Determining the Big Bang State Vector

Paul Bird
paulanbird@yahoo.co.uk

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1 Abstract

Based on several reasonable assumptions we construct possible candidates for the State Vector at the time of the Big Bang. We show how the possible histories of the Universe can be formulated mathematically in terms of the Feynman Path integral.

2 Introduction

If it is assumed that the Universe did not start in a singularity and that the laws of physics apply equally to the time 13.75 billion years ago then the Universe at that time should correspond to a wavefunction or state vector. We shall find that this state vector should be very special and highly symmetrical. By making some reasonable assumptions about the initial conditions of the Universe we can find state vectors that are candidates for this "Big Bang State Vector". Once the Big Bang State Vector is found this will help enormously in discussing the events following the Big Bang.

3 Assumptions of Big Bang conditions

Since we do not know for certain the events at the time of the Big Bang we shall need to make some assumptions. Most of these will seem reasonable enough. We shall use the following reasonable assumptions at the Big Bang (T=0)

1) The current laws of physics still apply
2) The Universe is at a maximum but finite density (i.e. no singularity [1][2])
3) Entropy is at a minimum (i.e. symmetry is at a maximum)
4) The masses of fermions become equal
5) The Universe is infinite and open and Euclidean
6) Gravity is described in terms of spin-2 graviton particles
7) The Universe has 3 spacial dimensions
8) The Universe primarily consists of matter (rather than anti-matter)
9) The Universe contains Standard Model particles and neutrinos have mass.

Some of these assumptions such as whether the Universe is an infinite size or whether it is open or closed are still open to debate but we choose these assumptions because they are the simplest and they lead to something we can formulate mathematically.

4 Properties of the Big Bang State Vector

Since entropy is always increasing with time and entropy is a measure of "disorder", the Big Bang State Vector should be a state which has the most symmetry possible of any state. Note that most states in nature have neither translational nor rotaional symmetry. Let us see if we can find a State Vector that satisfies the following global symmetries:

1) 3D Translational Invariance
2) 3D Rotational Invariance
3) Invariance under a global gauge transformations of all the particles
4) CPT symmetry of time evolved state vector at T=0

We shall not require any local symmetries. Local symmetries require gauge fields (bosons) be present. We shall only be considering the matter states (fermions) particularly because the quantum nature of the gauge fields (gravity, Yang-Mills) are not fully known.

The first two conditions says that no direction or position at time T=0 should be singled out as special. The state could be made out of a superposition of states that do not have these symmetries but the state itself could still be symmetric.

The CPT symmetry condition says that evolving the Big Bang State forward in time is identical to evolving the state backwards in time replacing particles with anti-particles and a parity change. A cosmological model consistent with this is the "Big Bounce" model[3]. However, since entropy is at a minimum at T=0 no information can pass from before T=0 and so there is not any real way that the Universe exists before T=0. We can say that T=0 is a boundary in time and thus it is physically correct to consider the state at T=0 as the "first" sate that all other states evolve from. In electrostatics problems when we are calculating the field lines from a source near a flat boundary it is often simpler to put another imaginary source on the opposite side of the boundary. We can think of the Universe before the Big Bang in these terms; as a reflection of our Universe which is useful mathematically to define a boundary at T=0.

Let our Big Bang State Vector be represented by $\mid O\rangle$ and the time evolution operator $U(t)$. Then the CPT condition requires:

$$U(t) \mid O\rangle = (U(-t) \mid O\rangle)^* \tag{1}$$

But since $U(t) = U(-t)^*$ by definition this implies that that $\mid O\rangle = \mid O\rangle^*$. That is to say the Big Bang State Vector is *real* valued.

5 Feynman Path Integral Formalism

In order to formulate the Big Bang State Vector we shall use the formalism of Feynman Path Integrals[4] which we shall briefly review. The probability amplitude for an initial state A to evolve to a final state B is (up to a normalisation constant):

$$F[A,B] = \int A[\psi]B[\psi]\exp(iS[\psi])[D\psi] \tag{2}$$

where $\psi(x,t)$ are particle fields and S is the action which is a functional of the fields and contains the content of the physical laws. This can also be written as:

$$F[A,B] = A\left[\frac{\delta}{\delta J}\right] B\left[\frac{\delta}{\delta J}\right] Z[J]|_{J=0} \tag{3}$$

where Z is the "partition function":

$$Z[J] = \int \exp(iS[\psi] + \psi.J)[D\psi] \tag{4}$$

The partition function Z contains every possible interaction expanded in terms of incoming/outgoing fields J.

6 Big Bang Partition Function

The partition function which contains the states of the Universe evolved from the Big Bang state (O), which we shall discuss later, is:

$$Z_o[J] = \int O[\psi]\exp(iS[\psi] + \psi.J)[D\psi] \tag{5}$$

Z_o is the partition function which represents the superpositions of every possible Universe that can evolve from the Big Bang state. The J's now only correspond to outgoing particles as all the incoming particles have been tied to the Big Bang state. Z_o is a remarkable functional since it contains all the laws of physics *including* the initial conditions of the Universe. Having said that there is no current consensus on what the action, S, or initial conditions, O, contain so Z_o is not much more than a mathematical template in which to insert the laws of physics. However, for our purposes we shall assume that S corresponds to the Standard Model action and see what initial conditions, O, this leads to. Let U be a possible state of the Universe. Then the amplitude for this possible universe is:

$$U\left[\frac{\delta}{\delta J}\right] Z_o[J]|_{J=0} \tag{6}$$

7 Big Bang Particle Lattice

To satisfy all the conditions that we have set out we shall take our Big Bang State Vector to be an infinite 3 dimensional lattice of particles (Λ_3) on the scale of the Plank distance. To make the state rotationally and translationally symmetric in 3 dimensions we integrate over all positions (Q) in the fundamental domain and all orientations (Ω) of the lattice. This is a finite integration area because of the spacial repetition of the lattice.

$$O[\psi] = \int \int \prod_{X \subset \Lambda_3} \psi(Rot_\Omega X + Q, 0) d\Omega dQ^3 \tag{7}$$

Thus even though we have started with a lattice which is orientated, the integral over all the orientations gives a highly symmetric state. We have only "smeared out" each lattice point over a finite domain but because every electron, for example, is identical to every other electron essentially this smears out the electrons over the entire space. But due to the finite domain each portion of space has a finite probability of an electron being there.

8 Which Lattice?

Which lattice should we choose to model the Big Bang state? Or should indeed the state be a superposition of many different lattices? Since the state is the state with highest entropy it should therefore be the most symmetrical state.

An obvious lattice to choose would be one of the densest lattice packings such as the face-centred-cubic packing[5]. We can call this the SU(4) lattice since it corresponds to the root vectors of SU(4). This has the highest degree of symmetry of any 3 dimensional lattice.

Now we need to assign flavours and polarisations to each particle in the lattice in such a way that a gauge transformation will leave the state vector invariant. First we assign flavours to each of the lattice points in a repeating pattern such that the particles match those we observe today and then we have to integrate over possible gauge rotations to make it gauge invariant.

9 Lattice Colourings

So far we have ignored the fact that the particles in the Universe come in many different types. At time T=0 we expect that all fermions acquire the same mass and so can be thought of as different flavours and polarisations of a single particle. Let us take the first generation of particles $[e, u_r, u_g, u_b]$ and $[v, d_r, d_g, d_b]$. These can fit in an SU(4) lattice as shown in Figure 1.

Let us now see how this would be accomplished mathematically as a state vector. As a simple example, let us take a single 2D layer of the 3D lattice which is a hexagonal lattice or an "SU(3) lattice". Let u_0 and u_1 be two root

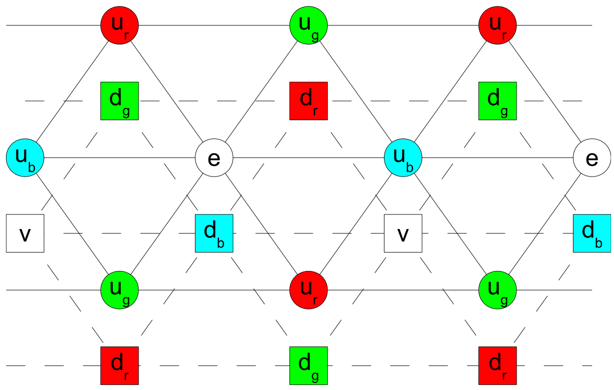

Figure 1: Possible arrangement of fermions in infinite lattice at time T=0

vectors of the lattice at a 60 degree angle from each other. The lattice can be written as:

$$\Lambda_2 = \{Nu_0 + Mu_1 : \{N, M\} \subset \mathbb{Z}^2\} \tag{8}$$

So we can write the Big Bang State Vector as:

$$O[\psi] = \sum_{Generations} \int \prod_{\{N,M\}\subset\mathbb{Z}^2} IU^N V^M W_{SU(3)\times U(1)} \psi(Rot_\Omega(Nu_0 + Mu_1) + Q, 0) d\Omega dQ^2 \tag{9}$$

where U and V are gauge rotation matrices given by:

$$U = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \tag{10}$$

$$V = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \tag{11}$$

and the ψ is a vector of fermion fields and I is a unit vector:

$$\psi = \begin{bmatrix} e \\ d_r \\ d_g \\ d_b \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

$W_{SU(3)\times U(1)}$ are general global gauge rotations that we can apply to all the particles in the lattice. We need to integrate over all possible global gauge rotations and spins for maximum symmetry. The same principles can be applied to the full 3 dimensional lattice. What we have set out is just a candidate for the Big Bang State Vector based on what we know about the Standard Model which can be used for model building, the exact form of it must wait until we know the exact gauge group and particle content of Nature.

10 Mathematical Difficulties

There is a slight mathematical problem in that the partition function Z_o is not well defined. This is because expanding in terms of J we are asking for the amplitude of the Universe containing 1, 2, etc. particles when it started with an infinite number of particles. For practical purposes, therefore, we should restrict the integral to within the light-cones of the events that we interested in because particles outside the light-cone can't have any effect. That way the Big Bang State Vector is only a finite product of fields defining a lattice over a finite volume and so is mathematically sound.

11 Cosmology

When considering the history of the Universe in terms of path integrals we must remember that according to Feynman's sum over history interpretation:

The history of the Universe is the superposition of every possible history that could lead to the Universe that we observe today.

Now this does not necessarily mean that the set of possible histories of the Universe are wildly different to each other. In fact in the large scale where quantum mechanical effects are less important the histories will presumably look very similar.

12 Gravity

It may seem odd to discuss the Big Bang without mentioning gravity. However, unlike fermions, the number of bosons in the Universe need not be conserved and so for simplicity we have assumed that there are no bosons at the time T=0. An instant after the big bang of course gravitons and Yang-Mills bosons can be emitted by the fermions and can have a big effect on the sates of the Universe. To discuss the states coming after the Big Bang of course we need to include the gravitational effects and this requires having a workable model of quantum gravity. Fortunately for our discussion, the exact state at T=0 does not seem to require gravity (or gravitons) at all.

13 Expanding Universe

We know that one of the possible histories of the Universe is an expanding Universe. We know this because the wavefunction of the Universe has collapsed from the superposition of every possible Universe into (the superposition of Universes consistent with) the one we observe today. Hence we need to prove that the Big Bang State Vector that we have created contains this history when it is evolved with the time evolution operator. Let E be the universe that we observe today. In other words we require:

$$E\left[\frac{\delta}{\delta J}\right] Z_o[J]|_{J=0} \neq 0 \tag{13}$$

What does this mean? What it means is that starting with the Universe today and rewinding time can we end up with the Big Bang State Vector that we have derived? If we can then it proves that our State Vector is at least consistent. It is not important, in some respects, how probable our particular expanding Universe is, just that it is *possible*. When we do this we find that as we rewind the universe it is compressed into a quark-lepton-gluon plasma. Now it is, at least feasible, that when compressed further, just as compressed coal becomes diamond in a highly ordered lattice, the quark-lepton-gluon plasma also becomes highly ordered in a similar way but in an infinite lattice. We can think of this as an infinite "fermion-crystal" filling all of 3 dimensional space. But importantly, we didn't derive this state by evolving the expanding Universe backwards, we derived it merely from assumptions of symmetry. The fact that the Universe is expanding was not needed to derive the Big Bang State Vector but it is consistent with it.

14 Singularities and Hyperbolic Matter Distributions

The assumptions that we started with lead us to think of the Big Bang State as existing on an infinite 3 dimensional plane at T=0. Let us change some of the original assumptions to show that this leads to other state vectors. Instead of 3D translational invariance at T=0 let us instead find a state that has 4D Lorenz invariance at a point at T=0. The Big Bang State Vector would then be of the form (before integrating over the 6 types of Lorenz transformations):

$$O[\psi] = \prod_{X\subset\Lambda_3} \psi(\alpha \sinh(|X|)\hat{X}, \alpha \cosh(|X|)) \tag{14}$$

So now that (considering just 1 of the spacial dimensions) the lattice is no longer a lattice on a line at T=0 but on a hyperbola which begins at $T = \alpha$ (proper-time). This is perfectly reasonable since no point on the hyperbola is contained in another point's light-cone. The lattice points are spaced by Lorenz transformations of multiples of a common angle.

Note the special case when $\alpha = 0$. This corresponds to the Universe starting off at a "singularity". (By this, we don't mean a singuarity in the sense of General Relativity, we mean that the wavefunction of all the particles is concentrated at a single point.) However, this is not possible with just the Standard Model particles because of the Pauli-exclusion principle. The only way this would be possible if at T=0 there existed an infinite number of *different* particles all at the same place. Some theories such as superstring theory would allow this. Slightly after the Big Bang these would have to decay into the quarks and leptons that we observe today.

There is another possibility and that is that $\alpha = \varepsilon$ for ε is an infinitesimly small number above zero. What this means is that two fermions *can* start off in the same place (despite the Pauli exclusion principle) but only for one infinitessimal point in space-time, the Big Bang. But for our mathematics to make sense we separate out the fermions by a small distance ε. The wave function for two fermions would be $|x, y\rangle = \delta'(x - y)$, i.e. the derivative of the delta function, a curious function which is antisymmetric yet 0 for $x \neq y$.

The Universe that this would correspond to would be one in which the distribution of galaxies becomes more dense the further out into the Universe we look and the Universe would be infinitely dense at the edge, rather like hyperbolic space when represented in a sphere. The entire universe would be enclosed inside the light cone of the starting "singularity" and hence would be finite in size but have an infinite number of particles. (See figure 2).

This model has the advantage of *requiring* an expanding universe. Assuming linear expansion the inertial frame of each galaxy is found by a Lorenz transformation about the hypothetical "singularity" at T=0. An observer on any galaxy will see the same average matter distribution of the universe. Hence in practical terms the Universe is infinite, you could travel forever and never get to an edge.

The two models, the plane model and the hyperbolic model can be distinguished from each other by observing the distribution of galaxies in deep space.

15 Matter/Anti-matter Asymmetry

The question of why there is more matter than anti-matter in the Universe is based on the (I would say "flawed") assumption that all matter was created at the Big Bang in matter/anti-matter pairs. This assumes that at the moment of the Big Bang the energy was all contained in curved space in the form of gravitons or as Yang-Mills fields such as photons which then decayed into matter/anti-matter pairs. However, it is perfectly possible that the Big Bang State Vector contains only matter or only anti-matter fermions. The question of "where is all the antimatter" could then be answered by saying that if we reflect the Universe in time at T=0 we would have a Universe consisting only of anti-matter and so the Universe really is symmetric in with respect to

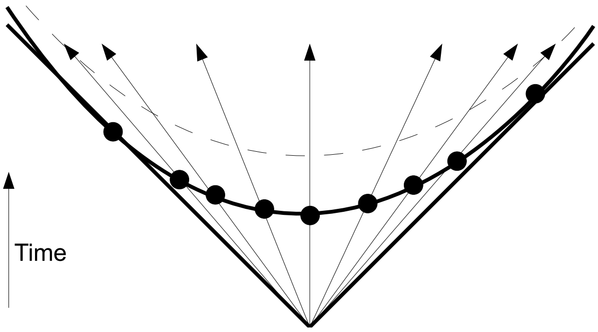

Figure 2: Showing the Big Bang state as a lattice on a hyperbola potentially starting from a singularity. The hyperbolae show slices of equal proper-time since the Big Bang.

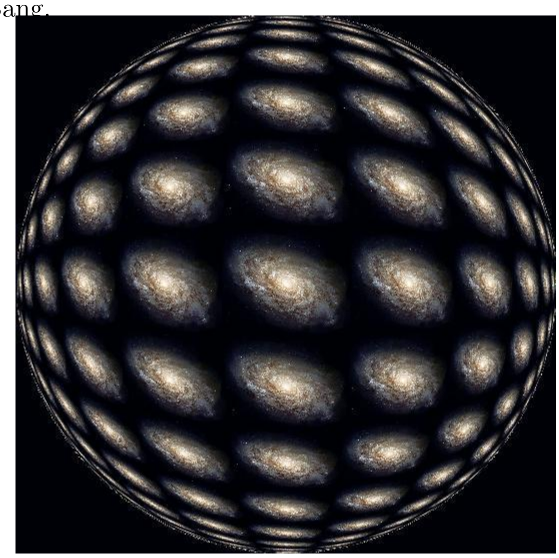

Figure 3: Showing the spread of matter in the Universe in which the Big Bang state is a hyperbola or singularity. Fast moving galaxies are smaller due to length contraction. The Universe is contained in a sphere.

matter/anti-matter. In other words this is no longer a paradox if we reject the assumption that matter was created at the Big Bang and simply assume that it was always there.

16 Higher Dimensions

So far we have been considering models based on 4 dimensional space-time. Some theories of the Universe require more dimensions such as 11 for supergravity/ M-Theory[7] with 7 closed dimensions. These models also need initial conditions. If the curled up dimensions are of the order of the Plank Length then we would still only need to only consider 3 dimensional lattices, the extra dimensions simply appearing as (an infinite tower of) flavours, polarisations and masses of the particles. Similarly in superstring theory, the modes of the string can be interpreted as an infinite tower of particles of different masses hence the lattice model still applies but with infinitely more "colourisations" of the lattices.

17 Supersymmetry

For models with supersymmetry (which, at this time, has not been observed in nature[6]), we would expect that the lattice to consist of fermions *and* bosons since this would add additional symmetry to the lattice and the Big Bang State is required to be the most symmetric state. If we evolve the Universe back in time from the present day until we get to the quark-gluon plasma, although the Pauli exclusion principle requires only that fermions be separated and hence be likely to form a lattice, supersymmetry would lessen the distinction between fermions and bosons and so bosons being included in the lattice does not seem so far fetched. However, at present, supersymmetry is only a theoretical possibility.

18 The Empty Universe or 'Why is there something rather than nothing?'

Another possibility for the Big Bang State Vector that we have not considered so far is that of a Universe consisting of no particles at all. We could add this to the state vector so that there is a non-zero amplitude for an empty universe. Does this have any meaning? In terms of the Copenhagen interpretation we would simply have to appeal to the Anthropic Principle and say that because we have observed the Universe in it's current state, the wave function collapsed into it's current form which does not include the history containing no particles. In the many worlds interpretation we say that there exists a parallel universe which contains no particles. We should leave it up to philosophers to decide if a Universe which contains nothing really exists!

19 Conclusion

One might be tempted to say that something as significant as the Big Bang and the start of the Universe should not have such a reasonably simple state vector. But on the contrary, the laws of thermodynamics, *demand* that the state Vector at time T=0 is incredibly simple and symmetrical. Another criticism might be that gravity does not have a prominent place in this state vector whereas it is of fundamental importance to cosmology but it can be argued that at time T=0 the matter particles have not "had the chance" yet of emitting gravitons. On philosophical grounds one may wish to believe that all matter is created at the Big Bang. Yet, the evidence tells us that if this were true (and that there is matter/antimatter symmetry) there would be an equal amount of matter and anti-matter which we do not see. One may, of course, object on religious grounds that to describe the Big Bang State Vector of the Universe is to remove the hand of "God" from the act of Creation. There could be many objections but ultimately the only way to see if the Big Bang State Vector is the correct one is to see if it consistent with observations.

Finally, we must remember that we have only constructed a single candidate for the Big Bang State Vector. The correct formulation for it depends on first determining the particle content of the Universe and secondly finding the most symmetrical lattice formation which satisfies the most symmetries.

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