Apportionment of Seats to *Althingi*, the Icelandic Parliament

Analysis of the Elections
on
May 10, 2003,
May 12, 2007,
April 25, 2009
and April 27, 2013

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Introduction

1. Short history of the Icelandic election process

A Royal Decree, issued by King Christian VIII of Denmark on March 8, 1843, declared that Iceland should have its own consultative assembly, which was to be known as the Althingi. It was to be composed of 20 popularly elected representatives elected in single-member constituencies.

All the way up to 1959 the size of the Parliament and its election process changed gradually. At the end of this period there were 52 MPs, 21 still elected in single-member constituencies, 12 in 6 double-member constituencies and finally 8 members in the Reykjavik district. In addition there were 11 adjustment seats[1] allocated to the parties on a national basis in an attempt to even out discrepancies in the proportionality. The candidates to fill these seats were chosen from the constituency lists by a rather complicated method. Only those parties that had gained at least one constituency seat were eligible for the adjustment seats. This had been so since their introduction in 1934.

These changes to the election system of the Althingi reflected the ever increasing migration from the countryside to the towns, mainly to the Reykjavik area. However, adjustment of the Election Act always lagged behind the migration process. This was causing two types of distortion to the composition of the Parliament. First, the rural constituencies were over-represented by a factor of up to ten or so. Secondly, this over-representation caused disproportionality among the parties as a certain party that drew the vast majority of its votes from the farming communities was constantly over-represented.

In 1959 drastic changes in the election clauses of the Constitution were made and a new election act passed. After these changes there were only 8 constituencies with 5 or 6 members each with the exception of Reykjavik which had 12 members. Additionally, there were 11 adjustment seats again. Thus the total number of seats was increased from 52 to 60.

Right after these amendments, in the elections in the autumn of 1959, the over-representation of the rural constituencies had been reduced somewhat down to a maximum discrepancy between two constituencies of 3:1. Once more one of the parties was over-represented but only by one seat.

This act was used up to 1983, including the elections that year. In the meantime the migration to the Southwest area continued. The discrepancy in the voting power was up to 4:1. Simultaneously the disproportionality increased giving one of the parties two extra seats on average.

The next main change in the Election Act came into force in the elections in 1987. The constituencies were not changed but some seats were transferred from the rural constituencies to Reykjavik and the surrounding constituency. The total number of seats was increased from 60 to 63, mainly to increase the number of adjustment seats

[1] Different names have been used for this type of seats in English, such as “equalization seats“ or “compensatory seats“.

from 11 to 13. The over-representation of the rural areas was brought down to a ratio of 2:1 or so in the beginning. But the speed of the migration increased once more so that in the elections of 1999 this ratio was once again up to the level of 4:1. However, the parties were proportionally fairly represented (at least those that passed the eligibility requirement for the adjustment seats) in each of the four elections in which the Act of 1987 was applied.

2. The new election system of 1999/2000

The provisions on elections in the Constitution were changed again in 1999, providing for a minimum of six and a maximum of seven constituencies reducing the number from eight. Consequently the Election Act was amended in 2000 to provide for these changes. [2].

The main change was that the constituencies which are now six were made equal in size in the sense that they had the same number of constituency seats, 9 seats each. The remaining 9 seats are adjustment seats used as before to enhance proportionality in the final allocation. The three Greater-Reykjavik constituencies have 2 of these seats each; the remaining three have 1 each.

A new provision of the Act requires the National Electoral Commission (NEC) to ensure that the discrepancy in the voting power between any two constituencies does not exceed the ratio 2:1. For this purpose NEC has the authority to relocate one or more (constituency) seat right after each election.

Adjustment seats are apportioned to those parties – and only those – that have received at least 5% of the national vote. This provision is new. The method used for assigning the adjustment seats to the individual lists was entirely revised; cf. subsection 10.

A final new provision thoroughly changes how candidates are to be selected from the individual party lists. It applies to the assignment of the seats that have already been allocated to the constituency lists to their individual candidates. Ever since the introduction of list elections in 1915, lists have been nominated in ranked order by the candidature. However, the voters have always had the right to alter this order by renumbering individual candidates or to cross out those that they do not like. What has varied from time to time is the weight of these changes. In the period 1915-1959 the power of the voters in this respect was considerable. Consequently it happened twice in that period that a candidate lost his prospective Parliament seat due to the changes made by the voters. From 1959-2000 it was rather difficult for the voters to have any influence on the rankings of the lists, apart from party primaries that were widely introduced in this period. With the new Election Act of 2000 the voters regained more or less the power they had prior to 1959.

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[2] The current Election Act no. 24/2000 can be found on the web-site http://eng.innanrikisraduneyti.is/laws-and-regulations/nr/6713

Electors and Election Results

3. Electors and seats

The four elections to the Icelandic Parliament, the *Althingi*, in this century so far, on May 10, 2003, May 12, 2007, April 25 2009 and April 27 2013, were based on a Constitutional Amendments Act (no. 77/1999) and a new Election Act (no. 24/2000) whereby the number of constituencies was reduced so that the country is now divided into six constituencies, the Northwest and Northeast, South and Southwest Constituencies and Reykjavik divided into two constituencies, Reykjavik North and Reykjavik South.

Figure 3.1 and table 3.1 show the number of seats in each constituency according to the election act. There is certain flexibility in the allocation of the seats to the constituencies; see below. As postulated by the Election Act each constituency had in the 2003 elections nine constituency seats. These seats are apportioned on the basis of the results in the constituency alone. In addition, there are nine supplementary seats, so-called "adjustment seats", which are first allocated to the parties to enhance fair proportional representation but are finally assigned to the constituencies, one or two in each of them.

Figure 3.1: Constituencies and seats as postulated by the 2000-act and used in the 2003 elections

The constituency boundaries are decided by law; however, the National Electoral Commission (NEC) is authorized to draw the boundaries between the two Reykjavik constituencies prior to each election in such a way that the number of electors is approximately the same in both.

Table 3.1:	*Number of seats as postulated by law and used in the 2003 elections*		
Constituencies	Constituency seats (in 2003)	Adjustment seats	Total number of seats
Northwest	9	1	10
Northeast	9	1	10
South	9	1	10
Southwest	9	2	11
Reykjavik South	9	2	11
Reykjavik North	9	2	11
Totals	54	9	63

If the number of voters on the electoral register represented by each Parliament seat, including adjustment seats, in any constituency is more than twice that of another constituency, the NEC shall adjust the number of constituency seats to bring this difference below this ratio. However, the number of constituency seats must never become less than six in any constituency. The re-allocation first enters into force in the elections following the one where the inequality has been identified.

Table 3.2 shows the number of registered electors per seat in the four elections 2003-2013. It shows that the condition for a re-allocation of the seats was fulfilled after two of these elections. Thus one constituency seat was transferred from the Northwest Constituency to the Southwest Constituency and again in the aftermath of the elections 2007 and 2009. The number of registered electors 2013 does not call for further changes prior to the next elections which will be held at the latest in April 2017.

Table 3.2: Registered voters on the electoral register and number of voters per seat

Constituencies	Elections 2003			Elections 2007				Elections 2009			Elections 2013			
	Reg. electors	Seats	Electors per seat	Reg. electors	Re-allo-cation	Seats (after reallo-cation)	Electors per seat	Reg. electors	Seats	Electors per seat	Reg. electors	Re-allo-cation	Seats (after reallo-cation)	Electors per seat
Northwest	21,247	10	2 125	21 126	-1	9	2 347	21 293	9	2 366	21 318	-1	8	2 665
Northeast	27,298	10	2 730	27 881		10	2 788	28 352	10	2 835	29 035		10	2 904
South	28,344	10	2 834	30 592		10	3 059	32 482	10	3 248	33 633		10	3 363
Southwest	42,812	11	3 892	54 584	1	12	4 549	58 202	12	4 850	63 125	1	13	4 856
Reykjavik S	48,842	11	4 440	43 391		11	3 945	43 747	11	3 977	45 189		11	4 108
Reykjavik N	42,761	11	3 887	43 756		11	3 978	43 767	11	3 979	45 529		11	4 139
Totals	211,304	63	3 354	221 330		63	3 513	227 843	63	3 617	237 829		63	3 775
		Maximum	4 440			Maximum	4 549		Maximum	4 850			Maximum	4 856
		Minimum	2 125			Minimum	2 347		Minimum	2 366			Minimum	2 665
		Ratio	2.090			Ratio	1.938		Ratio	2.050			Ratio	1.822

4. Election results in 2003

Six parties were represented in the 2003 elections in all constituencies. In addition there was a list with independent candidates in the South Constituency. The election results are shown in table 4.1.

Table 4.1: Number of votes in the 2003 elections									
	Pro-gressive Party	Indepen-dence Party	Liberal Party	New Force	Social demo-cratic Alliance	Inde-pendent list	Left-Green Move-ment	**Valid votes in total**	Partici-pation
Party symbol:	B	D	F	N	S	T	U		
Northwest	4 057	5 532	2 666	122	4 346		1 987	**18 710**	89.3%
Northeast	7 722	5 544	1 329	136	5 503		3 329	**23 563**	87.5%
South	5 934	7 307	2 188	166	7 426	844	1 167	**25 032**	88.9%
Southwest	6 387	16 456	2 890	399	14 029		2 671	**42 832**	88.5%
Reykjavik South	4 185	14 029	2 448	504	12 286		3 438	**36 890**	87.3%
Reykjavik North	4 199	12 833	2 002	464	13 110		3 537	**36 145**	85.5%
Votes in total	**32 484**	**61 701**	**13 523**	**1 791**	**56 700**	**844**	**16 129**	**183 172**	**87.7%**

Table 4.2 shows the same for the 2003 elections but in relative figures within each constituency and nationwide.

Table 4.2: Relative distribution of votes in the 2003 elections							
Party symbol:	B	D	F	N	S	T	U
Northwest	21.7%	29.6%	14.2%	0.7%	23.2%	-	10.6%
Northeast	32.8%	23.5%	5.6%	0.6%	23.4%	-	14.1%
South	23.7%	29.2%	8.7%	0.7%	29.7%	3.4%	4.7%
Southwest	14.9%	38.4%	6.7%	0.9%	32.8%	-	6.2%
Reykjavik South	11.3%	38.0%	6.6%	1.4%	33.3%	-	9.3%
Reykjavik North	11.6%	35.5%	5.5%	1.3%	36.3%	-	9.8%
National outcome	17.7%	33.7%	7.4%	1.0%	31.0%	0.5%	8.8%

5. Election results in 2007

In the elections 2007 two parties had left the scene, those labeled with N and T in the 2003 elections. A new movement, *The Icelandic Movement* (I), candidated for the first time. Note that the party symbol of the *Left-Green Movement* was changed from U to V (at the Movement's request).

The 2007 election results are shown in tables 5.1 and 5.2. Note that the participation in the elections 2007 declined somewhat from the traditional high participation as was the case in 2003.

Table 5.1: Number of votes in the 2007 elections								
	Pro-gressive Party	Indepen-dence Party	Liberal Party	The Icelandic Move-ment	Social demo-cratic Alliance	Left-Green Move-ment	**Valid votes in total**	Partici-pation
Party symbol:	B	D	F	I	S	V		
Northwest	3 362	5 199	2 432	255	3 793	2 855	**17 896**	86.0%
Northeast	5 726	6 522	1 378	278	4 840	4 558	**23 302**	84.8%
South	4 745	9 120	1 771	435	6 783	2 498	**25 352**	84.3%
Southwest	3 250	19 307	3 051	1 599	12 845	5 232	**45 284**	84.3%
Reykjavik South	2 081	13 846	2 385	1 680	10 234	5 065	**35 291**	82.6%
Reykjavik North	2 186	12 760	2 216	1 706	10 248	5 928	**35 044**	81.4%
Votes in total	**21 350**	**66 754**	**13 233**	**5 953**	**48 743**	**26 136**	**182 169**	**83.6%**

Table 5.2 shows the results for the 2007 elections in relative figures within each constituency as well as nationwide.

Table 5.2: Relative distribution of votes in the 2007 elections

Party symbol:	B	D	F	I	S	V
Northwest	18.8%	29.1%	13.6%	1.4%	21.2%	16.0%
Northeast	24.6%	28.0%	5.9%	1.2%	20.8%	19.6%
South	18.7%	36.0%	7.0%	1.7%	26.8%	9.9%
Southwest	7.2%	42.6%	6.7%	3.5%	28.4%	11.6%
Reykjavik South	5.9%	39.2%	6.8%	4.8%	29.0%	14.4%
Reykjavik North	6.2%	36.4%	6.3%	4.9%	29.2%	16.9%
National outcome	11.7%	36.6%	7.3%	3.3%	26.8%	14.3%

6. Election results in 2009

In the elections 2009 *The Icelandic Movement* (I) did not candidate. However, two new political movements, candidated for the first time: *The Civic Movement* (O) and *The Democratic Movement* (P).

The 2009 election results are shown in tables 6.1 and 6.2. The relatively low participation in the elections 2007 mostly returned to the previous level.

Table 6.1: Number of votes in the 2009 elections

	Progressive Party	Independence Party	Liberal Party	The Civic Movement	The Democratic Movement	Social democratic Alliance	Left-Green Movement	**Valid votes in total**	Participation
Party symbol:	B	D	F	O	P	S	V		
Northwest	3 967	4 037	929	587	66	4 001	4 018	**17 605**	88.1%
Northeast	5 905	4 079	384	690	61	5 312	6 937	**23 368**	86.3%
South	5 390	7 073	838	1 381	127	7 541	4 615	**26 965**	88.3%
Southwest	5 627	13 463	741	4 428	302	15 669	8 473	**48 703**	87.7%
Reykjavik South	3 435	8 211	700	3 076	226	11 667	8 106	**35 421**	86.3%
Reykjavik North	3 375	7 508	556	3 357	325	11 568	8 432	**35 121**	84.5%
Votes in total	**27 699**	**44 371**	**4 148**	**13 519**	**1 107**	**55 758**	**40 581**	**187 183**	**86.7%**

Table 6.2 shows the results for the 2009 elections in relative figures within each constituency as well as nationwide.

Table 6.2: Relative distribution of votes in the 2009 elections

Party symbol:	B	D	F	O	P	S	V
Northwest	22.5%	22.9%	5.3%	3.3%	0.4%	22.7%	22.8%
Northeast	25.3%	17.5%	1.6%	3.0%	0.3%	22.7%	29.7%
South	20.0%	26.2%	3.1%	5.1%	0.5%	28.0%	17.1%
Southwest	11.6%	27.6%	1.5%	9.1%	0.6%	32.2%	17.4%
Reykjavik South	9.7%	23.2%	2.0%	8.7%	0.6%	32.9%	22.9%
Reykjavik North	9.6%	21.4%	1.6%	9.6%	0.9%	32.9%	24.0%
National outcome	14.8%	23.7%	2.2%	7.2%	0.6%	29.8%	21.7%

7. Election results in 2013

Eleven political parties or movements nominated lists in all constituencies in the elections on April 27, 2013. This is a record number since the entire election system was revolutionized in 1959. In addition three local movements nominated lists in a few constituencies.

The 2013 election results are shown in tables 7.1 and 7.2.

Table 7.1: Number of votes in the 2009 elections

	Bright Future	Pro-gressive Party	Indepen-dence Party	The Right Green People's Party	The Human-ist Party	The House-holds' Party	The Rainbow Party	The Sturla Jóns-son's Party	The Iceland Democra tic Party	Provin-cial Party	The People's Front of Iceland	Social Demo-cratic Alliance	Dawn	Left-Green Move-ment	The Pirate Party	
Party symbol:	A	B	D	G	H	I	J	K	L	M	R	S	T	V	Þ	Totals
Northwest	792	6 104	4 282	208		161	774		251	326		2 122	328	1 470	537	17 355
Northeast	1 537	8 173	5 327	296		241	306		313			2 505	460	3 733	716	23 607
South	1 202	9 262	7 594	702		786	412		431			2 734	904	1 581	1 268	26 876
Southwest	4 687	10 944	15 608	925		1 838	188		1 241			6 932	1 927	3 995	2 541	50 826
Reykjavik S	3 790	5 931	9 464	575	55	1 394	161	222	1 025		54	5 007	1 163	4 279	2 179	35 299
Reykjavik N	3 576	5 759	8 180	556	71	1 287	180		1 397		64	4 994	1 073	5 488	2 407	35 032
National outcome	15 584	46 173	50 455	3 262	126	5 707	2 021	222	4 658	326	118	24 294	5 855	20 546	9 648	188 995

Observe that two of the new movements (I and T) have party symbols used by other parties in previous elections.

Table 7.2 shows the results for the 2013 elections in relative figures within each constituency as well as nationwide.

Table 7.2: *Relative distribution of votes in the 2009 elections*															
Party symbol:	A	B	D	G	H	I	J	K	L	M	R	S	T	V	Þ	Totals
Northwest	4.6%	35.2%	24.7%	1.2%		0.9%	4.5%		1.5%	1.9%		12.2%	1.9%	8.5%	3.1%	100.0%
Northeast	6.5%	34.6%	22.6%	1.3%		1.0%	1.3%		1.3%			10.6%	2.0%	15.8%	3.0%	100.0%
South	4.5%	34.5%	28.3%	2.6%		2.9%	1.5%		1.6%			10.2%	3.4%	5.9%	4.7%	100.0%
Southwest	9.2%	21.5%	30.7%	1.8%		3.6%	0.4%		2.4%			13.6%	3.8%	7.9%	5.0%	100.0%
Reykjavik S	10.7%	16.8%	26.8%	1.6%	0.2%	4.0%	0.5%	0.6%	2.9%		0.2%	14.2%	3.3%	12.1%	6.2%	100.0%
Reykjavik N	10.2%	16.4%	23.4%	1.6%	0.2%	3.7%	0.5%		4.0%		0.2%	14.3%	3.1%	15.7%	6.9%	100.0%
National outcome	8.2%	24.4%	26.7%	1.7%	0.1%	3.0%	1.1%	0.1%	2.5%	0.2%	0.1%	12.9%	3.1%	10.9%	5.1%	100.0%

The participation was exceptionally low in the elections 2013; cf. table 7.3.

Table 7.3: *Participation in the 2013 election*			
	Registered electors	*Ballots cast*	*Participation*
Northwest	21 318	17 825	83.6%
Northeast	29 035	24 227	83.4%
South	33 633	27 531	81.9%
Southwest	63 125	52 048	82.5%
Reykjavik South	45 189	36 228	80.2%
Reykjavik North	45 529	35 963	79.0%
Nationally	237 829	193 822	81.5%

Apportionment of Seats

8. Outline of the apportionment

The apportionment of seats is carried out in three main steps. In the first step the constituency seats are apportioned on the basis of the election results in each individual constituency. In the second step the adjustment seats are allocated to the eligible parties based on the national outcome in order to enhance full proportionality between the parties irrespective of different voting power in the constituencies. Furthermore, in this second step the adjustment seats are apportioned to the party lists, i.e. to the constituencies in such a way that each party gets all the seats allocated and at the same time the seats of the constituencies are filled. It is well known that such an apportionment with two sets of constraints (party and constituency constraints) is mathematically difficult to implement.[3]

This entire process will be illustrated in this chapter on the basis of the outcome of the 2003 elections.

Having apportioned all 63 Parliament seats to the party lists the third and final step in the process is to find out which candidates from these lists take the seats.

9. Apportionment of constituency seats in 2003

Constituency seats are allocated on the basis of d'Hondt's rule. In table 9.1 this process is shown for one of the constituencies, the Northwest Constituency.

[3] See e.g. Thorkell Helgason and Kurt Jörnsten: Entropy of Proportional Matrix Apportionments, Norges Handelshøyskole, working paper 4/94 (1994).

Table 9.1:	*Apportionment of constituency seats in the Northwest Constituency in 2003*						
Party symbol:	B	D	F	N	S	T	U
Votes	4057	5532	2666	122	4346		1987
	Votes divided by 1, 2 , 3 etc.						
Outcomes of the division							
	B	D	F	N	S	T	U
Votes	4057.0	5532.0	2666.0	122.0	4346.0		1987.0
divided by 2	2028.5	2766.0	1333.0	61.0	2173.0		993.5
divided by 3	1352.3	1844.0	888.7	40.7	1448.7		662.3
divided by 4	1014.3	1383.0	666.5	30.5	1086.5		496.8
divided by 5	811.4	1106.4	533.2	24.4	869.2		397.4
divided by 6	676.2	922.0	444.3	20.3	724.3		331.2
divided by 7	579.6	790.3	380.9	17.4	620.9		283.9
divided by 8	507.1	691.5	333.3	15.3	543.3		248.4
divided by 9	450.8	614.7	296.2	13.6	482.9		220.8
Highest outcomes prior to each allocation. The maximum number in each line, being the basis for each allocation, is shown in bold.							
	B	D	F	N	S	T	U
Seat 1	4057.0	**5532.0**	2666.0	122.0	4346.0		1987.0
Seat 2	4057.0	2766.0	2666.0	122.0	**4346.0**		1987.0
Seat 3	**4057.0**	2766.0	2666.0	122.0	2173.0		1987.0
Seat 4	2028.5	**2766.0**	2666.0	122.0	2173.0		1987.0
Seat 5	2028.5	1844.0	**2666.0**	122.0	2173.0		1987.0
Seat 6	2028.5	1844.0	1333.0	122.0	**2173.0**		1987.0
Seat 7	**2028.5**	1844.0	1333.0	122.0	1448.7		1987.0
Seat 8	1352.3	1844.0	1333.0	122.0	1448.7		**1987.0**
Seat 9	1352.3	**1844.0**	1333.0	122.0	1448.7		993.5

The apportionment of constituency seats is shown in total in table 9.2.

Table 9.2:	*Apportionment of the constituency seats in 2003*							
Party symbol:	B	D	F	N	S	T	U	Totals
Northwest	2	3	1	-	2	-	1	9
Northeast	4	2	-	-	2	-	1	9
South	2	3	1	-	3	-	-	9
Southwest	1	4	-	-	4	-	-	9
Reykjavik South	1	4	-	-	3	-	1	9
Reykjavik North	1	3	-	-	4	-	1	9
Constituency seats in total	11	19	2	-	18	-	4	54

10. Apportionment of adjustment seats in 2003

The apportionment of adjustment seats involves two sub-steps: First they have to be allocated to the eligible parties on the basis of the national results and secondly they must be allocated to the individual party lists in the constituencies. However, these two steps are intertwined.

Prior to this allocation process eligible parties have to be identified. Those, and only those parties, that receive at least 5% of the votes on a national basis are eligible to

acquire adjustment seats. The candidacies with the symbols N and T fall short of this threshold.[4]

The first step in the process is the allocation of the adjustment seats on the basis of the national outcome. This again is based on d'Hondt's rule. For that purpose the 54 allocated constituency seats are taken as given and the allocation of the 9 adjustment seats is continued with d'Hondt's rule.

Let us take the Progressive Party (B) as an example. They have already won 11 constituency seats. To find out the party's merits for further seats their total number of votes is divided by 11+1=12, 11+2=13, 11+3=14 etc. These quotients are called "national ranking numbers" in the election act and will be referred to under that name. These numbers in the 2003 elections are shown in table 10.1.

Table 10.1: "National ranking numbers" in the 2003-elections

National ranking numbers are calculated by dividing the national outcome by the number of apportioned constituency seats plus 1, 2, 3 etc. The 9 highest figures are shown in bold.

Party symbol:	B	D	F	S	U
National votes:	32 484	61 701	13 523	56 700	16 129
No. of constituency seats:	11	19	2	18	4
Highest ranking numbers	**2 707** 2 499	**3 085** **2 938** **2 805** 2 683	**4 508** **3 381** 2 705	**2 984** **2 835** 2 700	**3 226** 2 688

The national ranking numbers are referred to in descending order. Thus the first adjustment seat goes to the Liberal Party (F) as that party has the highest national ranking number, 13523/(2+1)=4508.

As postulated by the Election Act an adjustment seat has to be allocated to a list of the relevant party in one of the constituencies. The act postulates that the list with the relatively highest standing in a (theoretical) continuation of the apportionment of constituency seats shall receive this seat. Thus in each constituency the number of votes cast for each party list has to be divided by the number of its seats that the list has already received plus one. This outcome is then divided by the total number of eligible votes in the respective constituency. These relative outcomes are shown for all the lists in table 10.2 and are referred to as "Relative position". In those three constituencies with two adjustment seats each, the relative position of the second best candidate of each list is also shown.

[4] However, even if the law would have allowed these two candidacies to take part in the allocation process they would not have won any of the adjustment seats.

Table 10.2:		*Relative position of the candidates for adjustment seats in 2003*				
Party symbol:		B	D	F	S	U
Northwest		7.228%	7.392%	7.125%	7.743%	5.310%
Northeast		6.554%	7.843%	5.640%	7.785%	7.064%
South		7.902%	7.298%	4.370%	7.417%	4.662%
Southwest	First	7.456%	7.684%	6.747%	6.551%	6.236%
	Second	4.971%	6.403%	3.374%	5.459%	3.118%
Reykjavik South	First	5.672%	7.606%	6.636%	8.326%	4.660%
	Second	3.782%	6.338%	3.318%	6.661%	3.107%
Reykjavik North	First	5.809%	8.876%	5.539%	7.254%	4.893%
	Second	3.872%	7.101%	2.769%	6.045%	3.262%

Let us again have a look at the calculations for the Liberal Party (F) in the Northwest Constituency. According to table 4.1 it won 2666 votes in that constituency and has already received one (constituency) seat. Thus d'Hondt's quotient of its next candidate in a continuation of the constituency apportionment would be 2666/2 = 1333. This number has to be viewed relatively with respect to the total number of eligible votes cast in the constituency which is 18710; cf. again table 4.1. The crucial outcome is therefore 1333/18710 = 7.12%.

According to the column for the F-list in table 10.2 the relative position of the F-party is highest in the Northwest. Therefore, as the F-party is eligible for the first adjustment seat according to tables 10.1 and 10.4, the seat is allocated to that list of that party. Note that, according to table 10.2, there are three stronger candidates in this constituency, but they belong to other parties. The national outcome dictates that the F-party should receive this seat as its first adjustment seat.[5] Simultaneously the Northwest Constituency has now received its one and only adjustment seat. Therefore that constituency will not be taken further into account when the relative positions of party lists are considered.

[5] The current Norwegian Election Act is almost identical to the Icelandic one, except for this item. In Norway the first step of the allocation process of the adjustment seats – i.e. the calculation of the "national ranking numbers" and the corresponding allocation of seats to the parties on a nationwide basis – is carried out first. The second step – the allocation of adjustment seats to individual party lists – is based on the relative highest position of the lists nationally, not respecting the ranking of the national numbers as in current Icelandic Law. According to the Norwegian Election Law the first adjustment seat would go to the D-party in Reykjavik North as it has the highest relative position among all lists in the country, 8.88%, cf. table 10.2.

Table 10.3 shows the allocation process of the nine adjustment seats in total.

Table 10.3: Allocation of adjustment seats in 2003

Seat No.	"National ranking numbers"	Highest relative position	Party list	Constituency where allocated
1	4 508	7.125%	F	Northwest
2	3 381	6.747%	F	Southwest
3	3 226	7.064%	U	Northeast
4	3 085	8.876%	D	Reykjavik North
5	2 984	8.326%	S	Reykjavik South
6	2 938	7.684%	D	Southwest
7	2 835	7.417%	S	South
8	2 805	7.606%	D	Reykjavik South
9	2 707	5.809%	B	Reykjavik North

The first five seats are incidentally allocated to the candidates of the corresponding party with the highest relative position. In allocating the 6th seat the “national ranking numbers” demand that it shall go to party D. The list of that party with the highest relative position is in the Northeast (7.84%), but the only adjustment seat of that constituency has already been taken. The highest relative position in those constituencies still with vacant seats is in the Southwest constituency (7.68%). Therefore this allocation takes place there.

In table 10.4 this allocation of adjustment seats is shown in total and finally the combined apportionment of constituency and adjustment seats in the 2003 elections is shown in table 10.5.

Table 10.4: Apportionment of adjustment seats 2003

Party symbol:	B	D	F	S	U	Totals
Northwest	-	-	1	-	-	1
Northeast	-	-	-	-	1	1
South	-	-	-	1	-	1
Southwest	-	1	1	-	-	2
Reykjavik South	-	1	-	1	-	2
Reykjavik North	1	1	-	-	-	2
Adjustment seats in total	1	3	2	2	1	9

Table 10.5 shows the combined results of tables 10.2 and 10.4.

Table 10.5: Apportionment of seats in 2003

Party symbol:	B	D	F	N	S	T	U	Totals
Northwest	2	3	2	-	2	-	1	10
Northeast	4	2	-	-	2	-	2	10
South	2	3	1	-	4	-	-	10
Southwest	1	5	1	-	4	-	-	11
Reykjavik South	1	5	-	-	4	-	1	11
Reykjavik North	2	4	-	-	4	-	1	11
Seats in total	12	22	4	-	20	-	5	63

It should be noted that the final national assignment of seats (last row in table 10.5) shows a proportionally proper allocation of the seats, i.e. if all 63 seats were allocated

to the parties on the basis of the sum of votes from all the constituencies (applying d'Hondt's method) the allocation would be the same.

11. Apportionment of seats in the 2007 elections

Only the outcome, not the calculations, of the allocation steps will be shown in the case of the 2007, as well as in later elections; cf. tables 11.1-3.

Table 11.1: Apportionment of constituency seats in 2007

Party symbol:	B	D	F	I	S	V	Totals
Northwest	1	3	1	-	2	1	8
Northeast	2	3	-	-	2	2	9
South	2	4	-	-	2	1	9
Southwest	1	5	-	-	3	1	10
Reykjavik South	-	5	-	-	3	1	9
Reykjavik North	-	4	-	-	3	2	9
Constituency seats in total	6	24	1	-	15	8	54

Table 11.2: Apportionment of adjustment seats in 2007

Party symbol:	B	D	F	S	V	Totals
Northwest	-	-	1	-	-	1
Northeast	1	-	-	-	-	1
South	-	-	1	-	-	1
Southwest	-	1	-	1	-	2
Reykjavik South	-	-	1	-	1	2
Reykjavik North	-	-	-	2	-	2
Adjustment seats in total	1	1	3	3	1	9

Table 11.3: Apportionment of seats in 2007

Party symbol:	B	D	F	I	S	V	Totals
Northwest	1	3	2	-	2	1	9
Northeast	3	3	-	-	2	2	10
South	2	4	1	-	2	1	10
Southwest	1	6	-	-	4	1	12
Reykjavik South	-	5	1	-	3	2	11
Reykjavik North	-	4	-	-	5	2	11
Seats in total	7	25	4	-	18	9	63

As in the 2003 elections the final allocation of all 63 seats is proportionally fair given the 5%-threshold which excludes party I from sharing adjustment seats.

12. Apportionment of seats in the 2009 elections

The outcome of the allocation steps are shown in the case of the 2009 elections in tables 12.1-3.

Table 12.1:	*Apportionment of constituency seats in 2009*							
Party symbol:	B	D	F	O	P	S	V	Totals
Northwest	2	2	-	-	-	2	2	8
Northeast	2	2	-	-	-	2	3	9
South	2	3	-	-	-	3	1	9
Southwest	1	3	-	1	-	3	2	10
Reykjavik South	1	2	-	1	-	3	2	9
Reykjavik North	1	2	-	1	-	3	2	9
Constituency seats in total	9	14	-	3	-	16	12	54

Table 12.2:	*Apportionment of adjustment seats in 2009*							
Party symbol:	B	D	F	O	P	S	V	Totals
Northwest	-	-	-	-	-	-	1	1
Northeast	-	-	-	-	-	1	-	1
South	-	-	-	1	-	-	-	1
Southwest	-	1	-	-	-	1	-	2
Reykjavik South	-	1	-	-	-	1	-	2
Reykjavik North	-	-	-	-	-	1	1	2
Adjustment seats in total	-	2	-	1	-	4	2	9

Table 12.3:	*Apportionment of seats in total in 2009*							
Party symbol:	B	D	F	O	I	S	V	Totals
Northwest	2	2	-	-	-	2	3	9
Northeast	2	2	-	-	-	3	3	10
South	2	3	-	1	-	3	1	10
Southwest	1	4	-	1	-	4	2	12
Reykjavik South	1	3	-	1	-	4	2	11
Reykjavik North	1	2	-	1	-	4	3	11
Seats in total	9	16	-	4	-	20	14	63

As in both previous elections under the current Act, in 2003 and 2007, the final allocation of all 63 seats is proportionally fair, given the 5%-threshold. In the 2009 elections parties F and P are excluded from taking part in the allocation of the adjustment seats. However, even if the threshold is eliminated neither of these two parties would have gained any seats at all.

13. Apportionment of seats in the 2013 elections

Again only the outcome, not the calculations, of the allocation steps will be shown in the case of the 2013 elections. Furthermore only those parties will be shown that gained seats; cf. tables 13.1-3.

Table 13.1:	*Apportionment of constituency seats in 2013*						
Party symbol:	A	B	D	S	V	Þ	Totals
Northwest	-	-	-	-	1	-	1
Northeast	1	-	-	-	-	-	1
South	1	-	-	-	-	-	1
Southwest	-	-	1	-	-	1	2
Reykjavik South	1	-	-	-	-	1	2
Reykjavik North	-	-	-	1	-	1	2
Constituency seats in total	3	-	1	1	1	3	9

Table 13.2:	*Apportionment of adjustment seats in 2013*						
Party symbol:	A	B	D	S	V	Þ	Totals
Northwest	-	4	2	1	-	-	7
Northeast	-	4	2	1	2	-	9
South	-	4	4	1	-	-	9
Southwest	1	3	4	2	1	-	11
Reykjavik South	1	2	3	2	1	-	9
Reykjavik North	1	2	3	1	2	-	9
Adjustment seats in total	3	19	18	8	6	-	54

Table 13.3:	*Apportionment of seats in total in 2013*						
Party symbol:	A	B	D	S	V	Þ	Totals
Northwest	-	4	2	1	1	-	8
Northeast	1	4	2	1	2	-	10
South	1	4	4	1	-	-	10
Southwest	1	3	5	2	1	1	13
Reykjavik South	2	2	3	2	1	1	11
Reykjavik North	1	2	3	2	2	1	11
Seats in total	6	19	19	9	7	3	63

It should be noted that now for the first time since 1983 there is a discrepancy in the perfect proportionality in the total assignment of seats. Party B gets one more seat than justified by the national outcome. All the seats of that party are constituency seats so the adjustment seats cannot compensate for this. This extra seat of party B is at the expense of party V.

Furthermore some of the new parties are now “victims” of the 5%-threshold. These are parties G, I, L and T that would have acquired one seat each if the threshold had not existed. The other parties H, J, K, M and R would not have gotten any seats irrespective of the threshold. All these new parties received 11.8% of the total votes so these voters did not get any representation in the Parliament.

14. How seats are assigned to individual candidates

As mentioned in section 2, lists are nominated in ranked order by the candidature. However, the voters have the right to alter this order by renumbering individual candidates or to cross out those that they do not like. How this can be done is illustrated by an (constructed) example on the left side of table 14.1.

Table 14.1:		*Example of a changed ballot*			
Nomi-nation order	Re-ordering by the elector	Name of candidate The elector has crossed out one candidate	New ranking after changes made by the elector	Personal vote If no re-ranking	 After re-ranking
1		*Jón Jónsson*	2	1.00	0.75
2		~~*Anna Sigurðardóttir*~~		0.75	0.00
3	*1*	*Guðrún Magnúsdóttir*	1	0.50	1.00
4		*Pétur Guðmundsson*	3	0.25	0.50
5		*Sigríður Björnsdóttir*	Candidates in places 5, 6 etc. are ignored (see text)		
6	*2*	*Magnús Jónsson*			
...		...			

The changes made by the voter apply, according to the Election Act, only to the candidates in first places on the list equal to twice the number of seats allocated to the list (but with a minimum of three). It is assumed in the example of table 14.1 that the particular list has won 2 seats; hence the first 4 candidates are affected by the manipulations of the voter. The voter's re-ranking of the candidate in place 6 is thus ignored. The ranking of these top 4 candidates is evaluated on the basis of the *Borda*-method. The top candidate is assigned a value of 1 full so-called *personal vote*, the next one 1/4 less or 0.75 of a vote etc., since the ranking applies to 4 candidates. The personal votes of these candidates are shown in the two last columns of table 14.1, first as if the voter would not have made any changes on the ballot. The last column shows these vote-values taking care of the changes made by the voter. Note in particular that the candidate that the voter crossed out gets no part of a vote.

The personal votes thus calculated are summed up for each of the eligible candidates on the list and form the basis for the assignment of the seats acquired by the list to these candidates.

Unfortunately the only effective way for the voters to change the order of candidates is to use the "negative" method of crossing out candidates. Table 14.2 shows how a united group of voters can achieve for a candidate to be re-ranked up one place by crossing out the candidate above him and placing the favored candidate in place 1. The minimum size of this group needed to lift a candidate up one step depends on the number of seats acquired by the list and ranges from 25% down to 7.7% if the list has won six seats (which is the actual maximum in the elections in 2007 that are being considered here.)

Table 14.2:	*Minimum relative size of a group of electors of a particular list needed to lift a candidate up one place by crossing out the one above him and ranking the favored candidate as no. 1*
Number of seats allocated to the list	Portion of the voters of the list
1	25.0%
2	20.0%
3	14.3%
4	11.1%
5	9.1%
6	7.7%

15. Assignment of seats to individual candidates in the 2003 elections

The new rules of assigning seats to individual candidates, whereby the voters were given more power to influence the ranking of the candidates, were first applied in the elections 2003. On 4.8% of the ballots the ranking of the candidates was altered. Although comparative information on earlier elections is not available this ratio was definitely higher in 2003 than in any elections since 1959. However, in no instance did the changes result in re-ranking of the candidates on the lists in the 2003 elections.

16. Assignment of seats to candidates in the 2007 elections

In the elections 2007 changes made by the voters made a difference for the first time since 1946. It should, however be noted that in the period 1959-2000 the rules of calculating the personal votes were different making it very difficult for the voters to have any influence.

In the 2007 elections two candidates (incidentally) from the same party but in different constituencies dropped down one place on their lists due to ballot changes (mainly by being crossed out). However, neither of them lost their seat in Parliament.

Table 16.1 shows these personal votes in South Constituency for the nine constituency seats. Candidate no. 3 on the list of the *Independence Party* (D) was lifted up to place 2 whereas the candidate ranked by the party in place 2 dropped down to place 3. The party, i.e. its list, got 4 seats so no harm was done!

Table 16.1:				*Personal votes of candidates in South Constituency in 2007 with and without changes made by the electors*	
Allo-cation order of seats	Name of candidate	Party symbol	Nomi-nation order on the lists	Personal votes taking into account changes made by the electors	Personal votes with these changes being ignored
1	Árni M. Mathiesen	D	1	8904	9120
2	Björgvin G. Sigurðsson	S	1	6737	6783
3	Guðni Ágústsson	B	1	4700	4745
4	Kjartan Þ. Ólafsson	D	**3**	7054	6840
5	Lúðvík Bergvinsson	S	2	4958	5087
6	Árni Johnsen	D	**2**	6284	7980
7	Atli Gíslason	V	1	2493	2498
8	Bjarni Harðarson	B	2	3554	3559
9	Björk Guðjónsdóttir	D	4	5965	5700

The voters of list D enforced another switching of candidates in Reykjavik South between candidates no. 2 and 3 on that list; cf. table 16.2. Again this did not change the composition of the Parliament.

Table 16.2:				*Personal votes of candidates in the Reykjavik South in 2007 with and without changes made by the electors*	
Allo-cation order of seats	Name of candidate	Party symbol	Nomi-nation order on the lists	Personal votes taking into account changes made by the electors	Personal votes with these changes being ignored
1	Geir H. Haarde	D	1	13822	13846
2	Ingibjörg Sólrún Gísladóttir	S	1	10090	10234
3	Illugi Gunnarsson	D	**3**	11310	11077
4	Ágúst Ólafur Ágústsson	S	2	8523	8528
5	Kolbrún Halldórsdóttir	V	1	4941	5065
6	Björn Bjarnason	D	**2**	10187	12461
7	Ásta Möller	D	4	9866	9692
8	Ásta Ragnheiður Jóhannesdóttir	S	3	6826	6823
9	Birgir Ármannsson	D	5	8551	8308

17. Assignment of seats to individual candidates in the 2009 elections

Again in the 2009 elections two candidates on the party lists of the *Independence Party* (D), as in 2007, dropped down one place on their lists due to ballot changes (mainly by being crossed out). However, as in 2007, neither of them lost their seat in Parliament.

Table 17.1 shows these personal votes in the South Constituency for the nine constituency seats. Just as in 2007 candidate no. 3 on the list of the *Independence Party* (D) was lifted up to place 2 whereas the candidate ranked by the party in place 2 dropped down to place 3.

Table 17.1: Personal votes of candidates in South Constituency in 2009 with and without changes made by the electors

Allo-cation order of seats	Name of candidate	Party symbol	Nomi-nation order on the lists	Personal votes taking into account changes made by the electors	Personal votes with these changes being ignored
1	Björgvin G. Sigurðsson	S	1	6925	7541
2	Ragnheiður Elín Árnadóttir	D	1	6904	7073
3	Sigurður Ingi Jóhannsson	B	1	5363	5390
4	Atli Gíslason	V	1	4580	4615
5	Oddný G. Harðardóttir	S	2	6344	6284
6	Unnur Brá Konráðsdóttir	D	**3**	4898	4715
7	Eygló Þóra Harðardóttir	B	2	3994	4043
8	Róbert Marshall	S	3	5094	5027
9	Árni Johnsen	D	**2**	4781	5894

The voters of the list D caused another switching of candidates in Reykjavik South between candidates no. 1 and 2 on that list; cf. table 17.2. Again this did not change the composition of Parliament.

Table 17.2:				*Personal votes of candidates in Reykjavik South in 2009 with and without changes made by the electors*	
Allo-cation order of seats	Name of candidate	Party symbol	Nomi-nation order on the lists	Personal votes taking into account changes made by the electors	Personal votes with these changes being ignored
1	Össur Skarphéðinsson	S	1	10363	11667
2	Ólöf Nordal	D	**2**	6999	6843
3	Svandís Svavarsdóttir	V	1	8062	8106
4	Sigríður Ingibjörg Ingadóttir	S	2	10310	10209
5	Guðlaugur Þór Þórðarson	D	**1**	6269	8211
6	Lilja Mósesdóttir	V	2	6048	6080
7	Skúli Helgason	S	3	8807	8750
8	Vigdís Hauksdóttir	B	1	3391	3435
9	Birgitta Jónsdóttir	O	1	3018	3076

18. Assignment of seats to individual candidates in the 2013 elections

In 2013 the voters made fewer changes to the order of the candidates on the lists than both in 2007 and 2009 so that now the changes had no influence on the ranking of the candidates.

19. Magnitude of changes made by the voters on the party lists

As previously stated the new Election Act of 2000 gives the voters more power than before to implement changes in the order of names on the party lists. Table 19.1 shows an indicator of the magnitude of these changes.[6]

The table shows that the changes have been increasing in the three elections held since the new Law entered into force. Furthermore it shows that the *Independence Party* (D) is the main "victim" of changes. There are certainly several explanations, like the turmoil in the society in the wake of the collapse of the banks shortly before the last elections, increased dissatisfaction with the candidates at least within some of

[6] The „indicator" shows the number of cases where changes made on a ballot have had the effect that a candidate is moved down on the list. If this happens to two candidates they are counted as two cases. Usually re-ranking a candidate down on the list has the effect that another candidate is moved upwards. In order not to count the same case twice this is ignored. The data allows only those candidates to be counted that are (originally) filling seats that lead to seats in Parliament or to adjustment seats. Only those parties that nominated lists in all constituencies are taken into account. Finally these numbers are made comparable by evaluating them in relation to the number of votes cast for these particular parties in the respective constituency or nationwide for the respective party. (The definition of „ballot changes" has been altered slightly from an earlier version of this analysis.)

the parties or that the voters were gradually becoming aware of the increased power given to them to make changes.

Table 19.1: Ratio of changes made by voters on the lists of candidates

By constituencies	**2003**	**2007**	**2009**	**2013**
Northwest	6.6%	4.4%	5.1%	2.0%
Northeast	2.9%	2.3%	7.9%	1.2%
South	3.2%	10.7%	10.2%	3.0%
Southwest	4.7%	3.2%	12.1%	6.3%
Reykjavik South	4.9%	10.0%	21.7%	6.3%
Reykjavik North	6.3%	4.0%	15.7%	4.1%
Nationally	4.8%	5.7%	13.2%	4.4%

By parties	**B**	**D**	**S**	**V (2003 U)**	Other parties	In total
2003	2.1%	8.4%	4.4%	1.8%	0.3%	4.8%
2007	1.7%	11.3%	3.2%	1.9%	0.7%	5.7%
2009	4.3%	20.2%	17.0%	9.3%	3.5%	13.2%
2013	1.7%	8.6%	3.7%	3.2%	2.5%	4.4%

On the Quality of the Apportionments

20. Optimal assignments

The Icelandic election system is an example of an assignment problem with two sets of constraints:

- Constituency constraints: Prior to the election it is known how many seats are to be assigned within each of the six constituencies. This is the first set of constraints.
- Party constraints: Article 31 of the Icelandic Constitution states one of the main objectives of any Election Act as follows: *"[Adjustment] seats … shall be distributed … with the purpose of providing individual political parties with representation reflecting to the fullest possible extent each party's total [i.e. national] number of votes … [however, respecting the 5% threshold]."* This dictates the national assignment of the adjustment seats to the individual parties which together with the assignment of the constituency seats determines the total number of seats to be assigned to each party. This constitutes the other dimension of the sets of constraints.

Given these constraints, the problem facing the lawmakers is to construct a method to allocate seats to the individual lists (generally one list per party in each constituency) in such a way that the two sets of sums mentioned above are respected. But, of course, at the same time the assignment must in some sense be proportional to the votes of the individual lists.

Mathematically, this is a challenging problem. Balinski and Demange have set up axioms for such apportionment methods.[7] From a democratic point of view, these axioms are self-evident as minimum requirements. Nevertheless the authors can prove that there is only one solution, only one method, satisfying these axioms or requirements, given that the apportionment is to be based on a specific so-called divisor method, e.g. d'Hondt's rule.

This unique optimal method can be easily formulated in the case d'Hondt's rule is used as a basis. The solution is found by maximizing the product of votes per seat. Technically the solution can be found with a standard spreadsheet package like Excel.[8]

Table 20.1 shows the differences between the official and the optimal allocation of seats in the 2003 elections. The difference is minimal as any two different allocations can never involve less than four lists or cells in the table.

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[7] "An axiomatic approach to proportionality between matrices"; M.L. Balinski, G. Demange - Mathematics of Operations Research, 1989, Vol. 14, No. 4, November 1989.

[8] The task can be formulated as a linear programming problem. Due to the so-called unimodularity of the constraint sets the solution is always a proper integer solution. The objective function, i.e. the product, can be made linear by taking the logarithm of the product.

Table 20.1:	*Deviations of the official apportionment from the optimal one in 2003:* *+1 (-1) means that a seat should be added to (subtracted from) the official allocation to get the optimal one*							
Party symbol:	B	D	F	N	S	T	U	Sums
Northwest	2	3	2	-	2	-	1	10
Northeast	4	2	-	-	2	-	2	10
South	2+1	3	1	-	4-1	-	-	10
Southwest	1	5	1	-	4	-	-	11
Reykjavik South	1	5	-	-	4	-	1	11
Reykjavik North	2-1	4	-	-	4+1	-	1	11
Seats in total	12	22	4	-	20	-	5	63

In table 20.2 it is shown why the official allocation is improved by the four changes shown in table 20.1. The product of votes per seat is increased by 42% by the changes, showing at least an improvement, although more mathematics is needed to prove that these changes make it the optimal solution.

Table 20.2:	*Changes in votes per seat caused by the changes shown in table 20.1*				
The colored cells show votes per the last seat of the lists. Green cells refer to lists with one seat too many in the official allocation whereas the orrange ones refer to the opposite situation					
	Official allocation		Optimal allocation		
	B	S	B	S	Increase of the product of votes per seat
South	2 967	1 857	1 978	2 475	
Reykjavik North	2 100	3 278	4 199	2 622	
Product of the four relevant cells	37 902 781 266 699		53 906 177 801 528		42%

In the 2007 elections the official allocation of seats was further away from the optimal allocation than was the case in 2003 as can be seen in table 20.3.

Table 20.3:	*Deviations of the official apportionment from the optimal one in 2007:* *+1 (-1) means that a seat should be added to (subtracted from) the official allocation to get the optimal one*						
Party symbol:	B	D	F	I	S	V	Totals
Northwest	1+1	3	2-1	-	2	1	9
Northeast	3	3	0	-	2	2	10
South	2	4	1-1	-	2+1	1	10
Southwest	1-1	6	0+1	-	4	1	12
Reykjavik South	0	5	1	-	3	2	11
Reykjavik North	0	4	0+1	-	5-1	2	11
Seats in total	7	25	4	-	18	9	63

It should be noted that the optimal assignment for the elections in 2007 calls for changes of two constituency seats, not only a reshuffling of the adjustment seats. This would not be allowed given the current constitutional framework for an Election Act.

In 2009 the official allocation of seats was again closer to the optimal allocation than was the case in 2007 but not as good as in 2003 as can be seen in table 20.4.

Table 20.4:	*Deviations of the official apportionment from the optimal one in 2009:* *+1 (-1) means that a seat should be added to (subtracted from) the official allocation to get the optimal one*							
Party symbol:	B	D	F	O	P	S	V	Totals
Northwest	2	2+1	-	-	-	2	3-1	9
Northeast	2	2	-	-	-	3	3	10
South	2	3	-	1-1	-	3	1+1	10
Southwest	1	4-1	-	1+1	-	4	2	12
Reykjavik South	1	3	-	1	-	4	2	11
Reykjavik North	1	2	-	1	-	4	3	11
Seats in total	7	25	4	4	-	18	9	63

In 2013 the official allocation of the adjustment seats was identical to the optimal allocation. However, in this subsection it is being tested whether the allocation of all the seats is optimal, given the number of seats required in each constituency and already apportioned between the parties either as constituency seats or as adjustments seats in total. Here it must be borne in mind that all the seats of party B were constituency seats. In table 20.5 all the internal cells in the table are under consideration for improvements in the assignment given the boundary sums as constraints. Now, just as in 2007, the assignment can be improved by reshuffling two constituency seats, both of course involving party B.

Table 20.5:	*Deviations of the official apportionment from the optimal one in 2013:* *+1 (-1) means that a seat should be added to (subtracted from) the official allocation to get the optimal one*						
Party symbol:	A	B	D	S	V	Þ	Totals
Northwest	-	4-1	2+1	1	1	-	8
Northeast	1	4	2	1	2	-	10
South	1-1	4+1	4	1	-	-	10
Southwest	1+1	3	5-1	2	1	1	13
Reykjavik South	2	2	3	2	1	1	11
Reykjavik North	1	2	3	2	2	1	11
Seats in total	6	19	19	9	7	3	63

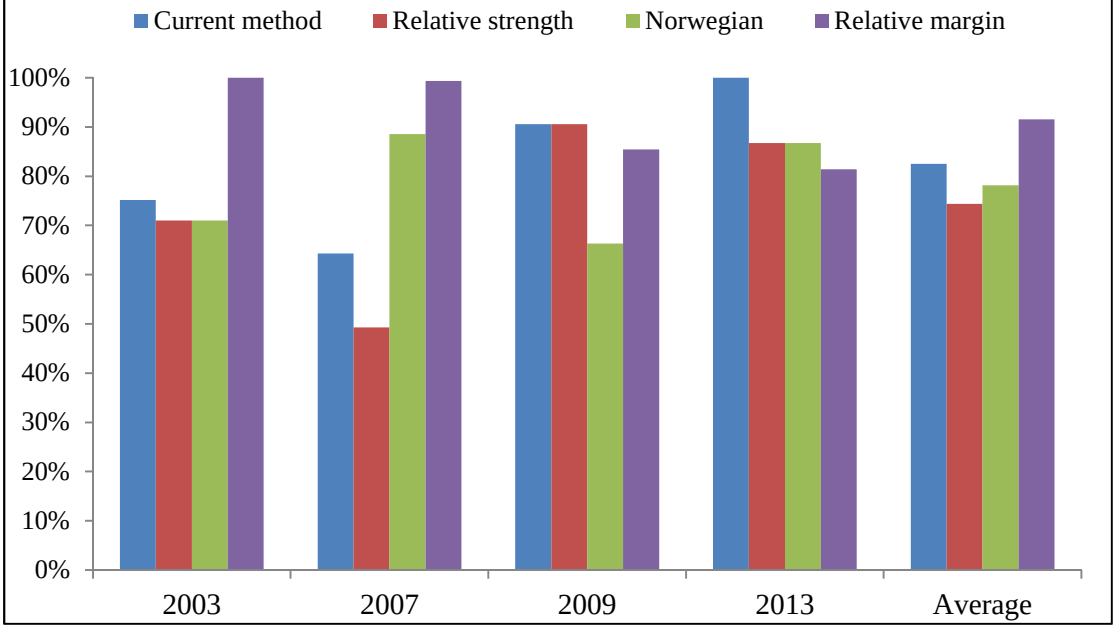
21. Comparison of different allocation methods

Several methods have been tested to see which of them seems to come closest to the optimal allocation. Graph 21.1 illustrates the outcome of these different methods relative to the optimal one. The measure used is the product of votes per seat relative to that of the optimal one which is set to 100%. In all cases only the adjustment seats are considered eligible for re-allocation. This is different from sub-section 20. Now all the seats are under consideration, also constituency seats, and not only adjustment seats.

The different methods tested are the following:

- The *current method* as described in sub-section 10 and in the appendix.
- *Relative strength*: A modification of the current Act whereby the order created by the "ranking numbers" is ignored, cf. sub-section 10.
- *Norwegian*: The adjustment seats are allocated on the basis of the same principles as used in the elections to the *Stortinget.* However everything in Iceland is based on d'Hondt's rule and not on the (modified) Sainte-Laguë rule as in Norway, cf. sub-section 10, footnote 5.
- *Relative margin*: The adjustment seats are allocated on the basis of a list's proportional margin over the second-best list in the same constituency, i.e. the number of votes behind the corresponding seat of the list relative to the same number of the next-best list.[9]

Graph 21.1: Comparison of different assignment methods.



[9] The idea, developed by the author of this analysis paper, is akin to the Vogel-approximization for the so-called transportation problem.

Appendix: Mathematical Formulation of the Apportionment of Seats according to the Current Icelandic Election Act

22. Terminology

Let SCC_i be the number of constituency seats in constituency *i* and SAC_i be the number of adjustment seats attributed to the same constituency.

The following is determined by the Election Act:

(1) $SCC_i = 9$ for all *i*
(in the 2003 election, other numbers in later elections)
and $SAC_i = 1$ for $i = 1,2,3$ and $SAC_i = 2$ for $i = 4,5,6$.

Let V_{ij} be the number of votes cast for list *j* in constituency *i*.

Furthermore let VC_i be the total number of valid votes cast in constituency *i* and VP_j be the total number of votes cast for party *j* and VN the grand total of the number of valid votes cast nationally.

Obviously, the following holds:

(2) $$\sum_j V_{ij} = VC_i\,,\ \sum_i V_{ij} = VP_j \text{ and } \sum_j VP_j = \sum_i VC_i = VN.$$

Let x_{ij} be the number of seats to be apportioned to party *j* as constituency seats in constituency *i* and y_{ij} the corresponding number of adjustment seats. The apportionment has to respect the following constraints:

(3) $$\sum_j x_{ij} = SCC_i \text{ and } \sum_j y_{ij} = SEC_i\,.$$

Later we will refer to the following sums

(4) $$Cx_j = \sum_i x_{ij}$$

i.e. the number of constituency seats apportioned to party *j* in total.

23. Apportionment of constituency seats

The constituency seats are apportioned on the basis of the d'Hondt quotients:

(5) V_{ij}/d where $d = 1, 2, 3, \ldots$

The seats are allocated within constituency *i* by picking the SCC_i largest quotients. This allocation yields values to x_{ij}.

24. Allocation of adjustment seats

First it must be decided which parties are eligible for taking part in the allocation of adjustment seats. These are the parties that exceed a 5% threshold on a national basis, i.e. that fulfill the requirement:

$$VP_j \geq 0.05VN. \qquad (6)$$

In the following, when referring to a party with index *j*, we will omit those parties that do not fulfill requirement (6).

Next the so-called national ranking numbers for those parties that are eligible according to (6) have to be calculated as follows:

$$VP_j / (Cx_j + d) \text{ where } d = 1,2,3, \ldots \qquad (7)$$

The 9 largest of these are chosen. Let $j_1, j_2, \ldots, j_9$ be the indices of these 9 largest national ranking numbers.

For later use we have to calculate the following reference numbers:

$$\frac{V_{ij} / (x_{ij} + d)}{VC_j} \text{ with } d = 1 \text{ for } i = 1, 2, 3 \text{ and } d = 1 \text{ and } 2 \text{ for } i = 4,5,6. \qquad (8)$$

The first adjustment seat is allocated to party j_1. To find out which of the lists of party j_1 should receive this seat the constituency with the highest reference number, see (8), must be identified. Label this constituency as i_1. Thus the first adjustment seat goes to the list of party j_1 in constituency i_1. This process now continues by finding the highest reference number for party j_2 taking notice of the fact that one (and possibly the only one) adjustment seat has already been allocated in constituency i_1.

This is continued until the last seat is allocated to party j_9 in the only constituency that has one adjustment seat missing.[10] [11]

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[10] It may, in exceptional cases, happen that a party is not represented in any of those constituencies which still have vacant adjustment seats. In this case that party will not get allocated any further seats and the list of national numbers is extended as needed to replace the vacancy caused by the exclusion of the particular party.

[11] The previously mentioned Norwegian Election Law differs from the Icelandic one in the following items:

- It is based on Sainte-Laguë's rule instead of that of d'Hondt.
- The "national ranking numbers" are only used to distribute the adjustment seats among the parties; not for governing the order of which these seats are allocated to the party lists as is the case in Iceland.
- The reference numbers (8) are in Norway $\frac{V_{ij} / (2x_{ij} + 1)}{VC_i / SCC_i}$. The numerator, although looking different, is principally the same as in the Icelandic system. In Norway it is the Sainte-Laguë

25. Assignment of seats to individual candidates

Let us consider a particular constituency list which has won S seats (constituency and/or adjustment seats). Let us call R the ranking number of the list. It is defined as

(9) $R = \max(3;2S)$

Let $a_{ijk} = 1$ if on the k-th ballot of the list a candidate nominated for seat i is placed directly or indirectly in seat j. Otherwise let $a_{ijk} = 0$. In this connection let a crossing out of a seat be reflected as assigning $a_{ijk} = 0$.

The sum of personal votes of candidate i is calculated as

(10) $$P_i = \sum_k \sum_{j=1}^{R} a_{ijk}(R+1-j)/R$$

These personal votes in descending order govern the final order of the candidates of the list.

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quotient for the next candidate following those that have already received a constituency seat. The denominator however is not the total number of votes as in Iceland but instead the average number of votes behind each constituency seat.

- Finally the adjustment seats are allocated to the constituency lists on the basis of the reference numbers; the national ranking numbers have no influence here, as said before.