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# Behaviour of Masonry Vaults and Domes: Geometrical Considerations

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ABSTRACT: In historic buildings, constructed of stone and brick masonry, the forms used to span between vertical supports and enclosing walls are arches, vaults and domes of various profiles and surfaces. The geometry of these forms and their force distribution has different characteristics. Stone, brick and mortar as building materials are weak in tension and can only withstand compression. Cracks are the visible signs of damage at the weaker zones. The geometry of such curved surfaces constituting the structure has important role in safety evaluation. The deep knowledge of the geometry and surface characteristics can help predicting the possible weaker zones and implement appropriate modelling in numerical analysis.

## 1 INTRODUCTION

Most historic vaulted and domes buildings are constructed of masonry blocks as stone and brick that are not resistant to tension and can only withstand compression. Due to the high ratio between the specific weight and the bearing capacity, masonry structures bear loads in massive and rigid forms. Aging, increased loads, exposure to environmental factors often result in deterioration of such structures. To understand the causes and mechanism of decay, diagnostic studies are carried out. Preliminary survey, before any intervention to a historic masonry structure include historical survey, geometric survey and a detailed survey of existing cracks and other damages the structure had developed during its life. This phase of investigation generally highlights the major structural problems and gives information about the present level of safety of the building.

The knowledge on how structures actually perform under imposed loads is very important. A building's resistance to its dead load and the imposed loads depends on the geometry of the structure. Any defect on the geometry shows itself as cracks indicating the presence of possible weaker zones. It is important to establish relationship between crack pattern and the force distribution. Cracks propagate perpendicular to the tension lines where the stresses exceed the strength. So, the crack patterns may show the distribution of the forces and of the stresses on a structure. But it should be kept in mind that with the increase of the stresses, crack pattern and the original tension line may change that the crack pattern may progress to different directions.

In historic buildings, the forms used to span between vertical supports and enclosing walls have great influence on overall structural behavior and are important on determining what is required of their supports. The structural elements allowing masonry to have more free and open spaces are arches, vaults and domes. The dominant profile used for these elements are semicircle, parabola and elliptic. The load bearing capacity and deformation behavior of the surfaces generated with these profiles and distribution of forces on such variety of geometry show different characteristics and it is important to understand the surface in order to predict possible weaknesses.

## 2 THE MASONRY ARCH

All curved surfaces of vaulted and domed structures are derived from arches. So it is important to understand the arch forms and behavior. An arch is a curved member, resisting principally to compression. An arch form, with or without a keystone, can be flat, segmental, circular, elliptical, parabolic or formed by the intersection of different curves, see Fig. 1. No matter how it is formed, it transmits the loads to the two end supports by means of inclined thrusts.

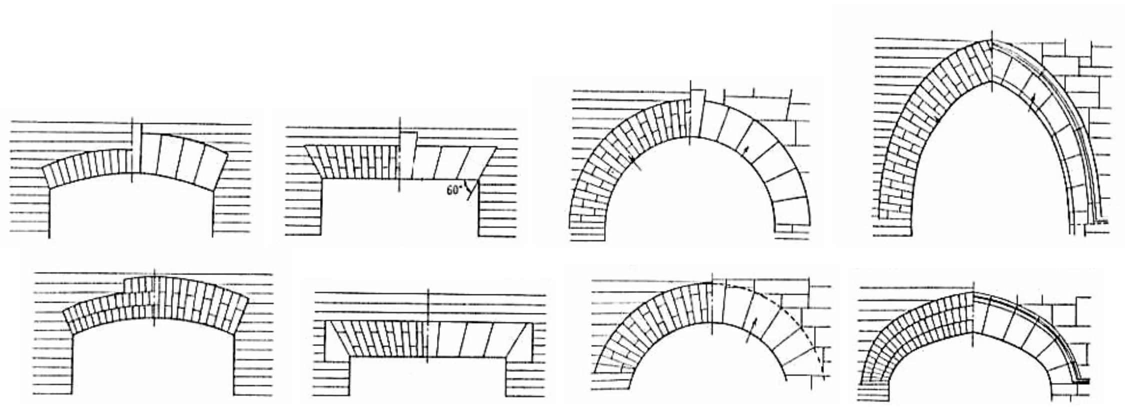



Figure 1 : Arch forms with or without a keystone.

As an arch gets lower with small rise, the compressive forces within the arch and the inclined thrust acting upon the supports will become greater. If the voussoirs and their abutting faces are deep enough and fitted perfectly, the thrust line would pass within the depth of the arch. Depending on its depth and friction in the joints between the voussoirs, arch can accommodate small inward and outward movements of the supports by producing small cracks near the quarter-points of the span and act as a hinged frame, see Fig. 2. Because of the masonry material, such hinging rotations may be irreversible and cracks may get larger in time.

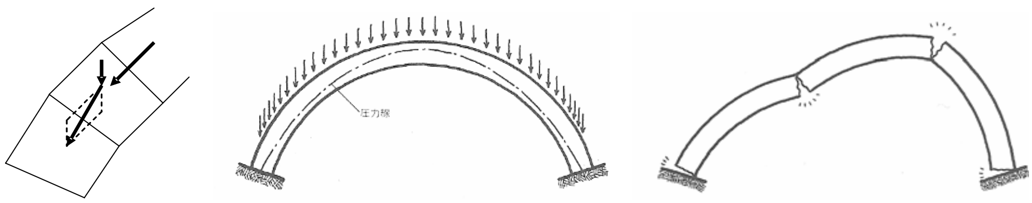


Figure 2 : Arch mechanism.

An arch always requires enough mass of wall or anchorage at its supports, to resist the horizontal thrust. Spandrel above the arch is important in distributing point loads. Typical crack patterns for an arch as longitudinal cracks near the edge of the arch, cracking of the wing walls and diagonal cracks near the sides of the arch springing towards the apex of the arch can be due to differential settlement of either supports. The bulging of the spandrels can indicate excessive vertical loading. When the opposing thrust at its supports is lost, arch collapses. Repetition of arch side by side increases the resistance to buckling under the primary compression as compared with the free standing arch.

## 3 MASONRY VAULTS

In historic masonry buildings, vaults are used as roof or floor to enclose space. The strength of a vault depends on how the units forming the vault are assembled. The construction of a vault may be of arch assemblies, each arch leaning back against the previous one or enchainment of masonry units making a continuous vault surface (Torroja 1962). In either case, due to their three dimensional extension, they have great strength in distributing imposed loads laterally.

The stress state of a vault can be defined by how the line of thrust emerges in the surface. Although the vault forms look similar, the surfaces that constitute a vault may have different characteristics.  The behavior of each surface is different whether the form is cylindrical, conic, torus, conoid or elliptic parabolic. These are singly or doubly curved surfaces.

### *3.1 Singly curved vault surface*

Cylindrical and conic surfaces are singly curved surfaces where one of the principle curvatures is a curve, like an arch, and the other is a straight line, see Fig. 3.

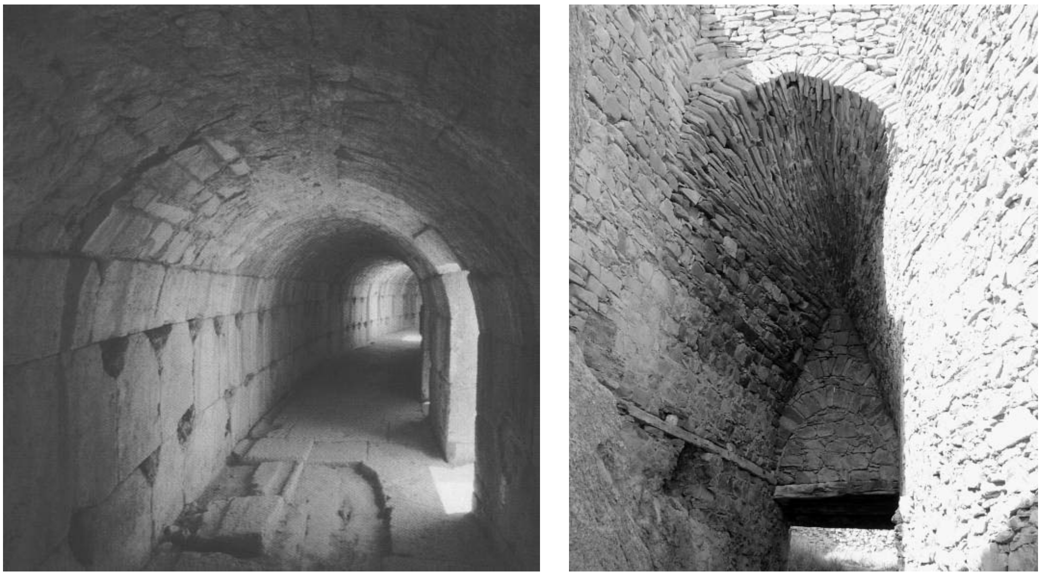


Figure 3 : Singly curved vaults.

A cylindrical surface is a translational surface where a curve profile, generetrix, moves parallel to itself, along a line, directrix. It is as if an arch extended laterally. Such a surface in masonry buildings is called simple barrel vault. The rising profile constituting the surface may be circular, parabolic, elliptic, pointed or derived from any other kind of a curve.

A Conic vault is a rotational surface generated by a generetrix of a tilted straight line moving around a fixed axis. So, one of the principle curvatures of this surface will always be a circular.

In singly curved vault surfaces, the principle stresses along the curve will always be compressive and the inclined thrusts at the edge require enough mass of supporting system. The behavior of such vault depends upon its support condition. If the vault is supported continuously along its longitudinal axis, it behaves like a series of independent arches parallel to one another and the principle stresses on the surface will be one-directional. The continuity of the vault along the longitudinal axis makes the structure resistant to bending in that direction. If the vault is supported at the ends instead of along the longitudinal edges, the vault will act like a beam. The longitudinal stresses will be compressive at the top, tensile at the base and the loads of the vault will be transmitted to the ends as a continuous shear. End walls, arches or quadrant domes are necessary to take this shear and increase the lateral rigidity.

When the supporting system moves slightly, the vault with enough thickness accommodates the new position by developing cracks. Longitudinal cracks may indicate an increase in the span while the transverse cracks along the curvature may indicate movement at the end supports, most probably due to soil settlement.

### *3.2 Doubly curved vault surfaces*

Doubly curved vaults may be a torus, an elliptic paraboloid or a conoid. Doubly curved surfaces are stiffer than singly curved forms, see Fig. 4.

Torus vault Elliptic paraboloid vault Conoid vault

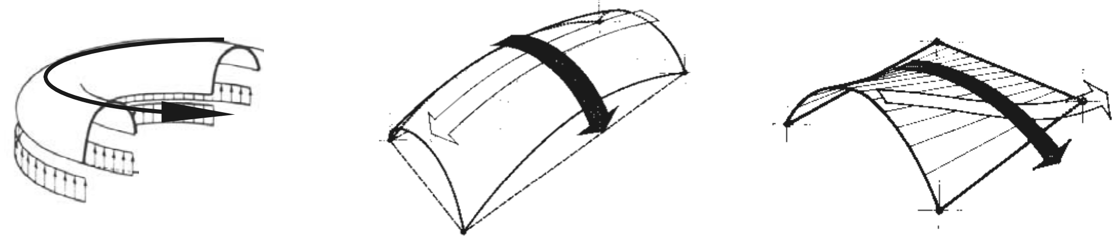


Figure 4 : Doubly curved vaults.

Torus surface is generated by rotation of a curve around an axis not in the same plane. Its rotating generetrix is in compression where its curved directrix is in compression at the top and tension at the base. The surface necessitates continuous supports to take the arch thrust. The cracks along the curvature may indicate the movement of the end supports as end walls, arches or quadrant domes. The cracks running along the curved directrix may show the movement of one of the supports at both sides of the span.

Elliptic parabolic surface is a translational surface where a curve profile moves parallel to itself, over another curve that has the centre of curvature at the same side. Such vaults are used generally to cover square spaces. The principal stresses of the surface are compression on both of its principal curvature (Arun 1983). Cracks appear due to the displacement of their supports.

Conoid is a doubly curved surface obtained by sliding a line segment with one end on a horizontal line and the other on a vertical curve having double curvature. The compression and tension lines of such surface are diagonal at each side. For this reason, any deficiency in the resistance constitutes diagonal crack patterns running from the longitudinal side towards the apex.

Vaults in general, of any shape, produce inclined forces upon its supports. This thrust has to be countered by a thick wall which diverts the lines of thrust downward, by thinner walls buttressed by perpendicular projecting vaults or walls at intervals, by an adjacent quadrant vault where a part of the load is transmitted within which the thrust line becomes progressively more vertical as it nears the ground or by ties in tension that link the walls together, see Figure 5. If the masonry vault has enough thickness, the large vault distortions and associated tilts of their supports can be accommodated in masonry.

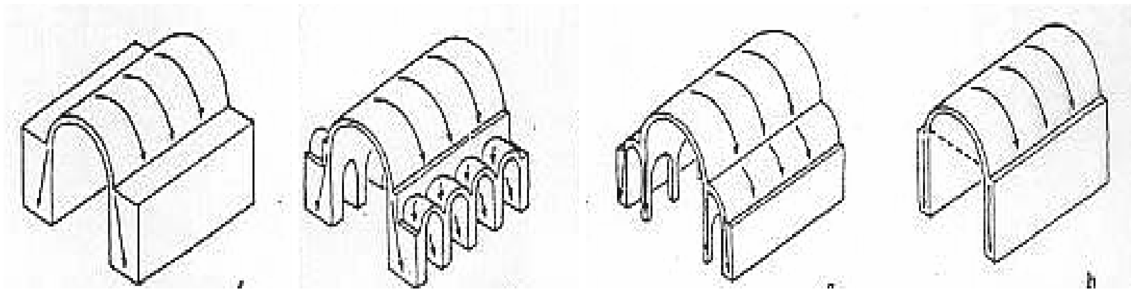
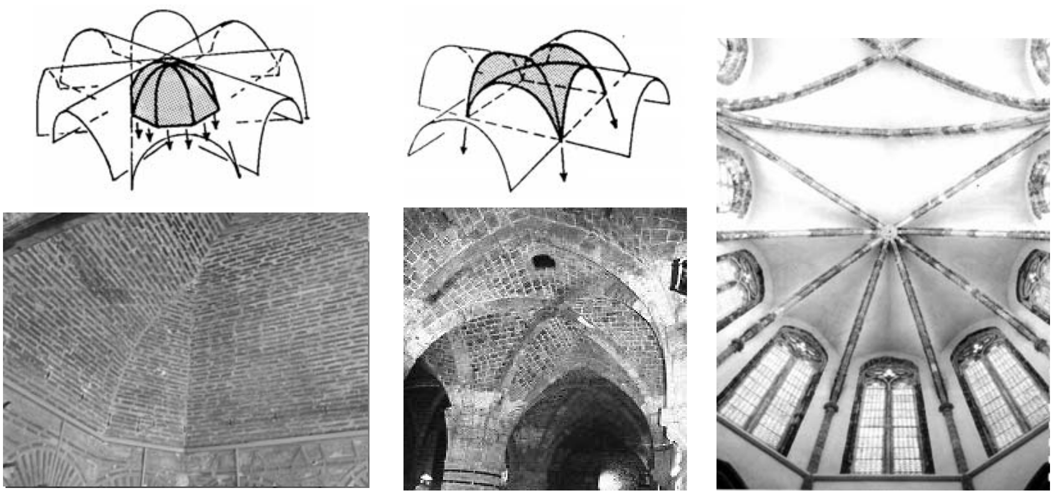


Figure 5 : Vault supports (Feilden 1982).

## 4 MASONRY DOMICAL VAULTS

To span over rectangular bays, the vault domes or rotational surfaced domes were used in historical structures. The vault domes were obtained with the intersection of vaults as cloister vault and groin vault or with ribbed domes, see Fig. 6.



Cloister Vault Groin Vault Ribbed Vault

Figure 6 : Domical Vaults.

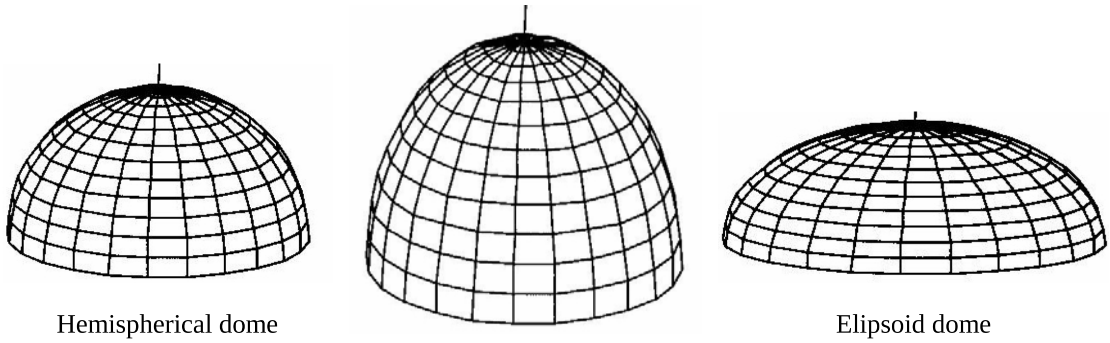
Cloister vault is formed by the intersection of two or more vaults forming a ridge at the intersection. Composition of a cloister vault may be of one of the vault surfaces or may be composed of different forms. No matter how it is composed, each surface carries its own characteristic. Dome of Sts Sergios and Bacchos (527-536 AD) in Istanbul is a type of cloister vault composed of 16 slices formed with an alternating lay of cylindrical and elliptic parabolic surfaces (Arun and Aköz 2003). Crack development in the parabolic surface may be due to the movement of the supports. Such domical vaults necessitate a continuous wall support. If the vault constituting the dome is cylindrical, an arched opening at the base can encounter the tension of the longitudinal stresses of the vault. When cloister vaults are built in series, one after the other, the components of the thrusts are cancelled out by equal thrusts in adjacent bays except the two ends of the series.

Groin vault is obtained by the intersection of two or more cylindrical vaults forming diagonal arches over the space to be covered. So, instead of continuous boundaries, they have widely spaced corner supports. The open periphery of the groin vault necessitates a stiffener as an arch or a quadrant dome. The arch action of each barrel vault brings all the loads as compressive forces down to the springings with an outward thrust at the supports. Buttressing forces are required to stabilize these diagonal ribs. When they are aligned one after another, the components of the thrusts are cancelled out by equal thrusts in adjacent bays except the two ends.

The ribbed vaults with vault webs in between also bring all loads to separated points. The ribs that stiffen and carry the web vaults act as a free standing arch. The vault web between the adjacent ribs behaves as groin vaults. The web vault's form may be of any vault surface.

## 5 MASONRY DOMES

Rotational domes are surfaces derived by rotating a meridian curve around a fixed vertical axis. A variety of forms can be obtained by rotating a circular arch, a half-ellipse, a parabolic sector or any other kind of a curve around the vertical axis. The surface generated by the rotation of semi circle makes hemispherical, parabola makes paraboloid and ellipse makes elipsoid domes, see Fig. 7. Because their horizontal sections are all circles, parallels, these are often called circular domes. These doubly curved surfaces have one set of principal stresses radially from crown to base, and another set circumferentially.



Hemispherical dome Elipsoid dome

Paraboloid dome

Figure 7 : Rotational Domes.

Under gravitational loads, the meridianal stresses in hemispherical domes are all in compression while the ring stresses are compressive in the upper part down to the $51.8^0$ from the top and tensile below this angle. The outward thrust of meridians and the tensile stresses of the parallels at the bottom require buttressing around the periphery. The arching action of the dome is usually stiffened by thickening the masonry lower down. The circumferential tensile stresses are usually countered by continuous boundaries as walls or closely spaced columns tied with small arches changing the tensile ring forces at the base into compression.

Principal stresses of a paraboloid dome are compression both radially and circumferentially. Due to the good rise of their curvature, such domes are stiffer than the flat hemispheric and ellipsoid domes.

In rotational domes, cracks depend on how the supports move under the thrust. Radial cracks around the tensile stress zone at the bottom may be because of arching action. Circumferential cracks may be due to the movement of the supports.

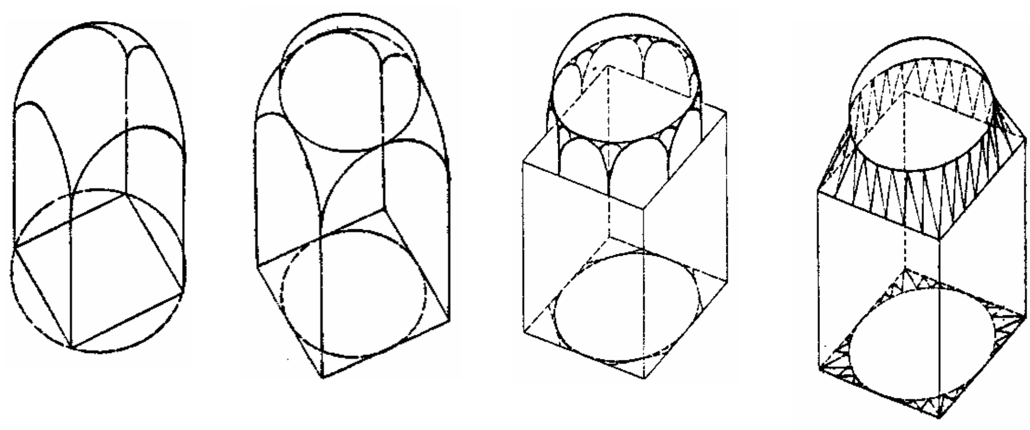
All rotational domes end up with circular base. When spanning over a square or polygonal plan, either pendentive dome or transition elements were used to construct such circular based domes over, see Fig. 8. The transition elements may be of many forms as pendentives, squinches or corbelled triangles. All forms involve some change in the direction of the resultant thrusts from the dome and add something to resist the total outward thrust of the dome.

When the hemispherical dome is cut with vertical planes to fit into a square plan, pendentive dome is obtained. Here, the tensile forces have to be resisted by outer walls or a next domical vault. In an eye inspection, such domes are easily confused with elliptic parabolic vaults. Clear identification of the surface is necessary for the elliptic parabolic vault contains no tensile forces in the surface.

Pendentive is a triangle segment of masonry sphere used as transition element. The masonry pendentive dome is stable enough to make openings on the top without any collapse allowing the construction of a dome with a smaller diameter equaling to the edge of the square plan. St Sophia is the first building where the pendentives were adopted as transition element. The pendentives necessitate buttresses as arches, vaults or quadrant domes.

Squinch is constructed as multiple arches or lintels one over another at the corner of the rectangular bays. Most squinch forms are closer to pendentives in their overall surface geometry and contain pendentive form within their thickness.

Masonry corbelling as a transition element is formed by horizontal lay of bricks from the wall of a square hall. Till it gets to the circular base of a dome, each successive layers project out, usually less than a quarter of a brick length, forming triangle surfaces. Corbelling triangle wall elements as transition element were often used in Seljuk and Ottoman monuments.



Pendentive dome Pendentive Squinch Corbelled triangles

Figure 8 : Forms to span rectangular plans.

## 6 CONCLUSIONS

For safety evaluation of historic masonry structures, the geometry of the elements constituting the structure is important. The variety of different forms used to span between vertical supports and enclosing walls are of basic surfaces. When a space is covered by forms using combination of these surfaces, it is important to identify these basic surfaces for each surface carry its own characteristics. It is important to understand the force distribution of these forms in order to predict possible weaknesses during preliminary survey of a damaged masonry building.

Deep knowledge of the geometry of the structure and of its elements can also help implementing appropriate three-dimensional geometry modeling in the analysis of historical masonry structures.

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