SHORT CUTS FOR ANALYZING PLANETARY GEARING

The analysis of planetary gearing often is unwieldy because components can be connected in various combinations of drive inputs and outputs. Moreover, the planet gears rotate about a moving axis. To simplify matters, a governing kinematic equation can be developed that not only yields all possible velocity ratios, but also provides a basis for evaluating torque and efficiency.

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THE VELOCITY relationships among components in planetary drives often prove difficult to evaluate because planet gears can rotate about a moving axis. Moreover, planetary components usually can be connected in various combinations to provide a number of velocity-reduction ratios.

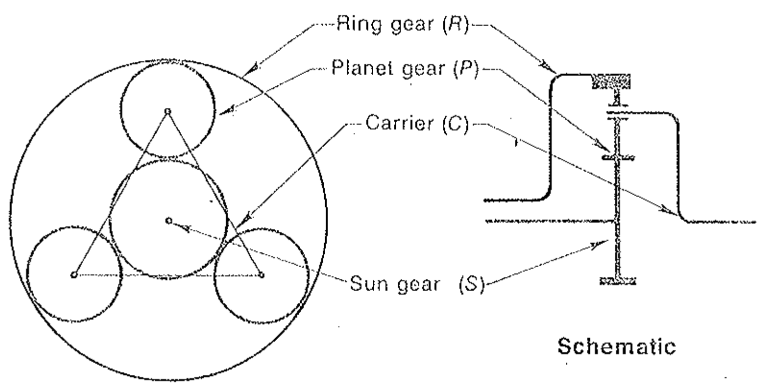
It is not widely realized that all the possible combinations or connections for a given planetary configuration need not be analyzed to determine all velocity ratios. For any given planetary configuration, however complex, a single governing kinematic equation can be developed, which then is manipulated to determine all possible velocity ratios. In addition, by relying on certain assumptions, both drive torque and efficiency can be determined from the data on planetary component velocities.

Angular velocities

The governing kinematic equation for any planetary configuration defines the angular

One configuration, many combinations

A simple planetary with a single carrier, sun gear, and ring gear can be connected in six different ways, each with a unique velocity-reduction ratio. The velocity of only two drive components can be controlled independently, thereby providing two inputs. The drive output velocity is a function of these two input velocities. Usually, one drive input is fixed stationary for speed-reduction applications.



Simple planetary configuration

Schematic

Drive components			
Input	Fixed input	Output	Velocity-reduction ratio*
S	C	R	$-\alpha$
R	C	S	$-1/\alpha$
C	S	R	$\alpha/(1+\alpha)$
R	S	C	$(1+\alpha)/\alpha$
S	R	C	$(1+\alpha)$
C	R	S	$1 (1+\alpha)$

*α = Number of ring-gear teeth/Number of sun-gear teeth = R/S

velocity of a drive component to be a function of the angular velocities of all the other drive components controlled independently. Generally, the number of velocity terms in this expression is one greater than the number of independently controlled components. Independent control simply means that any velocity can be applied freely to the component. For a drive with N independently controlled components,

$$\omega_{E1} = k_1\omega_{E2} + k_2\omega_{E3} + \ldots + k_N\omega_{E(N+1)} \quad (1)$$

where ω_{E1} = angular velocity of a given component $E1$; ω_{E2}, ω_{E3}, . . . $\omega_{E(N+1)}$ = angular velocities of the N independently controlled components; and k_1, k_2, . . . k_N = constants that depend on planetary drive geometry. The constants are evaluated by analyzing certain drive combinations that fix components at zero angular velocity.

The method of developing a governing kinematic equation for any planetary drive can be best explained by considering the case of a simple planetary with a single carrier, sun gear, and ring gear — all of which can be connected as drive inputs and outputs. If the angular velocities of any two of these three drive components are specified (that is, two components are selected as inputs), then the velocity of the third component (the output) is also established. From Equation 1, this relationship can be expressed as

$$\omega_{E1} = k_1\omega_{E2} + k_2\omega_{E3} \quad (2)$$

Although the simple planetary can be connected six different ways, all six connections need not be analyzed to develop a governing kinematic equation. Any drive connection can be considered. For instance, let the ring gear and carrier be the drive inputs, thereby establishing the sun gear as the drive output. Then from Equation 2,

$$\omega_S = k_1\omega_R + k_2\omega_C \quad (3)$$

where ω_S, ω_R, and ω_C are the absolute angular velocities of the sun gear, ring gear, and carrier.

Constant k_1 can be evaluated by analyzing a drive connection with zero carrier velocity. Thus, given that $\omega_C = 0$, Equation 3 becomes

$$k_1 = \omega_S/\omega_R$$

From the geometry, it can be shown that

$$\omega_S = (P/S)(-\omega_P)$$

$$\omega_R = (P/R)(\omega_P)$$

where ω_P = planet angular velocity and S, R, and P are the gear radii or numbers of teeth on the sun, ring, and planet. It follows that

$$k_1 = \omega_S/\omega_R = \frac{(P/S)(-\omega_P)}{(P/R)(\omega_P)} = -R/S$$

The remaining constant k_2 is evaluated by considering a drive connection that fixes the ring at zero angular velocity. Thus, given that $\omega_R = 0$, Equation 3 can be arranged as

$$k_2 = \omega_S/\omega_C$$

Because the carrier is in motion for this case, k_2 cannot be evaluated directly from gear geometry. Instead, the well-known tabular method of

Applying kinematic equations to complex planetaries

Problem: Determine the governing kinematic equation for a load-dividing double planetary.

Solution: The easiest way to develop a governing kinematic equation is by treating the drive configuration as two separate planetaries coupled in tandem. From the drive schematic, it can be seen that components $C1$ and $R2$ are coupled so their angular velocities are identical. Moreover, if α_1 and α_2 are the ratios of ring to sun gear radii for the two planetaries, then the governing kinematic equation for each planetary is

$$\omega_{S1} = -\alpha_1\omega_{R1} + [(1+\alpha_1)\omega_{C1}]$$

$$\omega_{S2} = -\alpha_2\omega_{R2} + [(1+\alpha_2)\omega_{C2}]$$

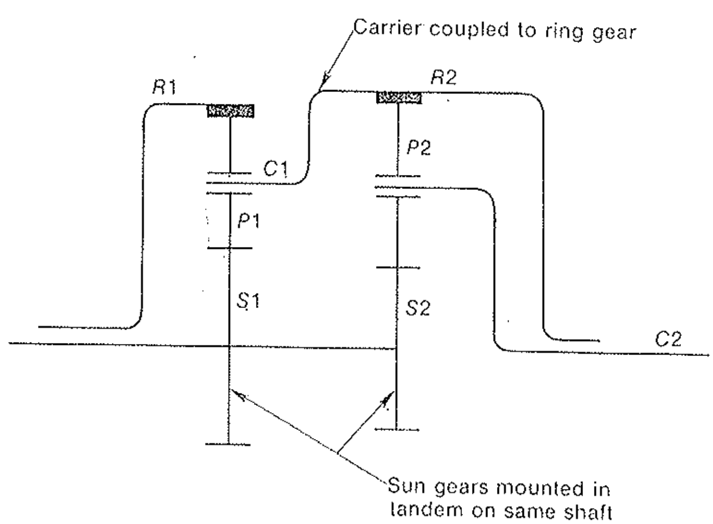
$$= -\alpha_2\omega_{C1} + [(1+\alpha_2)\omega_{C2}]$$

The above two equations can be solved for ω_{C1} and set equal to one another. The resulting expression then can be reduced to

$$(1+\alpha_1+\alpha_2)\omega_{S1} = (-\alpha_1\alpha_2)\omega_{R1} + [(1+\alpha_1)(1+\alpha_2)\omega_{C2}]$$

which is the governing kinematic equation for the double planetary drive.

Load-dividing double planetary



superposition from kinematics must be used.

This method essentially converts the drive into a fixed-axis geartrain for purposes of analysis. Typical results of applying the tabular method are shown here. Note that the entries in the last row of the table have meaning only relative to each other. Thus, for one revolution of the carrier, the sun rotates $1 + (R/S)$ revolutions. In turn, the angular velocities are related as

$$\omega_S/\omega_C = [1 + (R/S)]/1$$

which means that

$$k_2 = \omega_S/\omega_C = 1 + (R/S)$$

If α is defined equal to R/S, then the governing kinematic equation for the simple planetary drive is

$$\omega_S = k_1\omega_R + k_2\omega_C$$
$$= (-R/S)\omega_R + [1 + (R/S)]\omega_C$$
$$= -\alpha\omega_R + [(1 + \alpha)\omega_C] \quad (4)$$

This expression can be used to evaluate the velocity ratios across the planetary drive for any possible choice of fixed component. (See *One configuration, many combinations*.) Moreover, Equation 4 applies for drive connections where components either rotate at different velocities or are locked together.

For a complete description of drive velocities, the velocity of the idlers or planets must be known as well. A general velocity expression also can be written for the planets in terms of ω_R and ω_C, as was done for the sun gear. For the planet,

$$\omega_P = k_3\omega_R + k_4\omega_C \quad (5)$$

For the connection where $\omega_C = 0$, the constant k_3 is

$$k_3 = \omega_P/\omega_R$$

From drive geometry, it can be seen that the gear radii are related so that

$$R = 2P + S$$

Given the above expression for R, it can be shown that

$$k_3 = \omega_P/\omega_R = 2\alpha/(\alpha - 1)$$

Relative angular velocities for planetary with fixed ring gear

	Planet components			
Procedure	Ring gear	Planet gear	Sun gear	Carrier
1. Assume all gears locked together. Rotate carrier one revolution clockwise.	+1	+1	+1	+1
2. Fix the carrier stationary. Rotate the ring gear one revolution counter-clockwise.	−1	$-R/P$	$+R/S$	0
3. Assuming superposition, add the resulting revolutions from the first two steps.	0	$1 - (R/P)$	$1 + (R/S)$	1

R, S, and *P* denote gear radii or number of teeth for the ring, sun, and planet gears.

Next, for the drive connection where $\omega_R = 0$, constant k_4 is

$$k_4 = \omega_P/\omega_C$$

From the table of relative angular velocities for the fixed-ring drive, it is determined that

$$\omega_P/\omega_C = 1 - (R/P)$$

and that

$$k_4 = \omega_P/\omega_C = 1 - (R/P)$$
$$= (1 + \alpha)/(1 - \alpha)$$

Finally, the absolute velocity of the planets for any possible choice of fixed component can be summarized as

$$\omega_P = [2\alpha/(\alpha - 1)]\omega_R + [(1 + \alpha)/(1 - \alpha)]\omega_C \quad (6)$$

Taken together, Equations 4 and 6 provide a complete description of the angular velocities for a simple planetary. Similarly, comparable expressions can be developed for more complex planetary configurations.

Torque

Unlike the case for the equations for angular velocity, it is impossible to develop similar expressions that yield component torques for every possible drive connection. However, there are several rules that can be applied to aid torque analysis of planetaries.

For instance, the ratio of ring torque to sun torque in a simple planetary is equal to the ratio of the ring and sun gear radii. Moreover, the torque ratio for rotating input and output drive components with external torque is equal to the reciprocal of the velocity ratio.

Generally, it can be assumed that the algebraic sign of the velocity ratio also applies to the torque ratio if the drive input and output shafts are parallel and point in opposite directions. This assumption implies that the Right Hand Rule can be used to establish a sign convention for torque sense. Therefore, the fingers of the right hand indicate a positive torque sense if the thumb points in the shaft direction.

Finally, the reaction torque of a drive can be determined simply by choosing a positive torque direction and summing all the external torques acting on the drive system so that

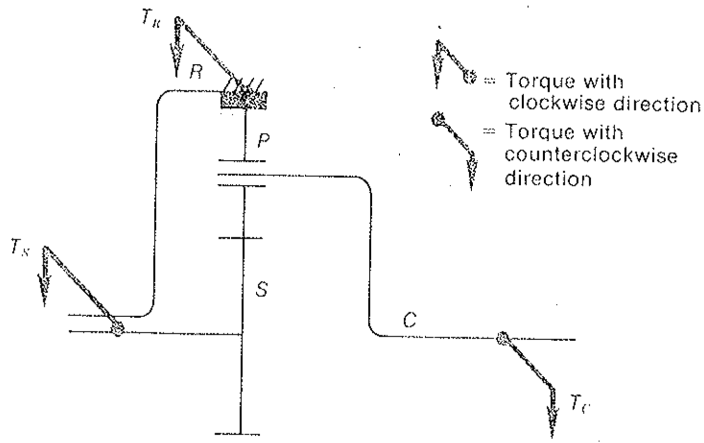
$$\Sigma T_{external} = 0$$

Efficiency

Given data on component velocities and torques, drive efficiency also can be evaluated. Most analyses of efficiency consider the concept of power flow whereby gear mesh losses occur at some constant percentage of the power transferred across each mesh. For instance, a parallel-shaft geartrain with four consecutive meshes will

Evaluating torque from velocity data

Planetary with fixed ring gear



Problem: Determine the torque ratio and reaction torque for a simple planetary with a fixed ring gear.
Solution: From Equation 4, the velocity ratio for this drive is

$$\omega_S/\omega_C = (1 + \alpha)$$

when $\omega_R = 0$, ω_S = drive input velocity, and ω_C = drive output velocity. Because the sun and carrier both rotate and have external torque, the torque ratio is equal to the reciprocal of the velocity ratio. Thus,

$$T_C/T_S = \omega_S/\omega_C = 1 + \alpha$$

Moreover, because the velocity ratio is positive, the input and output torques are also positive according to the Right Hand Rule. Finally, the reaction torque T_R acting at the ring gear and housing is

$$\Sigma T_{external} = 0$$

$$T_S - T_C + T_R = 0$$

$$T_S - [(1 + \alpha)T_S] + T_R = 0$$

$$T_R = \alpha T_S$$

have an overall efficiency of $(0.99)^4 = 0.961$ or 96.1%, assuming a 1% loss at each mesh.

Unfortunately, the application of power flow concepts to planetary drives usually requires difficult-to-derive mathematical expressions. Generally, it is simpler to just add up the assumed losses at each mesh and then compute the efficiency. This simplication can be applied to either parallel-shaft or planetary gearing. For instance, consider the four-mesh parallel-shaft drive. A 1% loss at the four meshes gives a system efficiency of $[1 - (4 \times 0.01)] = 0.96$ or 96.0%. The difference between the results of both efficiency-prediction methods is essentially insignificant. In fact, an unrealistically high power loss of 5% must be assumed for a drive with eight consecutive meshes before the difference in estimated efficiency approaches 10%.

Therefore, the following simplified procedure for estimating efficiency can be used for most drives with negligible impact on accuracy. First, the angular velocity and torque of one gear in each mesh must be determined to compute the transmitted power. Then, a certain percentage of transmitted power is attributed to mesh losses. For instance, a 1% loss can be assumed for internal meshes and a 2% loss for external meshes. Finally, the computed losses then are summed and subtracted from the input power to determine the overall efficiency.

The procedure is quite straightforward if the actual velocity of tooth engagement, which for planetaries is the velocity relative to the carrier, is used for determining the transmitted power losses. Also, absolute values must be used for the torque and velocity quantities so as to avoid the possibility of negative power losses, resulting from algebra inherent in the power loss equations. MD

Estimating efficiency from velocity data

Problem: Determine the efficiency of a fixed-ring simple planetary.

Solution: Assuming a 1% loss at the internal mesh between the ring and planet and a 2% loss at the external mesh between the planet and sun, the drive power losses can be estimated as

$$L = 0.01\left|(\omega_R - \omega_C)T_R\right| + 0.02\left|(\omega_S - \omega_C)T_S\right|$$

where the absolute values for power losses guarantee positive quantities.

Also, the gear velocities are determined relative to the carrier. Given that for a fixed-ring planetary the torque ratio is $T_C/T_S = 1 + \alpha$ and that the reaction torque is $T_R = \alpha T_S$, it can be shown that

$$L = (0.03\alpha\omega_C T_C)/(1 + \alpha)$$

Thus, the drive efficiency η is

$$\eta = (\omega_C T_C - L)/(\omega_C T_C) = (1 + 0.97\alpha)/(1 + \alpha)$$

where $\omega_C T_C$ is the input power.