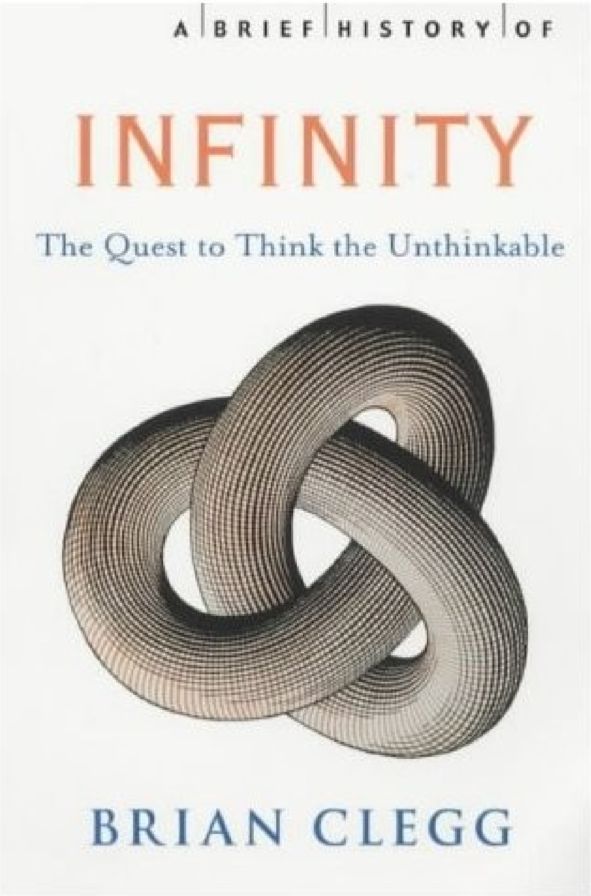
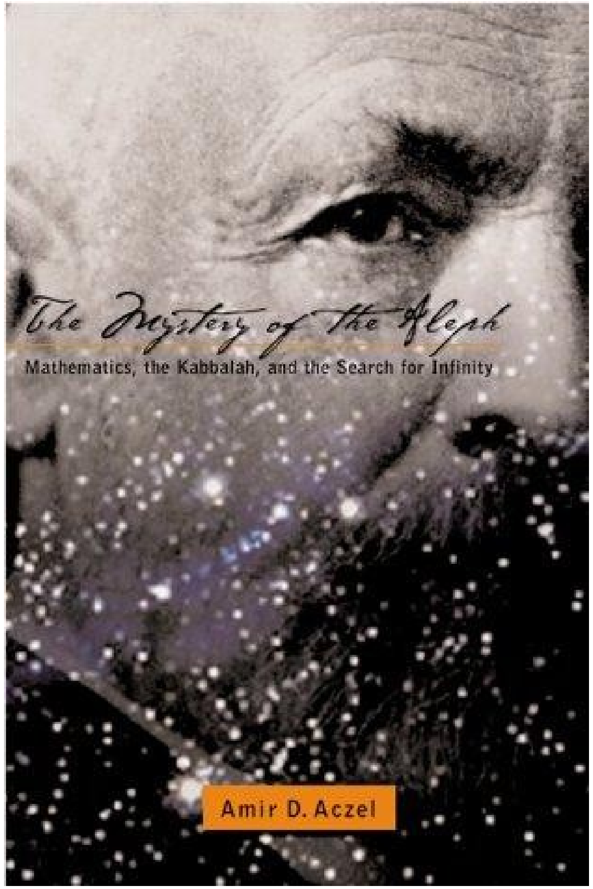
Direct Proofs

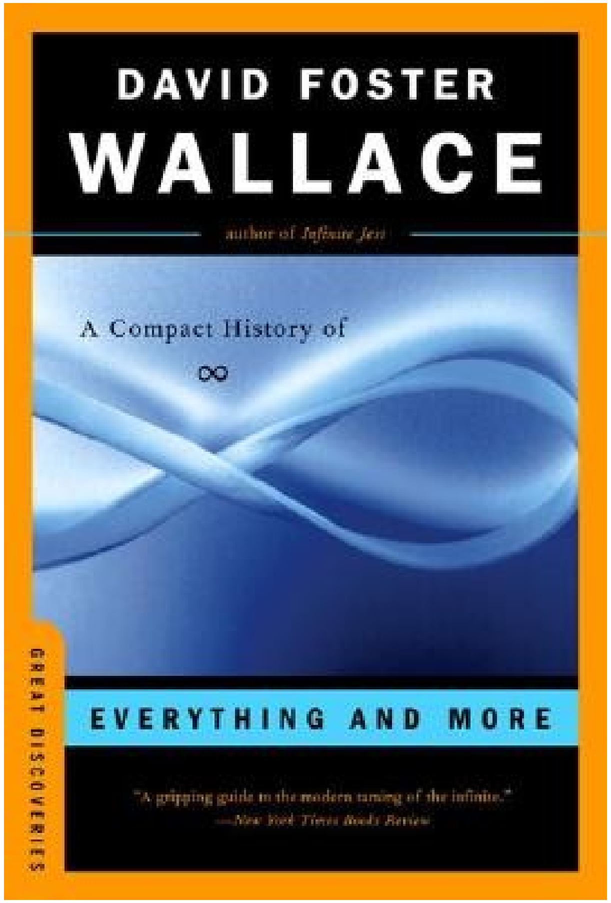
Recommended Reading



A Brief History of Infinity



The Mystery of the Aleph



Everything and More

Recommended Courses

Math 161: Set Theory

What is a Proof?

Induction and Deduction

- In the sciences, much reasoning is done **inductively**.
  - Conduct a series of experiments and find a rule that explains all the results.
  - Conclude that there is a general principle explaining the results.
  - Even if all data are correct, the conclusion might be incorrect.
- In mathematics, reasoning is done **deductively**.
  - Begin with a series of statements assumed to be true.
  - Apply logical reasoning to show that some conclusion necessarily follows.
  - If all the starting assumptions are correct, the conclusion necessarily must be correct.

Structure of a Mathematical Proof

- Begin with a set of initial assumptions.
  - Some will be explicitly stated, others assumed as background knowledge.
- Apply logical reasoning to derive the final result from those initial assumptions.
- Assuming all intermediary steps follow sound logical reasoning, the final result necessarily follows from the assumptions.
- It is a secondary question whether the initial assumptions are correct; that's the domain of the *philosophy of mathematics*.

Direct Proofs

Direct Proofs

- A **direct proof** is the simplest type of proof.
- Starting with an initial set of assumptions, apply simple logical steps to derive the result.
  - *Directly* prove that the result is true.
- Contrasts with **indirect proofs**, which we'll see on Friday.

Two Quick Definitions

- An integer n is **even** if there is some integer k such that $n = 2k$.
  - This means that 0 is even.
- An integer n is **odd** if there is some integer k such that $n = 2k + 1$.
- We'll assume the following for now:
  - Every integer is either even or odd.
  - No integer is both even and odd.

A Simple Direct Proof

Theorem: If n is an even integer, then n^2 is even.

Proof: Let n be an even integer.

Since n is even, there is some integer k such that $n = 2k$.

This means that $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$.

Since $2k^2$ is an integer, this means that there is some integer m (namely, $2k^2$) such that $n^2 = 2m$.

Thus n^2 is even. ■

This symbol means “end of proof”

A Simple Direct Proof

Theorem: If n is an even integer, then n^2 is even.

Proof: Let n be an even integer.

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such tha

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).

Since $2k$
there is
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that n^2 =

Thus n^2 i

To prove a statement of the form

"If *P*, then *Q*"

Assume that ***P*** is true, then show that ***Q*** must be true as well.

A Simple Direct Proof

Theorem: If n is an even integer, then n^2 is even.

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there is some uch

that $n^2 = 2m$.

Thus n^2 is eve

This is the definition of an even integer. When writing a mathematical proof, it's common to call back to the definitions.

A Simple Direct Proof

Theorem: If n is an even integer, then n^2 is even.

Proof: Let n be an even integer.

Since n is even, there is some integer k such that $n = 2k$.

This means that $n^2 = (2k)^2 = 4k^2 = 2(2k^2)$.

Since $2k^2$
there is so n
that $n^2 =$

Thus n^2 is

Notice how we use the value of ***k*** that we obtained above. Giving names to quantities, even if we aren't fully sure what they are, allows us to manipulate them. This is similar to variables in programs.

A Simple Direct Proof

Theorem: If n i n.

Proof: Let n b

Since n such th

This me ^{2}).

Our ultimate goal is to prove that n^2 is even. This means that we need to find some m such that $n^2 = 2m$. Here, we're explicitly showing how we can do that.

Since $2k^2$ is an integer, this means that there is some integer m (namely, $2k^2$) such that $n^2 = 2m$.

Thus n^2 is even. ■

A Simple Direct Proof

Theorem: If n is an even integer, then n^2 is even.

Proof: Let n be an even integer.

Since n is even, there is some integer k such that $n = 2k$.

This mea).

Since $2k$
there is s ch
that $n^2 = 2m$.

Hey, that's what we were trying to show! We're done now.

Thus n^2 is even. ■

An Important Result

- Set equality is defined as follows

  $A = B$ precisely when every element of A belongs to B and vice-versa

- This definition makes it a bit tricky to prove that two sets are equal.
- It's often easier to use the following result to show that two sets are equal:

  For any sets A and B,
  if $A \subseteq B$ and $B \subseteq A$, then $A = B$.

Theorem: For any sets A and B, if $A \subseteq B$ and $B \subseteq A$, then $A = B$.

How do we prove that this is true for *any* choice of sets?

Proving Something Always Holds

- Many statements have the form

For any *X*, P(*X*) is true.

- Examples:

For all integers n, if n is even, n^2 is even.

For any sets A and B, if $A \subseteq B$ and $B \subseteq A$, then $A = B$.

For all sets S, $|S| < |\wp(\mathrm{S})|$.

Everybody's looking forward to the weekend, weekend.

- How do we prove these statements when there are (potentially) infinitely many cases to check?

Arbitrary Choices

- To prove that P(*x*) is true for all possible *x*, show that no matter what choice of *x* you make, P(*x*) must be true.
- Start the proof by making an **arbitrary choice** of *x*:
  - “Let *x* be chosen arbitrarily.”
  - “Let *x* be an arbitrary even integer.”
  - “Let *x* be an arbitrary set containing 137.”
  - “Consider any *x*.”
- Demonstrate that P(*x*) holds true for this choice of *x*.

Theorem: For any sets A and B, if $A \subseteq B$ and $B \subseteq A$, then $A = B$.

Proof: Let A and B be arbitrary sets such that $A \subseteq B$ and $B \subseteq A$.

We're showing here that regardless of what **A** and **B** you pick, the result will still be true.

Theorem: For any sets A and B, if $A \subseteq B$ and $B \subseteq A$, then $A = B$.

Proof: Let A and B be arbitrary sets such that $A \subseteq B$ and $B \subseteq A$..

To prove a statement of the form

“If *P*, then *Q*”

Assume that ***P*** is true, then show that ***Q*** must be true as well.

Theorem: For any sets A and B, if $A \subseteq B$ and $B \subseteq A$, then $A = B$.

Proof: Let A and B be arbitrary sets such that $A \subseteq B$ and $B \subseteq A$.

By definition, $A \subseteq B$ means that for all $x \in A$, we have $x \in B$.

By definition, $B \subseteq A$ means that for all $x \in B$, we have $x \in A$.

Thus whenever $x \in A$ we have $x \in B$ and whenever $x \in B$ we have $x \in A$.

Consequently, $A = B$. ■

An Incorrect Proof

Theorem: For any natural number n, the sum of all the positive divisors of n is always no greater than $2n$.

Proof: Consider an arbitrary natural number, say, 16. 16 has positive divisors 1, 2, 4, 8, and 16. Note that $1 + 2 + 4 + 8 + 16 = 31 \leq 2 \cdot 16$. Since our choice of n was arbitrary, we see that for an arbitrary natural number n, the sum of all the divisors of n is no greater than $2n$. ■

ar·bi·trar·y

adjective /ˈärbiˌtrerē/

Not this one!

1. Based on random choice or personal whim, rather than any reason or system - *“his mealtimes were entirely arbitrary”*

2. *(of power or a ruling body)* Unrestrained and autocratic in the use of authority - *“arbitrary rule by King and bishops has been made impossible”*

3. *(of a constant or other quantity)* Of unspecified value

Use this definition

Source: Google

To prove something is true for all x,
don't choose an x and base the proof
off of your choice.

Instead, leave x unspecified
and show that no matter what x is,
the specified property must hold.

Another Incorrect Proof

Theorem: For any sets A and B, $A \subseteq A \cap B$.

Proof: We need to show that if $x \in A$, then $x \in A \cap B$ as well.

Consider any arbitrary $x \in A \cap B$. This means that $x \in A$ and $x \in B$, so $x \in A$ as required. ■

If you want to prove that P implies Q,
assume P and prove Q.

Don't assume Q and then prove P!

An Entirely Different Proof

Theorem: **There exists** a natural number $n > 0$ **such that** the sum of all natural numbers less than n is equal to n.

This is a fundamentally different type of proof that what we've done before. Instead of showing that every object has some property, we want to show that some object has a given property.

Universal vs. Existential Statements

- A **universal statement** is a statement of the form

For all *x*, P(*x*) is true.

- We've seen how to prove these statements.
- An **existential statement** is a statement of the form

There exists an *x* for which P(*x*) is true.

- How do you prove an existential statement?

Proving an Existential Statement

- We will see several different ways to prove “there is some *x* for which P(*x*) is true.”
- Simple approach: Just go and find some *x* for which P(*x*) is true!
  - In our case, we need to find a positive natural number *n* such that that sum of all smaller natural numbers is equal to *n*.
  - Can we find one?

An Entirely Different Proof

Theorem: There exists a natural number $n > 0$ such that the sum of all natural numbers less than n is equal to n.

Proof: Take $n = 3$.

There are three natural numbers smaller than 3: 0, 1, and 2.

We have $0 + 1 + 2 = 3$.

Thus 3 is a natural number greater than zero equal to the sum of all smaller natural numbers. ■

Extended Example: **XOR**

Logical Operators

- A **bit** is a value that is either 0 or 1.
- The set $\mathbb{B} = \{0, 1\}$ is the set of all bits.
- A **logical operator** is an operator that takes in some number of bits and produces a new bit as output.
- Example: Logical NOT, denoted $\neg x$:

$$\neg 0 = 1 \qquad\qquad \neg 1 = 0$$

Logical XOR

- The **exclusive OR** operator (**XOR**) operates on two bits and produces 0 if the bits are the same and 1 if they are different.
  - Since XOR operates on two values, it is called a **binary operator**.
- We denote the XOR of a and b by $\boldsymbol{a \oplus b}$.
- Formally, XOR is defined as follows:

$$0 \oplus 0 = 0 \qquad 0 \oplus 1 = 1$$

$$1 \oplus 0 = 1 \qquad 1 \oplus 1 = 0$$

Fun with XOR

- The XOR operator has numerous uses throughout computer science.
  - Applications in cryptography, data structures, error-correcting codes, networking, machine learning, etc.
- XOR is useful because of four key properties:
  - XOR has an **identity element**.
  - XOR is **self-inverting**.
  - XOR is **associative**.
  - XOR is **commutative**.

Identity Elements

An **identity element** for a binary operator ★ is some value z such that for any a:

$$a \star z = z \star a = a$$

In math-speak, the term
“***for any a***“ is synonymous
with “for every a“ or
“***for every possibly choice of a.***“
It does not mean
“***for some specific choice of a.***“

Identity Elements

- An **identity element** for a binary operator $\star$ is some value z such that for any a:

$$a \star z = z \star a = a$$

- Example: 0 is an identity element for +:

$$a + 0 = 0 + a = a$$

- Example: 1 is an identity element for ×:

$$a \times 1 = 1 \times a = a$$

Theorem: 0 is an identity element for ⊕.

Proof: We will prove that for any $b \in \mathbb{B}$ that $b \oplus 0 = b$ and that $0 \oplus b = b$. To do this, consider an arbitrary $b \in \mathbb{B}$. We consider two cases:

Case 1: $b = 0$.

Case 2: $b = 1$.

This is called a **proof by cases** (alternatively, a **proof by exhaustion**) and works by showing that the theorem is true regardless of what specific outcome arises.

Theorem: 0 is an identity element for ⊕.

Proof: We will prove that for any $b \in \mathbb{B}$ that $b \oplus 0 = b$ and that $0 \oplus b = b$. To do this, consider an arbitrary $b \in \mathbb{B}$. We consider two cases:

Case 1: $b = 0$. Then we have

$b \oplus 0 = 0 \oplus 0$ $\qquad$ $0 \oplus b = 0 \oplus 0$

In a proof by cases, after demonstrating each case, you should summarize the cases afterwards to make your point clearer.

Case 2

$b \oplus$ $\qquad$ 1

$= b$ $\qquad$ $= b$

In both cases, we find $b \oplus 0 = 0 \oplus b = b$.

Theorem: 0 is an identity element for ⊕.

Proof: We will prove that for any $b \in \mathbb{B}$ that $b \oplus 0 = b$ and that $0 \oplus b = b$. To do this, consider an arbitrary $b \in \mathbb{B}$. We consider two cases:

Case 1: $b = 0$. Then we have

$$\begin{aligned} b \oplus 0 &= 0 \oplus 0 \\ &= 0 \\ &= b \end{aligned} \qquad \begin{aligned} 0 \oplus b &= 0 \oplus 0 \\ &= 0 \\ &= b \end{aligned}$$

Case 2: $b = 1$. Then we have

$$\begin{aligned} b \oplus 0 &= 1 \oplus 0 \\ &= 1 \\ &= b \end{aligned} \qquad \begin{aligned} 0 \oplus b &= 0 \oplus 1 \\ &= 1 \\ &= b \end{aligned}$$

In both cases, we find $b \oplus 0 = 0 \oplus b = b$. Thus 0 is an identity element for ⊕. ■

Self-Inverting Operators

- A binary operator $\star$ with identity element z is called **self-inverting** when for any a, we have

$$a \star a = z$$

- Is + self-inverting?
- Is – self-inverting?

XOR is Self-Inverting

Theorem: $\oplus$ is self-inverting.

Proof: Since $\oplus$ has identity element 0, we will prove for any $b \in \mathbb{B}$ that $b \oplus b = 0$. To do this, consider any $b \in \mathbb{B}$. We consider two cases:

Case 1: $b = 0$. Then $b \oplus b = 0 \oplus 0 = 0$.

Case 2: $b = 1$. Then $b \oplus b = 1 \oplus 1 = 0$.

In both cases we have $b \oplus b = 0$, so $\oplus$ is self-inverting. ■

Associative Operators

- A binary operator ★ is called **associative** when for any *a*, *b* and *c*, we have

$$a \star (b \star c) = (a \star b) \star c$$

- Is + associative?
- Is – associative?
- Is × associative?

Theorem: $\oplus$ is associative.

Proof: Consider any $a, b, c \in \mathbb{B}$. We will prove that $a \oplus (b \oplus c) = (a \oplus b) \oplus c$. To do this, we consider two cases:

Case 1: $c = 0$. Then we have that

$$\begin{aligned} a \oplus (b \oplus c) &= a \oplus (b \oplus 0) \\ &= a \oplus b && \textit{(since 0 is an identity)} \\ &= (a \oplus b) \oplus 0 && \textit{(since 0 is an identity)} \\ &= (a \oplus b) \oplus c \end{aligned}$$

Case 2: $c = 1$. Then we have that

$$\begin{aligned} a \oplus (b \oplus c) &= a \oplus (b \oplus 1) \\ &= \textbf{?} \end{aligned}$$

When You Get Stuck

- When writing proofs, you are bound to get stuck at some point.
- When this happens, it can mean multiple things:
  - What you're proving is incorrect.
  - You are on the wrong track.
  - You're on the right tack, but you need to prove an additional result to get to your goal.
- Unfortunately, there is no general way to determine which case you are in.
- You'll build this intuition through experience.

Where We're Stuck

- Right now, we have the expression

  $$a \oplus (b \oplus 1)$$

  and we don't know how to simplify it.
- Let's focus on the $(b \oplus 1)$ part and see what we find:
  - $\mathbf{0} \oplus 1 = \mathbf{1}$
  - $\mathbf{1} \oplus 1 = \mathbf{0}$
- It seems like $b \oplus 1 = \neg b$. Could we prove it?

Relations Between Proofs

- Proofs often build off of one another: large results are almost often accomplished by building off of previous work.
  - Like writing a large program – split the work into smaller methods, across different classes, etc. instead of putting the whole thing into **`main`**.
- A result that is proven specifically as a stepping stone toward a larger result is called a **lemma**.
- Our result that $b \oplus 1 = \neg b$ serves as a lemma in our larger proof that $\oplus$ is associative.

Lemma: For any $b \in \mathbb{B}$, we have $b \oplus 1 = \neg b$.
Proof: Consider any $b \in \mathbb{B}$. We consider two cases:

Case 1: $b = 0$. Then

$$\begin{aligned} b \oplus 1 &= 0 \oplus 1 \\ &= 1 \\ &= \neg 0 \\ &= \neg b. \end{aligned}$$

Case 2: $b = 1$. Then

$$\begin{aligned} b \oplus 1 &= 1 \oplus 1 \\ &= 0 \\ &= \neg 1 \\ &= \neg b. \end{aligned}$$

In both cases, we find that $b \oplus 1 = \neg b$, which is what we needed to show. ■

Theorem: $\oplus$ is associative.

Proof: Consider any $a, b, c \in \mathbb{B}$. We will prove that $a \oplus (b \oplus c) = (a \oplus b) \oplus c$. To do this, we consider two cases:

Case 1: $c = 0$. Then we have that

$$\begin{aligned} a \oplus (b \oplus c) &= a \oplus (b \oplus 0) \\ &= a \oplus b && \textit{(since 0 is an identity)} \\ &= (a \oplus b) \oplus 0 && \textit{(since 0 is an identity)} \\ &= (a \oplus b) \oplus c \end{aligned}$$

Case 2: $c = 1$. Then we have that

$$\begin{aligned} a \oplus (b \oplus c) &= a \oplus (b \oplus 1) \\ &= a \oplus \neg b && \textit{(using our lemma)} \\ &= \textbf{??} \end{aligned}$$

Lemma 2: For any a, $b \in \mathbb{B}$, we have $a \oplus \neg b = \neg(a \oplus b)$.
Proof: Consider any a, $b \in \mathbb{B}$. We consider two cases:

Case 1: $b = 0$. Then

$$\begin{aligned} a \oplus \neg b &= a \oplus \neg 0 \\ &= a \oplus 1 \\ &= \neg a && \textit{(using our first lemma)} \\ &= \neg(a \oplus 0) && \textit{(since 0 is an identity)} \\ &= \neg(a \oplus b) \end{aligned}$$

Case 2: $b = 1$. Then

$$\begin{aligned} a \oplus \neg b &= a \oplus \neg 1 \\ &= a \oplus 0 \\ &= a && \textit{(since 0 is an identity)} \\ &= \neg(\neg a) \\ &= \neg(a \oplus 1) && \textit{(using our first lemma)} \\ &= \neg(a \oplus b) \end{aligned}$$

In both cases, we find that $a \oplus \neg b = \neg(a \oplus b)$, as required. ■

Theorem: ⊕ is associative.

Proof: Consider any $a, b, c \in \mathbb{B}$. We will prove that $a \oplus (b \oplus c) = (a \oplus b) \oplus c$. We consider two cases:

Case 1: $c = 0$. Then we have that

$$\begin{aligned} a \oplus (b \oplus c) &= a \oplus (b \oplus 0) \\ &= a \oplus b && \textit{(since 0 is an identity)} \\ &= (a \oplus b) \oplus 0 && \textit{(since 0 is an identity)} \\ &= (a \oplus b) \oplus c \end{aligned}$$

Case 2: $c = 1$. Then we have that

$$\begin{aligned} a \oplus (b \oplus c) &= a \oplus (b \oplus 1) \\ &= a \oplus \neg b && \textit{(using lemma 1)} \\ &= \neg(a \oplus b) && \textit{(using lemma 2)} \\ &= (a \oplus b) \oplus 1 && \textit{(using lemma 1)} \\ &= (a \oplus b) \oplus c \end{aligned}$$

In both cases we have $a \oplus (b \oplus c) = (a \oplus b) \oplus c$, and therefore ⊕ is associative. ■

Commutative Operators

- A binary operator ★ is called **commutative** when the following is always true:

$$a \star b = b \star a$$

- Is + commutative?
- Is – commutative?

Theorem: $\oplus$ is commutative.

Proof: Consider any $a, b \in \mathbb{B}$. We will prove $a \oplus b = b \oplus a$. To do this, let $x = a \oplus b$. Then

$$\begin{aligned}
&x = a \oplus b \\
&x \oplus b = (a \oplus b) \oplus b \\
&x \oplus b = a \oplus (b \oplus b) && \textit{(since } \oplus \textit{ is associative)} \\
&x \oplus b = a \oplus 0 && \textit{(since } \oplus \textit{ is self-inverting)} \\
&x \oplus b = a && \textit{(since 0 is an identity of } \oplus\textit{)} \\
&x \oplus (x \oplus b) = x \oplus a \\
&(x \oplus x) \oplus b = x \oplus a && \textit{(since } \oplus \textit{ is associative)} \\
&0 \oplus b = x \oplus a && \textit{(since } \oplus \textit{ is self-inverting)} \\
&b = x \oplus a && \textit{(since 0 is an identity of } \oplus\textit{)} \\
&b \oplus a = (x \oplus a) \oplus a \\
&b \oplus a = x \oplus (a \oplus a) && \textit{(since } \oplus \textit{ is associative)} \\
&b \oplus a = x \oplus 0 && \textit{(since } \oplus \textit{ is self-inverting)} \\
&b \oplus a = x && \textit{(since 0 is an identity of } \oplus\textit{)}
\end{aligned}$$

This means that $a \oplus b = x = b \oplus a$. Therefore, $\oplus$ is commutative. ■

Theorem: ⊕ is commutative.

Proof: Consider any $a, b \in \mathbb{B}$. We will prove $a \oplus b = b \oplus a$. To do this, let $x = a \oplus b$. Then

$$
\begin{aligned}
&x = a \oplus b \\
&x \oplus b = (a \oplus b) \oplus b \\
&x \oplus b = a \oplus (b \oplus b) \\
&x \oplus b = a \oplus 0 \\
&x \oplus b = a \\
&x \oplus (x \oplus b) = x \oplus a \\
&(x \oplus x) \oplus b = x \oplus a \\
&0 \oplus b = x \oplus a \\
&b = x \oplus a \\
&b \oplus a = (x \oplus a) \oplus a \\
&b \oplus a = x \oplus (a \oplus a) \\
&b \oplus a = x \oplus 0 \\
&b \oplus a = x
\end{aligned}
$$

This means that $a \oplus b = x = b \oplus a$. Therefore, ⊕ is commutative. ■

The only properties of ⊕ that we used here are that it is associative, has an identity, and is self-inverting. This same proof works for any operator with these three properties!

Binary operators that have this property give rise to **boolean groups** (but you don't need to know that for this class).

Application: **Encryption**

Bitstrings

- A **bitstring** is a finite sequence of 0s and 1s.
- Internally, computers represent all data as bitstrings.
  - For details on how, take CS107 or CS143.

Bitstrings and $\oplus$

- We can generalize the $\oplus$ operator from working on individual bits to working on bitstrings.
- If A and B are bitstrings of length n, then we'll define $A \oplus B$ to be the bitstring of length n formed by applying $\oplus$ to the corresponding bits of A and B.
- For example:

$$\begin{array}{r} \mathtt{110110} \\ \oplus\ \mathtt{011010} \\ \hline \mathtt{101100} \end{array}$$

Encryption

- Suppose that you want to send me a secret bitstring M of length n.
- You should be able to read the message, but anyone who intercepts the secret message should not be able to read it.
- How might we accomplish this?

$\oplus$ and Encryption

- In advance, you and I share a randomly-chosen bitstring K of length n (called the **key**) and keep it secret.
- To send me message M secretly, you send me the string $C = M \oplus K$.
  - C is called the **ciphertext**.
- To decrypt the ciphertext C, I compute the string $C \oplus K$. This is

$$\begin{aligned} C \oplus K &= (M \oplus K) \oplus K \\ &= M \oplus (K \oplus K) \\ &= M \end{aligned}$$

$\oplus$ and Encryption

- Suppose that you don't have the key and get the message $M \oplus K$.
- If K is chosen to be truly random, then every bit in $M \oplus K$ appears to be truly random.
- Intuition: Let b be a original bit from the message and k be the corresponding bit in the key.
  - If $k = 0$, then $b \oplus k = b \oplus 0 = b$.
  - If $k = 1$, then $b \oplus k = b \oplus 1 = \neg b$.
- Since the key bit is truly random, the bits in the original string are flipped totally randomly.
- Can formalize the math; take CS109 for details!

An Example

PUPPIES

M	01010000010101010101000001010000010010010100010101010011
K	11011100101110111100010011010101111001101111011111000010
C	10001100111011101001010010000101101011111011001010010001

Œî”…©² ‘

An Example

Œî”…©² ‘

C	**10001100111011101001010010000101101011111011001010010001**
K	**11011100101110111100010011010101111001101111011111000010**
M	**01010000010101010101000001010000010010010100010101010011**

PUPPIES

An Example

Œî”…©² ‘

C	**10001100111011101001010010000101101011111011001010010001**
K?	**01011100010101010101000001010000010010010100010101010011**
M?	**01001100010011110100110001000110010000010100100101001100**

LOLFAIL

Some Caveats

- This scheme is **very insecure** if you encrypt multiple messages using the same key.
  - Good exercise: Figure out why this is!
- This scheme guarantees security if the key is random, but it isn't tamperproof.
  - You'll see why this is on the problem set.
- General good advice: ***never implement your own cryptography!***
- Take CS255 for more details!

Next Time

- **Indirect Proofs**
  - Proof by contradiction.
  - Proof by contrapositive.