Conversion of formulae and quantities between unit systems

Nikolai G. Lehtinen

November 4, 2010

1 cgs→SI

SI system has additional constants ε_0 and μ_0, which satisfy $c^2\varepsilon_0\mu_0 = 1$ and $\mu_0 = 4\pi\times10^{-7}$ N A^{-2} by definition. It is "rationalized" in the sense that there are no factors 4π or c in Maxwell's equations. In SI units, we define magnetic charge and related quantities according to the Ampere-meter convention (i.e., magnetic charge is measured in Ampere-meters). The magnetic dipole energy is $U = -\mathcal{M}\cdot\mathbf{B} = -\mu_0\mathcal{M}\cdot\mathbf{H}$. The electric dipole energy, in contrast, is $U = -\mathbf{d}\cdot\mathbf{E}$, where the field instead of induction is used. This means we have to put factor μ_0 in Maxwell's equations to multiply all magnetic charges and currents. The completely electric-magnetic symmetrical convention is the Weber convention (i.e., magnetic charge unit is Weber), the use of which is not recommended.

1.1 Convert all quantities

To convert a cgs formula to SI, multiply the symbols by a corresponding factor in Table 1.

The derivative quantities are obvious. E.g., volume is r^3, therefore is multiplied by 10^6. The charge density is Q/r^3 and is multiplied by $10c/10^6 = 10^{-5}c$, etc. We have also given the dimensionalities in SI system (the power of length, time, mass and charge). The quantities ε_0, μ_0 do not, of course, occur in cgs formulas and are listed only for their dimensionality.

After this, the appearing factors of 10 are changed to $(4\pi c^2\varepsilon_0)^{1/7}$ or $(4\pi/\mu_0)^{1/7}$.

Table 1: Conversion table

Physical quantity	Symbol	**factor**	L^a	T^b	M^c	Q^d	coef
Mechanical quantities							
Distance	r	10^2	1	0	0	0	
Time	t	1	0	1	0	0	
Mass	m	10^3	0	0	1	0	
Velocity	v, c	10^2	1	-1	0	0	
Force	F	10^5	1	-2	1	0	
Energy	$\mathcal{E}$	10^7	2	-2	1	0	
Energy flux	S	10^3	0	-3	1	0	
Electromagnetic quantities							
Vac. permittivity	ε_0	N/A	-3	2	-1	2	N/A
Vac. permeability	μ_0	N/A	1	0	1	-2	N/A
Charge	Q	$10c$	0	0	0	1	$1/\sqrt{4\pi\varepsilon_0}$
Electric moment	d	10^3c	1	0	0	1	”
Current	I	$10c$	0	-1	0	1	”
Charge density	ρ	$10^{-5}c$	-3	0	0	1	”
Polarization (d dens.)	P	$10^{-3}c$	-2	0	0	1	”
Current density	J	$10^{-3}c$	-2	-1	0	1	”
Electric field	E	$10^4/c$	1	-2	1	-1	$\sqrt{4\pi\varepsilon_0}$
Electric displacement	D	$4\pi \times 10^{-3}c$	-2	0	0	1	$\sqrt{4\pi/\varepsilon_0}$
Magnetic field	H	$4\pi \times 10^{-3}$	-1	-1	0	1	$\sqrt{4\pi\mu_0}$
Magnetic induction	B	10^4	0	-1	1	-1	$\sqrt{4\pi/\mu_0}$
Conductivity	σ	$10^{-7}c^2$	-3	1	-1	2	$1/(4\pi\varepsilon_0)$
Ampere-meter convention							
Magnetic moment	$\mathcal{M}$	10^3	2	-1	0	1	$\sqrt{\mu_0/(4\pi)}$
Magnetic charge	Q_m	10	1	-1	0	1	”
Magnetization ($\mathcal{M}$ dens.)	M	10^{-3}	-1	-1	0	1	”
Weber convention							
Magnetic moment	$\mathcal{M}$	$(4\pi)^{-1}10^{10}$	3	-1	1	-1	$1/\sqrt{4\pi\mu_0}$
Magnetic charge	Q_m	$(4\pi)^{-1}10^8$	2	-1	1	-1	”
Magnetization ($\mathcal{M}$ dens.)	M	$(4\pi)^{-1}10^4$	0	-1	1	-1	”

Note that to convert *values*, one should *divide* by the value in the third column, using c in SI system of units. E.g., electron charge $e = 4.8 \times 10^{-10}$ ecgs $= 4.8 \times 10^{-10}/(10 \times 3 \times 10^8)$ C $= 1.6 \times 10^{-19}$ C.

1.2 Shortcut

We use the fact that the formulas connecting mechanical quantities are the same in cgs and SI. Then we can use the following shortcut. Notice that ε_0 contains the unit of charge (Coulomb) in 2 power and μ_0 contains it in (-2) power. Then in formulas for quantities containing p-th power of Coulomb and n-th power of c in the above conversion table, change them to same multiplied by $\sqrt{\mu_0/(4\pi)}$ to p-th power, then multiply by c to n-th power. E.g., $Q \to Qc\sqrt{\mu_0/(4\pi)}$. One can also change c to $(\varepsilon_0\mu_0)^{-1/2}$. Then in the above example we have $Q \to Q/\sqrt{4\pi\varepsilon_0}$. For a quantities like D and H, we have an additional coefficient 4π (see the Table). The result is Table 3 on page 819 in Jackson, "Classical Electrodynamics", 2nd edition; or the last column in Table 1.

2 SI→cgs

To convert an SI formula to cgs:

- Change ε_0 to $1/(4\pi \times 10^{-3}c^2)$ and μ_0 to $4\pi \times 10^{-7}$. The "strange" factor $4\pi \times 10^{-3}c^2$ is just $4\pi \times 10^{-7}c^2$ in SI system.

- Change the physical quantities to the same divided by the value in the third column of the table. Note that c present in the table value must be first converted to cgs, i.e. $c \to 10^2c$.

The factors of 10 should go away after this procedure (if the units in the formula match), therefore it is not necessary to consider them from the very beginning. Thus do the same as above but drop all powers of 10. This leads to the following shortcuts:

- Mechanical units (not involving Coulombs) don't change

- Charge/current units are divided by c

- $E \to Ec$; B stays the same

- $Q_1 Q_2/(4\pi\varepsilon_0) \to Q_1 Q_2$.
- $Q/(4\pi c\varepsilon_0) \to Q$

To convert values, multiply by the value in the third column of the table in Seciton 1, taking c in SI system ($\approx 3 \times 10^8$).

3 SI→natural

The natural system of units is such in which $c = 1$ and $\hbar = 1$. Otherwise, it is like SI, except that ε_0 and μ_0 should also be changed to 1. There are no factors 4π in Maxwell's equation, like in SI. The conversion is therefore very easy.

4 cgs→natural

First, convert to SI and then to natural as in previous section. One can just convert to SI and simultaneously put $c = \hbar = \varepsilon_0 = \mu_0 = 1$.

5 natural→SI

Quantities of dimensionality $L^a T^b M^c Q^d$ (they have dimensionality T^{a+b-c} in the natural system, since in it $L = T$, $M = 1/T$, and $Q = 1$) are divided by $c^{a-2c+d/2}\hbar^{c+d/2}\varepsilon_0^{d/2}$. Examples:

1. Energy $[\mathcal{E}] = L^2 T^{-2} M^1 Q^0$, therefore it is divided by $c^0 \hbar^1 \varepsilon_0^0 = \hbar$. From $\mathcal{E} = \omega$ we get $\mathcal{E} = \hbar\omega$.
2. Q is divided by $\sqrt{\varepsilon_0 \hbar c}$. Thus the fine structure constant $e^2/(4\pi)$ becomes $e^2/(4\pi\varepsilon_0\hbar c)$.

6 natural→cgs

First convert to SI, then to cgs.