A Theory of Microwave Propulsion for Spacecraft

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Abstract

A new principle of electric propulsion for spacecraft is introduced, using microwave technology to achieve direct conversion of d.c. power to thrust without the need for propellant.

A simplified illustrative description of the principles of operation is given, followed by the derivation, from first principles, of an equation for the thrust from such a device. The implications of the law of conservation of energy are examined for both static and dynamic operation of the device. A summary of the experimental work, that has been carried out to verify the theory, is then given.

1. Introduction

The technique described in this paper uses radiation pressure, at microwave frequencies, in an engine which provides direct conversion from microwave energy to thrust, without the need for propellant.

The concept of the microwave engine is illustrated in fig 1. Microwave energy is fed from a magnetron, via a tuned feed to a closed, tapered waveguide, whose overall electrical length gives resonance at the operating frequency of the magnetron.

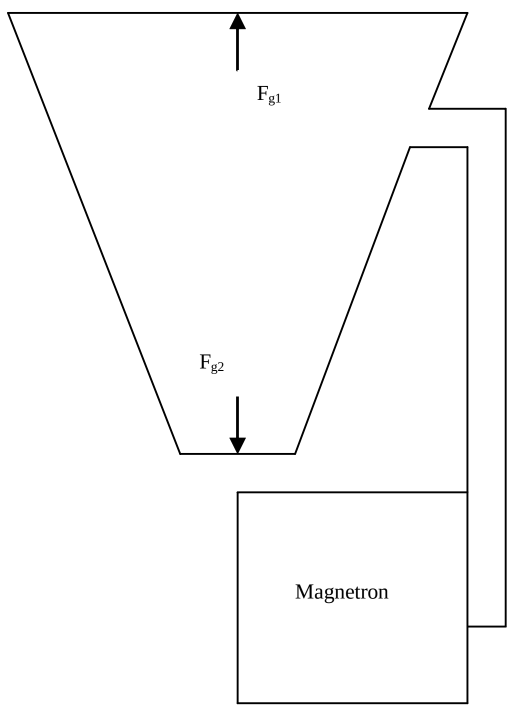

Fig 1 Diagram of Engine Concept

The group velocity of the electromagnetic wave at the end plate of the larger section is higher than the group velocity at the end plate of the smaller section. Thus the radiation pressure at the larger end plate is higher that that at the smaller end plate. The resulting force difference (F_{g1} -F_{g2}) is multiplied by the Q of the resonant assembly.

This force difference is supported by inspection of the classical Lorentz force equation (reference 1).

$$F = q(E + vB). \qquad (1)$$

If v is replaced with the group velocity v_g of the electromagnetic wave, then equation 1 illustrates that if v_{g1} is greater than v_{g2}, then F_{g1} should be expected to be greater than F_{g2}.

However as the velocities at each end of the waveguide are significant fractions of the speed of light, a derivation of the force difference equation invokes the difference in velocities and therefore must take account of the special theory of relativity.

Relativity theory implies that the electromagnetic wave and the waveguide assembly form an open system. Thus the force difference results in a thrust which acts on the waveguide assembly.

2. Principles of Operation

2.1. Free Space Propagation

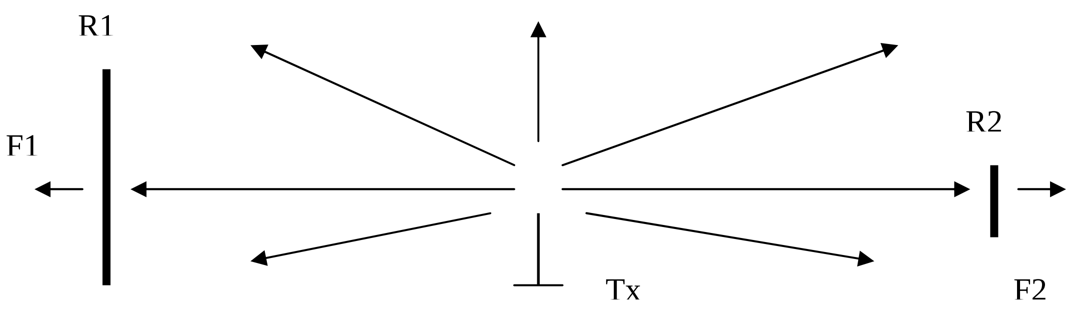

Fig 2.1.

Assume two reflective plates R1 and R2 are placed equidistance from a transmitter Tx in free space. The area of R1 is greater than R2, therefore the power incident on R1 is greater than R2, and therefore the force F1 is greater than F2. If the two plates are connected together, the resultant force F1-F2 will cause the assembly to move, in accordance with Newtons laws.

If the movement is small compared to the distance between the plates there will be negligible effect on reflected powers or forces. In a similar manner, small movement of the transmitter will have negligible effect.

This independence of transmitter and plate movement, leads to the conclusion that if the transmitter and the plates are connected, the whole assembly will move. Whatever the final speed of movement achieved, the propagation velocity remains c, the velocity of light in free space.

2.2. <u>Propagation in a wide waveguide</u>

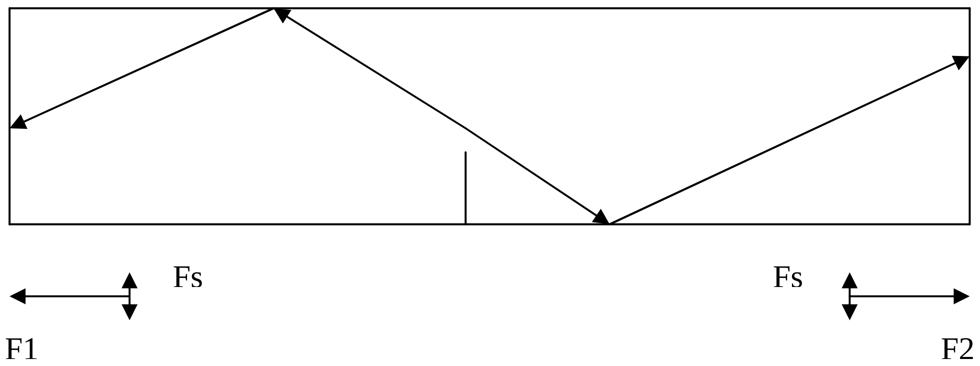

Fig 2.2.

Assume the two reflective plates, now the same size, form the end plates of a wide waveguide. The propagation path is constrained by reflection from the side walls of the waveguide and forms a sawtooth pattern as shown. The velocity of the microwave energy in the axial direction, termed the group velocity, is now below the value of c due to the longer, sawtooth path travelled by the wave.

Resolution of the forces at the two ends of the waveguide give equal opposing forces F1 and F2 and small orthogonal side wall forces Fs.

2.3 Propagation in a narrow waveguide

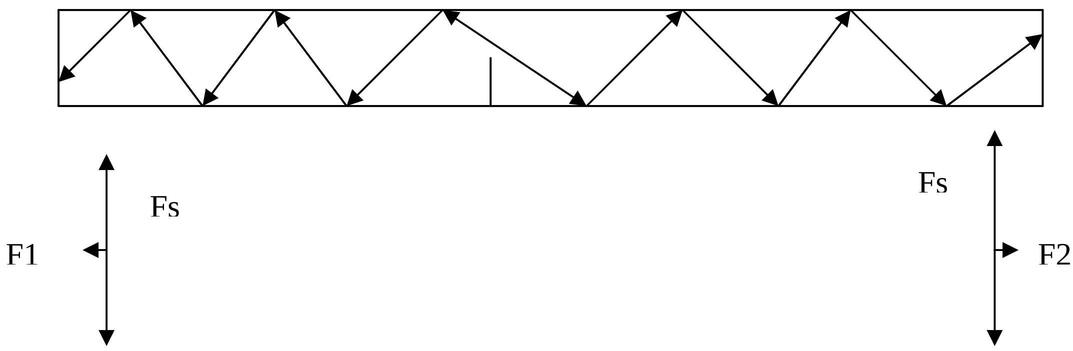

Fig 2.3

Assume the waveguide is now narrow compared to 2.1.2. with the result that the sawtooth becomes sharper. In this case the group velocity in the axial direction along the waveguide becomes an even small fraction of c, as the sawtooth path the wave must travel is longer than in 2.1.2.

Resolution of the forces now gives a smaller value of F1 and F2, due to the lower axial velocity at which the energy is being reflected. However a much higher value of sidewall force Fs is experienced.

2.4 Propagation in a tapered waveguide

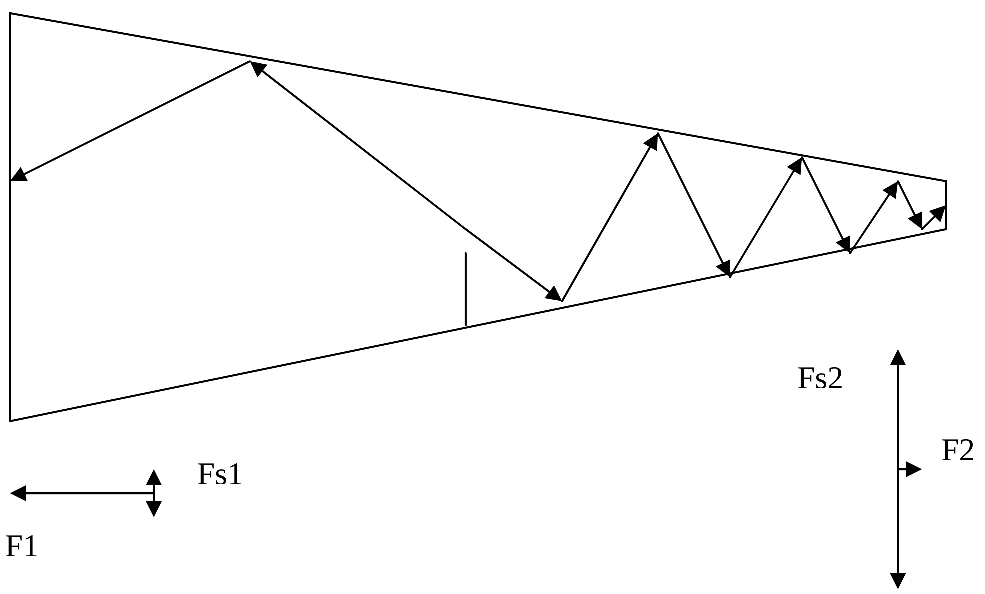

Fig 2.4

Assume that the waveguide is now tapered as shown. In this case the group velocity is higher at the wide end than at the narrow end. Thus resolution of the forces shows F1 is greater than F2, whereas the sidewall force Fs2 is higher than Fs1.

As with the free space example in 2.1.1, the force difference F1-F2 will give rise to movement of the waveguide, but this will have no effect on the propagation velocities within the waveguide.

2.5 Propagation in a resonant tapered waveguide

As with any microwave cavity, if the axial path length is a multiple of half the mean guide wavelength, at the frequency of operation, then the waveguide will form a resonant cavity. The electrical and magnetic fields at each end plate will add in phase, to give instantaneous powers equal to Q times the transmitted power. This will give rise to a force difference equal to Q(F1-F2). Note that in a well designed cavity the unloaded Q can reach values greater than 50,000. It is this high multiplication of the small force difference that gives the engine a useful net force.

3. Derivation of the thrust equation.

Consider a beam of photons incident upon a flat plate perpendicular to the beam. Let the beam have a cross-sectional area *A* and suppose that it consists of *n* photons per unit volume. Each photon has energy *hf* and travels with velocity *c*, where *h* is Planck's constant and *f* is the frequency. The power in the incident beam is then

$$P_0 = nhfAc\,. \tag{2}$$

The momentum of each photon is *hf/c* so that the rate of change of momentum of the beam at the plate (assuming total reflection) is *2nhfA*. Equating this change of momentum to the force F_0 exerted on the plate, we find

$$F_0 = 2nhfA = \frac{2P_0}{c}\,, \tag{3}$$

which is the classical result for the radiation pressure obtained by Maxwell (reference 2). The derivation given here is based on Cullen (reference 3). If the velocity of the beam is *v* then the rate of change of momentum at the plate is *2nhfA(v/c)*, so that the force F_g on the plate is in this case given by

$$F_g = \frac{2P_0}{c}(v/c)\,. \tag{4}$$

We now suppose that the beam enters a vacuum-filled waveguide. The waveguide tapers from free-space propagation, with wavelength λ_0, to dimensions that give a waveguide wavelength of λ_g and propagation velocity v_g . This is the group velocity and is given by

$$v_g = \frac{c}{\sqrt{\mu_r e_r}} \frac{\lambda_0}{\lambda_g}. \tag{5}$$

Then from (4) and (5) (with $\mu_r = e_r = 1$) the force on the plate closing the end of the waveguide is

$$F_g = \frac{2P_0}{c}\left(v_g / c\right) = \frac{2P_0}{c}\frac{\lambda_0}{\lambda_g}; \tag{6}$$

see Cullen (p.102 Eq. (15)).

Assume that the beam is propagated in a vacuum-filled tapered waveguide with reflecting plates at each end. Let the guide wavelength at the end of the largest cross-section be λ_{g1} and that at the smallest cross-section be λ_{g2}. Then application of (6) to each plate yields the forces

$$F_{g1} = \frac{2P_0}{c}\frac{\lambda_0}{\lambda_{g1}}, \qquad F_{g2} = \frac{2P_0}{c}\frac{\lambda_0}{\lambda_{g2}}.$$

Now $\lambda_{g2} > \lambda_{g1}$, due to the difference in cross-section, and hence $F_{g1} > F_{g2}$. Therefore the resultant thrust T will be

$$T = F_{g1} - F_{g2} = \frac{2P_0}{c}\left(\frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_0}{\lambda_{g2}}\right). \tag{7}$$

We note that if the forces had been the mechanical result of a working fluid within the closed waveguide assembly, then the resultant force would merely introduce a mechanical strain in the waveguide walls. This would be the result of a closed system of waveguide and working fluid.

In the present system the working fluid is replaced by an electromagnetic wave propagating close to the speed of light and Newtonian mechanics must be replaced with the special theory of relativity. There are two effects to be considered in the application of the special theory of relativity to the waveguide. The first effect is that as the two forces F_{g1} and F_{g2} are dependent upon the velocities v_{g1} and v_{g2}, the thrust T should be calculated according to Einstein's law of addition of velocities given by

$$v = \frac{v_1 + v_2}{1 + \left(v_1 v_2\right)/c^2}.$$

The second effect is that as the beam velocities are not directly dependent on any velocity of the waveguide, the beam and waveguide form an open system. Thus the reactions at the end plates are not constrained within a closed system of waveguide and beam but are reactions between waveguide and beam, each operating within its own reference frame, in an open system.

From (7) and (5) we find

$$T = \frac{2P_0}{c}\left(\frac{v_{g1}}{c} - \frac{v_{g2}}{c}\right),$$

where

$$v_{g1} = c\lambda_0 / \lambda_{g1}, \qquad v_{g2} = c\lambda_0 / \lambda_{g2}.$$

Applying the above addition law of relativistic velocities we obtain

$$T = \frac{2P_0}{c^2}\left(\frac{v_{g1} - v_{g2}}{1 - v_{g1}v_{g2}/c^2}\right) = \frac{2P_0 S_0}{c}\left(\frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_0}{\lambda_{g2}}\right), \tag{8}$$

where the correction factor S_o is

$$S_0 = \left\{1 - \frac{\lambda_0^2}{\lambda_{g1}\lambda_{g2}}\right\}^{-1}.$$

The concept of the beam and waveguide as an open system can be illustrated by increasing the velocity of the waveguide in the direction of the thrust, until a significant fraction of the speed of light is reached. Let v_w be the velocity of the waveguide. Then as each plate is moving with velocity v_w, the forces on the plates, given by equation 6, are modified as follows:

$$F_{g1} = \frac{2P_0}{c^2}\left(\frac{v_{g1} - v_w}{1 - v_{g1}v_w / c^2}\right) = \frac{2P_0}{c^2}v_{ga}$$

and

$$F_{g2} = \frac{2P_0}{c^2}\left(\frac{v_{g2} + v_w}{1 + v_{g2}v_w / c^2}\right) = \frac{2P_0}{c^2}v_{gb}$$

The thrust is then given by

$$T = \frac{2P_0}{c^2}\left(\frac{v_{ga} - v_{gb}}{1 - v_{ga}v_{gb} / c^2}\right) \tag{9}$$

Thus as the velocity of the waveguide increases in the direction of thrust, the thrust will decrease until a limiting velocity is reached when $T=0$. This limiting value of v_w is reached when $v_{ga} = v_{gb}$. Fig 3 illustrates the solution to equation 9 for values of v_w

from 0 to c, where v_{g1} = 0.95 c, v_{g2} = 0.05c. It can be seen that if v_w is increased beyond the limiting value of 0.7118c, the thrust reverses.

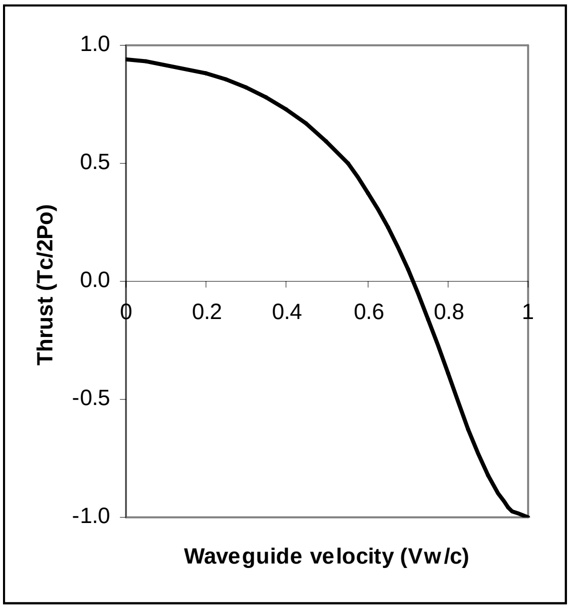

Fig 3. Solution to equation 9.

Returning to a stationary waveguide, we now let the waveguide include a dielectric-filled section at the smaller end of the taper and choose the dimensions to ensure a reflection-free transmission of the beam from the vacuum-filled section to the dielectric-filled section. Note that the reflection-free interface, with matched wave impedances , will ensure no forces are produced at the interface. We also suppose that the dielectric medium has μ_r = 1. Then the velocity v_{g2} is replaced by v_{g3}

$$v_{g3} = c\frac{\lambda_d}{\sqrt{e_r}\lambda_{g3}}$$

where λ_d is the wavelength in the unbounded dielectric medium and λ_{g3} is the guide wavelength at the end plate of the dielectric section. It then follows that the thrust takes the form

$$T = \frac{2P_0 S_d}{c}\left(\frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_d}{\sqrt{e_r}\lambda_{g3}}\right) \tag{10}$$

where

$$S_d = \left\{1 - \frac{\lambda_0 \lambda_d}{\sqrt{e_r}\lambda_{g1}\lambda_{g3}}\right\}^{-1}.$$

We suppose that the composite waveguide is resonant at the frequency of the microwave beam and that the conductive and dielectric losses are such that there are Q return paths (each at power P_0). Then the total thrust is finally given by

$$T = \frac{2P_0 Q S_d}{c}\left(\frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_d}{\sqrt{e_r}\,\lambda_{g3}}\right). \qquad (11)$$

4. Dynamic operation of the engine.

We now examine the implications of the principle of the conservation of energy when the thrust is first measured on a static test rig, and then when an engine is used to accelerate a spacecraft.

With the microwave engine mounted on a static test rig all the input power P_0 is converted to electrical loss. In this case the Q of the engine may be termed Q_u, the unloaded Q.

Now
$$Q_u = \frac{P_c}{P_0} = \frac{P_c}{P_e},$$
where P_c is the circulating power within the resonant waveguide and P_e is the electrical loss. From (11) we find
$$T = \frac{2P_0 D_f Q_u}{c},$$
Where D_f is the design factor
$$D_f = S_d\left(\frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_d}{\sqrt{e_r}\,\lambda_{g3}}\right).$$

Then
$$T = \frac{2D_f P_c}{c}. \qquad (12)$$

Thus if the circulating power remains constant, for instance in a superconducting resonant waveguide, then T will remain constant. This will be important in non spacecraft applications where very high values of Q_u could be employed to provide a constant thrust to counter gravitational force.

If the engine is mounted in a spacecraft of total mass M and is allowed to accelerate from an initial velocity v_i to a final velocity v_f in time Δt , then by equating kinetic energies we obtain:

$$P_k \Delta t = \frac{M}{2}\left(v_f^{\,2} - v_i^{\,2}\right)$$

where P_k is the output power transferred to the spacecraft. From this we obtain

$$P_k \Delta t = \frac{M}{2}\left(v_f - v_i\right)\left(v_f + v_i\right),$$

so that
$$P_k = M\bar{v}a\,, \tag{13}$$

where $\bar{v}$ is the average velocity over time Δt and a is the acceleration of the spacecraft.

Now $M.a$ is the force due to the acceleration of the spacecraft, which opposes the thrust of the engine. Then

$$P_k = \frac{2P_0 Q_l D_f \bar{v}}{c} \tag{14}$$

where Q_l is the loaded Q of the engine when it is delivering an output power P_k.

The electrical power losses P_e are assumed to be I^2R losses and thus for any value of Q ,

$$P_e = Q^2 P_{e0}$$

where P_{e0} is the loss for $Q=1$. From the static case, we have

$$P_{e0} = \frac{P_0}{{Q_u}^2}\,,$$

so that
$$P_e = P_0\left(\frac{Q_l}{Q_u}\right)^2. \tag{15}$$

For an accelerating spacecraft,

$$P_0 = P_e + P_k\,.$$

Substitution of (14) and (15) into this last equation then yields

$$\left(\frac{Q_l}{Q_u}\right)^2 + \frac{2Q_l D_f \bar{v}}{c} = 1\,. \tag{16}$$

Fig 4.1 shows the solution to (16) for values of $\bar{v}$ up to 10km/sec and for values of Q_u equal to $5\text{x}10^3$, $5\text{x}10^4$ and $5\text{x}10^5$. The value of D_f is taken to be 0.945, resulting from the same group velocities as those used in fig 3.

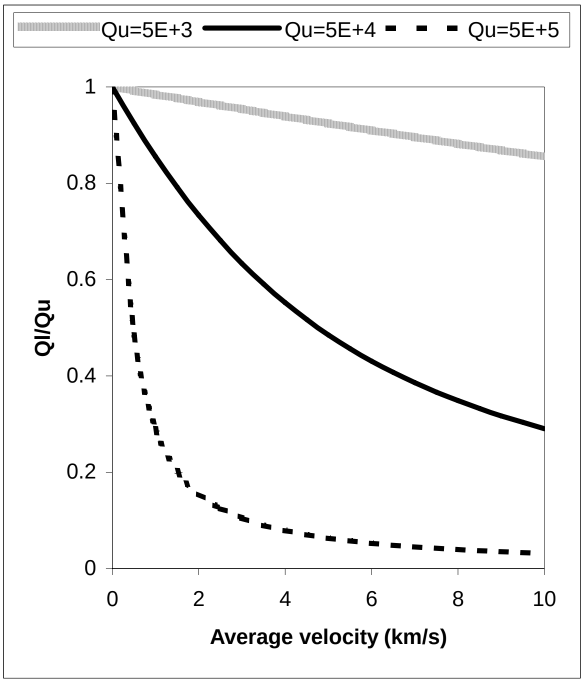

Fig 4.1 Solution to equation 16.

For D_f equal to 0.945 and an average velocity of 3 km/s, the specific thrust is obtained from (11) and (16) and is given in fig 4.2. This illustrates that the specific thrust increases to a maximum of 333 mN/kW at this velocity.

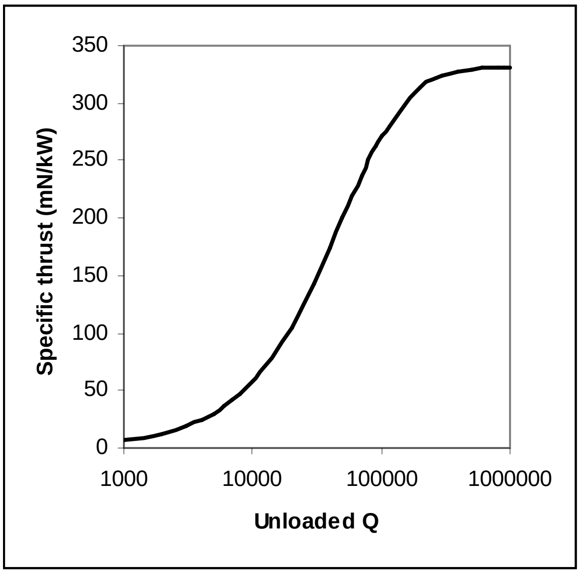

Fig 4.2 Specific thrust at 3km/s.

5 Summary of Experimental Work

5.1 Experimental Thruster

An experimental thruster with a tapered circular air-filled section and a cylindrical dielectric section was designed and manufactured. The thruster operated at 2540 MHz and was powered by an 850 W magnetron.

The thruster was designed with the aid of a computer program, which modelled the waveguide assembly as a series of 250 short sections. The program was verified by small signal testing. A maximum Q of 5,900 was measured by mechanical tuning with a fixed frequency source.

An extensive test programme was carried out on the experimental thruster. In each series of tests, the test runs were repeatedly carried out with the thruster in both a nominal position (thrust direction vertically up, thus measured thrust is negative) and an inverted position (thrust direction vertically down, thus measured thrust is positive). In all tests the thruster gave the correct thrust direction.

The experimental thruster, which weighs 15.5 kg, was subjected to tests using three different balance configurations to measure the thrust. One series of tests employed a counterbalanced beam and load cell. A second series replaced the load cell with a 110 g commercial precision balance having a resolution of 1 mg. In the third series of tests the thruster was directly mounted on a 16 kg precision balance with a resolution of 100 mg. Tests were also carried out with the thruster sealed into an airtight enclosure to calibrate out thermal buoyancy effects.

A total of 450 test runs were carried out using 5 different magnetrons. Input and resonant tuning positions were varied to give a range of thrust outputs from a maximum of 16mN, at optimum tuning, down to zero when totally detuned.

Substitution of the design parameters and measured Q into equation 10 gives a theoretical thrust output of 16.6 mN which is in close agreement with the thrust measured.

5.2 Demonstrator Engine

The design software was updated and used to design a higher Q thruster, which was incorporated into a complete Demonstrator Engine. The engine was powered from a water cooled, 2450 Mhz, magnetron, with variable output power up to 1.2kW. The engine incorporated a coolant loop with pump and radiators, a stepper motor tuning mechanism, and full on-board control and telemetry. A maximum Q of 35,000 was measured in initial small signal tests, where a small degradation of Q was attributed to the loading effect of the measurement probe.

A test programme was carried out to measure thrust on a vertical test rig with the thrust vector up, down and horizontal, and on a horizontal rig, with the thrust vector horizontal. In all test configurations the measured thrust was in the correct direction. Test runs were carried out with both fixed and swept resonance tuning. Maximum thrust was measured at the tuning position corresponding to the operating frequency and the design point, calculated from the as-built dimensions.

The microwave input feed assembly, from magnetron to thruster, incorporated tuning elements which enabled tests to be carried out over the full envelope of input match conditions. A maximum specific thrust of 214mN/kW was achieved, which when substituted in equation 10, together with the design parameters, gives a calculated Q of 38,000.

6. Conclusions

A theory has been developed for a propulsion technique, which for the first time, allows direct conversion from electrical energy at microwave frequencies to thrust. An expression has been developed from first principles to enable the thrust for such a technique to be calculated. This expression has been verified by the results of two test programmes carried out on an Experimental Thruster and a Demonstrator Engine.

Consideration of the principle of the conservation of energy has led to the derivation of an equation for the loaded Q of an engine used to accelerate a spacecraft at orbital velocities, which allows the prediction of thrust. The spacecraft system advantages in eliminating the need for propellant, together with the predicted specific thrust, offer a major improvement in overall mission performance.

Acknowledgements

The author is grateful for the assistance given by colleagues in SPR Ltd, by Dr R B Paris of Abertay University, Dundee, by J W Spiller of Astrium UK Ltd and by Professor J Lucas of The University of Liverpool. The theoretical work and experimental programmes were carried out with support from the Department of Trade and Industry under their SMART award scheme, and then under a Research and Development grant.

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