Parameters of Chelyabinsk and Tunguska Meteoroids

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Abstract

This paper describes application of mathematical model that establishes relationships between parameters of celestial bodies motion in the spheres of activity of the Sun and the Earth with mass-energy characteristics of these objects and their explosion modes during destruction in the Earth atmosphere, that in turn are linked with phenomena observed on underlying surface. This model was applied to calculate the characteristics of the objects that caused Chelyabinsk and Tunguska explosions with using its trajectory parameters described in scientific publications. It turned out that the size of Chelyabinsk meteoroid was equal to 180 – 185 meters, and its mass was close to 1.8 megatons. Energy of its explosion was equal to 57 megatons of TNT. Size of Tunguska meteoroid was equal to 105 m, mass – 0.35 megatons, while energy of explosion was about of 14.5 megatons of TNT. Due to the common origin of these two celestial bodies their average densities turned to be equal and were about of 570 kg/m^3.

Keywords: Chelyabinsk meteoroid, Tunguska meteoroid, cometary fragments, trajectory, explosion, energy, shock wave, overpressure peak

I. Introduction

Within one and a half month after fall of Chelyabinsk meteoroid that occurred February 15, 2013, there was created a mathematical model that relates parameters of celestial bodies motion in spheres of activity of the Sun and the Earth with mass-energy characteristics of these objects and their explosion modes during the destruction in the Earth atmosphere, that in turn are linked with phenomena observed on the underlying surface [1]. This model was used then for calculation of characteristics of Chelyabinsk and Tunguska meteoroids that in this paper, as well as in subsequent articles of the author (see below), were linked by their origin and, therefore, the proximity of their orbits. Thus, main data were obtained which fully describe these remarkable phenomena through regular physical and mathematical procedure without any speculative hypotheses and/or assumptions.

However, those calculations were based on preliminary information obtained during first 2 – 3 weeks after the Chelyabinsk incident, and author's own estimations of certain key parameters of this phenomenon, which, for obvious reasons, couldn’t have a high accuracy at that time. Furthermore, one of the internal parameters of the mathematical model – a nominal height of the atmosphere has been updated also. Instead of the standard value of 100 km the height has been used that is more closely associated with the conditions of the problem when the object enters the Earth's atmosphere. Thus, it was refined the influence of the Earth atmosphere on meteoroid trajectory in framework of numerical model at the cost of some complication of computational procedure.

In addition, the characteristic height in calculating the trajectory was the same (h = 8.00 km) as the corresponding parameter in «external» interactive module [2], which describes the destruction of meteoroids in the atmosphere, and was developed a few years earlier by other researchers (see [2, 3]). These, in principle, relatively small changes in the internal parameters of the model have some influence on the calculating results. Moreover, due to significant non-linearity of the model there may be that at a certain set of parameters the solution not exists under the old, rougher model, and exists in the model refined.

Two big review articles [4, 5] about Chelyabinsk incident were published in November 2013 in famous journals – Nature and Science by two large scientific groups (33 and 59 co-authors, respectively). These articles were given sufficiently reliable and relatively detailed data, derived from video images and eyewitness reports, about the parameters of meteoroid's trajectory and the phenomena which accompanied its passage and explosion. After this the opportunity came to recalculate the characteristics of Chelyabinsk and Tunguska meteoroids on these materials, refining the observed pattern of the phenomenon in the sky at Chelyabinsk in comparison with data from express reports in the media and fragmentary personal reports of eyewitness used in paper [1]. Calculation results of the characteristics of Chelyabinsk and Tunguska meteoroids on these data are presented in this paper, preprint of which was posted in Arxive.org [6]. The results were presented also at XL Academic Space Conference, dedicated to the memory of academician S. P. Korolev [7].

II. Computational model

Constructed mathematical model links all the essential characteristics of the described event: from the parameters of the object trajectory through its physical characteristics up to the phenomena observed in the atmosphere and underlying surface. In other words, full model of this phenomenon is analyzed. Just these features of the model radically distinguish it from fragmentary studies of individual components of phenomenon without linking them to each other, as was in all known author of publications on this topic.

Full and detailed description of this model isn't purpose of this work, as absolutely incompatible with the limited scope of the article, which describes, in author's opinion, much more important question – the results of its application to calculation of main parameters of Chelyabinsk and Tunguska meteoroids. Therefore, the mathematical model consisting of an interconnected set of computational units, most of which are quite trivial, or have already been described (see [2, 3]), is presented here in fragments, so that the reader can make a general idea about it. The most not trivial fragments are described in more detail.

A model for calculation of parameters of celestial bodies' motion is rather traditional. For a given orbit of the object and the known orbit of the Earth, which, due to its very small eccentricity is assumed for simplicity a circular, the parameters of the object on elliptical orbit around the Sun are determined at any point from laws of conservation of energy and impulse-momentum. Then, from geometrical considerations are calculated angles and speeds in the Sun coordinate system. Further, when the object approaches the Earth, there is a transition to the calculation of its movement within the sphere of activity of the Earth. In this case, the sphere having a zero dimension on the scale of the solar system is infinite in near-Earth space, and solutions in different coordinate systems are sewn through the geometric relationships and mechanical recalculations of speed and energy. The principles which are based in such asymptotic approach to the description of bodies' motion in the central gravitational fields are described, for example, in book [8].

In considering the movement of the object in the gravitational field of the Earth there is the problem of computing the so-called impact parameter – the length of the perpendicular drawn from the object velocity vector to a straight line parallel to it and passing through the center of the Earth (see [8]). This parameter determining the motion of the object relative to the Earth can be calculated in this model through known geographical coordinates of the point in which ends its flight and geodesic azimuth of the trajectory. To do this, we need to provide twice the pivoting of the initial system of geographical coordinates. The first twist is performed to account for the inclination of the axis of rotation of the Earth relative to the plane of the object trajectory. The second twist is performed so that the plane of the object trajectory was in the equatorial plane of the new coordinate system. Then, the problem of spatial movement of the object near the Earth passes into the problem of its flat hyperbolic motion, in which flight radius of the object in polar coordinate system, with zero point, located in the center of the Earth, can be described through one angular parameter – polar angle φ. Since there are 2 branches of the hyperbola, there are 2 sets of angles that are providing a second pivoting of the coordinate system. However, from the condition, that the point of object explosion is closer to perigee than the point of entry, we may select the only solution that meets the conditions of the problem.

After this remains only problem of action of the Earth atmosphere on the object motion in the final part of its trajectory. This problem is significant merely for small entry angles of the object with relatively long trajectories, one of which was in the Chelyabinsk incident. The solving of this problem is the least trivial part of the algorithm. Because of this, unlike the rest, it is described in this paper in more detail.

There was chosen the simplest embodiment of the computational method of the effect the Earth's atmosphere – parameters of trajectory have been calculated for average speed of flight. What is the «average speed» and how it to calculate – this was a major issue in the making of this module of calculation. Based on the data about the explosion of the object (that may be obtained after the calculation of the trajectory), we can calculate the ratio of its remaining kinetic energy just before the explosion E_f, equal to the explosive energy E_e, to the initial energy E_0. Then neglecting the loss in mass of the object in the atmospheric motion

$$v_f = v_0 \left(\frac{E_f}{E_0} \right)^{\frac{1}{2}}, \qquad (1)$$

where v_f is the speed of the object before the explosion (final speed), v_0 is its speed at the inlet to the atmosphere (initial speed). It should be noted that this a priori assumption has been fully validated through the estimates of trace of Chelyabinsk meteoroid [9, 10] – loss of its mass in the trail was not more than 1 – 1.5 % of the initial mass.

For approximate calculation of the object's «average speed» was also used the asymptotic approach – the search for solutions was realized for flat trajectories at low entrance angles. In this case, the difference between point of the explosion of the meteoroid and perigee of its trajectory is small. In the limit, they coincide. Then (at small impact angles and small changes in object's speed) we may easily obtain that the height exceeding of the trajectory Δz above its perigee to a first approximation is proportional to the square of the change in the polar angle φ:

$$\Delta z \sim \varphi^2 \qquad (2)$$

Density of the air is the only parameter which is highly changing during hypersonic flight because of its exponential dependence from altitude. Therefore from formula (2) we provide that the braking acceleration in a first approximation is described as follows:

$$a \approx -c_1 \exp\left(-\xi^2\right),$$

$$\xi = \left(\frac{z - z_f}{h}\right)^{\frac{1}{2}} \approx \frac{\varphi - \varphi_f}{\varphi_0 - \varphi_f},$$

where c_1 is a function of the constants determining the aerodynamic forces and mass of object, h is characteristic height of the atmosphere, where air density is changed in e times, the index 0 corresponds to the parameters entering the atmosphere, the index f – to the finish of the flight at the point of explosion.

Thus, to a first approximation, after integration over the angle φ we receive a reduction in the object speed Δv in the atmosphere:

$$\Delta v(\xi) \approx -\Delta v_f\left[1 - \mathrm{erf}(\xi)\right],$$

$$\Delta v_f = v_0 - v_f \ ,$$

where erf (ξ) is the probability integral or error function. It is known that in the most part of the interval $0 \le \xi \le 1$, the function erf (ξ) is close to linear f (ξ) = ξ, and if $\xi > 1.5$, it almost goes to the asymptote of f (ξ) = 1. Only in a relatively small neighborhood of $\xi = 1$, there is a smooth transition of function erf (ξ) from one a nearly linear mode depending on the argument ξ to another [11]. So it's a good approximation for using the corresponding piece-wise linear function, and at that case changes in the rate of loss of the object speed along the path approximately may be described as follows:

$$\Delta v(\xi) \approx -\Delta v_f(1 - \xi) \text{ при } 0 \le \xi \le 1,$$

$$\Delta v(\xi) \approx 0 \text{ при } 1 \le \xi \le \xi_0$$

This means that on the part of the trajectory from the upper edge of the atmosphere $\xi_0 \ge \xi \ge 1$ the flight speed is constant and equal to the initial speed v_0, and at $1 \ge \xi \ge 0$ it is linearly changes from v_0 to v_f, and its average value is equal to half the sum of the initial and end values. Then, in the interval $0 - \xi_0$ is easy to determine the average speed of the object $<v>$ on the atmospheric trajectory through statistical weighting coefficients α and β and through the start and end speed values:

$$< v > = \alpha v_0 + \beta v_f, \qquad (3)$$

where

$$\alpha = 1 - \frac{1}{2\xi_0}, \qquad (4)$$

$$\beta = \frac{1}{2\xi_0} \qquad (5)$$

For example, for Chelyabinsk meteoroid at the entry speed into the atmosphere 18.85 km/s, at the height of the object explosion 28 km (see further) and at the characteristic height of the atmosphere h = 8.00 km (see [2, 3]), ξ_0 = 2.78, and weight coefficients are as follows: $\alpha = 0.820$, $\beta = 0.180$.

Despite the fact that the formulas (3) – (5) are derived under rather restrictive assumptions imposed on the trajectory, due to the exponential rise in atmospheric density the main share of the change in speed of an object always falls on the last part of its trajectory, commensurate with the value of the characteristic height h. Therefore, the approximate calculations with the formulas (3) – (5) may well be used far beyond the limits that were imposed during their derivation. In addition, if the trajectory of the object becomes more and more steep, the loss of speed during braking, at least, for enough to large objects (class of Tunguska meteoroid) tends to 0 ($v_f \rightarrow v_0$). Therefore, in such a situation, any inaccuracies in the determination of average speed are almost no longer having any significant value.

Algorithms for calculation the consequences of a meteoroid impact on the Earth have been described in some detail in reference [3]. Thus, all parameters of the process are defined through a required number of equations. And at the known orbit of the object, height of the explosion and coordinates of its epicenter we may uniquely identify all basic parameters of the object and the burst, which is generated through its destruction. The speed and the angle of inclination of the object's trajectory are determined at any point. We may easily to define the length of the trajectory from the entry point into the atmosphere to the point of air burst and its height. And for a given speed and the angle of entry (path angle at the point of entry), together with the known height of explosion and a peak overpressure on a shock wave at a given distance from the epicenter, we may clearly define the characteristics of the explosion caused by the destruction of this object.

However, from the description of the module that calculates the speed at the atmospheric part of the trajectory it is clear that in order to apply all these relationships, it is necessary to know the parameters of the explosion, which are required to calculate the final speed of the object through the formula (1). In this case, for their determination, in turn, we need to know the parameters of the trajectory. And besides, the polar angle of the object's entry point into the atmosphere isn't known beforehand, as well as the impact distance or length of the atmospheric portion of the trajectory. At the same time, in early of calculation of any event, isn't clear even the level of parameters with which should to begin the process of decision. That is why the procedure, which, as can be seen from the analysis of module descriptions of speed calculations, should be the procedure of successive approximations. And all used algorithms should be simple and quick, so we could make a lot of embedded computing cycles on several parameters. Consequently, all computational modules, including the module, in which is determined the braking of the object in the atmosphere, should be as simple to use as possible, that is achieved using the procedure described above.

Note that the meteoroid impact module [2] meets all these requirements. And only for this reason such module of computation of the trajectory's atmospheric portion at low angles of entry ($\delta \leq 10^{\circ} - 15^{\circ}$) is used, as simplest from adequate variants at the first phase of the numerical model development. But because of this simplicity we should to pay for the proximity of solutions. The main source of possible errors except the simple model of object motion in the atmosphere is that in «external» interactive part of the model, describing the destruction of meteoroids in the atmosphere [2, 3], its motion is considered without the Earth's curvature that leads to inaccuracies in determining of the entry angles. Therefore, for these trajectories this parameter is a conditional, and can't be used without correction to compare with observations because of the deviation of the normal to the surface within the length of the trajectory.

Thus, the above-described mathematical model allowed moving from full of uncertainty and speculations about incidents with the inputs of celestial bodies into the atmosphere to regular solution of completely certain physical and mathematical problem. If necessary, on the basis of this model and initial approximation of the solutions obtained with its help, we may create computational modules, more accurately describing any of the elements of these phenomena, and get more accurate results. It should also be noted, as follows from comparison with other data, some of the most important parameters, such as, for example, the energy of the explosion, in the framework of this model are determined practically exact. However, in reality, their accuracy will be limited as well as the precision of the parameters of the phenomena observed in the underlying surface. And this accuracy usually is not too high. Therefore, the excessive refinement of a computational model may be virtually useless.

III. Way to obtain the most accurate and precise trajectory parameters of Chelyabinsk meteoroid

It is known that the orbital parameters of Chelyabinsk meteoroid which have been received in the first 2 – 3 weeks after the incident and were the input data for the calculations presented in paper [1], characterized by considerable scatter (see, for example, [12]). Data available now are more accurate, however, comparison of the results of several more recent sources shows that the variation of parameters decreased, but remained quite noticeable (see [4, 5, 13, 14]). Therefore, when new numerical calculations were executed, main attention should be directed on the reliability and accuracy of the input parameters for the numerical algorithm. For this, after consultations with experts-astronomers there was chosen the speed of the object's entry into the atmosphere as one of three required input parameters describing the orbit of Chelyabinsk meteoroid before the collision with the Earth (not counting the known data on the intersection of orbits of the object and the Earth). This parameter was obtained directly from the videos and, therefore, in principle, contains minimal errors from algorithms for computing and converting.

The second quantity – value of semi-major axis with a high degree of accuracy may be obtained from resonance 13:6 with the Earth (see [1]) – almost all the later sources give values that are very close to this resonant period of revolution of this meteoroid. In our solar system are known to large number of orbital resonances of planets, dwarf planets, satellites and asteroids [9]. In addition, typical meteor showers are also in resonance with the Earth. Thus, the value of semi-major axis of Chelyabinsk meteoroid a = 1.674 AU (astronomical unit) is known from its period of revolution of $\tau = 13/6 = 2.167$ years ($a \sim \tau^{2/3}$).

At zero angle of inclination of meteoroid's orbit plane and with known point of its intersection with the Earth orbit of these two parameters are enough to determine the orbit of the object. As the angle of orbit inclination was little according to all data sources, its effect on input parameters in the atmosphere is relatively small. Therefore, a little rounded value i = 5.00° was just taken from source [5].

Therefore, it remains to analyze the latest published data of Chelyabinsk meteoroid entry speed v in the Earth atmosphere. And here again there were some problems in connection with noticeable differences even these new and revised data. The smallest value of the speed and the smallest error were stated by G. Ionov – v = 18.85 ± 0.09 km/s for two series of measurements [13]. There were also given the following values for this parameter: v ≈ 19.0 km/s [4], v = 19.16 ± 0.15 km/s [5], and v = 19.3 ± 0.9 km/s [14]. Taking into account that the accuracy of the results of the source [14], in which has been described a new method of handling the trajectory measurements, as quite clearly states the author of this work, is currently lower than in other studies mentioned here, this value was excluded from this comparative review. Yet even from data sources [4, 5, 13] is followed that the accuracy of the determination of the object's entry speed into the atmosphere is not better than 0.3 km/s – v = 19.0 ± 0.3 km/s.

Such variations in the speed when errors mentioned in sources [5, 13] not exceed ± 0.15 km/s, suggests that at least one of these results has a systematic error not less than 0.15 km/s. Probably that it is associated with application to a little sloping and long trajectory of Chelyabinsk meteoroid standard algorithms for sufficiently steep and/or short trajectories («flat» Earth, the trajectory in the atmosphere is a straight line segment), which are fully adequate only for objects with the scale much smaller than the scale of Chelyabinsk meteoroid. Other noticed problems and inconsistencies in the data of paper [5], which are discussed in a separate articles devoted exclusively to criticism of sources [9, 10], allowed us to conclude that this error was there. It follows that the most accurate data of entry speed were obtained by G. Ionov, who has made a video of Chelyabinsk bolide flight almost from the doorstep of his house and then repeatedly made a photographs of night sky with the same position, having reduced the random errors of measurements to level of ± 0.09 km/s [13].

Thus, the entry speed of Chelyabinsk meteoroid was 18.85 km/s in the numerical calculations according to the source [13]. Than the perihelion of the meteoroid's orbit was 0.746 AU, aphelion – 2.603 AU, eccentricity – 0.554 and the value of semimajor axis was equal to 1.674 AU. It may be noted that the value of semi-major axis of Chelyabinsk meteoroid used in this paper differs on – 1.6 % from the average value based on data from four papers given in source [5] (a = 1.70 ± 0.05 AU), and from the result of source [14] on + 0.2 % (a = 1.67 ± 0.10 AU). Values of perihelion are follows: average value from source [5] – q = 0.77 ± 0.05, that is, the difference amounts + 3.2 %, from source [14] – q = 0.73 ± 0.01, and the difference amounts – 2.1 %. So, there is a good agreement between all of these data.

In addition, the basic version of computational results with v = 18.85 km/s was recalculated to a speed equal to 19.00 km/s, and the consequences of this possible increase of speed were analyzed in the article.

IV. Way to obtain the most reliable and accurate data describing Chelyabinsk explosion

Now it is necessary to clarify the quantitative characteristics of the phenomena associated with the meteoroid's approach to the «point» of explosion, and with the explosion itself. The explosion of Chelyabinsk object was occurred because of its destruction down to small crumbs and dust, and very sharp braking of debris avalanche. From observations follows that all these process didn’t occur instantly. Two peaks of electromagnetic radiation were observed during this explosive process. From data of source [5] follows that the second main peak of emission was recorded at an altitude of about 30 km, and the completion of fireball formation before its conversion to relatively little luminous cloud occurred at the altitude of approximately 27 km. From the description of nuclear explosions is known that the shock wave separation from fireball occurs in that moment (see, for example, [15]), and the shock wave of meteoroid broke away from the fireball at this height range in the vicinity of its lower border.

The mathematical model of the explosion used in the described computational method is simplified. The explosion of a celestial body is similar to nuclear burst, and this explosion is spherically symmetric and happens instantly. Therefore, we should specify «point» of explosion, which allows the best way to approximate much more complicated and lengthy process of explosion to this simplified model of burst. This may be done only by varying the main parameters of the explosion point in a reasonable range of variation, in process of comparing the calculated and the observed parameters characterizing the propagation of a shock wave, to give the best agreement between the calculated and experimental data. We may assume that the height of the explosion point lies in the range 27 – 30 km.

The average value of the geographical coordinates for the trajectory of Chelyabinsk meteoroid corresponding to the middle of this very short section of the path is very close to the data from sources [1, 5, 14]. North latitude is equal to 54.87° and east longitude is 61.20°. Deviation in latitude from the data used in paper [1] is 0.02°, and for longitude differences there were no quite. Estimated time of the explosion hasn’t changed – 9:20:30, February 15, 2013. Estimates of the geodesic trajectory azimuth were 283.2° [5] – this means that the object was moving from east to west, shifting northward at 13.2°, which is 1.3° less than in earlier calculations, with the azimuth determined according to the meteoroid's trace [1].

In order to start the numerical calculations in the framework of this model, we should to determine one more parameter – anywhere but far enough from the epicenter of the explosion is necessary to know the overpressure peak on a shock wave. As in earlier paper by the author [1], so in articles [4, 5] published much later, parameters of such type were determined through a state of glass windows in the zone of destruction, that is, the presence and/or part of shattered windows in the area of the shock wave action. Such map is given in reference [5] and its main part is shown in Fig. 1. Orange points (according to the Emergency Department data), as well as red points (from field surveys of source [5] co-authors) are shown for regions of Chelyabinsk and for localities of the homonymous province, where the window glasses were shattered. The open points mark locations where noticeable quantities of shattered glass were not registered. Yellow points on this map show drops of small meteoroid's fragments, and there is no interest to them for us in the present context.

It can be seen that the zone with shattered windows is something like a few rounded rectangle. Black line is a projection of the meteoroid trajectory on the Earth surface. The distance on interactive Yandex-map from the epicenter to the maximum distant settlement with shattered windows in the direction perpendicular to the projection of the flight path was 105 km. That is, the maximum size of the zone of broken glass in this direction was about 210 km, and its size was 2.2 times less – about 95 km along the trajectory.

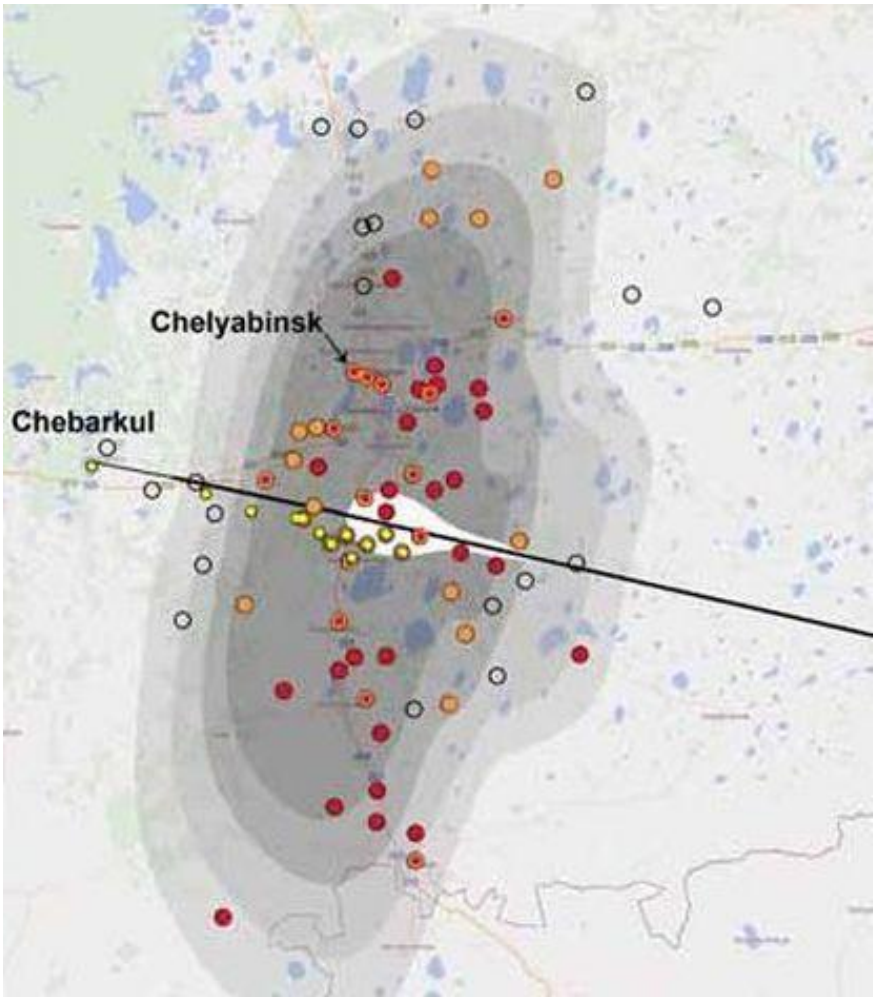

Fig. 1

The white area in Fig. 1 illustrates the change in the emission intensity of the fireball on the flight path that is light curve of bolide [5]. Gray areas varying intensities show borders of zones with constant values of overpressure peak on the shock wave, obtained from calculations of explosion by using the numerical gas-dynamic code [16], provided that the energy of the explosion was distributed along the flight path proportionally to the light curve [5]. There is if not quantitative, then at least a qualitative similarity between these boundaries and the limit of zone with broken glass. At explosion in a point, despite the small slope of meteoroid trajectory, this gasdynamical code as well as other similar methods gives almost circular picture of boundaries with constant values of overpressure peak on the shock wave (see [1, 5]).

Data presented in [5] suggest that here, unlike of Tunguska explosion, in which explosion height was in more than three times lower (see [1]), the impact of ballistic shock wave on the underlying surface was almost indistinguishable. Further, from the fact that the slope of the trajectory of Tunguska meteoroid was in several times larger than for Chelyabinsk object, we may conclude that the distance of the explosive energy release in the first case have to be much shorter and, therefore, Tunguska burst was considerably closer to the explosion in the point. Therefore, there is every ground to conclude that the reasons for deviations from circular symmetry of destruction zones at Tunguska and Chelyabinsk incidents are different, and to understand why they are so different in their forms. From this also follows that the boundary condition for Tunguska explosion (overpressure peak on the shock wave for tree felling is equal to 30 kPa at the distance of 20 km from the epicenter [1]) doesn't need to change.

And here, at Chelyabinsk, the approximation of real border of destruction zone with the aid of circular symmetrical region, required for the algorithm used in the calculation module [2], leads to the following boundary condition – the overpressure peak on the shock wave, which is required for certain multiple glass broken, equal to 5.0 kPa [9, 10], is achieved at a distance of 80 km from the epicenter, what is on 10 km less than in the original calculations (see [1]). This distance was obtained by equating of areas of real and circular symmetric approximation of destruction zones, what is a fairly obvious way to do such computational assessment. Test calculations then were carried out at values of the radius of this zone from 74 to 86 km.

But another source gives much more accurate information about the overpressure peaks on the shock wave at the points with known exact coordinates. In reference [4] was reported that part of broken glass in the area around the Chelyabinsk Zinc Plant is indicated on «overpressures close to 7 – 8 kPa». Location of explosion epicenter (mark 1) on the map of Chelyabinsk and its environs and plant's storehouse of zinc concentrate (mark 2), where there was destruction of the roof, is shown in Fig. 2. The distance between them is 39.5 km. It should be noted that the Ice Palace «Urals Lightning» (mark 3), in which in the morning February 15, 2013 one supporting beam has collapsed, several beams were curved, and cladding from the facade was destroyed (not to mention the broken glass) [17], was located at a distance of 35 km from the epicenter, and almost on the same line, which connects the epicenter of the explosion with a storehouse of zinc concentrate (deviation from this line isn't more than 0.65 km, see Fig. 2).

The level of overpressure on the shock wave «in the Chelyabinsk urban area based on all forms of window damage» was estimated in source [4] as 3.2 ± 0.6 kPa, and the overpressures of 7.5 ± 0.5 kPa in the region, which is extending from the epicenter of the explosion on a few kilometers further than center of the city, were surprise for authors of paper [4]. They tried to explain this fact using such terms as «caustic» and «constructive acoustic interference». However, words, no matter how are profound they would be on their own, without disclosing the real mechanisms of phenomenon can't explain anything. For a person who is familiar with reflection and interference of nonlinear shock waves, which are qualitatively different from reflection and interference of linear acoustic waves, these mechanisms are transparent enough. But, there wasn't such person of 33 co-authors of the article, apparently. In this context, it is worth noting that first article of the author of this work (written with V. V. Keldysh) was devoted to the three dimensional interference of shock waves [18].

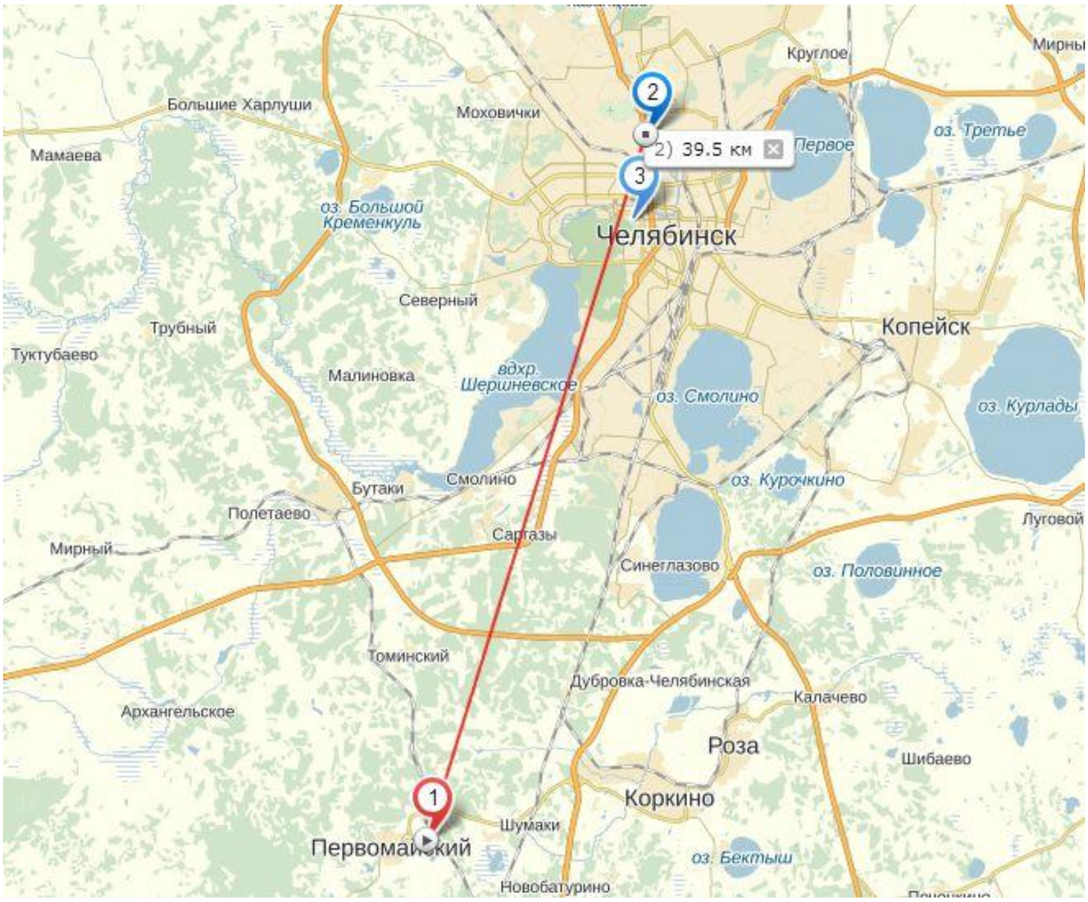

Fig. 2

Elementary acquaintance with features of propagation of shock waves leads to a quite obvious thought that a shock wave from the airburst, height of which is comparable and even greater than the distance to the target, interacts with it not so, as a shock wave from low-altitude explosion, when the removal of the target in many times greater than the height of the explosion. It is also obvious that all nuclear explosions, data of which were used to receive the dependence of amounts of shattered glass from nominal overpressure of the shock wave, were made at low altitudes. So that the distance from them to the areas where was possible to consider the broken glass on the walls of intact buildings, was many more than the heights of these explosions (typical altitude of airburst for warhead with the energy of about 1 Mt TNT was approximately 1.5 km with a characteristic radius of glass breaking about 20 – 40 km, see [15]). In such a case, into facets, which are turned towards to the explosion, such as the walls of houses with windows, straight shock wave falls, plane of which is parallel to the plane of these facets. If the burst is high-altitude, as at Chelyabinsk, then right along the surface of the earth runs oblique shock wave (inclined to the surface), which in this scale is almost flat part of the shock wave of explosion. And, the farther away from the epicenter, the greater becomes its slope.

The straight shock wave falls down on rooftops at the epicenter of the explosion, and to a first approximation, slides practically along their vertical walls interacting with them relatively weakly. At more detailed examination of this process should be taken into account a turn of the shock wave on the facets formed by a flat roof of house, or on a set of oblique edges formed by a peaked roof. This unfolded and oblique shock wave is reflected from the ground to form new oblique shock waves. And, in principle, the impact of these waves on walls of buildings and its windows can be determined by numerical simulations for a specific geometry and arrangement of buildings standing close. However, it's well known even without gasdynamics calculations that any oblique shock wave is a «weaker» than a straight, and its impact on the obstacle should be less significant. That is why the proportion of broken glass near epicenter was far from absolute [5]. And only snow load on the roofs in this Russian region is commensurate with the maximum impact assessments of the shock wave from the explosion of Chelyabinsk meteoroid in the epicenter area, so that the safety of flat roofs near the epicenter doesn’t seem inexplicable [9, 10].

Such interaction, which occurs in the epicenter of explosion between a shock wave and a peaked roof of house, consisting of two or three facets, is implemented between wave and facets of standard multi-storey building with flat roof for the average foreshortening of wave propagation, when height of explosion and distance to it are commensurate. Therefore, when the oblique shock wave runs along the earth, part of broken window panes will be significantly less than that of for a straight shock wave from low-altitude explosion of the same energy.

So, oblique shock wave from high-altitude powerful explosion runs along the earth surface at a considerable distance from the epicenter, reflecting from the solid surface as another oblique shock wave. This type of reflection is called regular, and it can be represented schematically in the form of V letter, where the slanting dashes depict the incident and reflected shock waves. At the time as the distance from the epicenter grows, the angle of the incident wave grows also, and at some point it leads to the inability to implement the regular reflection. From this point it becomes so-called Mach reflection [19], which can be schematically represented as Y letter – between the point (in the planar case) or line of intersection of oblique shock waves (in three dimensions) and a solid surface, so-called «Mach stem» arises (vertical dash of Y letter), which is a straight shock wave. From that moment, a high-altitude airburst in its impact on the underlying surface becomes equivalent to a low-altitude explosion, and only then one can begin to compare part of broken glass from all previously existing sources with that, what happened after the explosion of Chelyabinsk meteoroid. And that is why the boundary conditions in computational model should be set at such a great distance, where were guaranteed the irregular or Mach reflection of the shock wave.

All of the above illustrates the simple and obvious fact that to determine the energy of high-altitude explosion through the data of breakage of ground objects (including the part of broken glass), obtained at propagation of shock waves from low-altitude nuclear explosions, we should consider data from regions of Mach (irregular) reflection only. Glass shattered by waves can only be compared for comparable shock waves. Hence it becomes clear that where in source [4] was found «abnormal» level of overpressure in «caustic zone», which is not known for shock waves, there was realized the picture of the interaction of waves with obstacles, which alone can properly interpret the observed phenomena using available data of previously observed powerful explosions. For some desire this transition from regular to Mach reflection can be called «constructive interference» of the incident and reflected waves, resulting in a significant increase in real, not nominal, pressure, as it wanted to do the authors of paper [4].

It remains now to consider only one question: why this «constructive interference» was seen in only one area of Chelyabinsk? To answer this question we turn to maps and satellite images of the terrain. Even on a large-scale map shown in Fig. 2 is seen that on line of propagation of the shock wave from Chelyabinsk explosion at a distance of about 35 km up to Ice Palace «Urals Lightning» are almost entirely non built-up flat plains where are only fields or rare forests. This line pass then along the coastal part of Shershnevskiy (Hornets) water basin, which was covered with ice during the explosion, and then line pass through Chelyabinsk city forest (green spot in Figure 2, through which is crossed by the red line). If we move across this line, watching underlying areas with the aid of satellite images at full resolution, it could be seen that quantity of such obstacles as urban multi-storey buildings on this line can be counted up to Chelyabinsk Zinc Plant on the fingers of one hand.

Thus, the oblique shock wave from high-altitude explosion has propagated along the ground practically with no energy loss due to obstacles. And its high intensity was fixed through mass of broken glass only after transformation of the wave into a straight as waves of previously observed low-altitude explosions. To east, in the direction to the center of Chelyabinsk and its eastern regions on the way still an oblique shock wave has passed through large arrays of high multi-storey buildings, what was accompanied by the emergence of very large quantity of local incident and reflected waves and their interactions with each other and with new obstacles. Such process had to seriously affect the geometry of the shock wave near the ground, the average pressure levels and the picture of the destructions in those «shielded» areas of the city. And, apparently, the above-described model of transition on a smooth solid surface from regular to Mach reflection of shock wave had to transform into something much more complicated and chaotic.

It should also be noted that prior to the explosion over Chelyabinsk were only 2 cases of strong shock waves passing through a continuous urban development for a distance of several kilometers – in Hiroshima and Nagasaki, and in a much smaller scale, apparently, another 2 in Halifax and Texas City [20 – 22]. But the explosion energy in these cases was of a few thousand or tens of thousands times smaller than the explosion over Chelyabinsk, and path lengths of intense waves were at least by one or one and half order of magnitude smaller. Furthermore, the main part of buildings in these two Japanese cities was small one-storey or two-storey wooden houses [20]. This contrasts sharply with urban architecture in central and eastern districts of Chelyabinsk. Known correlations between the amount of broken glass and overpressure on shock wave were derived from data on the propagation of shock waves on the urban housing of Hiroshima and Nagasaki, or even on the bare steppe of Semipalatinsk test site with detached buildings when the attenuation of these waves due to obstacles was weak. Therefore, these data are not fully adequate for all districts of Chelyabinsk, but only for direction to «Urals Lightning» and Zinc Plant, where there are vast areas with no high-rise buildings. This is true even without differences of impact of straight and oblique shock waves on obstacles.

In this context, it should also be noted that there is another factor from which the altitude of explosion has significantly affected to the impact on the underlying surface. This is a decrease in density of the atmosphere at the point of explosion because of the growth of its height. For heights of 25 – 30 km, this factor may lead to a reduction in overpressure at the surface in several times. Influence of this factor is studied in detail in report and paper criticizing sources [9, 10].

Further calculations showed that the condition p = 7.5 ± 0.5 kPa at a distance of 39.5 km from the explosion epicenter is equivalent to the condition p = 5.0 kPa at a distance of 80 ± 6 km, which is in good agreement with previous estimates of the model for the affected area. Thus, the previous analysis shows that the overpressure peak on the shock wave of 7 – 8 kPa in the region of Chelyabinsk in the neighborhood of the Zinc Plant [4] is the most accurate from available boundary conditions for solving the problem.

V. Calculation results of Chelyabinsk and Tunguska meteoroids' parameters and their explosion modes

Several tens of calculations of inputs into the atmosphere and explosions of Chelyabinsk and Tunguska meteoroids were made, and the results of 11 of them as the most representative are shown in Tables 1 – 4. Tables 1 – 2 show the influence of height changing in explosion point to the characteristics of Chelyabinsk meteoroid (ChM), as well as to overpressure peaks on the shock wave from the explosion at several distances from the epicenter. As stated in section II of this paper, the entry speed of the object was 18.85 km/s. A minimum height of the explosion point, which was some approximation of the real height of blast, is, as shown above, 27.0 km. This value was increased in increments of 0.5 km in calculations with the boundary condition 5.00 kPa at distance of 80.0 km from the explosion epicenter. Nominal height of the atmosphere at this process assumed to be equal 90.0 km, which, according to [2, 3], approximately corresponds to the beginning of its impact on the object of this type.

In Table 1 are shown: var – variant of calculation of Chelyabinsk meteoroid, H is the height of explosion in kilometers, δ is the entry angle in degrees, ρ is the density of the object in kilograms per cubic meter, D is the diameter of the object in meters, m is the mass in megatons, E_0 is the kinetic energy of the object entering the atmosphere in megatons of TNT, E_e is the explosion energy of the object in the same units.

Table 1

var	H (km)	δ (°)	ρ (kg/m³)	D (m)	m (Mt)	E_0 (Mt)	E_e (Mt)
ChM-1	27.0	7.22	870	158.5	1.81	76.6	52.6
ChM-2	27.5	7.19	740	167.5	1.82	77.3	54.4
ChM-3	28.0	7.17	630	177	1.83	77.8	56.2
ChM-4	28.5	7.14	540	187	1.85	78.3	57.8
ChM-5	29.0	7.11	460	197.5	1.86	78.9	59.6

As shown in Table 1, increasing the height of the air blast from 27 to 29 km under these conditions results in a decrease in the angle of entry into the atmosphere at 0.1° and drop in the density of the object in 1.9 times – from 870 to 460 kg/m³. The diameter of it is growing at 25 % – about from 160 to 200 m, while increasing its mass and kinetic energy at the input into the atmosphere by 3 %: m = 1.81 – 1.86 Mt, E_0 = 76.6 – 78.9 Mt of TNT. Thus, this energy is increased by 2.3 Mt, while the explosion energy E_e increases more rapidly – from 52.6 to 59.6 Mt, that is by 7.0 Mt or 13 % of initial value. This is due to the reduction of energy losses less dense meteoroid during braking at higher altitudes that is at a lower density of the atmosphere.

Table 2 shows the values of the main factor of the shock wave from explosion on the ground obstacles at these distances – the overpressure peak. Here: var – variant, p is overpressure peak on the shock wave in kilopascals at a distance L from the explosion, measured in kilometers along the ground and demonstrated in the column to the left of the pressure.

Table 2

var	L_0 (km)	p_0 (kPa)	L_1 (km)	p_1 (kPa)	L_2 (km)	p_2 (kPa)	L_3 (km)	p_3 (kPa)	L_4 (km)	p_4 (kPa)
ChM-1	0	11.6	20	9.4	35	8.1	39.5	7.67	80	5.0
ChM-2	0	11.4	20	9.3	35	8.0	39.5	7.60	80	5.0
ChM-3	0	11.2	20	9.2	35	7.9	39.5	7.53	80	5.0
ChM-4	0	11.0	20	9.0	35	7.8	39.5	7.46	80	5.0
ChM-5	0	10.8	20	8.9	35	7.7	39.5	7.39	80	5.0

Maximum overpressure peak is attained on the shock wave at the epicenter (at L_0 = 0). The more powerful and higher is the explosion, the lower is this overpressure. The distance L_1 = 20 km is characteristic for Tunguska explosion, which is compared with Chelyabinsk blast, the distance L_2 = 35 km corresponds to the distance between its epicenter and the center of Chelyabinsk (as well as the Ice Palace «Urals Lightning»). At the distance of L_3 =

39.5 km is located Chelyabinsk Zinc Plant, the distance L_4 = 80 km is the length to the border of a circular area with overpressure of 5.0 kPa, which approximates the real zone of destruction.

Nominal overpressures on a straight shock wave have not reached 12 kPa even in the epicenter, at the distance of 35 km (in the center of Chelyabinsk) it was about 8 kPa and below, and in the area of Zinc Plant pressure level is near 7.7 – 7.4 kPa. As stated earlier, these magnitudes may be reached here after the realization of Mach reflection and arising of straight shock wave. These conditions are comparable to those that have realized for low-altitude explosions. Since exactly these conditions for straight shock wave are employed in all the data on broken windows, it is this area, which should be used to fine tune the parameters of the computational model that describes in the best way what happened in reality. When analyzing the data in Table 2 should be understood that the presented data are maximum pressures at the shock wave front incident on the surface disposed normal to the wave. It is assumed that the wave is not attenuated due to scattering and multiple reflections on the obstacles that lie closer to the epicenter of the explosion. As described above, more or less similar conditions in Chelyabinsk realized only in the direction from the epicenter to the Chelyabinsk Zinc Plant, but real values of the maximum pressure on the wave in other parts of the city due to the energy dissipation on numerous obstacles were to be lower than shown in Table 2.

It follows from Table 2 that the overpressure on the wave of 7.50 kPa at the distance of 39.5 km (average value in the area of Zinc Plant according to the source [4]) is realized when the height of the explosion is 28.2 km. This is perfectly consistent with the preliminary estimates: «the interval from 27 to 30 km, but closer to the lower boundary of heights», as well as with the data source [14]. Then the meteoroid explosions were calculated at the altitude of 28.2 km and at the overpressure peak on the shock wave from 7.0 to 8.0 kPa at the distance of 39.5 km. At the same time was also adjusted the nominal height of the atmosphere – it was increased from 90.0 to 91.2 km. Iterative computational procedure showed that this height of the atmosphere corresponds to the beginning of its impact for the basic version of Chelyabinsk meteoroid ChM-7 (see Table 3), which creates an explosion overpressure of 7.50 kPa at a specified distance. Further, all calculations were carried out precisely at such nominal height of atmosphere. It may be noted that the perigee of the computed trajectory was at the altitude of 28.07 km.

Calculated data for three variants of the Chelyabinsk meteoroid at the overpressures on the wave of 7.00 kPa, 7.50 kPa and 8.00 kPa at the distance L_3 = 39.5 km are shown in the first three lines of Tables 3 and 4. The parameters and the designations are the same as before. Bold fonts are used for basic variants of Chelyabinsk (ChM-7) and Tunguska (TM-1) meteoroids.

Table 3

var	v (km/s)	i (º)	H (km)	δ (º)	ρ (kg/m³)	D (m)	m (Mt)	E_0 (Mt)	E_e (Mt)
ChM-6	18.85	5.00	28.2	7.22	635	173	1.71	72.7	52.1
ChM-7	**18.85**	**5.00**	**28.2**	**7.22**	**570**	**182.5**	**1.82**	**77.4**	**56.8**
ChM-8	18.85	5.00	28.2	7.22	515	193	1.93	82.1	61.8
TM-1	**18.72**	**– 5.00**	**8.25**	**50.5**	**570**	**105**	**0.35**	**14.6**	**14.4**
ChM-9	19.00	5.00	28.2	7.25	580	181	1.80	77.5	56.7
TM-2	18.87	– 5.00	8.33	50.0	580	104.5	0.35	14.8	14.6

Entry angles δ in all three variants of calculations (ChM-6 – ChM 8) coincide with each other up to the third digit after the decimal point. The density of the object ρ at overpressure increasing at a predetermined range is decreased by 23 % – from 635 to 515 kg/m^3, and the diameter D is increased by 11.5 % from 173 to 193 m. Accordingly mass m is increased substantially by the same amount, and the explosion energy E_e, providing the required overpressure, is increased by 19 %. Thus, a dynamic equilibrium is reached at the point of the object destruction: greater but less dense object is destroyed at the same height as the smaller but denser body. Growth of the masses and energies of the object under increasing the overpressure on the shock wave at a predetermined distance from the explosion is quite natural process. At the overpressure of 7.50 kPa the calculated mass of basic variant of Chelyabinsk meteoroid ChM-7 is 1.82 Mt at a diameter of 182.5 m, and the energy of its explosion E_e is 56.8 Mt of TNT, which is only 1.0 Mt (1.8 %) less than in early calculations [1]. Its total energy E_0 was 2.5 % greater due to higher initial flight speed.

Table 4

var	L_0 (km)	p_0 (kPa)	L_1 (km)	p_1 (kPa)	L_2 (km)	p_2 (kPa)	L_3 (km)	p_3 (kPa)	L_4 (km)	p_4 (kPa)
ChM-6	0	10.3	20	8.5	35	7.3	39.5	7.00	73.9	5.0
ChM-7	**0**	**11.1**	**20**	**9.1**	**35**	**7.9**	**39.5**	**7.50**	**80.0**	**5.0**
ChM-8	0	12.0	20	9.8	35	8.4	39.5	8.00	85.5	5.0
TM-1	**0**	**82.9**	**20**	**30.0**	**35**	**11.8**	**39.5**	**9.8**	**63.4**	**5.0**
ChM-9	0	11.1	20	9.1	35	7.8	39.5	7.50	80.0	5.0
TM-2	0	81.6	20	30.0	35	12.0	39.5	9.9	63.8	5.0

We turn now to Tunguska meteoroid and to its explosion. Earlier in report [7] has been shown the generality of the origin of Tunguska and Chelyabinsk meteoroids, as they were members of the same family of cometary debris. This leads to proximity of orbits of these two objects. Therefore, to evaluate parameters of Tunguska meteoroid there was used the same orbit as for Chelyabinsk meteoroid with the sole exception – angle of inclination of the orbit plane has got the opposite sign (see Table 3). The estimations showed that, while maintaining a longitude of perihelion, this orbit's modification can approximately provide its intersection with the Earth orbit in late June – early July that is in the first window of rapprochement with cluster of close orbits of Tungus family [7]. After that, using the described model, it is possible to compare these results with those which are known from numerous papers devoted to Tunguska phenomenon, and figure out how is true this assumption.

The explosion of Tunguska meteoroid (TM in Tables 3 and 4) occurred June, 30 1908, in the first window of approach with another position of the Earth axis to ecliptic plane and with such speed vector position of the object, which leads to a mirror image of it relative to the speed vector of the planet compared to what it was in February 2013. The Tunguska explosion was considerably northerly of Chelyabinsk – its coordinates were: 60.89° north latitude and 101.90° east longitude [1]. Local time of the explosion was 7:14:30, solar time – 7:02:06. All of these factors combine to affect the increase of entry angle of Tunguska meteoroid, which at geodetic azimuth of 279° (9° inclination of the trajectory parallel to the north) [1] was equal to 50.5°.

There are two boundary conditions on the shock wave for Tunguska explosion: the overpressure of 30 kPa at the distance of 20 km from the epicenter of the explosion, which is an approximation of the border of tree-felling zone (excluding the «butterfly wings», which are zones, caused by a ballistic shock wave) [1], and the boundary of standard level about 5.0 kPa for broken glass – near 63.5 km. At such a distance from the epicenter (63.5 – 64 km) Vanavara village is located. Eyewitnesses – its residents have reported the following: «Then it turned out that many of the windows were broken» [23]. Thus, the speed and the angle of entry of the Tunguska meteoroid, which were derived from the astronomical module of numerical calculation method, and these two conditions on the shock wave, are sufficient to perform calculations similar to those for Chelyabinsk meteoroid.

Such short and steep tracks are computed much easier and faster than long and flat trajectories. On such trajectory this module has very small influence on the final result, as the effect of the atmosphere on the path up to the explosion of the object is minimal. This is evident from the fact that its energy has decreased on the atmospheric part of the trajectory by only 1.4 % (see line TM-1 in Table 3).

Interactive module, which describes the destruction of meteoroids in the atmosphere [2, 3], operates in the framework of «flat» Earth. That, as already mentioned above, introduces additional error in the results of calculations of long and flat trajectories. Therefore, the calculated entry angle of Chelyabinsk meteoroid is different from the real at high altitudes, and it can be regarded only as estimation (with taking into account the curvature of the Earth for the base variant ChM-7 input angle is 15.6°, which is close enough to the value of 18.3° from source [6]). The calculations of short and steep trajectory of Tunguska meteoroid are practically free from these errors. And all possible errors are determined only by the deviation of its computed orbit from real, that, of course, could and should be somewhat different from Chelyabinsk meteoroid's orbit not only by inclination angle with the ecliptic plane, but these deviations should be sufficiently small. And especially important in this context is the fact that the average densities of these two celestial bodies at these calculations are exactly equal – about 570 kg/m^3, see lines FM-7 and TM-1 in Table 3, that is a necessary condition for the recognition of common origin of Tunguska and Chelyabinsk meteoroids. Therefore, the value of density was enough correctly computed and for Chelyabinsk meteoroid. This means that there are no internal contradictions in this approach, and inevitable calculated errors are insignificant. It should also be noted here that the estimates of the energy of the Tunguska explosion from seismograms leads to the value of its energy 12.5 ± 2.5 Mt, and from barograms – 12 ± 2.5 Mt [24, 25], that is in a good agreement with obtained result, which was equal to 14.4 Mt (see line TM-1 in Table 3). Thus, the assumption of the unity of origin and the proximity of orbits of Chelyabinsk and Tunguska meteoroids leads to correct values of the explosion energy of the latter.

The diameter of Tunguska meteoroid was approximately in 1.75 times smaller than of Chelyabinsk object, its mass was in 5.2 times less, and the energy of the explosion – in 3.9 times less (see lines ChM-7 and TM-1 in Table 3). But, since its explosion occurred at the height of 3.4 times lower, the impact on the underlying surface was much stronger. The overpressure peak in the epicenter is estimated in 7.5 times more than at the explosion of Chelyabinsk meteoroid (see lines ChM-7 and TM-1 in Table 4). As reported, the roofs were not damaged in the vicinity of the epicenter of Chelyabinsk explosion, when the overpressure maximum at the earth surface was of about 10 – 11 kPa. Overpressure on the glass was significantly lower what was explained in detail in the previous section of this paper. And because of this there was relatively few of broken glass. In contrast, in taiga near Stony Tunguska River was the region of full tree-felling at the radius no less than 20 km except the epicenter, where there was a dead forest from tree trunks completely without branches [24]. Boundary of the overpressures equality lies from the epicenter at the distance of about 51.5 km away. At greater distances the stronger wave was for a much more powerful and much more high-altitude Chelyabinsk explosion.

Increase of entry speed of Chelyabinsk meteoroid from 18.85 km/s to 19.00 km/s had in general to a negligible effect on its characteristics (see lines ChM-7 and ChM-9 in Table 3). Slightly reduced size, slightly increased density and angle of entry, and the two concerned energy values have not practically changed. Overpressure peaks on the shock wave also remained virtually unchanged (see lines ChM-7 and ChM-9 in Table 4). Approximately similar but somewhat more significant is the influence on the characteristics of Tunguska meteoroid in the case of corresponding increase in speed with 18.72 to 18.87 km/s (see lines TM-1 and TM-2 in Tables 3 and 4). Its explosion energy is raised by 0.2 Mt to 14.6 Mt, which is greater by 1.4 % than at lower speed.

For anyone representing the processes of comet's nuclei and their debris evolution should be clear that known from other sources the value of density for Chelyabinsk meteoroid – 3300 kg/m^3 (see, for example, [5]) is the density of its external crust. This crust is formed due to solar ablation of snow-ice composite which is contaminated by chondrites. Therefore, the density of a relatively thin crust doesn't characterize the average density of the object before destruction. The thickness of this crust should be of order of meter that follows from parameters of the largest surviving fragment of Chelyabinsk meteoroid, see, for example, its photo [26]. It is obvious that only a small portion of this crust may survive after the explosion, while snow and ice – main part of the meteoroid material should to evaporate completely. And we should not to judge about the average density of large object watching only these insignificant residuals of this thin surface layer. It may be noted that the calculated average density of Chelyabinsk and Tunguska comet fragments is at level, which is consistent with the known data on the nuclei of comets (see, for example, [27 – 29]).

V. Discussion of results

Thus, in the morning February 15, 2013 some celestial body has exploded in the sky over Chelyabinsk at a height near 28 km (28.2 km for modeling point of blast). Its size was of approximately 180 – 185 m, density was of about 570 kg/m^3 and mass – of about 1.8 Mt. Energy of the explosion was 56.8 ± 4.9 megatons of TNT with accounting of maximum overpressure error on shock wave ± 0.5 kPa in the region of Zinc Plant. Hence, the explosion energy in the sky at Chelyabinsk was almost equal to the energy of the most powerful thermonuclear explosion of so-called Tsar Bomba, which amounted to 58 Mt (other designations – AN602, Kuzka's mother), produced by the Soviet Union October 30, 1961 at Novaya Zemlya [30]. Comparison of these two catastrophic events held in [9, 10]. Determination of Chelyabinsk explosion energy by acoustic methods leads to a value fully coinciding with this magnitude calculated there with lower error – 56.8 ± 1.1 Mt [31]. This seems to be the most accurate estimate of the energy of this explosion.

Over 104.5 years before this, June 30, 1908 some meteoroid has exploded on the Stony Tunguska River, which was much smaller, however, it is still considered as the largest celestial body that entered the Earth atmosphere in historic times. This celestial body had the same density, but its size was 105 m, and mass – 0.35 Mt. The energy of explosion was 14.4 Mt, but because of that the height at which this explosion has occurred was in 3.4 times less, that is 8.25 km, the impact on the underlying surface at that time was much stronger. The calculated data of Tunguska incident are in a good agreement with the results obtained previously by several generations of researchers for decades of work on this problem: the energy of the explosion from 7 to 17 Mt at the altitude of between 6.5 and 10.5 km [32]. The calculated explosion energy is also within the boundaries of 10 – 15 Mt defined by seismic data and barograms [24, 25]. The coincidence of these two densities of meteoroids as well as agreement of data received in this work with the most reliable estimates of energy of Tunguska explosion are the grounds to recognize the possibility of commonality of their origins and the proximity of their orbits.

That these objects were members of the same family of cometary debris was stated as a conjecture in paper [1] and was proved in report [7]. Owing to the orbits proximity of members of this group, which were named as Tungus family, the characteristics of any from these objects may be evaluated by the method of this work with high degree of accuracy at minimum information about them. Thus, this work confirms the concept that there are families of cometary debris which threatened to the world in the historical past and still are threatening up to the present time [7].

The difference between characteristics of Chelyabinsk meteoroid, which are presented here, and those, which has been published in other papers on this subject (see, for example, [4, 5]) and are propagated by the media in countless quantities, extremely great. However, these differences are thoroughly reviewed in [9, 10], where was proven falseness of the most widespread point of view.

Conclusions

1. The results of calculations by the mathematical model that relates the parameters of celestial bodies motion in the spheres of activity of the Sun and the Earth, with the mass-energy characteristics of these objects and their explosion modes during the destruction in the atmosphere, well matched with the data obtained from observations.
2. Calculations have shown that size of Chelyabinsk meteoroid was equal to 182.5 meters, and its mass was close to 1.82 megatons. Energy of explosion was 56.8 ± 1.1 megatons of TNT.
3. Size of Tunguska meteoroid was close to 105 m, its mass was 0.35 megatons, while the energy of the explosion was about 14.5 megatons of TNT.
4. Due to common origin of these two celestial bodies their average densities were equal about of 570 kg/m^3.
5. This mathematical model may also be used for calculating the characteristics of other celestial bodies entering the Earth atmosphere to replace guesses, assumptions and myths by scientific data.

References

1. Yu. I. Lobanovsky – Parameters of Chelyabinsk and Tunguska Objects and their Explosion Modes. *Arxiv.org*, 08.07.2013, pp. 1–13 // http://arxiv.org/abs/1307.1967
2. R. Marcus, H. J. Melosh, G. Collins – Earth Impact Effects Program. *Imperial College (London), Purdue University*, 2004 // http://impact.ese.ic.ac.uk/cgi-bin/crater.cgi?dist=20&diam=17&pdens=1000&pdens_select=0&vel=18&theta=30&tdens=&tdens_select=3000
3. G. S. Collins, H. J. Melosh, R. A. Marcus – Earth Impact Effects Program: A Web-based computer program for calculating the regional environmental consequences of a meteoroid impact on Earth. *Meteoritics & Planetary Science*, **40**, no 6, 2005, pp. 817–840 // http://impact.ese.ic.ac.uk/ImpactEffects/effects.pdf
4. P. G. Brown et al. – A 500-kiloton airburst over Chelyabinsk and an enhanced hazard from small impactors. *Nature Letter*, **503**, no 7475, 14.11.2013, pp. 238–241 // http://www.nature.com/nature/journal/v503/n7475/full/nature12741.html
5. O. P. Popova et al. – Chelyabinsk Airburst, Damage Assessment, Meteorite Recovery, and Characterization. *Science*, **342**, no 6162, 29.11.2013, pp. 1069–1073 // https://www.sciencemag.org/content/342/6162/1069
6. Yu. I. Lobanovsky – Refined Parameters of Chelyabinsk and Tunguska Meteoroids and their Explosion Modes. *Arxiv.org*, 28.03.2014, pp. 1–12 // http://arxiv.org/abs/1403.7282
7. Yu. I. Lobanovsky – Parameters of Chelyabinsk and Tunguska Meteoroids and Revaluation of Level of Celestial Bodies' Threat. *XL Academic Space Conference, dedicated to the memory of academician S. P. Korolev and other outstanding national scientists – pioneers of space exploration*, January, 2016 (in Russian)
8. V. I. Levantovsky – Mechanics of Space Flight in an elementary Presentation. Moscow, Science, 1980, 512 pp. (in Russian)
9. Yu. I. Lobanovsky – Chelyabinsk Meteoroid: Critique of Sources and Proving of Conclusions. *XL Academic Space Conference, dedicated to the memory of academician S. P. Korolev and other outstanding national scientists – pioneers of space exploration*, January, 2016 (in Russian)
10. Yu. I. Lobanovsky – Chelyabinsk Meteoroid: Critique of Sources and Proving of Conclusions. *Synerjetics Group*, February 15, 2016 // http://www.synerjetics.ru/article/springs_critique_eng.htm
11. L. C. Andrews – Special Functions of Mathematics for Engineers. Oxford University Press, 1998, 664 p.
12. J. Zuluaga, I. Ferrin – A Preliminary Reconstruction of the Orbit of the Chelyabinsk Meteoroid. *Arxiv.org*, 21.02.2013, pp. 1–10 // http://arxiv.org/pdf/1302.5377v1.pdf
13. G. V. Ionov – The Calculation of the Geometric Properties of the Chelyabinsk Bolide Trajectory by Car DVR Recordings. *International Scientific Conference «Asteroids and comets. Chelyabinsk event and the study of meteorite falling into the Chebarkul lake»*, June, 2013, pp. 89–90.
14. A. V. Golubaev – Main Parameters of Meteoroid Motion during the Fall of Chelyabinsk Meteorite Shower on February 15, 2013. *Solar System Research*, **49**, Issue 3, 2015, pp.147–158.
15. Atmospheric Nuclear Explosion. *Wikipedia* // http://ru.wikipedia.org/wiki/Атмосферный_ядерный_взрыв (in Russian)
16. V. V. Shuvalov – Multi-dimensional hydrodynamic code SOVA for interfacial flows: Application to thermal layer effect. *Shock Waves*, **9**, 1999, pp. 381–390.
17. Ice Palace «Ural Lightning» is officially opened today in Chelyabinsk. *Спорт@mail.ru*, 20.09.2013 // http://sport.mail.ru/news/skating/14851524/ (in Russian)
18. V. V. Keldysh, Yu. I. Lobanovsky – Some Features of three-dimensional Flows with bridge-like Shocks. *Proceedings of TsAGI*, **7**, N 5, 1976, pp. 1–8 // http:// cyberleninka.ru/article/n/nekotorye-osobennosti-prostranstvennyh-techeniy-s-mostoobraznymi-skachkami-uplotneniya (in Russian)
19. Fundamentals of Gas Dynamics, H. W. Emmons, ed. *Princeton University Press*: Princeton NJ, 1958, 784 p.

20. Atomic bombings of Hiroshima and Nagasaki. *Wikipedia* // http://en.wikipedia.org/wiki/Atomic_bombings_of_Hiroshima_and_Nagasaki
21. Halifax Explosion. *Wikipedia* // http://en.wikipedia.org/wiki/Halifax_Explosion
22. Texas City disaster. *Wikipedia* // http://en.wikipedia.org/wiki/Texas_City_disaster
23. V. Bronshtein – Tunguska Meteorite: History of Research. Selyanov AD, 2000, 312 p. (in Russian)
24. V. Bronshtein – Meteors, Meteorites, Meteoroids. Chapter 4. Tunguska Meteorite. Moscow, Science, 1987, 173 p. // http://tunguska.tsc.ru/ru/science/1/BronstenMMM/Glava4/ (in Russian)
25. A. Ben-Menahem – Source parameters of the Siberian explosion of June 30, 1908, from analysis and synthesis of seismic signals at four stations. *Physics of The Earth and Planetary Interiors*, **11**, no. 1, 1975, pp. 1–35 // http://65.54.113.26/Publication/40382391/source-parameters-of-the-siberian-explosion-of-june-30-1908-from-analysis-and-synthesis-of
26. I. Abramova – Meteorite: to see, and not to die. *Chelyabinsk State Regional Museum*. *News*, 24.10.2013 // http://www.chelmuseum.ru/news/uvidet-i-ne-umeret/?filter=past&page=2 (in Russian)
27. A. Ball, S. Gadomski et al. – An Instrument for in Situ Comet Nucleus Surface Density Profile Measurement by Gamma Ray Attenuation. *Planetary and Space Science*, **49**, 2001, pp. 961–976 // http://www.people.fas.harvard.edu/~planets/sstewart/ahrens/Papers_pdf/Seismo_2033.pdf
28. B. Davidsson, P. Gutieres – An Estimate of the Nucleus Density of Comet 19P/Borrelly. DPS 35th Meeting, 2003 // http://aas.org/archives/BAAS/v35n4/dps2003/43.htm
29. N. Samarasinha, P. Gutieres et al. – Bulk Density of Comet 9P/Tempel 1. *SAO/NASA ADS Astronomy Abstract Service*, p. 966 // http://adsabs.harvard.edu/abs/2010DPS....42.2834S
30. Tsar Bomba. *Wikipedia* // http://ru.wikipedia.org/wiki/Царь-бомба
31. Yu. I. Lobanovsky – Acoustic Methods for Evaluation of High Energy Explosions. *Arxiv.org*, 06.03.2014 // http://arxiv.org/abs/1311.6565
32. V. Svettsov, T. Potapova, V. Shuvalov – The age-old Mystery of Tunguska. *«In the World of Science»,* no 3, 2007, pp. 1–7 // http://www.smoliy.ru/view_alonetext.php?g=74&outpdf (in Russian)

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