Feature Detection and Matching

CS4243 Computer Vision and Pattern Recognition

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Outline

Feature Detection

For image registration, need to obtain correspondence between images.
Basic idea:

- detect feature points, also called keypoints
- match feature points in different images

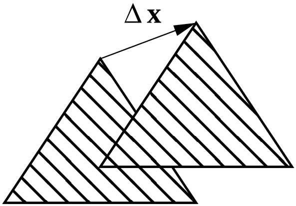
Want feature points to be detected consistently and matched correctly.

Many features available

- Harris corner
- Tomasi's "good features to track"
- SIFT: Scale Invariant Feature Transform
- SURF: Speeded Up Robust Feature
- GLOH: Gradient Location and Orientation Histogram
- etc.

Harris Corner

Observation:



- A shifted corner produces some difference in the image.
- A shifted uniform region produces no difference.
- So, look for large difference in shifted image.

Suppose an image patch W at $\mathbf{x}$ is shifted by a small amount $\Delta\mathbf{x}$. Then, the sum-squared difference at $\mathbf{x}$ is

$$E(\mathbf{x}) = \sum_{\mathbf{x}_i \in W} \left[I(\mathbf{x}_i) - I(\mathbf{x}_i + \Delta\mathbf{x}) \right]^2 . \tag{1}$$

That is,

$$E(x, y) = \sum_{(x_i, y_i) \in W} \left[I(x_i, y_i) - I(x_i + \Delta x, y_i + \Delta y) \right]^2 . \tag{2}$$

This is also called the auto-correlation function.

Apply Taylor's series expansion to $I(\mathbf{x}_i + \Delta\mathbf{x})$:

$$\begin{aligned} I(x_i + \Delta x, y_i + \Delta y) &= I(x_i, y_i) + \frac{\partial I}{\partial x}\Delta x + \frac{\partial I}{\partial y}\Delta y && (3) \\ &= I(x_i, y_i) + I_x \Delta x + I_y \Delta y && (4) \\ &= I(\mathbf{x}_i) + (\nabla I)^\top \Delta\mathbf{x} && (5) \end{aligned}$$

where $\nabla I = (I_x, I_y)^\top$.

Substituting Eq. 5 into Eq. 1 yields

$$E(\mathbf{x}) = \sum_W [I_x \Delta x + I_y \Delta y]^2 \tag{6}$$

$$= \sum_W \left[I_x^2 \Delta^2 x + 2 I_x I_y \Delta x \Delta y + I_y^2 \Delta^2 y\right] \tag{7}$$

$$= (\Delta \mathbf{x})^\top \mathbf{A}(\mathbf{x}) \Delta \mathbf{x} \tag{8}$$

where the auto-correlation matrix $\mathbf{A}$ is given by (Exercise)

$$\mathbf{A} = \begin{bmatrix} \sum_W I_x^2 & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y^2 \end{bmatrix}. \tag{9}$$

$\mathbf{A}$ captures intensity pattern in W.

Response $R(\mathbf{x})$ of Harris corner detector is given by [HS88]:

$$R(\mathbf{x}) = \det \mathbf{A} - \alpha(\operatorname{tr} \mathbf{A})^2. \tag{10}$$

Two ways to define corners:

(1) Large response
The locations $\mathbf{x}$ with $R(\mathbf{x})$ greater than certain threshold.

(2) Local maximum
The locations $\mathbf{x}$ where $R(\mathbf{x})$ are greater than those of their neighbors, i.e., apply non-maximum suppression.

Sample result (large response):

Many corners are detected near each other.
So, better to find local maximum.

Tomasi's Good Feature

Shi and Tomasi considered weighted auto-correlation [ST94]:

$$E(\mathbf{x}) = \sum_{\mathbf{x}_i \in W} w(\mathbf{x}_i) \left[I(\mathbf{x}_i) - I(\mathbf{x}_i + \Delta\mathbf{x}) \right]^2 \tag{11}$$

where $w(\mathbf{x}_i)$ is the weight.

Then, $\mathbf{A}$ becomes

$$\mathbf{A} = \begin{bmatrix} \sum_W w I_x^2 & \sum_W w I_x I_y \\ \sum_W w I_x I_y & \sum_W w I_y^2 \end{bmatrix}. \tag{12}$$

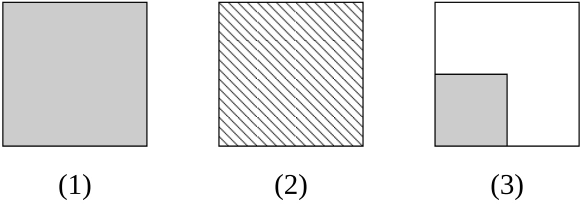
$\mathbf{A}$ is a 2×2 matrix. This means there exist scalar values λ_1, λ_2 and vectors $\mathbf{v}_1, \mathbf{v}_2$ such that

$$\mathbf{A}\,\mathbf{v}_i = \lambda_i \mathbf{v}_i \ , \quad i = 1, 2 \tag{13}$$

- $\mathbf{v}_i$ are the orthonormal eigenvectors, i.e.,

$$\mathbf{v}_i^\top \mathbf{v}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

- λ_i are the eigenvalues; expect $\lambda_i \geq 0$.



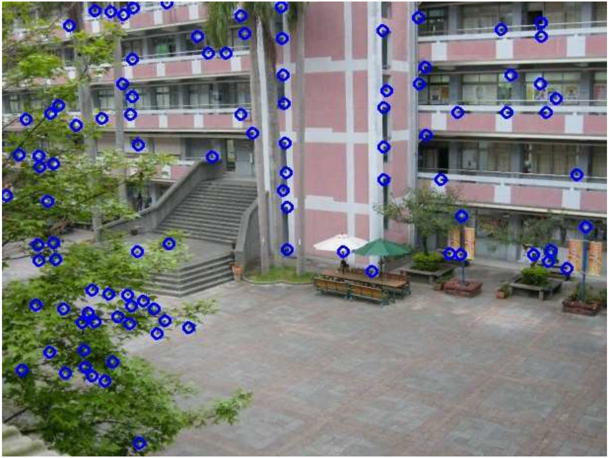
(1) If both λ_i are small, then feature does not vary much in any direction. $\Rightarrow$ uniform region (bad feature)

(2) If the larger eigenvalue $\lambda_1 \gg \lambda_2$, then the feature varies mainly in the direction of $\mathbf{v}_1$. $\Rightarrow$ edge (bad feature)

(3) If both eigenvalues are large, then the feature varies significantly in both directions. $\Rightarrow$ corner or corner-like (good feature)

(4) In practice, I has a maximum value (e.g., 255).
So, λ_1, λ_2 also have an upper bound.
So, only have to check that $\min(\lambda_1, \lambda_2)$ is large enough.

Sample results (local maximum):



With non-maximum suppression, detected corners are more spread out.

Comparison

- Tomasi's good feature uses smallest eigenvalue $\min(\lambda_1, \lambda_2)$.
- Harris corner uses $\det \mathbf{A} - \alpha(\operatorname{tr} \mathbf{A})^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$.
- Brown et al. [BSW05] use the harmonic mean

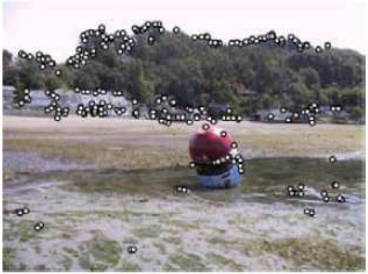
$$\frac{\det \mathbf{A}}{\operatorname{tr} \mathbf{A}} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}. \tag{15}$$

Subpixel Corner Location

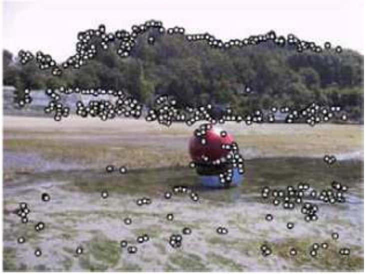
- Locations are detected keypoints are typically at integer coordinates.
- To get more accurate real-number coordinates, need to run subpixel algorithm.
- General idea: starting with an approximate location of a corner, find the accurate location that lies at the intersections of edges.

Adaptive Non-maximal Suppression

- Non-maximal suppression: look for local maximal as keypoints.
- Can lead to uneven distribution of detected keypoints.
- Brown et al. [BSW05] used adaptive non-maximal suppression:
    - local maximal
    - response value is significantly larger than those of its neighbors



(a) Strongest 250



(b) Strongest 500

(c) ANMS 250, $r = 24$

(d) ANMS 500, $r = 16$

Scale Invariance

In many applications, the scale of the object of interest may vary in different images.

Simple but inefficient solution:

- Extract features at many different scales.
- Match them to the object's known features at a particular scale.

More efficient solution:

- Extract features that are invariant to scale.

SIFT

Scale Invariant Feature Transform (SIFT) [Low04].

Convolve input image I with Gaussian G of various scale σ:

$$L(x, y, \sigma) = G(x, y, \sigma) * I(x, y) \tag{16}$$

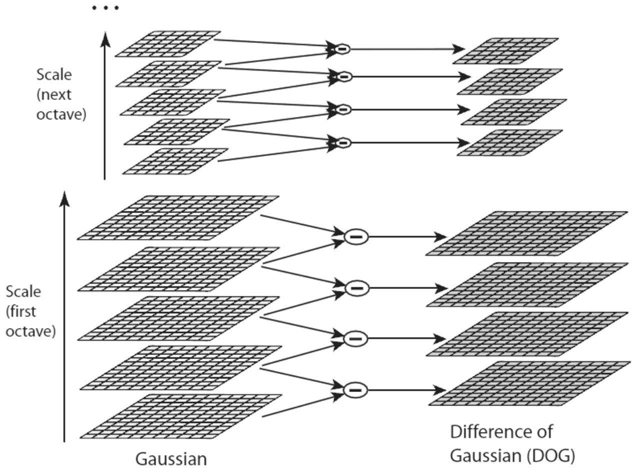
where

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right). \tag{17}$$

This produces L at different scales.

To detect stable keypoint, convolve image I with difference of Gaussian:

$$\begin{aligned} D(x, y, \sigma) &= (G(x, y, k\sigma) - G(x, y, \sigma)) * I(x, y) \\ &= L(x, y, k\sigma) - L(x, y, \sigma). \end{aligned} \tag{18}$$



- Have 3 different scales within each octave (doubling of σ).
- Successive DOG images are subtracted to produce D.
- D images in a lower octave are downsampled by factor of 2.

Detect local maximum and minimum of $D(x, y, \sigma)$:

- Compare a sample point with its 8 neighbors in the same scale and 9 neighbors in the scale above and below.
- Select it if it is larger or smaller than all neighbors.
- Obtain position x, y and scale σ of keypoint.

Orientation of keypoint can be computed as

$$\theta(x, y) = \tan^{-1} \frac{L(x, y+1) - L(x, y-1)}{L(x+1, y) - L(x-1, y)}. \tag{19}$$

Gradient magnitude of keypoint can be computed as

$$m(x, y) = \sqrt{(L(x+1, y) - L(x-1, y))^2 + (L(x, y+1) - L(x, y-1))^2}. \tag{20}$$

Keypoints detected may include edge points.
Edge points are not good because different edge points along an edge may look the same.

To reject edge points, form the Hessian **H** for each keypoint

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix} \tag{21}$$

and reject those for which

$$\frac{\operatorname{tr} \mathbf{H}^2}{\det \mathbf{H}} > 10. \tag{22}$$

Better Invariance

Rotation invariance

- Estimate dominant orientation of keypoint.
- Normalize orientation.

Affine invariance

- Fit ellipse to auto-correlation function or Hessian.
- Apply PCA to determine principal axes.
- Normalize according to principal axes.

For more details, refer to [Sze10] Section 4.1.1.

Feature Descriptors

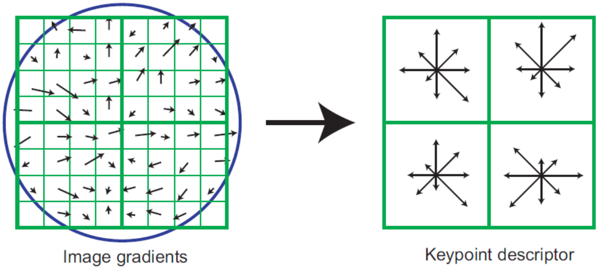
Why need feature descriptors?

- Keypoints give only the positions of strong features.
- To match them across different images, have to characterize them by extracting feature descriptors.

What kind of feature descriptors?

- Able to match corresponding points across images accurately.
- Invariant to scale, orientation, or even affine transformation.
- Invariant to lighting difference.

SIFT Descriptors



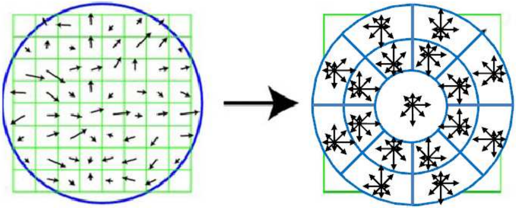
- Compute gradient magnitude and orientation in 16×16 region around keypoint location at the keypoint's scale.
- Coordinates and gradient orientations are measured relative to keypoint orientation to achieve orientation invariance.
- Weighted by Gaussian window.
- Collect into 4×4 orientation histograms with 8 orientation bins.
- Bin value = sum of gradient magnitudes near that orientation.

- Get $4 \times 4 \times 8 = 128$ element feature vector.
- Normalize feature vector to unit length to reduce effect of linear illumination change.
- To reduce effect of nonlinear illumination change, threshold feature values to 0.2 and renormalize feature vector to unit length.

Other Feature Descriptors

Variants of SIFT:

- PCA-SIFT [KS04]
  Use PCA to reduce dimensionality.
- SURF (Speeded Up Robust Features) [BTVG06]
  Use box filter to approximate derivatives.
- GLOH (Gradient Location-Orientation Histogram) [MS05]
  Use log-polar binning structure.

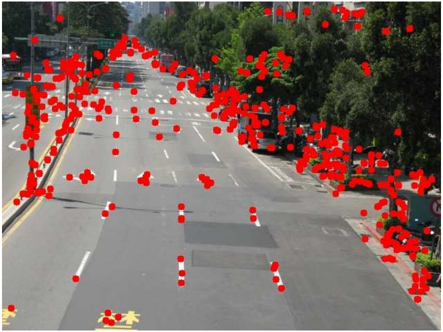


(a) image gradients (b) keypoint descriptor

[MS05] found that GLOH performs the best, followed closely by SIFT.

Sample detected SURF keypoints (without non-maximal suppression):

(a)

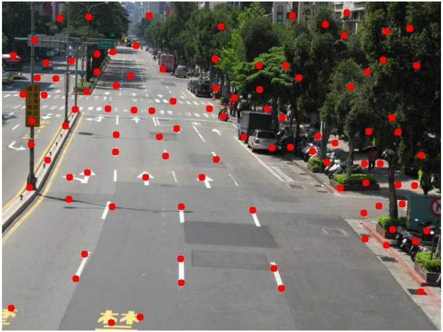


(b)

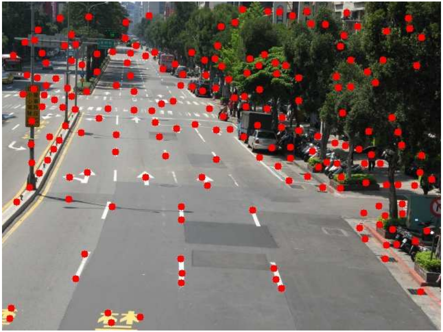
(a) Low threshold gives many cluttered keypoints.

(b) Higher threshold gives fewer keypoints, but still cluttered.

Sample detected SURF keypoints.
With adaptive non-maximal suppression, keypoints are well spread out.



(a)



(b)

(a) Top 100 keypoints.
(b) Top 200 keypoints

Feature Matching

Measure difference as Euclidean distance between feature vectors:

$$d(\mathbf{u}, \mathbf{v}) = \left(\sum_i (u_i - v_i)^2 \right)^{1/2} \tag{23}$$

Several possible matching strategies:

- Return all feature vectors with d smaller than a threshold.
- Nearest neighbor: feature vector with smallest d.
- Nearest neighbor distance ratio:

$$\text{NNDR} = \frac{d_1}{d_2} \tag{24}$$

  - d_1, d_2: distances to the nearest and 2nd nearest neighbors.
  - If NNDR is small, nearest neighbor is a good match.

Sample matching results: SURF, nearest neighbors with min. distance.

- Some matches are correct, some are not.
- Can include other info such as color to improve match accuracy.
- In general, no perfect matching results.

- Feature matching methods can give false matches.
- Manually select good matches.
- Or use robust method to remove false matches:
  - True matches are consistent and have small errors.
  - False matches are inconsistent and have large errors.
- Nearest neighbor search is computationally expensive.
  - Need efficient algorithm, e.g., using k-D Tree.
  - k-D Tree is not more efficient than exhaustive search for large dimensionality, e.g., > 20.

Summary

- Harris corner detector and Tomasi's algorithm find corner points.
- SIFT keypoint: invariant to scale.
- SIFT descriptors: invariant to scale, orientation, illumination change.
- Variants of SIFT: PCA-SIFT, SURF, GLOH.

Software Available

- SIFT code is available in Lowe's web site: www.cs.ubc.ca/∼lowe/keypoints
- Available in OpenCV 2.1:
  - Corner detectors: Harris corner, Tomasi's good feature.
  - Subpixel corner location.
  - Feature descriptors: SURF, StarDetector.
- New in OpenCV 2.2:
  - Feature descriptors: FAST, BRIEF.
  - High-level tools for image matching.
- SciPy supports k-D Tree for nearest neighbor search.
- Fast nearest neighbor search library: mloss.org/software/view/143/
- Approximate nearest neighbor search library: www.cs.umd.edu/∼mount/ANN/

Exercise

(1) Derive the auto-correlation matrix **A** given in Eq. 9.

Further Reading

- Subpixel location of corner: [BK08] p. 319–321.
- Orientation and affine invariance: [Sze10] Section 4.1.1.
- SIFT: [Low04]
- SURF: [BTVG06]
- GLOH: [MS05]
- Adaptive non-maximal suppression: [Sze10] Section 4.1.1, [BSW05].

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