APSU Math Problem of the Week

Problem #2: Ants on a Log

Ninety-nine ants are dropped randomly along a log, with each ant facing one end or the other. The log is 1 meter long from end to end. Each ant travels either toward the left or the right end with a constant speed of 1 meter per minute. When two ants meet, they bounce off each other and reverse directions, keeping their speed intact. When an ant reaches an end of the log, it falls off. At some point, all of the ninety-nine ants will fall off.

Over all the possible initial configurations, what is the longest amount of time you would need to wait to guarantee that the log will have no ants? Provide justification for your answer.

Solution:

While 99 ants bouncing off of each other make this seem too chaotic to solve, one key observation simplifies the problem, which can be seen more easily if you consider two ants rather than 99. When two ants bounce off each other and go in opposite directions, since their speeds are not changing, this is equivalent to the ants passing by one another in the sense that the positions of the ants in each case are identical. So you can ignore all the collisions and treat the 99 ants as acting with independent motions. Viewing it this way, the longest you would have to wait is potentially for one ant to move along the entire log, which would take 1 minute.

Feel free to take this printout, or find each Problem of the Week by scanning this:

Complete the problem each week for a chance to win a prize