## 12 Twelve XII



Corresponding ordinal: twelfth.

The number 12 is a pentagonal number (see the pattern above).

The number 12 is the seventh even number and the sixth composite number.

As a product of primes:  $12 = 2^2 \cdot 3$ .

The number 12 has 6 divisors: 1, 2, 3, 4, 6, 12. The number 6 is perfect (see **6**). The sum of the divisors of 12 is 28, also a perfect number. Such a number has been called *sublime*. The other known sublime number is

$$2^{126} (2^{61} - 1) (2^{31} - 1) (2^{19} - 1) (2^7 - 1) (2^5 - 1) (2^3 - 1)$$

which is  $6086\,555\,670\,238\,378\,989\,670\,371\,734\,243\,169\,622\,657\,830\,773\,351\,885\,970\,528\,324\,860\,512\,791\,691\,264$ . Note that this is a product of a power of 2 and a bunch of Mersenne primes. So is 12, which is  $2^2\,(2^2-1)$ . That is, if "one" qualifies as a bunch.

The number 12 is the first abundant number. A number is *abundant* if the sum of its proper divisors is greater than the number, and the sum of the proper divisors of 12 is s(12) = 1 + 2 + 3 + 4 + 6 = 16 > 12. Each number n is one of three types, depending on how it compares to the sum of its proper divisors. If s(n) < n, then n is *deficient*. If s(n) = n, then n is *perfect*. If s(n) > n, then n is *abundant*.

As the sum of four or fewer squares:  $12 = 2^2 + 2^2 + 2^2 = 1^2 + 1^2 + 1^2 + 3^2$ .

As the sum of nine or fewer cubes:  $12 = 1^3 + 1^3 + 1^3 + 1^3 + 2^3$ .

As the difference of two squares:  $12 = 4^2 - 2^2$ .

The number 12 appears in four Pythagorean triples: [5, 12, 13], [9, 12, 15], [12, 16, 20], [12, 35, 37]. The first and the last are primitive.

As the sum of two odd primes: 12 = 5 + 7.

The *thirteen spheres problem* was the subject of a dispute between Newton and Gregory. The problem is whether you can arrange thirteen spheres of the same size so that they each touch a central sphere of the same size. That you can't was proved in 1953. Thus you can fit 12 spheres, but no more, around a central sphere.

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There are 12 *pentominoes*. These are the figures you can make from five squares by joining them at edges. Here are three of them:



The twelfth President of the United States was Zachary Taylor.

The twelfth state to enter the Union was North Carolina.

The twelfth largest state in the United States is Minnesota.

Many clock faces show only 12 hours, so the phrase "4 o'clock" is ambiguous—it could be AM or PM. Similarly, "12 o'clock" can mean either noon or midnight.

There are 12 months in a year. From the names of the months, there seem to be only ten because September starts with *septem*, meaning seven, October starts with *octo*, meaning eight, November starts with *novem*, for nine, and December starts with *decem*, for ten. There were only ten months in the year when the months of January and February were introduced about forty years after the founding of Rome. Some seven hundred years later the names of the fifth and sixth months, *Quintilis* and *Sextilis*, were changed to *July* and *August* in honor of Julius and Augustus Caesar.1

Carbon 12 is an isotope of carbon of mass number 12. It is the most abundant carbon isotope and is used as a standard for measurements of atomic weight.

Jesus had 12 disciples.

Some people think that we should base our numeration system on twelve rather than on ten. This is called the *duodecimal* or the *dozenal* system. In the duodecimal system, there are the twelve digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, T, and E. The digits T and E are what we call "Ten" and "Eleven." To distinguish numerals written in different bases, we use a subscript. Thus,  $37_{10}$  means three tens plus seven while  $37_{12}$  means three twelves (or dozens) plus seven, or what we would denote by 43 in our base ten numeration system. For more information, see www.Twelve, or look up the Dozenal Society on the web.