, stability of time-varying systems is slightly more complex. From 22 we have,

$$x_k' Y_k x_k - x_k' C_k' C_k x_k \geq x_{k+1}' Y_{k+1} x_{k+1}$$

$$\frac{x_k' Y_k x_k - x_k' C_k' C_k x_k}{x_k' Y_k x_k} \geq \frac{x_{k+1}' Y_{k+1} x_{k+1}}{x_k' Y_k x_k}$$

$$1 \geq 1 - \frac{x_k' C_k' C_k x_k}{x_k' Y_k x_k} \geq \frac{x_{k+1}' Y_{k+1} x_{k+1}}{x_k' Y_k x_k} \geq 0.$$

If we define

$$\alpha_k := 1 - \frac{\underline{\lambda}(C_k' C_k)}{\overline{\lambda}(Y_k)} \qquad (23)$$

then we have

$$\alpha_k x_k' Y_k x_k \geq x_{k+1}' Y_{k+1} x_{k+1}$$

and

$$x_0' Y_0 x_0 \prod_{j=0}^{k-1} \alpha_j \geq x_k' Y_k x_k.$$

Finally, since $Y_k \geq C_k' C_k$,

$$\|x_0\|^2 \frac{\|Y_0\|}{\underline{\lambda}(C_k' C_k)} \prod_{j=0}^{k-1} \alpha_j \geq \|x_k\|^2 \qquad (24)$$

Thus uniform exponential stability can be guaranteed if $\alpha_k < 1 - \varepsilon$ and $\|Y_j\| \leq M < \infty$. From the definition of α, equation 23, and by inequality 24, it is clear that the smaller Y_k, the more stable the closed-loop system. Thus for $C'C > 0$ fixed, stability can be assessed by considering the size of Y_k.

From another point of view, Y_k is related to the quadratic cost. If $C'C > 0$ and is fixed, then a small cost means that $\|x_{k+j}\|$ becomes small slower. Furthermore, if $Y_k < Y_{k+1}$ then the cost of starting at stage k is smaller than starting at stage $k+1$. Thus, the system is more stable if it is started at stage k. If Y_k is much larger than $Y_{k\pm1}$, then one can conclude that the system at time k is difficult to stabilize. In this way one can pinpoint the regions of instability. This will be done in the example in section 10.

9.4 *Determining a Stabilizing Controllers for Time Varying Systems*

From the discussion above, it is clear that, given Y_{k+1}, we should look for F_k and Y_k such that inequality 22 is satisfied and Y_k is as small as possible. This suggests the following algorithm: Let K be the terminal time and consider the cost $\sum_{k=0}^{K-1} \|z_k\|^2 + x_K' Y_K x_K$. Set $Y_K = I$ (other values for Y_K are also possible and perhaps better.) For each $k < K$ solve

$$\min_{Y_k > 0, F_k} Y_k \text{ subject to}$$
$$0 \leq Y_k - (A_k + B_k F_k)' Y_{k+1} (A_k + B_k F_k) - F_k' D' D F_k - C'C.$$

The constraint can be posed as an LMI:

$$\min_{Y_k > 0, F_k} Y_k \quad \text{subject to}$$
$$0 \leq \begin{bmatrix} Y_k & (A_k + B_k F_k)' & (F_k D)' & C' \\ A_k + B_k F_k & Y_{k+1}^{-1} & 0 & 0 \\ F_k D & 0 & I & 0 \\ C & 0 & 0 & I \end{bmatrix}$$

However, Y_k is not a scalar objective function; thus popular LMI tools cannot be used. However, we have the following:

Theorem 5. If Y_k minimum and F_k are such that

$$\begin{bmatrix} Y_k & (A_k + B_k F_k)' & (F_k D)' & C' \\ A_k + B_k F_k & Y_{k+1}^{-1} & 0 & 0 \\ F_k D & 0 & I & 0 \\ C & 0 & 0 & I \end{bmatrix} \geq 0$$

and if Z and G satisfy

$$\begin{bmatrix} Z & (A_k + B_k G)' & (GD)' & C' \\ A_k + B_k G & Y_{k+1}^{-1} & 0 & 0 \\ GD & 0 & I & 0 \\ C & 0 & 0 & I \end{bmatrix} \geq 0$$

then

$$v' Y_k v \leq v' Z v \text{ for all } v.$$

Furthermore, Y_k is given by

$$\min_{Y_k > 0, F_k} Trace\,(Y_k) \quad \text{subject to} \qquad (25)$$
$$0 \leq \begin{bmatrix} Y_k & (A_k + B_k F_k)' & (F_k D)' & C' \\ A_k + B_k F_k & Y_{k+1}^{-1} & 0 & 0 \\ F_k D & 0 & I & 0 \\ C & 0 & 0 & I \end{bmatrix}$$

The proof of theorem 5 relies on the following lemma:

Lemma 6. If $Trace\,(Y_1) \leq Trace\,(Y_2)$ then $v' Y_1 v \leq v' Y_2 v$.

Proof: Y_1 and Y_2 can be simultaneously diagonalized so that $Y_1 = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$ and $Y_2 = \begin{bmatrix} \mu_1 & & \\ & \ddots & \\ & & \mu_n \end{bmatrix}$. Thus $Trace\,(Y_1) \leq Trace\,(Y_2)$ implies that $\sum_i (\mu_i - \lambda_i) \geq 0$, hence $\sum_i (\mu_i - \lambda_i) v_i^2 \geq 0$ and $\sum_i \mu_i v_i^2 \geq \sum_i \lambda_i v_i^2$. Therefore, $v' Y_2 v \geq v' Y_1 v$. *Q.E.D.*

Note that 25 is a minimization that is linear in the objective and linear in the constraints. This LMI can easily be solved by LMI tools.

Remark 1. Robustness and measurement output feedback can also be posed a minimization subject to LMI constraints. For this discussion we will only concentrate on LQ optimization.

9.5 *Linear Set Valued Dynamically Varying Systems*

With minor modification, the stabilizability techniques for time-varying systems developed above can be made to work for stabilizing linear dynamically varying systems with set valued dynamical shifts f. The method is illustrated with a simple example. Consider system 20 with

$$f(\theta) = \begin{cases} \theta + 1 \text{ for } \theta < 10 \\ \{20, 100\} \text{ for } \theta = 10 \\ \theta + 1 \text{ for } \theta \geq 20 \\ \theta + 1 \text{ for } \theta \geq 100 \end{cases},$$

where the system runs for a total of 20 steps. More specifically, the system runs for 10 steps, then a failure possibly occurs, and finally the system runs for 10 more steps. Define the terminal costs $Y_{30} = Y_{110} = I$. The minimization problem 25 is solved when the system is recovering from the failure, that is, for $k \in [10, 20]$, i.e., $\theta \in \{10, [20, 30]\}, \{10, [100, 110]\}$. To be specific, we solve

$$\min_{Y_\theta > 0, F_\theta} Trace(Y_\theta) \quad \text{subject to}$$

$$0 \leq \begin{bmatrix} Y_\theta & (A_\theta + B_\theta F_\theta)' & (F_\theta D)' & C' \\ A_\theta + B_\theta F_\theta & Y_{f(\theta)}^{-1} & 0 & 0 \\ F_\theta D & 0 & I & 0 \\ C & 0 & 0 & I \end{bmatrix}$$

at $\theta_{\cdot} = 10$. $Y_{f(\theta)}$ is not defined because $Y_{f(10)} \in \{Y_{20}, Y_{100}\}$. To accommodate for this, we find a Y_{10} and a F_{10} that solve the minimization problem

$$\min_{Y_{10} > 0, F_{10}} Trace(Y_{10}) \quad \text{subject to}$$

$$0 \leq \begin{bmatrix} Y_{10} & (A_{10} + B_{10} F_{10})' & (F_{10} D)' & C' \\ A_{10} + B_{10} F_{10} & Y_{20}^{-1} & 0 & 0 \\ F_{10} D & 0 & I & 0 \\ C & 0 & 0 & I \end{bmatrix}$$

$$0 \leq \begin{bmatrix} Y_{10} & (A_{10} + B_{10} F_{10})' & (F_{10} D)' & C' \\ A_{10} + B_{10} F_{10} & Y_{100}^{-1} & 0 & 0 \\ F_{10} D & 0 & I & 0 \\ C & 0 & 0 & I \end{bmatrix}$$

This is yet another minimization subject to two LMI constraints and it can easily be solved with LMI tools. In this way, a stabilizing controller is designed that can take into account failures and the time-varying nature of the system. This controller will, by design, be time-varying and, since it accounts for all possible failures, reconfigurable. The main difficulty is developing f, and keeping track of the possible large number of possible trajectories. This is a programming task and is not insurmountable.

10. RECONFIGURABLE CONTROL OF X-33

The X-33 is a reduced-scale, unhabited, suborbital demonstration vehicle for the new space shuttle, the "Venture Star." The technology is radically different from that of the current space shuttle. First of all, instead of an airplane configuration like the current space shuttle, the X-33 embodies the "lifting body" technology concept, that is, most of the lift is generated by the body itself rather than the wings. The main reason for this arrangement is the avoidance of hot spots during re-entry. The drawback of this concept is that aerodynamic control is drastically reduced—like an airplane with depleted hydraulics. Second, the engines of the X-33 are of the so-called "linear aerospike" type, instead of the bell-shaped exhaust nozzle concept of the current space shuttle. In the latter, the explosion occurs in a bell-shaped profile. The problem with this concept is that the explosion is not optimally located relative to the bell-shaped profile for all flight conditions, resulting in loss of efficiency. The aerospike engines, on the other hand, consist of two modules—an upper stage module above the C.G. and a lower stage module below the C.G. On the upper module, the gas are ejected on top of a "plug" profile while the gas of the lower module are ejected below the "plug" profile. It turns out that, with this arrangement, the combustion sticks to the profile all across the envelope hence improving efficiency and allowing for "Single Stage To Orbit." The drawback of the aerospike engines is that they cannot be gimballed for thrust vectoring. This, together with deficient aerodynamic control, calls for differential throttling attitude control, blended with some aerodynamic control, not unlike the scheme adopted by several airline captains, struggling to fly their crippled aircraft with a combination of conventional control and propulsion control.

To highlight the LSVDV reconfigurable control procedure, the ascent of the X-33 is examined under three possible configurations in the system: *(i)* no failure, *(ii)* no flaps nor elevators, and *(iii)* no differential thrust. These failures can occur at any time and may rectify themselves at any time as well. Hence, at each time step of the ascent, three configurations are possible for the next time step. Therefore, $f(\theta)$ takes three values. The designed controller begins 120 seconds after lift-off and runs until 275 seconds after lift-off, at which time the engines are shut off and the X-33 descends. It is assumed that the flight path does not change if a failure occurs.

Figure 1 shows the largest norm of the quadratic cost at each time step of the LSVDV approach. Figure 1 also shows the worst case cost that would result if a more standard gain scheduling approach was taken. That is, a controller is designed at each time step assuming that the system parameters are not changing. Since large changes in the system parameters can occur very quickly, the gain scheduling approach results in a very high cost.

Note that the cost at time step 160, for both the LSVDV approach as well as the classical gain scheduling approach, is quite high. Thus we can

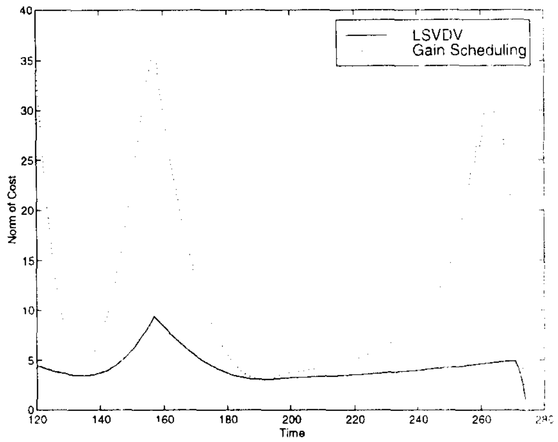

Fig. 1. This figure shows the worst case cost for the LSVDV approach versus the gain scheduling approach. Since the LSVDV approach takes into account the possible rapid variation of the parameters, the LSVDV approach has a far smaller cost.

conclude that the system is difficult to stabilize for the corresponding set of parameter values.

11. CONCLUSION

The main point of this paper has been that, in case of hydraulic failure on board an aircraft, it is not easy to control attitude by manually operating the throttles, so that some kind of "fly-by-wire" system—where the pilot's intentions (up, down, left, right,...) are fed to a controller which in turn synthesizes the throttle signal–would be welcome.

Along the same lines of investigation, similar concepts apply to scramjet and aerospike Reusable Launch Vehicles, since the newer engines cannot be gimballed and control surfaces are reduced to the bare minimum.

While propulsion controllers to emulate control surfaces can be designed using conventional H^∞ techniques, the underlying system theoretic problem is model matching. As illustrated on the "Winged Cone" example, the geometric theory of model matching reveals that a properly compensated differential thrust can reproduce the response of an elevon.

From a broader perspective, swapping such actuators as differential thrust and elevon can be considered as "control reconfiguration." A new approach based on Linear Set Valued Dynamically Varying systems has been developed. As illustrated on the X-33 example under possible failure of differential thrust, flaps and/or elevon, the dynamical aspect of this approach allows for better handling of the failure than a mere static parameter adjustment scheme.

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