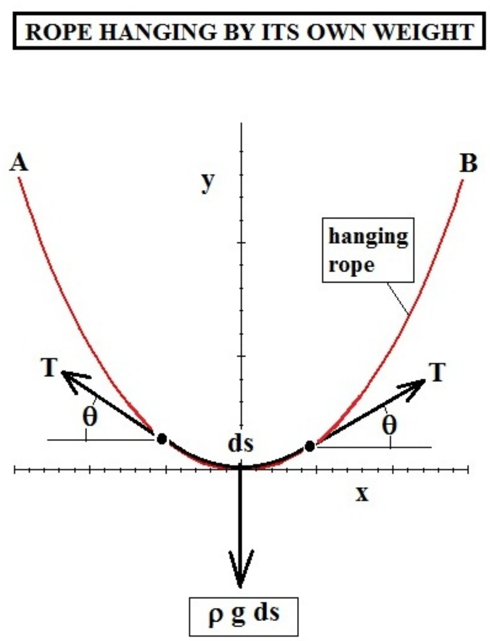
SHAPE OF A HANGING ROPE

One of the classic problems in statics is to determine the shape of a rope hanging between two fixed supports by its own weight. Also a second related problem deals with a weightless rope hanging between two fixed points and subjected to a constant downward force along its entire length. We want here to show how these problems lead to two simple solutions.

We begin with theproblem a uniform density rope of linear density ρ held between two fixed points A and B as shown-



Without loss of generality we can have the rope ends A and B be located at the same height and at equal distances from the y axis. By symmetry one would expect the slope y'=dy/dx=0 at the point where the rope crosses the y axis. Looking at the short segment $ds=dx\ \mathrm{sqrt}[1+(y')^2]$of the rope near y=0, we get from the equations for static equilibrium that –

$$T\cos(\theta)=\text{const.} \qquad \text{and} \qquad 2T\sin(\theta)=\rho g ds$$

with g being the acceleration of gravity. Combining these last two equations yields-

$$y''=\sqrt{1+(y')^2}$$

after setting Tcos(θ)=ρg/2 to simplify this last equation. To solve it we let z=y' to get-

$$\int_0^{y'} \frac{dz}{\sqrt{1+z^2}} = x$$

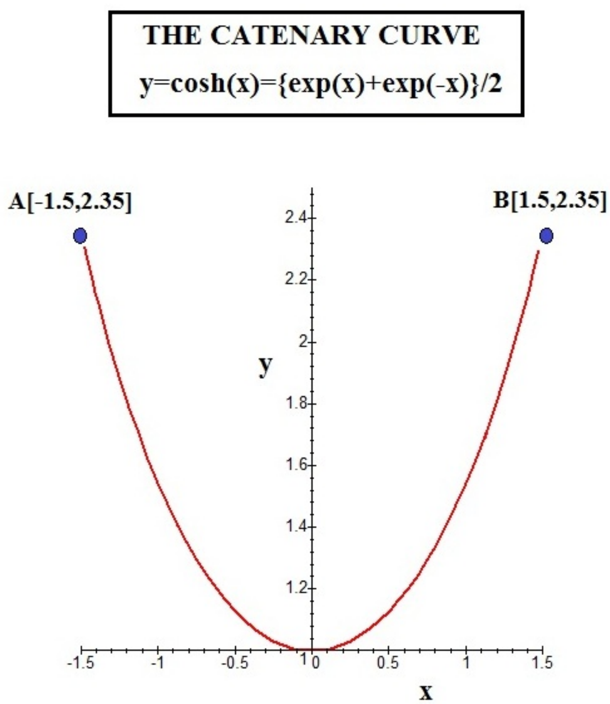
Integrating then produces the solution y'=sinh(x) which means-

$$y(x) = \cosh(x) + a$$

If we impose the extra condition that y(0)=1, we have the final result-

$$y(x)=\cosh(x)$$

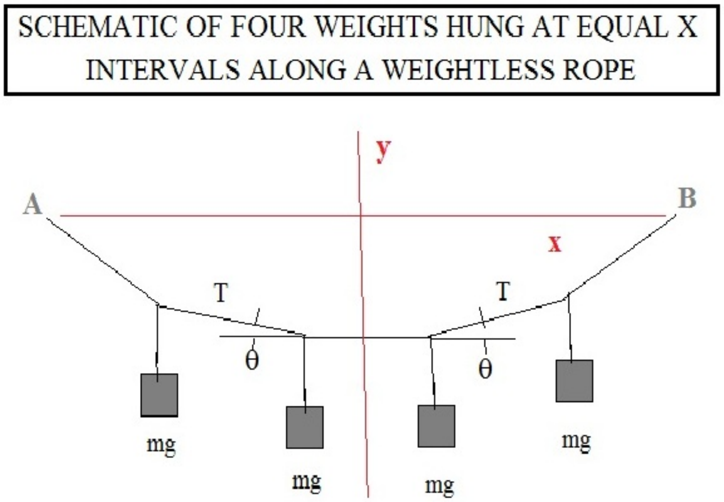
which represents the classic catenary curve as shown in the following graph-



Note the even symmetry about the y axis and its lowest point occurring at [x,y]=[1,0]. The shape is essentiasly that taken by a hanging telephone or power line . It is also the shape of a clothesline when no washing is hung on it. The English physicist Robert Hooke , famous for Hooke' law in mechanics and being a bitter rival of Newton, first pointed out that inverting the catenary produces the most stable of all arches. Since all compression forces lie strictly along the curve, it requires no flying buttress to support it unlike the requirement for a standard gothic arch. The Gateway Arch in St.Louis is close in shape to such an inverted catenary. If you get a chance to visit St Louis , I can highly recommend taking the internal elevator-escalator to the top. Its offers a spectacular view of the Mississippi River and the surrounding countryside. Also the silos intended to protect Assad's jet fighter airbase in Syria and which were recently attacked by US cruise missiles, very much resemble inverted catenaries. Here is a pre-strike photo-

The US government release of a photo of one such silo showing burns near its apex after the cruise missile attack is suspect. There is no way to take such a clear ground level photo from a satellite and Assad would certainly not want to reveal any damage.

A second type of rope suspension problem involves the shape of a weightless rope fixed at its ends A and B and hung at equally spaced horizontal intervals by equal weights mg. One has the following setup-



Treating the central two masses as one unit, we find-

$T\cos(\theta)=T\sin(\theta)(dy/dx)=\text{const.}$ and $2T\sin(\theta)=2mg$

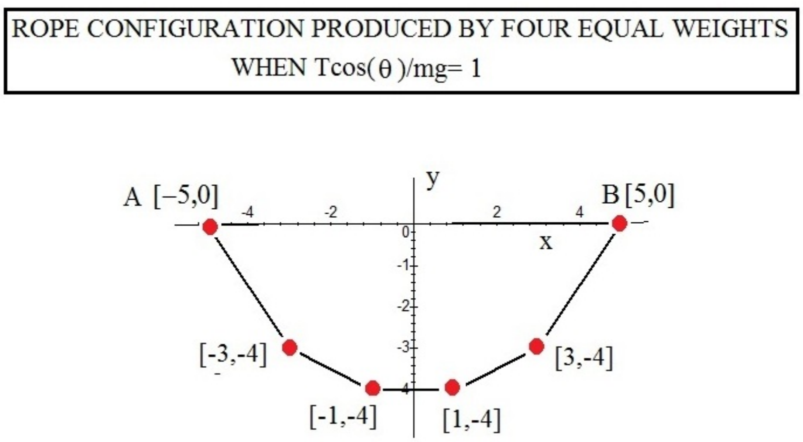
On combining this leads to-

$$\frac{dy}{dx}=\frac{T\cos(\theta)}{mg}= \text{a fixed number}$$

This means all sections of rope connecting two neighboring weights are straight lines given by-

y=a+bx

the horizontal line connecting weights two and three is just y=a. Now letting a=-4 and placing A and B at [- 5,0] and 5,0], respectively, we find that weights one and three ahave their attachments to the rope at [-3,-4] and [3,-4]. To have this happen one needs to set the additional constraint that Tcos(θ)/mg=1. Here is the picture of the configuration-



Note that this figure starts resembling the catenary. With more equally spaced constant masses the configuration will indeed yield a catenary. If just a single weight is placed at the rope center [x,y]=[0, -d] and the symmetric end points A and B are fixed at [-L/2,0] and [L/2,0], respectively, we have the rope walker problem. Typically one must make the tension on the rope sufficiently large so that the sag d produced by the rope walker remains small compared to L. Solving the equilibrium equations in this case yields-

$$d = \frac{(L/2)}{\sqrt{(\frac{2T}{W})^2 - 1}} \approx \frac{WL}{4T}$$

Thus for a L=60ft long rope which holds a 160 lb tight rope walker at its middle producing a 3 inch depression d, will require a rope tension of approximately 9600 lbs.The rope must be strong enough to withstand such a tension.

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