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Observations on the violin bow and the interaction with the string*

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Abstract

The bow influences a string performance in two principal ways: (1) by adding to the sound quality, primarily set by the violin body, and (2) by serving as the tool through which tones and phrases are formed. The influence on tone quality has been attributed to a modulation of either the relative velocity between bow and string, or the bow pressure, due to the resonances of the bow. New results indicate that certain regions in the string spectra are more influenced than others by these effects. The bow resonances are an individual signature of a bow, but the resonance frequencies and their relations seem not to be directly related to quality. In this respect, the damping ratios could be more revealing. In its second role, the bow serves as the interface between the player and the string by which phrasing and articulation are controlled. This puts additional demands on the bow, known as the playing properties, for which the mass and stiffness distributions along the stick seem to be the main parameters. Some recent findings on the properties of the bow will be described and the sparse literature on the subject will be reviewed.

Introduction

The violin bow as we know it today, was developed to perfection about 150 years ago. The work by the French bow makers of that time with François Tourte in the first place, followed by others like Peccatte, Sartory, Voirin, and Lamy, created the outstanding quality mark "a good French bow," which still is equivalent to the best bow you can get according to a majority of string players. Taste changes slowly over the centuries, however, and today the classical French bows are perhaps not "stiff" enough for some players. A redesign of these old bows in a similar manner as the old Italian violins is, however, hardly possible, nor desirable.

The differences in quality between bows are undoubtedly larger for the player than for the listener. The player is "inside the loop," and this role as an active and compensating part of the control system may affect the judgments unconsciously (Woodhouse, 1992). This phenomenon was observed very clearly by Weinreich (1994) when a violinist was testing instruments. The player remarked that a particular instrument was "scratchy." A closer look showed that the violin had a poor radiation so that the output was low. Unconsciously, the player compensated for this (bad) property and forced this instrument much harder than the other violins. Not surprising that the instrument felt "scratchy"! The player was of course right in his comments, but instead of a first-stage judgment like, "this is a weak instrument," he went one step further and concluded that it was "scratchy" when played at an appropriate level. Anyway it was not a good instrument.

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* Edited version of an invited paper at International Symposium on Musical Acoustics (ISMA 95), July 2-6, 1995, Dourdan, France.

In a similar way, quality ratings of bows are a delicate affair, as most properties are present only in the silent feedback to the player, and not in the sound output. A listening panel is not particularly helpful. However, players do agree that some bows are better than others, but the personal preferences seem to be more developed for bows than for the violin itself.

A look back

It is a very reasonable statement to say that the scientific study of the violin bow was started by Schumacher (1975). In a modest paper in one of the early Catgut Newsletters from 1975, many deep insights into the properties of the bow were presented, along with a series of measurements. Later, a series of pioneering papers by McIntyre & Wood (1979), Schumacher (1979), and McIntyre et al. (1981a,b and 1983) on the simulations of the bow-string interaction brought an immense amount of new knowledge about the seemingly simple Helmholtz motion. Also the many complicated processes which can occur on the bowed string - but are seldom used or are not wanted in normal playing - were elucidated. The need for "forgiving" mechanisms which makes violin playing possible after all emphasised the importance of the torsional motion, and the effects of the bow.

In parallel with this flow of new insights into the physics of the bowed string, Pickering (1991a) performed a continuous series of measurements on strings, bows, bow hair, and rosin, using his computerised bowing laboratory. All modesty set aside, Askenfelt (1986, 1989) reported the first measurements on the actual use of the bowing parameters in violin playing during the same period.

Despite the "working paper" character of the original paper by Schumacher (1975), no new measurements on bows appeared until the early 90's (Schumacher, 1991; Askenfelt, 1992). Recently, studies of the details of the string motion, for example the sensitivity of an instrument to changes in the bowing parameters (Woodhouse, 1993a,b), the small fluctuations in fundamental period - which is known to be an important characteristic of real string sound (Schumacher, 1992; 1994) - and the influence of the finite bow width (Pitteroff, 1994), have further increased the interest for the bow and its dynamic properties*. The unison evidence from professional players as to the large influence of the bow on the string sound feeds the continued search for explanations.

Playing properties - tonal properties

It is generally agreed among musicians that the quality of a bow can be rated in two areas, dealing with; (1) the way the bow can be controlled in playing ("playing properties"), and (2) the influence of the bow on the tone quality ("tonal properties"). It seems reasonable to assume that both of these quality aspects are basically defined by the mass and stiffness distributions along the bow stick. However, rather than relying on such descriptions of the bow on a micro level (which probably would be too detailed to allow any simple conclusions), we could hope that the characteristic

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* Shortly before this article went to press a comprehensive dissertation was published which relates to several of the topics discussed in the following (Pitteroff, 1995).

properties could be summarised in a few condensed measures. For example, the playing properties could possibly be summarised in a set of parameters like the position of center of gravity, the center of percussion (with respect to an axis through the frog), and resistance to bending for a well-defined load.

The tonal properties - which is a more surprising effect of the bow - have been assumed to be connected with the normal modes of the bow. These include transverse vibrations of the bow stick (bending modes), and longitudinal resonances in the bow hair (Cremer, 1984; Schumacher, 1975). A relatively strong coupling exists between the transversal and longitudinal modes. A model of the mechanism which accounts for the influence on the string vibrations has been missing, but a modulation of the normal bow pressure, as well as a variation in bow velocity during sticking, have been proposed as plausible explanations.

In the following, some of the properties of the bow, which can be assumed to influence the playing and tonal qualities, will be discussed.

Bow stick

Wood

The wood used in high quality bows is pernambuco exclusively. The trade name pernambuco refers to the heart wood of the Brazilian tree *Guilandinia echinata* Fam. *Caesalpinia* (synonym *Caesalpinia echinata.)* Investigations of wood properties state that it is a hardwood with high density (800 - 1000 kg/m^3), high Young's modulus (15 - 22 GPa along the grain), and low damping (logarithmic decrement 0.01 - 0.03), all values from seasoned wood (Barducci & Pasqualini, 1948; Haines, 1979, Protze, 1980; Wagenführ & Scheiber, 1985).

A thorough study of pernambuco wood (Protze, 1980), shows that the main quality parameter for its use in bow making is the Young's modulus. First class pernambuco have a Young's modulus above 20 GPa and a density above 970 kg/m^3. Surprisingly enough, the internal damping is not decisive for the quality according to this study.

The supply of wood for master bows is limited. Out of a set of 300 blanks, only about 100 can be used by a master violin maker, and of these only one or two will transform into an excellent stick (nominating it for gold mounting, or the alike).

Stiffness

The stiffness distribution along the stick seems to be of primary importance to the playing properties. Players rate bows using a dimension "stiff - soft," often accompanied by more detailed judgments like "a little too stiff at the tip," or similar comments.

A measurement of the bending stiffness of the bow stick was made by applying a force at equally spaced measuring points along the stick, and observing the deflection at the midpoint (see Fig. 1). By reciprocity, this measurement also gives the load needed at the middle of the stick to cause said deflection at the measuring points. The bow was supported at the tip and frog, and measured under different hair tensions, and without hair as well.

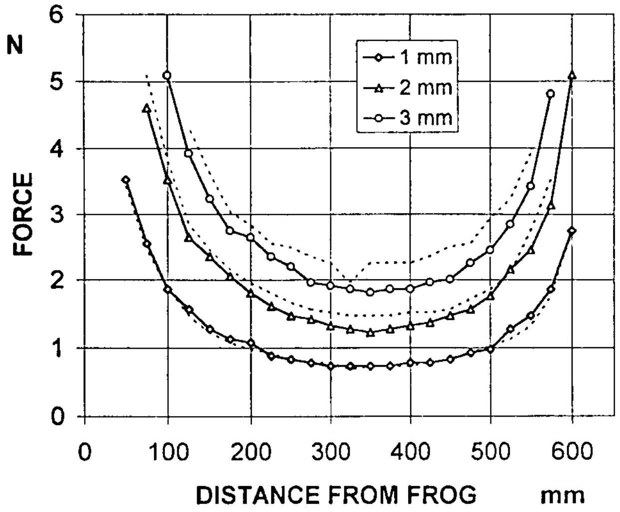

Fig 1. Stiffness curves for a violin bow, showing the force at a measuring point on the stick which causes a deflection of 1, 2, and 3 mm, respectively, at the midpoint. By means of reciprocity, this measurement also gives the force needed at the middle of the stick to cause said deflection at the measuring points. Student's bow of Chinese origin. With bow hair, normal tension (broad lines), without hair (dashed lines).

When measured in this way, the bow is stiffer near the tip and the frog than at the middle. Typical values are 2.5 N/mm (tip, frog), compared to 0.5 N/mm (middle). A little surprisingly at first, the bow was found to be softer with the hair under tension than without. The preloading by the hair tension, however, makes the bow stick more straight, and the resulting bending strain helps to restore the initial curvature of the stick, when loaded. Bows rated as "soft" and "stiff", respectively, gave clear differences in these measurements, but a discrimination between bows with high and poor quality ratings was not possible.

This is certainly not the only, or optimal, way of characterising the stiffness of a bow. For example, Pickering (1987) measured the stiffness by a computerised procedure, in which the deflection of the stick was recorded as a roller was moved along the bow under a constant (and substantial) force of 7 N. This method could be claimed to give a more correct measurement of the stiffness distribution of the stick, as the force and deflection are measured in the same point. No clear differences between a "good" and "bad" stiffness profile were reported.

From the player's point of view, a more relevant measure of the bow stiffness, or rather the bow compliance, might be the change in angular deflection of the stick at the player's fingers per unit increase in bow pressure. The bow is (simplified) considered as being held so that it can pivot round an axis through the frog at the position of thumb and the middle/ring fingers, and be pressed against the string by the index finger on top of the stick. An example of such a measurement is shown in Fig. 2.*

This set of curves shows that the bow was slightly more compliant at the tip (1.2 mm/N) than at the frog (1.0 mm/N). The compliance was essentially independent of the bow pressure except close to the tip, where the bow gets successively softer for higher bow pressures. This occurs, however, only for bow pressures above 1 N, which is approximately the upper limit for playing pressures at the tip (Askenfelt, 1989).

There are reasons to believe that there is an optimal range for the bow compliance measured in this way. If the compliance is too low, even minute changes by the

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[*] The measured property is of course not an actual compliance as force and deflection are measured at different points. A more appropriate term would be something like "bow force sensitivity."

player's hand will cause large changes in bow pressure. On the other hand, too high a compliance would make the bow "not responding."

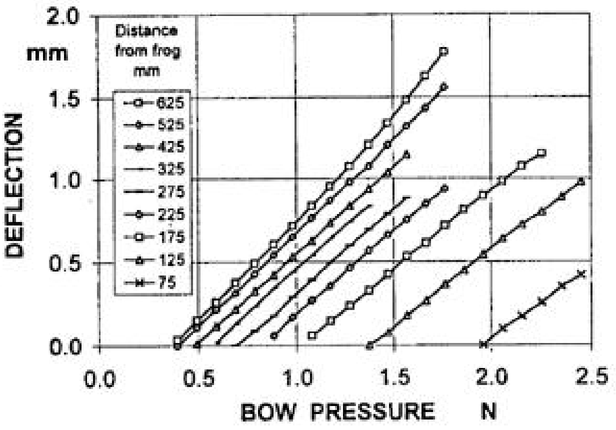

Fig. 2. "The bow compliance perceived by the player." Deflection at the player's index finger (58 mm from the pearl eye in the frog) when a force is applied at this point, versus bow pressure. The bow is pivoting around an axis through the frog. The set of curves represents different distances from the frog to a rigid support ("at the tip" - "at the frog"). Student's bow Wilfer, *normal hair tension.*

Stick modes

The transversal modes of the stick alone, without hair and frog, are basically those of a free-free bar. Typically a dozen, very clear, modes are seen below 2 kHz. The eight lowest mode frequencies are 60, 160, 300, 500, 750, 1000, 1300, and 1700 Hz, approximately (Askenfelt, 1992).

The damping ratio for the stick modes are between 0.2 - 0.6 % of critical damping (Q-values between 250 and 80), slightly increasing with mode number. When comparing bows rated in tonal quality by professional players, low damping turned out as favorable. A little puzzling, this observation has no direct support in the wood data reported above (Protze, 1980).

A tuning of the stick mode frequencies to some particular ratios in good bows - like in violins - was not possible to identify. Some bow makers, however, claim that they use tap tones when setting the final dimensions of the stick. It appears that the makers focus on mode # 4 at about 500 Hz when doing this test, which might assist in setting the absolute value of the stiffness of the stick. It is unclear whether the aim of this test is also to "line up" a couple of resonances.

With the hair and frog mounted to the stick some changes occur. Most mode frequencies are shifted only marginally compared to the free stick (< 7 %), but in contrast, the damping almost doubles. The influence of the hair tension is small, less than 2 % in frequency between "much too high" and "much too low" conditions. An additional mode is found in the assembled bow at about 60-75 Hz, depending on tension. This is the lowest transversal mode of the bow hair, which often couples to the lowest mode of the stick.

When the bow is held as for playing, a dramatic increase in damping is observed. This increase amounts to a factor 20 approximately, giving about 6% of critical damping. The parallels to measurements on the violin under laboratory and playing conditions, respectively, are obvious.

Bow hair

Mechanical properties

The bow is haired with horse hair of particular selection. Normally, only a small fraction of the tail hairs can be used for bows (<10%). The raw hair is "dressed" by cleaning, combing, and sorting in different qualities and lengths. It must be considered as a surprising and lucky coincidence that the length of a sweeping gesture of the human hand approximately equals that of a good horse tail. Today, much bow hair come from Mongolia, Siberia and China.

Studies of horse hair using electron microscopy reveals "sharply articulated scales" at the root end, which become degraded (due to wear) when approaching the older parts at the tip (Menzel & Marcus, 1979). The scale projection is, however, only about 0.4 µm, which is a factor 600 less than the diameter of a violin E-string. The scales are probably important for the ability of the hair to take rosin. Another mechanism for fixing rosin is due to its chemical structure. Horse hair is a proteinic fiber made up of the protein keratin. Like all high polymers, keratin shows a surface activity, manifested in secondary forces which are capable of attracting other substances like rosin (Rocaby, 1990). Synthetic materials such as nylon are much smoother than horse hair, and lack the surface activity. New polymers like kevlar might offer an interesting alternative in this respect, as they exhibit external forces due to their dipole character.

The diameter of a bow hair is typically 0.20 mm ± 0.05 mm. (The diameter of human hair is typically between 0.05 and 0.08 mm.) A complete bundle of hair with 160 - 190 hairs of normal length (650 mm) weighs around 5 g, which corresponds to a mass of about 0.03 g for a single hair, and a density between 1200 - 1400 kg/m^3.

The mechanical properties of bow hairs of different quality were examined in a tensile testing machine. A typical set of force-extension curves shows a clear division in an elastic and plastic deformation, respectively (see Fig. 3). The hair is elastic up to about 4 N, with a "spring constant" of about 0.2 N/mm for a single hair of normal length. Above this range, a pronounced elongation of the order of 30 - 40% takes place before the hair breaks at a force between 4 and 7 N. The Young's modulus in the linear range is between 4 and 7 GPa, which is of the same order as synthetic polymers like nylon.

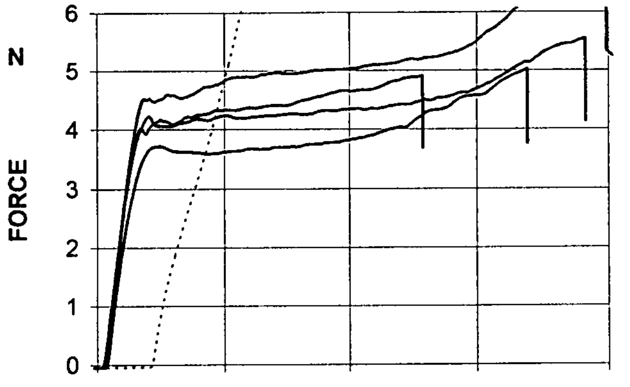

Fig. 3. Force - extension curves for four single bow hairs of Polish origin. Sample length 90 mm, and diameters between 0.18 and 0.21 mm. Young's modulus in the elastic range is about 5 GPa. The dashed line represents a high quality fly fishing leader with diameter 0.20 mm and guaranteed tensile breaking strength 38 N (curve displaced for clarity).

When testing a complete bundle of hairs, more and more hairs are successively engaged in taking up the load. At a full nominal load of 60 N the spring constant was about 16 N/mm, equivalent to about 80 fully engaged hairs. However, when superimposing a periodically varying load of ± 5 N, the spring constant increased to about 30 N/mm. This corresponds to about 150 fully active hairs, which means that almost all hairs in the bundle are sharing the load equally. Thus it seems that, in normal playing, each hair operates under a static load of approximately 60 N /150 = 0.4 N, which is well in the linear range.

When the periodic force was applied, a hysteresis loop was observed. This indicates that some losses occur due to a rubbing motion between hairs when the whole bundle is stretched and relaxed, in spite of the fact that all the hairs move in synchrony at the terminations. In playing, only one or two layers of hair can be driven directly by the string, the others being set in motion at the end points by the vibrating bow stick. Whether these conditions give greater or smaller losses than the measured case above seems difficult to answer without direct experiments.

When tensioning the hair, the motion of the frog is roughly 3 mm, as estimated from observing the number of turns of the frog screw. Of this, about 2 mm would be due to the elongation of the bow hair, and the remaining 1 mm to the displacement of the tip as the stick straightens. These estimations were confirmed by measuring the actual displacements of the frog and the tip, respectively, by dial gauges, as the hair was brought up to full tension.

The characteristic impedance for longitudinal wave propagation of a single hair amounts to about $Z_L = \mu_L c_L = 0.1$ kg/s, assuming a longitudinal propagation velocity c_L of 2300 m/s (see below), and a linear density $\mu_L = 0.03$ g/ 0.65 m = 0.05 g/m. The characteristic impedance in the longitudinal direction for a complete bundle of hair, Z_{HL}, is then about 17 kg/s. All bow hairs are not in contact with the string, however, and the effective characteristic impedance may well be lower than this value. If only one layer of hairs is exercised by the string (corresponding to 35 - 45 hairs), Z_{HL} would be about 4 kg/s.

Heat generation

When the bow is drawn across the string, heat is generated at the contact point. This rise in temperature is of interest as the viscosity of rosin falls drastically in a temperature range between 40 and 60°C (Smith, 1990). A raised temperature in the contact area would, thus, have a dramatic effect on the friction characteristic, and hence, on the bow-string interaction. Even a fast cyclic variation in temperature at audio frequencies has been proposed, in which the rosin partially melts during slip and resolidifies during stick (Smith, 1990).

The temperature of the string and bow during playing was examined by the use of an infrared camera system (see Fig. 4). Vigorous strokes were played on the G-string (wrapped steel), one by one, repeated up- and down-bows, and tremolo. The temperature rise was largest for a *tremolo furioso*, in which the temperature increased from 27°C (room temperature) to 33°C. The unknown emission factor of the rosin covered string gives some uncertainty in the data, but even an optimistic estimation of this coefficient would not raise the temperature above 36°C. Even at this hard playing, it was thus not possible to reproduce the temperature rise of 10 - 25°C in the bowing area earlier reported in the literature, also using an infrared camera system (Pickering, 1991b). Similar temperature shifts as for the violin was observed for the cello and double bass. For all instruments, the heat dissipated fast in the string but stayed much longer locally in the bow hair.

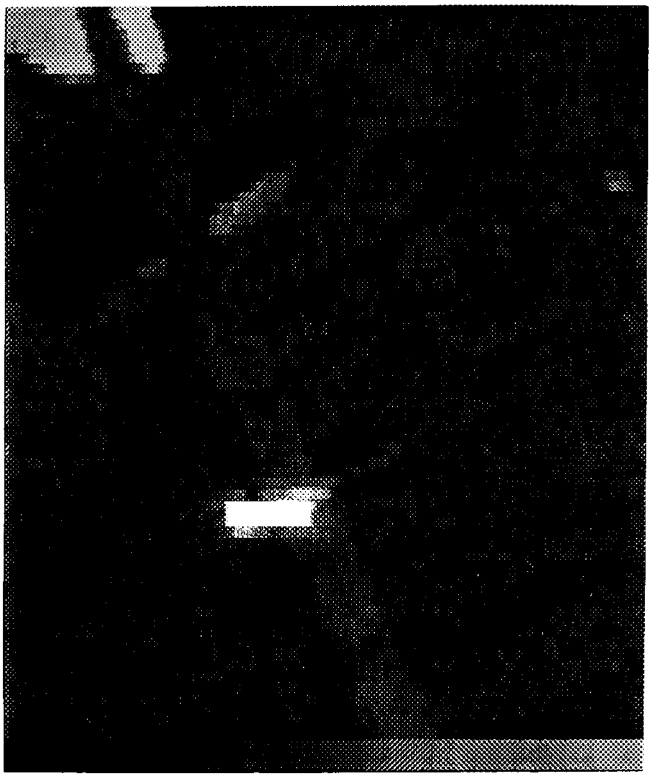

Fig. 4. Registration of the temperature distribution in a violin string and bow after some seconds of modest tremolo on the (steel) G-string. The violin is played with the bow under the strings. The player's fingers are seen at the top left corner. The full temperature span in the picture (black - white) is 3°C.

It is not possible to measure the spot temperatures at the actual contact point with this camera technique, simply because the string hides the view. This is a limitation as these temperatures might be considerably higher than the observed bulk temperatures.

If the bow-string interaction was strongly dependent on temperature, outdoor violin playing in the Tropics or in the Scandinavian countries in wintertime would hardly be possible. After all, experience tells us that playing under such conditions is often possible (although trying). The upper temperature limit for violin playing was estimated by heating a steel G-string by means of an electrical current. While it was still easy to draw a tone of normal quality at string temperatures between 40 and 50°C, it certainly became impossible above 60°C.

Longitudinal resonances

The longitudinal wave propagation in the bow hair was studied by mounting the bow hair to a rigidly clamped force transducer at one end, and to a spring-balance and turn buckle (rigging screw) at the other. Once the desired tension was reached, the termination at the spring balance was locked in a heavy vice (5 kg). With the ribbon of bow hair thus mounted between two rigid supports and under normal tension (about

60 N), a series of longitudinal resonances were observed when hitting one of the supports with a force hammer. For two different samples of hair, the frequency of the lowest longitudinal mode was 1660 and 1810 Hz, respectively. This corresponds to a longitudinal propagation velocity c_L in the range 2200 - 2400 m/s. These values were confirmed by measuring the time delay between the blow of the hammer and the arrival of the wave to the force transducer. The results agree well with earlier estimations (Schumacher, 1975; Cremer, 1984).

The longitudinal resonance frequencies were not influenced by tension as expected, but the Q-values were. For the lowest mode, the Q was about 20 at normal tension (60 N ± 5 N), rising to about 30 for high tension (80 N) and decreasing to about 15 at low tension (40 N), both of the latter cases being at the very extremes of the range of playing tensions.

Higher modes, close to multiples of the lowest longitudinal resonance frequency, were observed in this experiment. At normal tension, five equally prominent modes were observed within the measurement range up to 10 kHz.

Bow admittance

Schumacher's (1975) early work reported the first measurements on the input admittance of the bow, as seen by the string. The measurements were taken by shaking the hair via a needle and observing the motion by an optical device. Since then, there has been an increased need for additional measurements of the bow admittance for use in "physical modeling" of the bow-string interaction.

In the present experiment, a miniature accelerometer (1.1 g) was fastened in the bundle of bow hair (with all hairs in contact), and hit by a miniature force hammer. Measurements were taken both in the longitudinal and transversal direction of the bow hair, and at different positions along the bow (tip - middle - frog). The bow was flexibly mounted in the bow holder of a bowing machine, with the hair free from contact with the strings.

Longitudinal admittance

An example of a measurement of a student's bow of mediocre quality (unsigned), taken at the midpoint of the bow, is shown in Fig. 5 (a). The measurement is influenced by the accelerometer, the mass of which (1.1 g) is of the same magnitude as that of the bow hair (a complete bundle of bow hairs weighs roughly 5 g). A compensation has therefore to be made (cf. Fig 5 (a) and (b)). As the compensation is relatively large, even a small uncertainty in the compensation admittance will have a rather large influence on the result. We should therefore be somewhat cautious when considering the measurements above, let's say, 5 kHz.

As seen in Fig. 5 (b), the longitudinal admittance shows a set of resonance peaks below 4 kHz. The highest peaks, which for this bow falls in the range 1500 - 2000 Hz, reach an admittance level of 0.5 s/kg, approximately, in good agreement with the results of Schumacher.

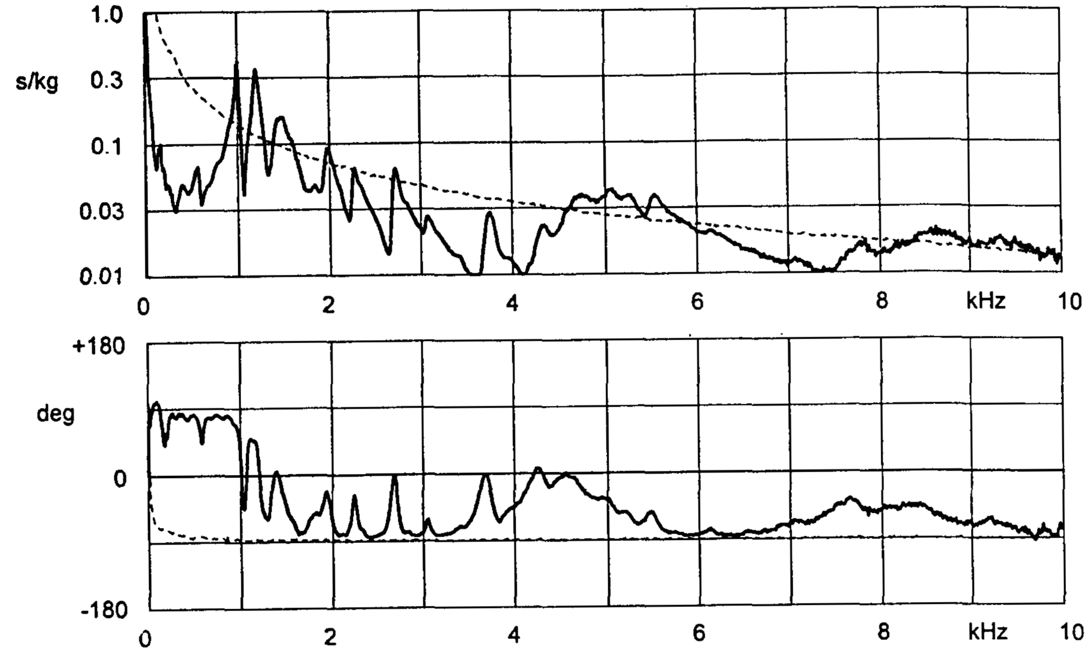

Fig. 5. (a) Measured input admittance in the longitudinal direction of the bow hair of a student's bow (unsigned) at the midpoint (broad line), and admittance of the freely suspended accelerometer used for compensation (dashed thin line).

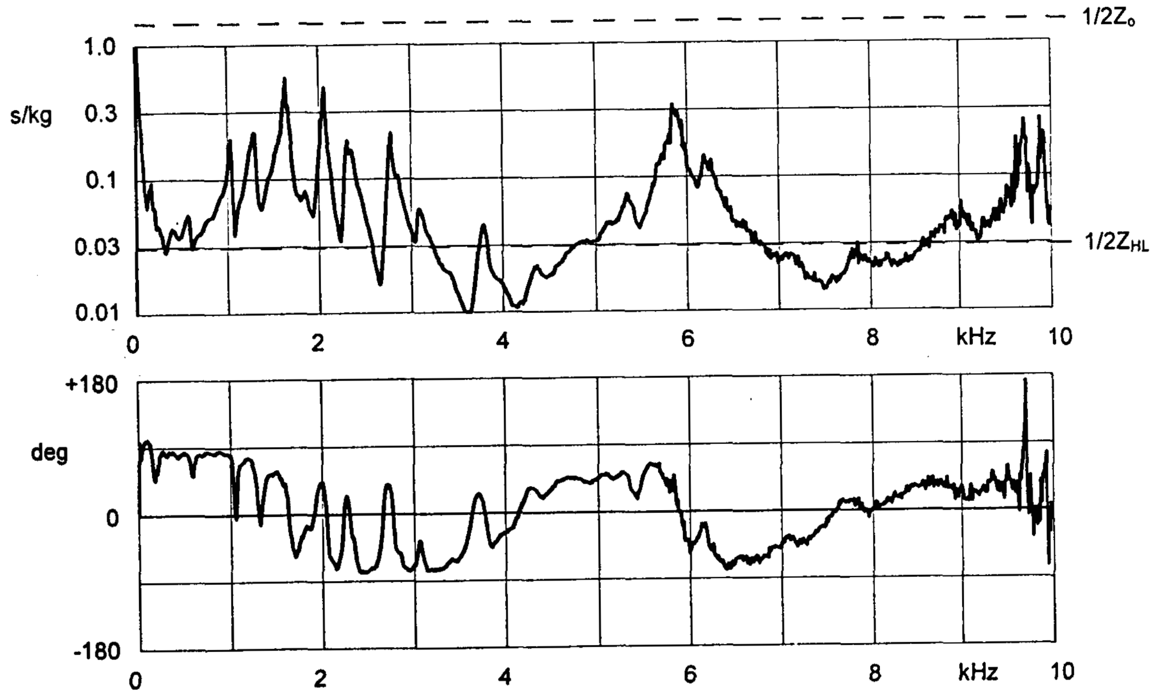

Fig. 5. (b) Compensated admittance. The lower horizontal dashed line indicates the longitudinal point impulse admittance of the bow hair $1/2Z_{HL} \approx 1/34$ *s/kg = 0.03 s/kg, and the upper line the transversal characteristic point admittance of a violin steel G-string* $1/2Z_0 \approx 1/0.6$ *s/kg = 1.7 s/kg.*

It may be tempting to sort the resonances into “stick modes” and “hair modes,” respectively, but it is obvious that such a division is not really appropriate. The bow hair is terminated at resonant supports defined by the stick, and what we see is the response of the complete system. Having said that, we could continue with some caution, however, and discuss some modes as primarily originating from the stick and others from the hair, respectively.

All resonances in the bow admittance in Fig. 5 (b) below 5 kHz are “stick modes,” (with the exception of the peak at about 1600 Hz), slightly shifted in frequency compared to the frequencies of the free stick. The Q-values are of the order of 40 - 60. Below 500 Hz, traces of transversal resonances can be seen. The highest peak at about 1600 Hz is a combination of a stick mode and the lowest longitudinal hair mode.

The admittance of the stick alone (without hair) as measured in the longitudinal direction at the tip (ivory plate), is surprisingly similar to the admittance measured in the bow hair. This applies both to admittance levels and resonance frequencies. A notable exception is, however, that the stick exhibits sharp resonances all the way up to 5 - 6 kHz. Besides that, many of the inherent properties of the bow stick - set by the bow maker - seems to be present also at the contact point with the string.

Above 8 kHz, the phase of the bow hair admittance fluctuates between 0 and + 45 ° (“springy”), see Fig. 5 (b). The admittance level approaches $1/2Z_{HL} \approx 1/34$ s/kg = 0.03 s/kg (“the longitudinal point impulse admittance of the bow hair” in the terminology used by Cremer, 1984) as might be expected. As mentioned, however, this is a frequency range in which the admittance curve is sensitive to even small changes in the compensation for the additional mass of the accelerometer. The admittance level in this range is therefore subject to some uncertainty.

The prominent peak around 6 kHz, which also falls in the somewhat less reliable frequency range, was present in the measurements on the rigidly supported bow hair as well. It seems to be connected with the third longitudinal resonance in the hair. The even hair resonances are not seen at this measuring point at the middle of the bow.

The tension of the bow hair did not influence the overall behavior of the admittance in a drastic way. Even for such large changes in tension as ± 20 N from the “normal” 60 N, only small changes in the admittance peaks were observed. As the characteristic longitudinal impedance of the hair does not change with tension, the effect is probably due to the changing preloading of the stick.

Holding the bow in the hand in a normal playing manner did not change the admittance dramatically. Some of the resonance peaks were slightly boosted or attenuated, but the average admittance level remained unchanged as did the resonance frequencies. In this respect, the bow holder seems to simulate the human bow grip rather satisfactorily.

Also when the bow was pressed against a string by the bowing machine (G-string, bow pressure 800 mN), no major changes were observed. The bow is a device of high impedance compared to that of the string, and consequently the string load does not introduce a major change.

When measuring closer to the tip and to the frog, respectively, essentially the same resonances were seen as at the midpoint of the bow, but with a slightly different amplitude distribution of the peaks. Also, the overall admittance level was slightly lower compared to the midpoint of the bow.

A comparison with three other bows were made, including a medium quality student's bow (*Wilfer*), a very stiff bow made of a carbon fiber tube (ex fishing rod), and a quite respectable bow (*Friedrich Glass,* early 20th century), see Fig. 6.

The Wilfer bow showed less clear resonances, but otherwise the peak levels were very similar to the unsigned student's bow in Fig. 5, except that the frequencies of the tallest peaks were shifted up to between 3 and 4 kHz (see Fig. 6a).

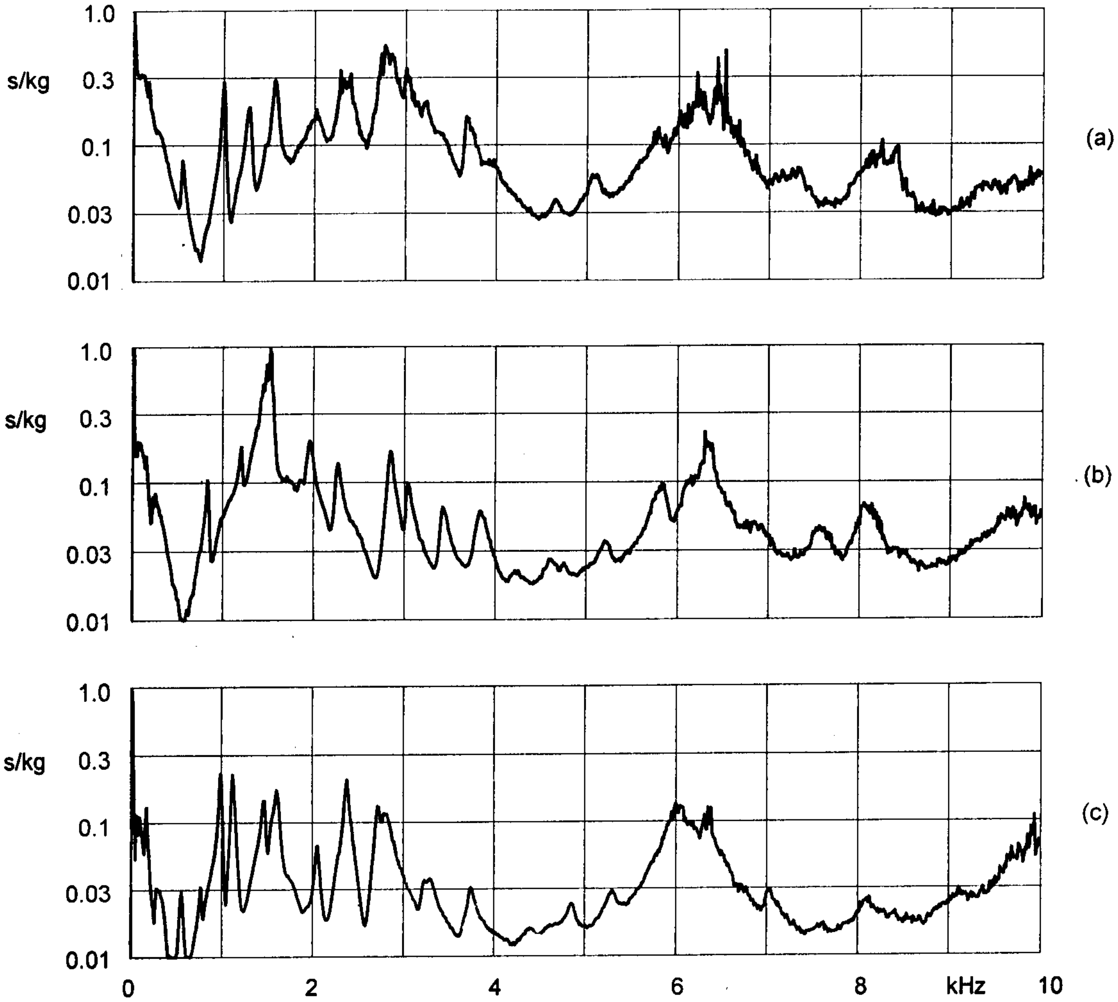

*Fig. 6. Comparison of the longitudinal admittance of three bows measured at the midpoint of the bow hair. (a) Medium quality student's bow (*Wilfer*); (b) carbon fiber bow (ex fishing rod); (c) high quality bow (*Friederich Glass*).*

The "fishing bow," which indeed is far from normal bow standards, showed an enhanced peak at 1500 Hz caused by the lowest longitudinal hair mode, which reached about 1 s/kg (see Fig. 6 b). This dominating resonance was surrounded by lower stick resonances at somewhat different frequencies than the other bows Although the admittance of this odd device deviates a little from that of the normal bows, it is tempting to cite Schumacher's (1975) wording when commenting on one of his measurements of a fiber-glass bow; “it is perhaps somewhat disappointing that this bow, a hardly respectable bow at all, looked so similar to the others.”

The quality bow by Glass showed a generally lower admittance level with clean resonance peaks, none exceeding 0.3 s/kg (see Fig. 6 c). The highest peaks were now found at lower frequencies, just above 1 kHz. The traces of the transversal resonances below 500 Hz appeared a little clearer as well.

It is difficult to draw any clear conclusions from this limited sample of bows, but “clean peaks” might be a quality characteristic (as for violins). The absolute admittance levels may also hide a quality parameter, but it does not seem quite clear how bows should be ranked according to this property. As mentioned, the bow adds to the torsional effects of the string, whose damping is known to be important in order to establish a safe Helmholtz motion. The optimal damping of the bow would then be dependent on the type of strings used, a fact which is not contradicted by experience.

Transversal admittance

In the transversal direction of the bow hair (in the direction of the bow force), the interesting frequency range is below 500 Hz (see Fig. 7). Here the admittance is dominated by the transversal (“string”) modes of the taut bow hair. Mainly the odd modes at about 70, 210, and 350 Hz are seen in this measurement, taken at the middle of the bow. The admittance level is much higher than in the longitudinal direction, reaching about 10 s/kg at the resonance peaks. The Q-values are in the range 30 - 50. Traces of stick modes can be seen, the most clear one for this bow at about 90 Hz. These are normally at least a factor four weaker than the hair modes, if well separated in frequency. Otherwise a coupling can occur with a corresponding increase in level. (In these cases our “suspect” division into hair and stick modes is of course less appropriate than ever). Above 400 Hz the transversal admittance flattens out at an average level of about 0.5 s/kg.

Naturally, the tension of the bow hair influences the transversal admittance in several ways. The frequency of the lowest hair mode changes from a normal value in the vicinity of 65 Hz to above 80 Hz and below 50 Hz, respectively, at the very extremes of the playing range tensions. While the overall admittance level changes only little with tension, the curve becomes more featureless when tension is brought down because the number of resonances decreases.

As for the longitudinal measurements, the transversal admittance changes only little when the bow is held by the hand in a normal playing manner. The influence is essentially a slight reduction in the peak levels.

When measuring at the tip and the frog, the “missing” even hair modes are seen in addition. Here, the peak admittance levels are even a little higher, up to 15 s/kg, but the stick modes remain at the same low level as at the midpoint.

With the bow resting on the string (or pressed by the bowing machine as for playing) the peak admittance levels of the hair modes were reduced to about 3 s/kg, except at some frequencies promoted by the division of the bow hair into two parts. Now, when the hair modes have been reduced (and shifted in frequency depending on the contact point with the string), the stick modes are at about equal level, and appear more influential.

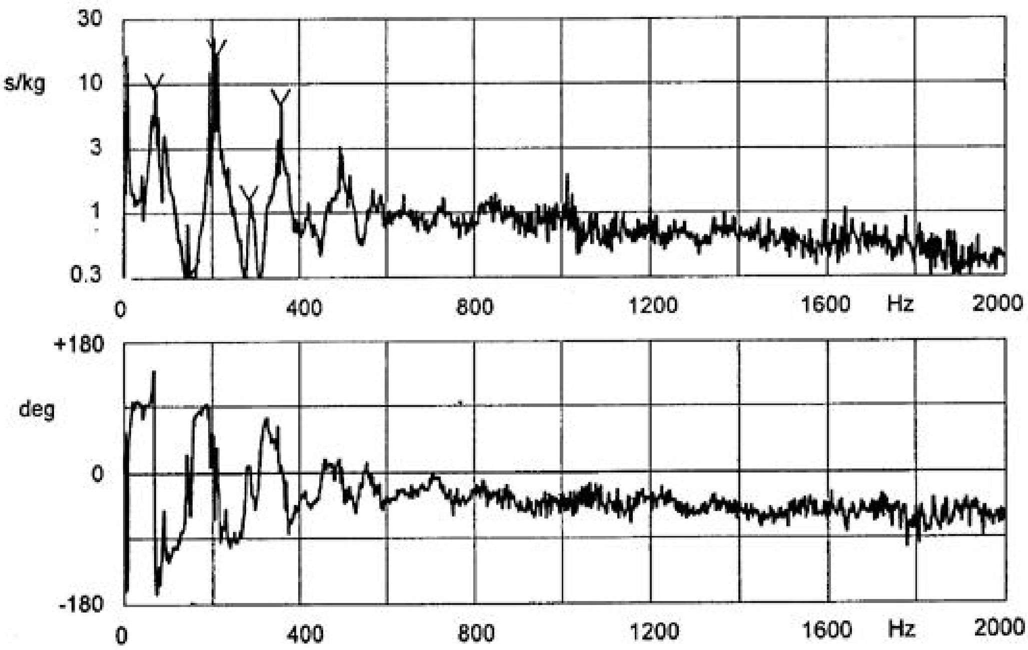

Fig. 7. Measured input admittance of a student's bow (unsigned) in the transversal direction of the bow hair (midpoint). The markers indicate harmonic components of the lowest transversal mode at 70 Hz.

A comparison with the three other bows showed no large differences (see Fig. 8). The admittance is dominated by the hair resonances seconded by much lower stick modes. The high-quality bow by Glass shows the highest admittance at the lowest transversal resonance, and the corresponding frequency was a little lower than for the other bows of normal design in Figs. 7 and 8 (58 Hz compared to 70 Hz and 63 Hz at normal playing tension). This observation is at least qualitatively in line with the opinion of bow makers, stating that the bow should be "vibrating much." (Schumacher, 1991). This is claimed to include a very low frequency mode at about vibrato rate (6 - 7 Hz).

Schumacher (1992) has reported a low frequency component at 9 Hz in bowed notes. An admittance peak in the range 7 - 10 Hz was also observed in all the present measurements, including the hand held case, but the origin of this mode is not clear. If the transversal resonances are excited by the unsteadiness of the player's hand, a modulation of the bow pressure may occur as proposed by Cremer (1984). In particular, the lowest transversal modes would be the most active in promoting this effect.

As mentioned, the frequency of the lowest transversal mode was somewhat different between the bows, when tightened to "normal" playing tension. A main factor setting this normal value is a sufficient distance between the bow hair and the stick at high bowing pressures, the actual value being determined by the curvature and stiffness of the stick. A generous frequency range covering most normal cases would be between

55 and 70 Hz. Here too a quality parameter may be hidden. The indications presently at hand, are that a low first transversal mode is favorable.

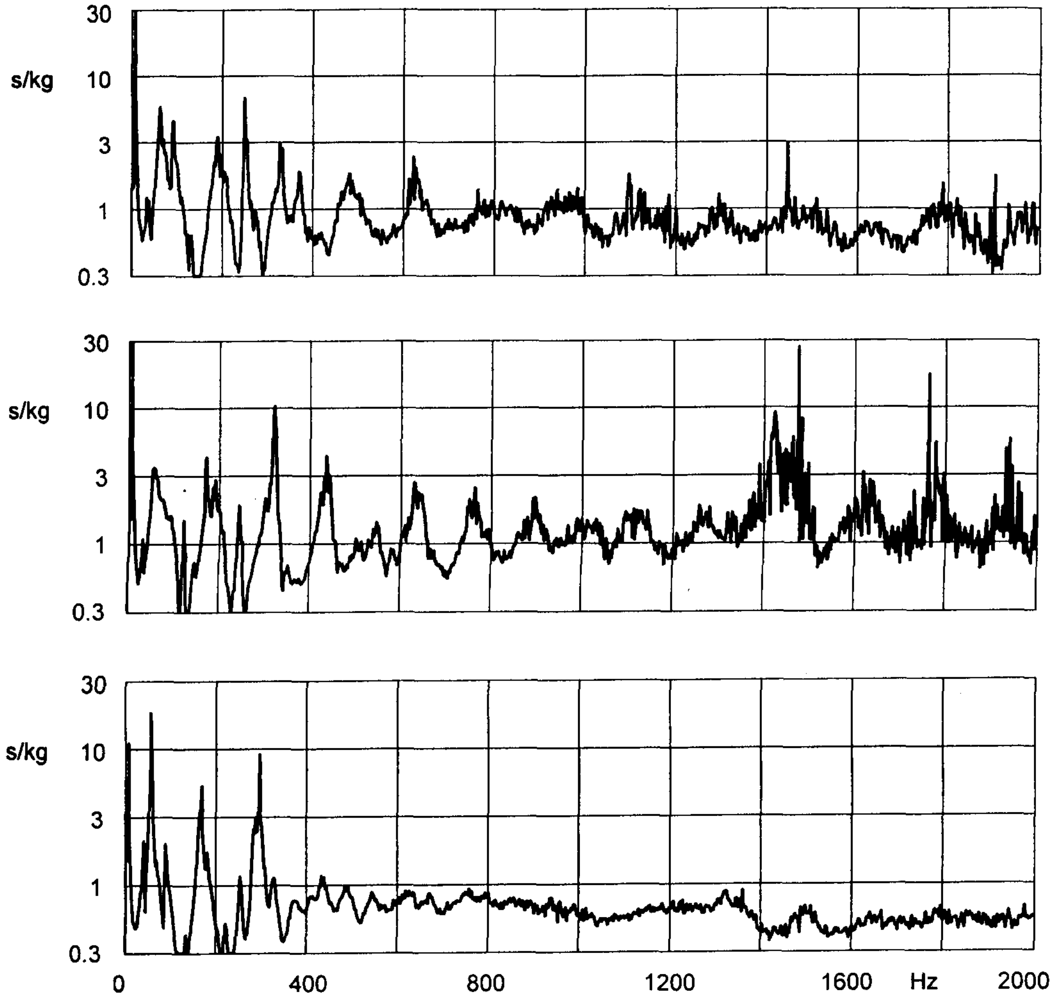

*Fig. 8. Comparison of the transversal admittance of three bows measured at the midpoint of the bow hair. (a) Medium quality student's bow (*Wilfer*); (b) carbon fiber bow (ex fishing rod); (c) high quality bow (*Friederich Glass*).*

Influence on string vibrations

Any influence of the bow on the string vibrations must be due to the fact that the frictional forces between bow and string set the bow in vibration, and the bow in turn interacts back on the string. The difficulty for this process to occur lies in the difference in impedance levels between the string and the bow. The bow has a high impedance compared to the string. Typically, the characteristic impedance of a steel G-string would be about 0.3 kg/s, while the characteristic impedance of the bow hair (longitudinal direction) is in the range 4-17 kg/s, depending on how many hairs are effectively in contact with the string. However, the string has a real opportunity to drive the bow if the bow operates close to a nodal point on the string where the string impedance is relatively high (Guettler & Askenfelt, 1995). The periodic forces at stick and slip, or even the much smaller disturbances during the sticking phase, can then excite the bow. For a given bowing distance βL from the bridge, the velocity spectrum of the bow hair will reach its highest values close to the dips in the string velocity spectra, the "node frequencies" set by 1/β (see Fig. 9).

Measurements confirm that the velocity of the bow hair contains a substantial periodic component, superimposed on the steady bow velocity supplied by the player (or bowing machine). In the simultaneous measurements of bow hair velocity and string velocity in Fig. 9, the bow hair velocity amounts to about 20% of the string velocity at the lowest node frequency at the 7th partial.

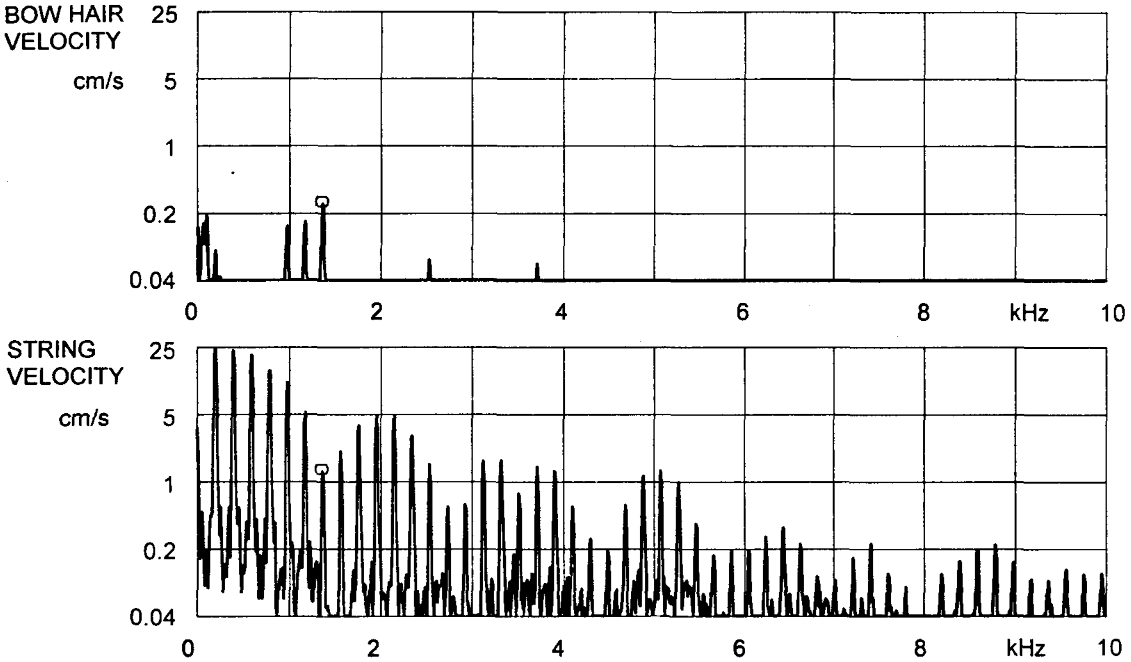

Fig. 9. Measurements of the velocity at the middle of the bow hair (top), and the string velocity at the contact point with the bow (bottom) during a stroke by a bowing machine. Relative bowing position $\beta \approx 1/7$*. Lowest "node frequency" indicated by circles.*

The premises for a fairly strong excitation of the bow are thus at hand at the node frequencies, particularly if some of the bow resonances (at which the bow has a relatively low impedance) happen to match reasonably in frequency. If so, these resonances might be driven sufficiently strongly to set their "signature" on the velocity of bow hair. However, there is no simple relation between the amplitude of the bow hair velocity and the details of the spectrum of the bridge velocity. The transfer function between bow and bridge is composed of several elements (Guettler & Askenfelt, 1995). Nevertheless, the chances for the relatively modest resonant fluctuations in the bow hair velocity to have an impact on the string and bridge velocity are always greatest near the node frequencies, as discussed above.

An evaluation of the influence of the bow on the string motion lends itself particularly well to computer simulations, by which comparisons between a rigid bow, and bows with resonances at different frequencies and with different Q-values can be performed. Such simulations verify that changes in the string spectrum do occur as the properties of the bow are changed (Guettler & Askenfelt, 1995). In line with expectations, the differences are found primarily around the node frequencies, but also as envelope shifts at higher frequencies. The differences are generally small but not insignificant, of the order of 1 to 5 dB.

Corresponding experimental evidence from measurements with regard to the effects of the bow has not been obtained as yet, only indications that an interaction would be possible under certain conditions. Possibly, the effects of the bow, if captured, might be smaller than in the clear-cut simulations. In any case, it is still fair to summarise by stating that "a lot goes on in the bow, but very little (if any!) is seen in the string or bridge motion" (Askenfelt, 1994).

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References

Askenfelt A (1986). Measurement of bow motion and bow force in violin playing, *Journal of the Acoustical Society of America*, 80: 1007-1015.

Askenfelt A (1989). Measurement of the bowing parameters in violin playing. II: Bow-bridge distance, dynamic range, and limits of bow force. *Journal of the Acoustical Society of America*, 86: 503-516.

Askenfelt A (1992). Observations on the dynamic properties of violin bows. *STL-QPSR* 4: 43-49.

Askenfelt A (1994). A look at violin bows. *Proc of SMAC 93* (Stockholm Music Acoustics Conference, July 28 - August 1, 1993), Royal Swedish Academy of Music, Stockholm, 341-345.

Barducci I & Pasqualini G (1948). Measurement of the internal friction and the elastic constants of wood. In: *Benchmark Papers in Acoustics, Vol. 5, Musical Acoustics, Part 1* (ed. Carleen Hutchins), Dowden, Hutchinson & Ross, Inc. Stroudsburg, 1975. Translation of the original article in Italian Misura dell'attrito interno e delle constanti elastiche del legno, *Il Nuovo Cimento* Vol. 5/5: 416-466.

Cremer L (1984). *The Physics of the Violin*, MIT Press, Cambridge, Massachusetts 1994.

Guettler K & Askenfelt A (1995). Relation between bow resonances and the spectrum of the bowed string, *STL-QPSR, this issue* and *Proceedings of ISMA 95* (International Symposium on Musical Acoustics, July 2 - 6 , 1995, Dourdan).

Haines D (1979). On musical instrument wood. *Catgut Acoustical Society Newsletter*, 31: 23-32.

McIntyre ME & Woodhouse J (1979). On the fundamentals of bowed-string dynamics, *Acustica,* 43: 93-108.

McIntyre ME, Schumacher RT & Woodhouse J (1981a). Aperiodicity in bowed-string motion, *Acustica,* 49: 13-32.

McIntyre ME, Schumacher RT & Woodhouse J (1981b). Aperiodicity in bowed-string motion: On the differential slipping mechanism, *Acustica,* 50: 294-295.

McIntyre ME, Schumacher RT & Woodhouse J (1983). On the oscillations of musical instruments, *Journal of the Acoustical Society of America,* 74: 1325-1345.

Menzel R & Marcus R (1979). The microtexture of horse hair, *Catgut Acoustical Society Newsletter,* 32: 14-20.

Pickering N (1987). Physical properties of violin bows, *J Violin Society of America,* 8/2: 41-57.

Pickering N (1991a). *The Bowed String,* Amereon, Mattituck, New York.

Pickering N (1991b). A new light on bow action, *J Violin Society of America,* 11/1: 83-92.

Pitteroff R (1994). Modelling of the bowed string taking into account the width of the bow, *Proc of SMAC 93,* Stockholm Music Acoustics Conference, July 28 - August 1, 1993, Royal Swedish Academy of Music, Stockholm, 407-410.

Pitteroff R (1995). *Contact mechanics of the bowed string.* Ph.D. diss., University of Cambridge.

Protze G (1980). *Die Verwendung von Pernambouc (Caesalpinia echinata) für den Streichbogen,* Abschlussarbeit, Fachhochschule Rosenheim.

Rocaby F (1990). The structure of bow hair fibres, *Catgut Acoust Soc J,* 1/6 (Series II): 34-36.

Schumacher RT (1975). Some aspects of the bow, *Catgut Acoustical Society Newsletter,* 24: 5-8.

Schumacher RT (1979). Self-sustained oscillations of the bowed string, *Acustica,* 43: 109-120.

Schumacher RT (1991). The influence of the bow on the aperiodicity of violin notes, *Journal of the Acoustical Society of America,* 89/2: 1926.

Schumacher RT (1992). Analysis of aperiodicities in nearly periodic waveforms, *Journal of the Acoustical Society of America,* 91: 438-451.

Schumacher RT (1994). Aperiodicity, subharmonics and noise in bowed string instruments, *Proc of SMAC 93,* Stockholm Music Acoustics Conference, July 28 - August 1, 1993, Royal Swedish Academy of Music, Stockholm, 428-431.

Smith JH (1990). *Stick-slip vibration and its constitutive laws,* Ph.D. diss. University of Cambridge.

Wagenführ R & Scheiber C (1985). *Holzatlas,* VEB Fachbuchverlag, Leipzig.

Weinreich G (1994). Radiativity revisited: Theory and experiments ten years later, *Proc of SMAC 93,* Stockholm Music Acoustics Conference, July 28 - August 1, 1993, Royal Swedish Academy of Music, Stockholm, 432-437.

Woodhouse J (1992). Physical modeling of bowed strings, *Computer Music Journal*, 16: 43-56.

Woodhouse J (1993a). On the playability of violins, Part 1. Reflection functions, *Acustica*, 78: 125-136.

Woodhouse J (1993b). On the playability of violins, Part 2. Minimum bow force and transients, *Acustica,* 78: 137-153.