Indian Calendars

Leow Choon Lian

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Supervisor: Associate Professor Helmer Aslaksen

Department of Mathematics

Faculty of Science

National University of Singapore

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Contents

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I am delighted to be the first student to work on a project about Indian Calendars. Initially, I was doubtful of my capability of understanding the reading materials and doing the programming in Mathematica. It was my supervisor Associate Professor Helmer Aslaksen who gave me confidence and helped me in the completion of my project. I would like to thank him for his patience and guidance.

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Introduction

India is a vast country consisting of altogether 35 states and union territories. With a long civilization and hence a rich history, the country naturally developed its own methods of time keeping. Due to cultural diversification, these methods vary in different regions and variations in the Indian (Hindu) calendar-making emerged.

Today, there are several calendars being used in India. The government uses the Gregorian calendar for administrative purposes. The Muslims use the Muslim (Islamic) calendar. The Indian solar and lunisolar calendars, including their variations, are used for both civil and religious purposes and hence exert great influence on the daily activities of the people of India. The aim of this thesis is to describe the workings of the Indian solar and lunisolar calendars. The rules and principles that guide the calendars will be explained in a simple and systematic way for readers to understand.

The first chapter introduces basic astronomical concepts required to understand the fundamental units of time, namely, the day, the month and the year. In the second chapter, we start to classify calendars into solar, lunar and lunisolar calendars. In the last chapter, we will focus on the conventions that are used for making the Indian solar and lunisolar calendars. We start with a brief introduction of the Indian calendars which are used in India. Then we proceed with the discussion of the Indian solar and lunisolar calendars respectively. Finally, we present a number of computer codes to generate the dates of some Indian solar and lunisolar calendars.

Statement of Author's Contributions

My thesis provides explanation for the rules behind Indian calendar-making. Honestly speaking, I have spent tremendous effort and time to write and organize my thesis so that it is easy for readers to follow.

In addition to the thesis, I have also written computer codes to produce the dates of some Indian calendars that will be mentioned in Chapter 3. I call them the calendar codes. These codes are included in the appendix section. They are basically modification and piecing of some of the computer codes obtained from the Mathematica package Calendrica.m written by Nachum Dershowitz and Edward M. Reingold. My supervisor and I worked together to discuss the ideas and approaches to write the algorithms for the calendar codes.

True longitudes of the Sun and the Moon are essential data that has to be included in the algorithms of the calendar codes. Calendrica.m contains computer codes to give the values of these longitudes. However, Nachum Dershowitz and Edward M. Reingold wrote their algorithms basing on old Siddhantic methods. This causes their outputs to differ from the longitudes obtained using modern methods. I will explain what Siddhantic and modern methods are in Chapter 3. As for a detailed description of their algorithms basing on the Siddhantic methods, it can be found in reference book (10).

Because of time constraint, I could not debug their computer codes for finding the longitudes. I have used them directly in my algorithms. Nevertheless, I have come up with codes to give, as accurate as possible, other information that are necessary to run the calendar codes correctly. They are the Indian Standard Time (IST) for sunrise, sunset, aparahna, new moon and full moon. Again, we will come to the definitions and the reasons as to why these times are important in Chapter 3. The following is a list of the computer and calendar codes that I have written:

ujjainSunrise
ujjainSunset
ujjainAparahna
orissaHinduSolar
tamilHinduSolar

malayaliMonth

malayaliYear

malayaliHinduSolar

bengalHinduSolar

IndianNewMoonAtOrBefore

amantaSouthHinduLunar

checkSkippedRasiForEasternRule

amantaEastHinduLunar

checkSkippedRasiForNorthWesternRule

amantaNorthWestHinduLunar

IndianFullMoonAtOrBefore

I have tested the codes above by comparing the outputs with some of the actual data obtained from reference books (7) and (11). I find that the codes to give the IST for sunrise, sunset and aparahna are accurate within about 5 minutes. For those on IST for new moon and full moon, the discrepancy is about a minute. However, the generated Indian calendar dates may differ from the actual date by a day. In addition, the occurrence of kshaya months, leap months and leap days may not tally with those indicated in the actual lunisolar calendars. I will give the definitions of kshaya and leap months and leap days later in Chapter 3.

Errors in generating the Indian calendar dates from the codes occur because for making the actual Indian calendars, modern methods are employed to measure longitudes of the Sun and the Moon. These values are more accurate than the ones obtained by Siddhantic methods which are being used in the calendar codes. Hence there is a need to produce algorithms to calculate the true longitudes of the Sun and the Moon using modern methods. With this correction, the accuracy of the calendar codes that I have obtained can be improved and perhaps some other codes can be written to determine the dates of important Indian festivals and religious events.

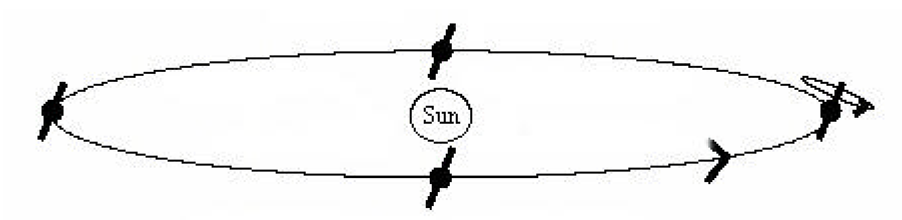
Chapter 1: Astronomical Bases of Calendars

A calendar is a system of organizing units of time for the purpose of reckoning time over extended period. The natural units of time are the day, the month and the year. They are based on the Earth's rotation on its axis, the Moon's revolution around the Earth and the Earth's revolution around the Sun respectively. In this chapter, we will look at the essential astronomical concepts that are needed to define these units of time. I obtain most information from the references (1) and (6) available in my supervisor's website. Reference book (3) contains more details on ancient astronomy.

The Earth and the Sun

The Earth revolves anticlockwise around the Sun in an elliptical orbit, the plane of which is called the *plane of the ecliptic*. At the same time, the Earth rotates anticlockwise on its own axis. This axis is tilted from the pole of the plane of the ecliptic by 23.5^0. The Earth's rotation will cause an observer on Earth to see the Sun as rising from the east and setting in the west.

Figure 1: The plane of the Ecliptic



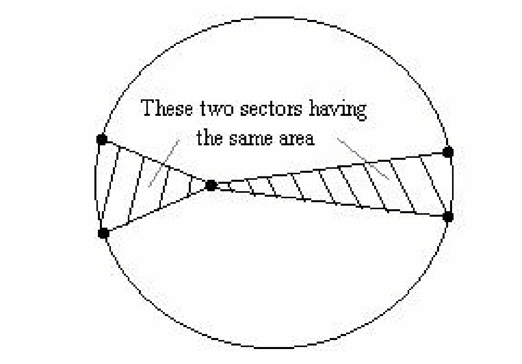
Kepler's Laws

The Earth's revolution around the Sun obey Kepler's first two laws of planetary motion, namely,

1. The orbit of a planet around the Sun is an ellipse with the Sun at one focus of the ellipse.
2. The radial line that joins a planet to the Sun sweeps out equal areas in equal intervals of time.

The first law explains the Earth's elliptical orbit around the Sun. The second law implies that the Earth's velocity along the elliptical orbit is not uniform. In fact, the Earth moves faster along the orbit around the *perihelion,* the point where it is closest to the Sun, and slower when it is around the *aphelion*, the point where the Earth is farthest away from the Sun.

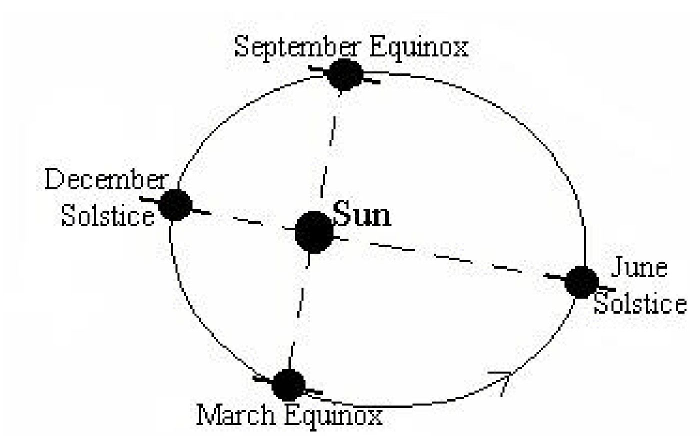
Figure 2: Kepler's first two laws



The Equinoxes and Solstices

Besides the perihelion and aphelion, the Equinoxes and Solstices are also important positions on the ecliptic. As the Earth revolves around the Sun, the two positions at which the projection of the Earth's axis onto the plane pointing directly towards the Sun are called the *June (Summer) and December (Winter) Solstices*. On the other hand, the two positions at which the radial line from the Sun to the Earth is perpendicular to the Earth's axis are the *March (Spring or Vernal) and September (Autumnal) Equinoxes*. These four positions are often known as the *seasonal markers*.

Figure 3: The Seasonal Markers

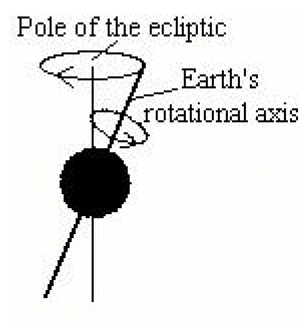


Some people make the mistake thinking that the June Solstice and aphelion (or the December Solstice and perihelion) should coincide all the time. This is not always true. To understand why, we need to know *precession of the Equinoxes*.

Precession of the Equinoxes

Under gravitational attractions of the Sun, the Moon and the planets, the Earth's axis undergoes a slow, conical clockwise motion, with a period of about 25800 years, around the pole of the ecliptic and maintains the same inclination to the plane of the ecliptic. This causes the March Equinox to slide westward on the ecliptic at a rate of about 50.2'' per year. We call this *precession of the Equinoxes*.

Figure 4: Precession of the Equinoxes



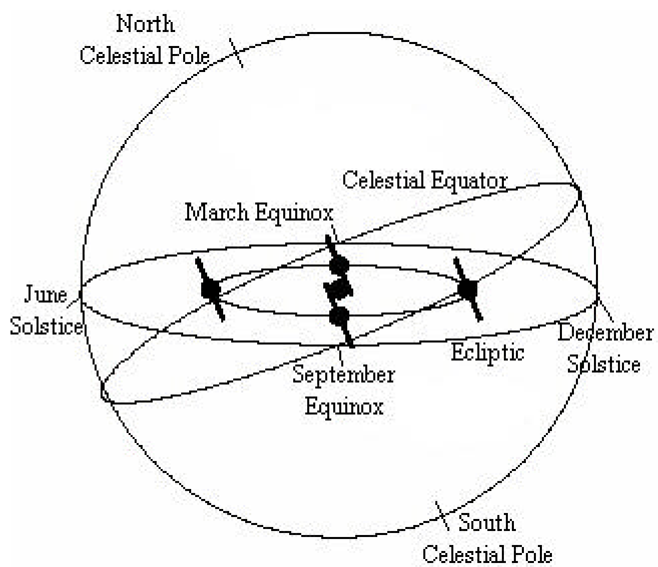
The Celestial Sphere

The model that is used above to describe the motions of the Earth and the position of the seasonal markers is a *heliocentric* model where the Sun is taken to be at the centre. Ancient astronomers, however, adopted the *geocentric* model which has the Earth placed in the middle of the *Celestial Sphere*. The *Celestial Sphere* is an imaginary sphere around the Earth. Stars appear on the inner surface of the sphere as points of light. The Sun, the Moon and the stars are seen to rotate from east to west.

The Earth's rotational axis extends to meet the sphere in the *north and south celestial poles*. The *celestial equator*, an extension of the Earth's equator to meet the sphere, is the great circle midway between the poles. The path of the Sun across the Celestial Sphere is a great circle called the *ecliptic*. It is an extension of the Earth's elliptical orbit around the Sun to meet the sphere. The plane of this ecliptic makes an angle of 23.5^0 with the celestial equator.

The points at which the ecliptic intersects the celestial equator are the Equinoxes while the points at which the ecliptic and the celestial equator are farthest apart are the Solstices. We will use either of these models for our calendar discussion.

Figure 4: The Celestial Sphere



The Moon

The Moon shines by reflected sunlight. Half of the Moon that faces towards the Sun is always illuminated. An observer on Earth will see the Moon as rising from the east and setting in the west like the Sun. At the same time, the Moon revolves anticlockwise around the Earth causing different lunar phases to occur.

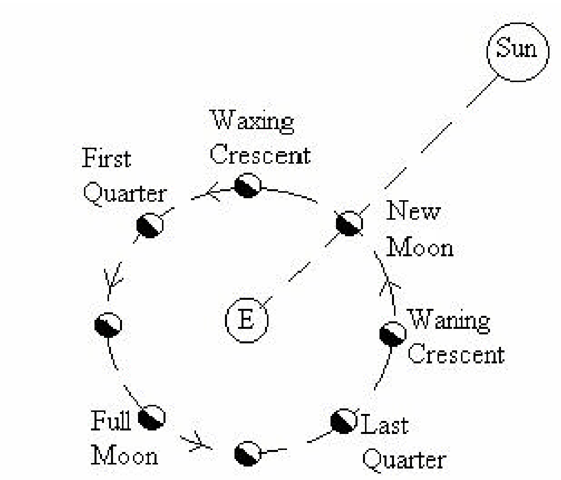
When the Moon is in *conjunction* with the Sun, that is, when the Moon is directly between the Sun and the Earth, the unilluminated half faces us. We called this *new moon*. At this time, the Moon rises and sets approximately at the same time as the Sun. A few days after conjunction, we can see the *waxing* crescent at night.

As the Moon moves anticlockwise in its orbit around the Earth, it begins to rise (and set) after the Sun does so we can gradually see more of the Moon. Then the Moon reaches its first Quarter.

As the Moon's orbit continues, there is a time when the Moon is in *opposition* to the Sun, meaning that the Moon is aligned with the Sun and the Earth but on the opposite side of the Earth. We called this *full moon*. At this time, the Moon rises at sunset and sets at sunrise.

Gradually, we see the Moon growing smaller, reaching its third Quarter and then its *waning* crescent before new moon occurs again. We can say that a lunar phase cycle has completed in this case.

Figure 6: Phases of the Moon



The Units of Time

As the Earth rotates with respect to the Celestial Sphere, the Sun, the Moon and the stars are seen to move across the sky from east to west. We see that the alternation of daylight and night happens much more frequently than the lunar phases and the seasons. Hence astronomers relate the day with a complete rotation of the Earth on its axis.

The Day

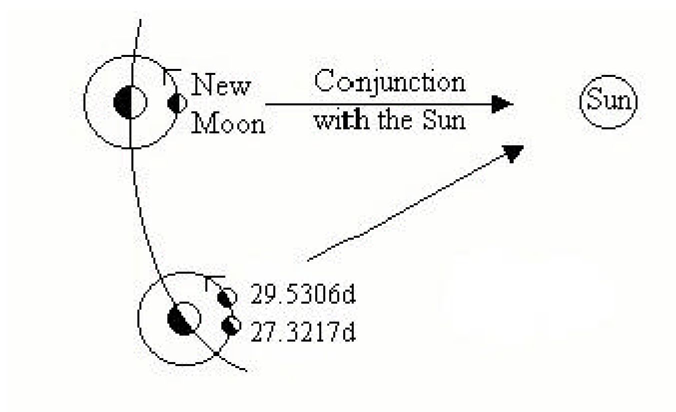
The day is taken to be the mean *solar day*. It is the average interval between two successive passages of the Sun over the meridian of a place. Sometimes, the meridian is taken to be the position when the Sun is directly above the place, that is, when it is at noon. Hence, we can also define the day, or equivalently the mean solar day, to be the mean time taken from one noon transit of the Sun to the next. The day, of length 24 h, is the smallest unit and is taken to be the fundamental unit of time. The lengths of months and years are expressed in terms of the day as the unit.

The Month

The *sidereal month* is the time in which the Moon completes one revolution around the Earth and returns to the same position in the sky. Its length is about 27d7h43m14.88s (27.3217 days). However, the Moon has not completed a revolution around the Earth with respect to the Sun because during this time, the Earth and the Moon have also revolved about 27^0 around the Sun.

For calendrical calculation, the *synodic month* is used. It is defined to be the time interval between two successive new moons. The mean length is about 29d12h44m3.84s (29.5306 days). The actual length can vary up to 7 hours owing to eccentricity of the moon's orbit and complicated interactions between the Earth, the Moon and the Sun.

Figure 7: Sidereal month and Synodic month



The Year

The *sidereal year* is the actual time taken for the Earth to revolve once around the Sun with respect to the stars. The stars are fixed with respect to the elliptical orbit. The mean length of a sidereal year is about 365d6h9m12.96s (365.2564 days).

The *tropical year* is the time interval between two successive March Equinoxes. Due to shortening effects of precession of the Equinoxes, the Earth makes a revolution of less than 360^0 around the Sun to return to the March Equinox. Hence the tropical year, of mean length about 365d5h48m46.08s (365.2422 days), is shorter than the sidereal year by about 20 minutes.

Chapter 2: Classification of Calendars

In Chapter 1, we have discussed the units of time, namely, the day, the month and the year. They serve as natural units of calendars. These units are incommensurable, meaning that none of them is an integral multiple of any of the others. Ancient astronomers tried to find relations and had come up with different ways to structure days into larger units of weeks, months, years and cycle of years. The relations obtained are only approximations because their related astronomical cycles change slowly with time. From this situation, three distinct types of calendars emerge.

Solar Calendar

A *solar calendar* is designed to approximate the tropical year using days. In order to synchronize with the tropical year and hence the seasons, days are sometimes added, forming leap years, to increase the average length of the calendar year. A solar calendar year can be divided into months but these months ignore the Moon.

The Gregorian calendar is a solar calendar with a common year having 365 days and a leap year having 366 days. Every fourth year is a leap year unless it is a century year not divisible by 400.

Lunar Calendar

A *lunar calendar* consists of a number of lunar months with each month covering the period between two successive new moons or full moons. We say that the lunar month follows, or depends on, the lunar cycle. Each calendar or lunar year has 12 lunar months. Each month has an average length of about 29.5 days. This amounts to about 12 x 29.5 = 354 days a year, around 11 days shorter than the tropical year. Hence a lunar calendar ignores the tropical year and does not keep in line with the seasons.

The Muslim calendar is a lunar calendar. We can see that the Hari Raya Puasa festival always falls about 11 days earlier than a year ago in the Gregorian calendar.

Lunisolar Calendar

A *lunisolar calendar* is designed to keep in phase with the tropical year using lunar months. A whole lunar month is occasionally added at every few years interval to help the calendar keep up with the tropical year. This additional month is known as the *leap month* or the *intercalary month*.

The Chinese calendar is a lunisolar calendar, consisting of 12 lunar months, each beginning at new moon. A normal calendar year has 12 months and a 13^{th} month is added according to certain rules to synchronize with the tropical year. In Chapter 3, we will see that the Indian lunisolar calendars are made to approximate the sidereal year instead of the tropical year.

The Metonic cycle

The *Metonic cycle* is a mathematical rule to determine when a leap month should be added to keep the lunisolar calendar in pace with the tropical or sidereal year. The mathematics behind it is shown below.

For the lunar months,

Mean length of the synodic month = 29.5306 days.

Mean length of a lunar year (making up of 12 lunar months) is (12 x 29.5306) days = 354.3672 days.

In 19 lunar years with 7 leap months, there are approximately (19 x 12 + 7) x 29.5306 days = 6939.6910 days. --- (A)

For the tropical year,

Mean length of the tropical year is 365.2422 days.

The lunar year is short of the tropical year by (365.2422 - 354.3672) days = 10.875 days.

In 19 tropical years, there are 19 x 365.2422 days = 6939.6018 days. --- (B)

For the sidereal year,

Mean length of the sidereal year is 365.2564 days.

The lunar year is short of the sidereal year by (365.2564 - 354.3672) days = 10.8892 days.

In 19 sidereal years, there are 19 x 365.2564 days = 6939.8716 days. --- (C)

From above, we see that the lunisolar calendar catches up with 19 tropical or 19 sidereal years by adding 7 leap months in every 19 lunar years interval. This can be seen from values (A), (B) and (C). On average, a leap month is added at a period of 19/7 = 2.7 years.

Although the Metonic cycle provides a way of determining occurrence of leap months, not all lunisolar calendars follow this cycle. The Indian lunisolar calendars rely on true positions of the Sun and the Moon to determine the occurrence of leap months.

Arithmetical and Astronomical calendars

There is another different way of grouping calendars. We can classify calendars that are operated by straightforward numerical rules as *arithmetical calendars*. The Gregorian calendar is an arithmetical calendar. A normal year has 365 days and a leap year having 366 days. Every fourth year is a leap year unless it is a century year not divisible by 400. Furthermore, the lengths of months in the calendar are fixed with Feburary having 28 days in normal year and 29 days in a leap year. We see that there is an arithmetical formula to determine which year is leap. Together with lengths of months being fixed, we can easily and accurately construct the Gregorian calendar for a year which is way ahead of our present year.

Calendars that are mainly controlled by astronomical events are *astronomical calendars*. These calendars do have some arithmetical components. However, they are really close approximations to their related astronomical events. The Indian solar calendars are astronomical calendars. Lengths of the calendar year and solar months are determined by the time taken for the Sun to travel along certain paths along the ecliptic. The process of rounding the lengths to whole numbers depends on a set of rules involving the occurrences of some astronomical events. Since the times of astronomical events vary from year to year, lengths of the calendar year and solar months also vary. Hence we cannot formulate any arithmetical rules to determine their lengths. We will discuss the Indian solar calendars in greater details later.

Chapter 3: The Indian Calendars

The history of calendars in India is complex oweing to the long history of Indian civilization and the diversity of cultural influences. It is known that the Indians used both the solar and lunisolar calendar. The modern Indian calendars, that are used in India today, are astronomical in nature because they are close approximations to *true* times of its related astronomical events such as the travelling of the Sun along certain paths on the ecliptic and lunar conjunctions.

However, before AD 1100, the old Indian calendars used the *mean* times. In the old solar calendar, mean length of a sidereal year is used to estimate the calendar year. A solar month is one twelfth of the calendar year. The times of sunrise and sunset were taken to be at 6am and 6pm respectively. With these few illustrations, we can see that old Indian calendars were arithmetical in character since rough approximations were used in the calendar-making. For more details on the old Indian calendars, one can look at reference book (10).

In my thesis, we will only touch on the modern Indian calendars. I obtain the materials mainly from reference books (7) and (8). Information indicating the various regions using the different Indian calendars can be found from references (7) and (9).

3.1: *A Brief Introduction*

The modern Indian solar and lunisolar calendars have many local variations, and hence their own characteristics, due to the difference in customs and astronomical practices adopted by calendar-makers in different regions of India. However, they are still based on common calendrical principles found mainly in an ancient astronomical treatise called the *Surya Siddhanta*.

3.1(a): *The Surya Siddhanta*

The *Surya Siddhanta* contains rules of calendrical astronomy to construct the Indian solar and lunisolar calendars. It also includes formulae and equations to find true values of astronomical events and to determine true positions of the Sun and other luminaries in the sky. It is not known when the treatise was originally written but its calendrical rules were believed to come into use as early as around AD 400 in some places of India.

The treatise appeared to be constantly revised. However, the knowledge of position astronomy was not that advanced as compared to now. Hence astronomical values and true positions obtained by Siddhantic methods are not very accurate. They differ from those obtained by modern methods. Modern methods refer to scientific, sophisticated techniques of recording and taking measurements that are used today. To see the difference, let's take the sidereal year as an example. The correct mean length of the sidereal year is about 365d6h9m12.96s (365.2564 days) but the value given by the revised Siddhanta is 365d6h12m36.52s (365.258756 days), longer than the modern value by about 3m23.56s.

Despite the inaccuracy in the astronomical values, the calendrical principles found in the Surya Siddhanta are still regarded with veneration by calendar-makers. The workings of the Indian calendars are based mainly on its calendrical principles. However, there seems to be a deviation in the use of astronomical values in the calendar-making.

3.1(b): *The Modern Panchangs and the Old Panchangs*

A *panchang* or *'panjika'* is an annual publication written by Indian calendar-makers, also known as panchang-makers. It contains calendrical information on celebration of festivals, performance of ceremonies or rites and on astronomical and astrological matters. Every family in India possesses a panchang.

Families in different regions of India may use different panchangs. This is due to the adopting of different conventions by calendar-makers in the calendar-making. Such inconsistency is unavoidable. However, even if a uniformed convention is applied, we can still group the calendar-makers into two schools. They are the Modern school and the Old school.

The Modern school uses modern methods to determine astronomical events and data needed for making the Indian calendars and hence their modern panchangs. The Old school believes that the Surya Siddhanta cannot be wrong and the inaccurate Siddhantic methods and results continue to be used to construct the calendars and their old panchangs. Although at present, the existence of two different schools causes additional confusion in the calendar or panchang-making, the Old school has come to realize the mistakes in the Siddhanta and is gradually adopting modern methods to correct them.

3.1(c): *Overview*

Before going into the rules and principles behind the making of the Indian calendars, we first present Table 1 below to show that for a given date, such as 1st Kartika Saka 1918, of an Indian calendar, it can indicate different days of the year of the Gregorian calendar.

Table 1: The date 1st Kartika Saka 1918 with reference to different days of the year of the Gregorian calendar

Name of calendar	Gregorian calendar date for 1st Kartika Saka 1918
The Orissa Calendar	16 Oct 1996
The Tamil calendar	17 Oct 1996
The Malayali calendar	17 Oct 1996
The Bengal calendar	18 Oct 1996
The National calendar	23 Oct 1996
The Chaitra (amanta) calendar	12 Nov 1996
The Chaitra (purimanta) calendar	27 Oct 1996

Notes:

1. The first four calendars in column 1 are the Indian solar calendars that will be introduced in Section 3.2. The last two calendars are the main types of the Indian lunisolar calendars and they will be discussed in Section 3.4.

2. For the Tamil calendar, the date 1st Arppisi Saka 1918 corresponds to the given date 1st Kartika Saka 1918 since the names of solar months are different from the other calendars. In the Malayali calendar, the date 1st Tula Kollam 1172 corresponds to the given date 1st Kartika Saka 1918 because the names of solar months and the solar era in used are different.

3. The given date 1st Kartika Saka 1918 represents (S) 1st Kartika Saka 1918 for the Chaitra (amanta) calendar and (K) 1st Kartika Saka 1918 for the Chaitra (purimanta) calendar.
   (S) refers to sukla paksha and (K) refers to krishna paksha. Their definitions are discussed in Section 3.4.

We also indicate on the map of India the regions where the Indian solar and lunisolar calendars are used. Note that the lunisolar calendars consist of the amanta and purimanta lunisolar calendars, which we will discuss in Section 3.4.

In the map, the solar calendars, which we will introduce in Section 3.2, are generally used for civil purposes. For the lunisolar calendars, we prefix the term 'religious' to the calendar if it is used mainly for religious purpose. Otherwise the calendar is used for civil dating. See Map 1.

Map 1: Different regions in India using the Indian solar and lunisolar calendars

Notes:

1. The regions in white colour within the boundary of India are the states in which we have no information on the types of Indian calendars being use.

2. The solar calendars comprise the Orissa, Tamil, Malayali and Bengal calendars. The Orissa calendar is followed in Orissa, Punjab and Haryana. The Bengal calendar is used in West Bengal, Tripura and Assam. The Tamil and Malayali calendars are used in Tamil Nadu and Kerala respectively.

3. There are three types of amanta lunisolar calendars, namely, the Chaitra, Kartika and Ashadha calendars. The Kartika calendar is followed in Gujarat. In a place called Kutch found in Gujarat, people use the Ashadha calendar.

Now we shall proceed with our calendar discussion in details. In Sections 3.2 to 3.4, we need to note the followings,

1. We will explain the Indian calendrical principles with true positions of the Sun and the Moon and the use of correct astronomical values obtained by modern methods. Astronomical data are taken at the Central station in Ujjain (Latitude: $23^0$11'E and Longitude: $75^0$46'6''N). The *Indian Standard Time (IST),* which is 5h30m ahead of universal time in Greenwich, is used for time recording. For IST correction, we replace the longitude of Ujjain with longitude $82^0$30'N.

2. For all the Indian calendars, a *civil day* is taken to run from sunrise to the next sunrise. The civil day is also known as the *panchang* or *savana day*.

3.2: The Solar Calendars

In this section, we will introduce four different Indian solar calendars. They are constructed using similar calendrical rules. In each solar calendar, the lengths of the calendar or civil year and solar months are expressed in numbers of civil days. In addition, the calendar is made to approximate the sidereal year rather than the tropical year. Let us begin with the basic structure of the Indian solar calendar.

The Nirayana Year

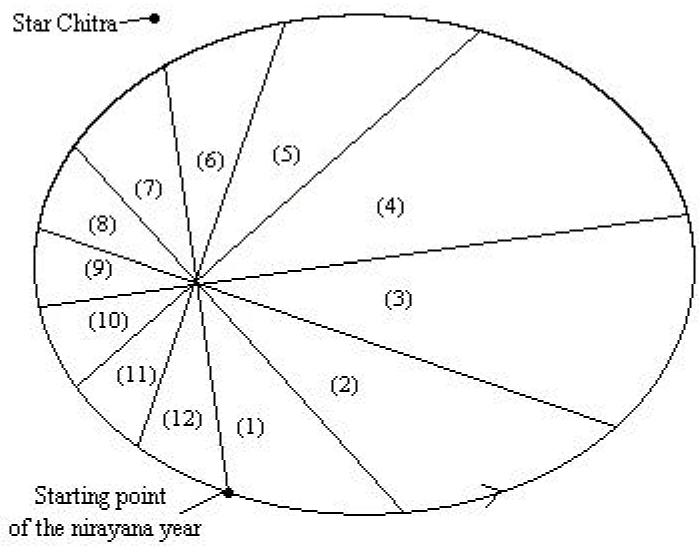
The *nirayana year* is the actual time required for the Earth to revolve once around the Sun with respect to a starting point on the ecliptic that is directly opposite a bright star called Chitra. The longitude of Chitra from this point is 180^0. The Indian solar calendar is made to keep in phase with the nirayana year. See Figure 8 for the starting point of the nirayana year.

In the year AD 285, the starting point of the nirayana year coincided with the March Equinox. The celestial longitude, as measured from the March Equinox, of Chitra was $179^0 59'52''$ at that time. For calendrical calculations, the longitude may be taken to be 180^0. Since the stars are fixed with respect to the ecliptic, the starting point remains unchanged. However, under precession of the Equinoxes, the March Equinox recedes on the ecliptic westward each year and by 1 January 2001, it has shifted nearly $23^0 51'26''$ from the starting point. Hence the nirayana year is really a sidereal year with mean length about 365d6h9m12.96s (365.2564 days). This is about 20m26.88s longer than the mean length of the tropical year which is about 365d5h48m46.08s (365.2422 days).

The Solar Month

From the geocentric point of view of the Sun-Earth motion, a solar month is determined by the entrance of the Sun into a *rasi*. A *rasi* is defined to be a division that covers 30^0 of arc on the ecliptic. The ecliptic is divided into 12 such rasis. The first rasi starts from the same point that starts the nirayana year. See Figure 8 for the starting point of the nirayana year.

Figure 8: The starting point of the nirayana year and the rasis



The length of a solar month is the time taken for the Sun to travel its linked rasi completely, that is, to pass 30^0 of its elliptical orbit. Hence a nirayana year has 12 solar months. Since the solar calendar has several local variations, the start of the nirayana year and names of the month may differ. For example, the Malayali calendar, to be introduced later, starts the calendar year at the solar month that corresponds to the Simha rasi. See Table 2 for the names of the rasis and their corresponding solar months in several solar calendars.

Table 2: Relationships between rasis and solar months

Rasi No.	Name of Rasi	Name of corresponding solar month in most solar calendars	Name of corresponding solar month in the Tamil solar calendar	Name of corresponding solar month in the Malayali (Kerala) solar calendar
1	Mesha	→ Vaisakha	→ Chittirai	Mesha
2	Vrisha	Jyaistha	Vaikasi	Vrisha
3	Mithuna	Ashadha	Ani	Mithuna
4	Karkata	Sravana	Adi	Karkata
5	Simha	Bhadra	Avani	→ Simha

6	Kanya	Asvina	Purattasi	Kanya
7	Tula	Kartika	Arppisi	Tula
8	Vrischika	Agrahayana (Margasirsha)	Karthigai	Vrischika
9	Dhanus	Pausha	Margali	Dhanus
10	Makara	Magha	Thai	Makara
11	Kumbha	Phalguna	Masi	Kumbha
12	Mina	Chaitra	Panguni	Mina

→ indicates the starting month of the nirayana year

Notes:

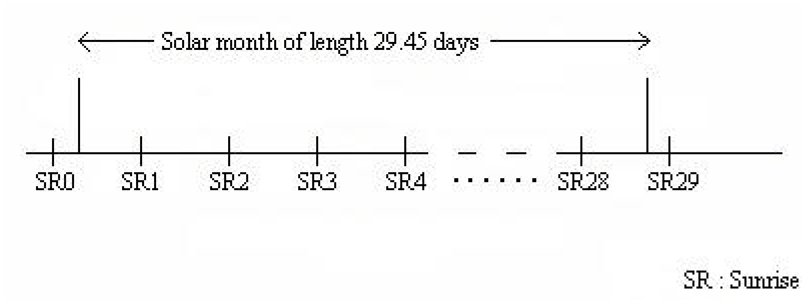
1. The rasi number in column 1 is as per notation in Figure 8.
2. The solar calendars mentioned in columns 3, 4 and 5 are some different Indian solar calendar.

From Kepler's law, the Earth's revolution around the Sun, or the Sun's orbit around the Earth, is not uniform. This causes the length of each solar month to vary. The mean length of a solar month is about 30d10h29m8.16s (30.4369 days) but its actual length can vary from 29d10h48m (29.45 days) to 31d10h48m (31.45 days). After knowing the actual length of a solar month, a point is still required to begin the month.

The first entry (ingress) of the Sun into a rasi is called a *samkranti*. Altogether, there are 12 samkrantis in a nirayana year. The samkranti can occur at any time of the day. Hence it is not convenient to start a solar month at the concerned samkranti. Instead, the beginning of a solar month is chosen to be from a sunrise that is close to its concerned samkranti. This will depend on certain rules of samkranti to be explained later. Consequently, the civil day becomes the basic unit of the Indian solar calendar.

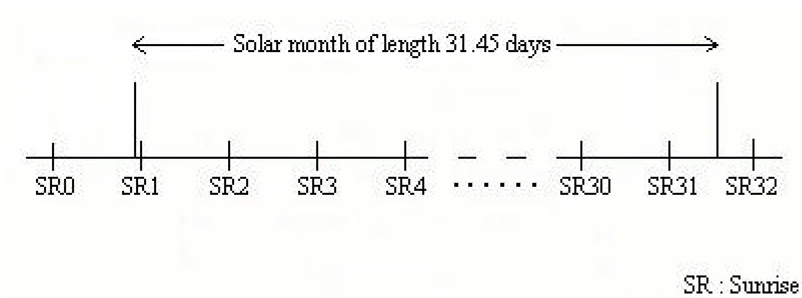
From the actual length of a solar month, we see that each solar month can have 29 to 32 days. Refer to Figure 9. Consider a solar month with its length to be 29d10h48m (29.45 days). If its concerned samkranti falls close to, but after, a sunrise (SR0), there will be 28 sunrises (SR1 to SR28) falling within the solar month. We see that the solar month ‘captures’ 28 days and also the day starting at (SR0). Altogether, there will be 29 days. This process is like ‘rounding down’ the actual length of the solar month.

Figure 9: Illustration on how a solar month can have 29 days



Similarly, if the solar month, of length 31d10h48m (31.45 days), falls close to but before (SR1), the solar month will ‘capture’ 31 days (SR1 to SR31) and also the day, starting at (SR0), in which its samkranti falls. Hence there will be 32 days for that solar month. In this case, we have the ‘rounding up’ process instead. See Figure 10 for this illustration.

Figure 10: Illustration on how a solar month can have 32 days



These explanations hold regardless of whichever rules of samkranti the calendar follows.

Solar months with their corresponding rasis near the aphelion will most probably have 32 days while solar months that are linked to rasis near the perihelion will likely to have 29 days. In other words, months with corresponding rasis Vrisha, Mithuna and Karkata can have 32 days while months with corresponding rasis Vrischika, Dhanus and Makara can have 29 days.

For determining the starting day of a solar month, there are several rules of samkranti that can be followed. We will talk about the four common rules.

Rules of Samkranti:

1. The Orissa rule

   Solar month begins on the same day as the samkranti.

2. The Tamil rule

   Solar month begins on the same day as the samkranti if the samkranti falls before the time of sunset on that day. Otherwise the month begins on the following day.

3. The Malayali rule

   Before stating the Malayali rule, we need to define what an *aparahna* for a particular day is. *Aparahna* is the point at $3/5^{th}$ duration of the period from sunrise to sunset. For example, suppose the times of sunrise and sunset are 6am and 6pm respectively. Then the time of the aparahna = [(3/5) x (18 – 6) + 6]am = 1.12pm. Now we state the Malayali rule.

   Solar month begins on the same day as the samkranti if the samkranti occurs before the time of aparahna on that day. Otherwise the month starts on the following day.

4. The Bengal rule

   When samkranti takes place between the time of sunrise and midnight on that day, the solar month begins on the following day. If it occurs after midnight, the month begins on the next following day, that is, the third day. This is the general rule. In some special circumstances, there are some deviations from this rule. However, we will focus on the general rule here.

We shall call the solar calendars following the four stated rules in the order above as the Orissa, Tamil, Malayali and Bengal calendars respectively. There exists other diversification but we will not discuss them here.

The Calendar or Civil Year and the Solar Eras

The Orissa, Tamil and Bengal calendars begin their civil year with the solar month that corresponds to the Mesha rasi. The Malayali calendar starts the year at the solar month that links with the Simha rasi.

The solar eras being used in the solar calendars are the Kali Yuga, the Saka traditional, the Saka national, the Bengali San and the Kollam eras. See Table 3 for their epochs with respect to the Gregorian calendar and the solar calendars in which the eras are used.

Table 3: Different solar eras in use and their epochs with reference to the Gregorian calendar

Solar Era	Epoch of the era with reference to the Gregorian calendar	Calendars using the era
Kali Yuga	AD year +3101 from mid-Apr to Dec AD year +3100 from Jan to mid-Apr	General era used in all India solar calendars
Saka traditional	AD year –78 from mid-Apr to Dec AD year –79 from Jan to mid-Apr	Orissa, Tamil and Bengal calendars
Saka national	AD year –78 from 22 Mar to Dec AD year –79 from Jan to 21 Mar	Only in the National calendar introduced by government of India in 1957
Bengali San	AD year –593 from mid-Apr to Dec AD year –594 from Jan to mid-Apr	Bengal calendar
Kollam	AD year –824 from mid-Aug to Dec AD year –825 from Jan to mid-Aug	Malayali (Kerala) calendar

By now, we can look at some starting day of the Mesha rasi as per four samkranti rules mentioned earlier. See Table 4.

Table 4: Time of transit of the Sun to the mesha rasi, length of the nirayana year and the starting day of the solar years as per four conventions for years Saka 1911 to 1916 (AD 1989 to 1995)

Year		Transit date and time of the Sun to the Mesha rasi			Length of the nirayana year			Starting day of the solar month corresponding to the Mesha rasi			
Saka	Gregorian (AD)	Date #	Time h	Time m	d	h	m	Bengal	Orissa	Tamil	Malayali
1911	1989-90	13 April 1989	21	45	365	6	12	14 April 1989	13 April 1989	14 April 1989	14 April 1989
1912	1990-91	14 April 1990	3	57	365	6	7	15 April 1990	13 April 1990	14 April 1990	14 April 1990
1913	1991-92	14 April 1991	10	4	365	6	3	15 April 1991	14 April 1991	14 April 1991	14 April 1991

1914	1992-93	13 April 1992	16	7	365	6	18	14 April 1992	13 April 1992	13 April 1992	14 April 1992
1915	1993-94	13 April 1993	22	25	365	6	6	14 April 1993	13 April 1993	14 April 1993	14 April 1993
1916	1994-95	14 April 1994	4	31	365	6	0	15 April 1994	13 April 1994	14 April 1994	14 April 1994

#: The IST for sunrise, aparahna and sunset ranges from 5h40m54.1902s to 5h41m48.0244s, 13h16m32.6652s to 13h16m40.2379s and 18h19m55.047s to 18h20m18.3152s respectively for the Gregorian dates of the day of Mesha samkranti. These times are obtained from the codes that I have written. They can be found in the Mathematica package IndianCalendar.m.

Notes:

1. The IST for sunrise, sunset and aparahna are taken at Ujjain (Latitude: $23^0$11'E, Longitude: $82^0$30'N (IST)) in India.

2. The time of Mesha samkranti is measured in IST.

3. Kollam year of the Malayali (Kerala) calendar starts on the Sun entering Simha rasi. The year of the remaining calendars starts on the Sun entering Mesha rasi.

Occurrence of Leap Years

The mean length of a nirayana year is about 365d6h9m12.96s (365.2564 days). Suppose we have a solar calendar to approximate the nirayana year and the basic calendrical unit used is the 24h day. Let each of its solar months has a fixed number of days. Then most calendar years will normally have 365 days. To synchronize with the mean length of the nirayana year, a leap day has to be added to the normal length of the calendar year at some intervals. Hence a leap year has 366 days.

Since every such normal year is short of the mean length of the nirayana year by 6h9m12.96s (0.2564 days), it takes 1/(365.2564 - 365) = 3.9002 years to accumulate the shortfall to a day. Hence approximately ten days must be added in 39 years to bring the solar calendar back with the nirayana year. This means that there must be ten leap years in every 39 years.

Referring back to the Indian solar calendar with the civil day as its basic unit, it has no fixed number of days for its months. The length of a civil year cannot be determined by arithmetical rule. Instead, it depends on the time of transit of the Sun to the first rasi that starts the year and the conventions used to fix the starting day of solar months. However by astronomical observation, it is found that there are normally 365 days in a civil year and 366 days in a leap year. Furthermore, leap years automatically occur at an interval of three or four years so that ten leap years occur in a period of 39 years for the solar calendar to keep in pace with the nirayana year.

If we refer to the leap year rule for the Gregorian calendar, we will generally have about 39/4 = 9.75 leap years in every 39 years. This value is smaller because the Gregorian calendar approximates the tropical year, not the nirayana year. Hence the occurrence of leap years for the Gregorian calendar is close to what is actually happening for the Indian solar calendar. See Table 5 for occurrence of leap years.

Table 5: Occurrence of leap years

Saka Year	Orissa Rule		Tamil Rule		Bengal Rule			Kollam year	Saka year	Malayali Rule	
	Starting date	Year length	Starting date	Year length	Bengali San	Starting date	Year length			Starting date	Year length
1900	13 Apr 78	366 (L)	14 Apr 78	365	1385	15 Apr 78	365	1154	1900-01	17 Aug 78	365
1901	14 Apr 79	365	14 Apr 79	365	1386	15 Apr 79	365	1155	1901-02	17 Aug 79	365
1902	13 Apr 80	365	13 Apr 80	366 (L)	1387	14 Apr 80	365	1156	1902-03	16 Aug 80	366 (L)
1903	13 Apr 81	365	14 Apr 81	365	1388	14 Apr 81	366 (L)	1157	1903-04	17 Aug 81	365
1904	13 Apr 82	366 (L)	14 Apr 82	365	1389	15 Apr 82	365	1158	1904-05	17 Aug 82	365
1905	14 Apr 83	365	14 Apr 83	365	1390	15 Apr 83	365	1159	1905-06	17 Aug 83	366 (L)
1906	13 Apr 84	365	13 Apr 84	366 (L)	1391	14 Apr 84	365	1160	1906-07	17 Aug 84	365
1907	13 Apr 85	365	14 Apr 85	365	1392	14 Apr 85	366 (L)	1161	1907-08	17 Aug 85	365
1908	13 Apr 86	366 (L)	14 Apr 86	365	1393	15 Apr 86	365	1162	1908-09	17 Aug 86	365
1909	14 Apr 87	365	14 Apr 87	365	1394	15 Apr 87	365	1163	1909-10	17 Aug 87	366 (L)
1910	13 Apr 88	365	13 Apr 88	366 (L)	1395	14 Apr 88	365	1164	1910-11	17 Aug 88	365
1911	13 Apr 89	365	14 Apr 89	365	1396	14 Apr 89	366 (L)	1165	1911-12	17 Aug 89	365
1912	13 Apr 90	366 (L)	14 Apr 90	365	1397	15 Apr 90	365	1166	1912-13	17 Aug 90	365
1913	14 Apr 91	365	14 Apr 91	365	1398	15 Apr 91	365	1167	1913-14	17 Aug 91	366 (L)

1914	13 Apr 92	365	13 Apr 92	366 (L)	1399	14 Apr 92	365	1168	1914-15	17 Aug 92	365
1915	13 Apr 93	365	14 Apr 93	365	1400	14 Apr 93	365 (L)	1169	1915-16	17 Aug 93	365
1916	13 Apr 94	366 (L)	14 Apr 94	365	1401	15 Apr 94	365	1170	1916-17	17 Aug 94	365

Notes:

1. (L) Occurrence of years of 366 days (leap years) in the solar calendars. Generally, there are 10 leap years in a period of 39 years.
2. Years of the Bengal, Orissa and Tamil calendars start from the Mesha rasi while that of the Malayali (Kerala) calendar starts from Simha rasi.

Regions in India where calendars are used

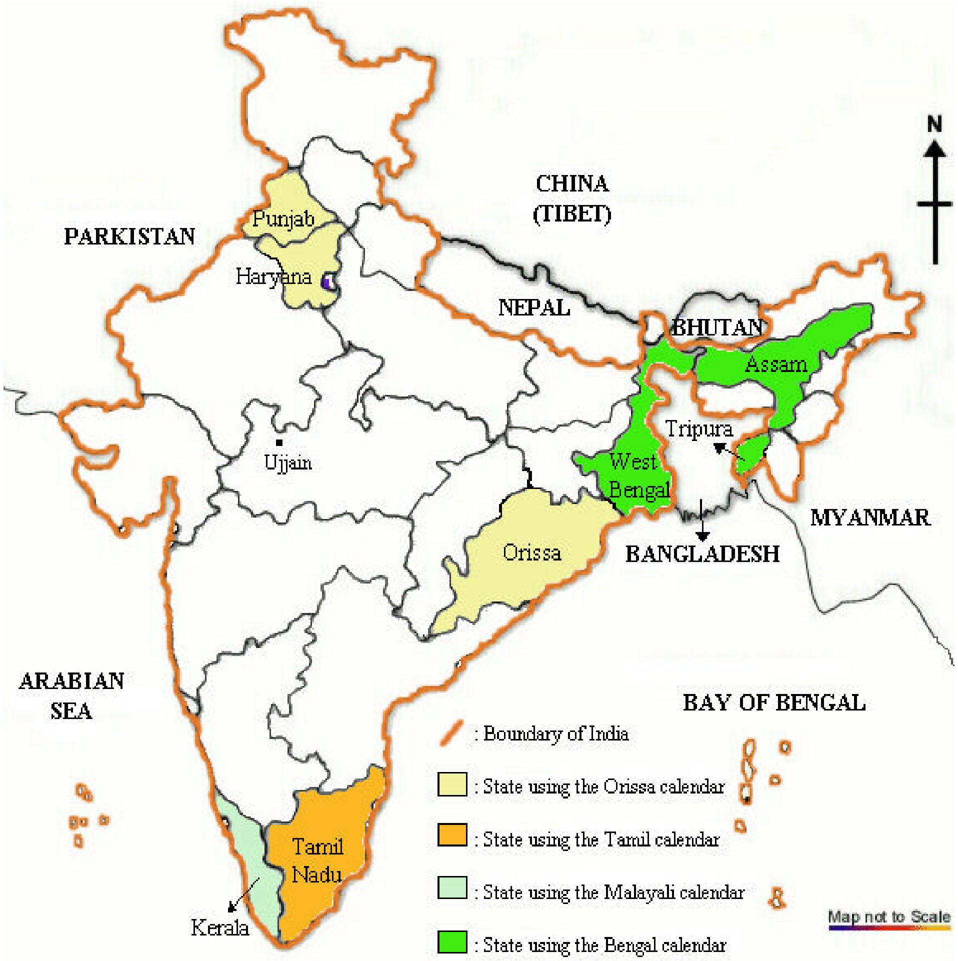
From Map 1, we have seen the regions in India using the solar calendars. On Map 2 below, we highlight the states in these regions where the Orissa, Tamil, Malayali and the Bengal calendars are used.

The Orissa calendar is mainly used in Orissa and partially in Punjab and Haryana. In Tamil Nadu and other Tamil speaking areas, people generally follow the Tamil calendar. The Malayali calendar is used in Kerala and the Bengal calendar is used in West Bengal, Assam and Tripura. The solar calendars in these states are used mainly for civil dating. See Map 2.

It will be observed that when the Indian solar calendars are used in different regions of India, several problems arise.

1. The starting day of the solar month may differ by one or two days in different parts of India.
2. The number of days of different solar months also varies from 29 to 32.
3. The length of the solar month by integral number of days is not fixed but changes from year to year.

Map 2: Areas in India using the different solar calendars



The Council of Scientific and Industrial Research for the Government of India appointed a Calendar Reform Committee in November 1952. The committee's objective is to examine all existing calendars in used in India and proposed an accurate and uniform all-India calendar for both civil and religious use. After close examination, the Committee recommended a unified solar calendar for civil use. The Government of India accepted the proposal and introduced it as the Indian national calendar with effect from 22 March 1957.

3.3: *The National Calendar*

The national calendar is a modification of the existing Indian solar calendars. The principle unit of the calendar is still the civil day. The solar era chosen is the Saka national era. See Table 3 for its epoch with reference to the Gregorian calendar.

The following shows the features that are different from the existing Indian solar calendars.

1. The national calendar is made to approximate the *sayana* year, not the nirayana year. The *sayana* year is a tropical year. As a result, the calendar year starts on the day after the March Equinox day.

2. The solar months have fixed number of days restricted to either 30 or 31 days. This would still depend on the time taken for the Sun to travel the concerned tropical rasi, where the starting point of the sayana year and hence, the first rasi, is the March Equinox. Referring to Table 6, the five months from the second to the sixth have mean lengths over 30.5 days and so their lengths are rounded up to 31 days. The remaining months have 30 days. Names for the solar months are kept the same as those of the Indian solar calendar listed in Table 2 colunm 3. However, the first month is named as Chaitra, followed by Vaisakha and so on. See Table 6.

Table 6: Lengths of different solar months reckoned from the March Equinox

Name of months for the general solar calendar as in Table 2 column 3	Arc measured from the March Equinox point covered by the true longitude of the Sun	Mean lengths of solar months			
		Modern value (AD1950)			Names of Months (as proposed)
		d	h	m	
Vaisakha	0^0 - 30^0	30	11	25.2	Chaitra
Jyaistha	30^0 - 60^0	30	23	29.6	Vaisakha
Ashadha	60^0 - 90^0	31	8	10.1	Jyaistha
Sravana	90^0 - 120^0	31	10	54.6	Ashadha
Bhadra	120^0 - 150^0	31	6	53.1	Sravana

Asvina	150^0 - 180^0	30	21	18.7	Bhadra
Kartika	180^0 - 210^0	30	8	58.2	Asvina
Agrahayana	210^0 - 240^0	29	21	14.6	Kartika
Pausha	240^0 - 270^0	29	13	8.7	Agrahayana
Magha	270^0 - 300^0	29	10	38.6	Pausha
Phalguna	300^0 - 330^0	29	14	18.5	Magha
Chaitra	330^0 - 360^0	29	23	18.9	Phalguna

3. The occurrence of leap years for this calendar is made to fall in the same leap year of the Gregorian calendar to keep the relation of the dates between these two calendars the same. When leap year occurs, Chaitra would have 31 days instead of 30 days. See Table 7.

Table 7: Names of the months of the national calendar, their lengths and the dates of the Gregorian calendar corresponding to the first day of its month

	Names of months and their lengths	Gregorian date for the 1st day of the month		Names of months and their lengths	Gregorian date for the 1st day of the month
1	Chaitra (30) Chaitra (L) (31)	22 March 21 March	7	Asvina (30)	23 September
2	Vaisakha (31)	21 April	8	Kartika (30)	23 October
3	Jyaishtha (31)	22 May	9	Agrahayana (30)	22 November
4	Ashadha (31)	22 June	10	Pausha (30)	22 December
5	Sravana (31)	23 July	11	Magha (30)	21 January
6	Bhadra (31)	23 August	12	Phalguna (30)	20 Feburary

However, most calendar-makers do not accept the National calendar mainly because the sayana system was adopted instead of the nirayana system. To them, this change was too drastic because the Surya Siddhanta had suggested that the solar calendars should be made to keep in line with the nirayana year and the calendar-makers would not want to abandon this principle. In the end, the existing solar calendars continue to be used actively in India up to today. By introducing this national calendar, it would only cause more confusion in determining the actual date of a particular day.

3.4: *The Lunisolar Calendars*

As we know, the basic unit of a lunisolar calendar is the lunar month. There are two kinds of lunar months being used in India. They are the new moon ending lunar month and the full moon ending lunar month, resulting in the amanta and the purimanta lunisolar calendars.

3.4(a) *The Amanta Lunisolar Calendar*

The amanta lunisolar calendar is based on the new moon ending lunar month, also known as the *amanta month*. The calendar is constructed to keep in phase with the nirayana year by adding leap months.

The Amanta Month

The *amanta month* refers to a lunar month that runs from new moon to the next new moon. Each amanta month and hence the lunar year are expressed in integral number of civil days. The numbering of days in the amanta month will be explained later when we come to define a *tithi*.

In general, the amanta month is named after the solar month in which the moment of its defining initial new moon falls. This procedure may change when the amanta month is too close to a *kshaya* month. We will explain what a kshaya month is and how it can affect the naming later.

In order to assign names to amanta months, a solar month is taken to start from the exact moment of the concerned samkranti to the next samkranti. Hence an amanta month can start from any day of the concerned solar month.

The amanta month is divided into two half-months, the *sudi* and *vadi* halves. The sudi half is also known as the *sukla paksha* or the bright half-month, covering the time period from new moon to the next full moon (the waxing phases). The vadi half, also known as the *krishna paksha* or the dark half-month, covers the period from full moon to the next new moon (the waning phases).

The Calendar Year and the Lunisolar Eras

For the amanta lunisolar calendar, the lunar year starts at the amanta month of Chaitra. We call this lunisolar calendar the Chaitra calendar for convenience. In some states of India, calendar-makers prefer the lunar year to begin with the amanta months of Kartika or Ashadha. We call these variations the Kartika and Ashadha calendars respectively.

In our discussion, we will focus on the Chaitra calendar. The calendrical principles behind the Chaitra calendar are the same for the Kartika and Ashadha calendars. See Table 8 for the names of the amanta months.

Table 8: The 12 months of the Chaitra (or Kartika/Ashadha) calendar year that are named after solar months

1. Chaitra	2. Vaisakha	3. Jyaishtha
4. Ashadha	5. Sravana	6. Bhadra
7. Asvina	8. Kartika	9. Agrahayana or Margasirsha
10. Pausha	11. Magha	12. Phalguna

The lunisolar eras that are used in the Indian lunisolar calendars are the Salivahana Saka, Vikram Samvat (Chaitradi), Vikram Samvat (Kartikadi) and Vikram Samvat (Ashadadi) eras. See Table 9 for their epochs with reference to the Gregorian calendar and the lunisolar calendars in which the eras are used.

Table 9: Different lunisolar eras in use and their epochs with reference to the Gregorian calendar

Lunisolar Era	Epoch of the era with reference to the Gregorian Calendar	Calendars using the era
Salivahana Saka	AD year –78 from Mar/Apr to Dec AD year –79 from Jan to Mar/Apr	General era used in the Chaitra calendar
Vikram Samvat (Chaitradi)	AD year +57 from Mar/Apr to Dec AD year +56 from Jan to Mar/Apr	Used in the Chaitra calendar
Vikram Samvat (Kartikadi)	AD year +57 from Oct/Nov to Dec AD year +56 from Jan to Oct/Nov	Era used in the Kartika calendar

Vikram Samvat (Ashadadi)	AD year +57 from June/July to Dec AD year +56 from Jan to June/July	Era used in the Ashadha calendar

Occurrence of Leap or Adhika Month

The lunar calendar year, which consists of 12 amanta months, is shorter than the nirayana year and hence leap months are occasionally added at intervals so that the amanta lunisolar calendar is kept adjusted to approximate the nirayana year. The occurrence of leap months in the Chaitra calendar does not follow the Metonic cycle and cannot be determined by any arithmetical rules. Leap month occurs when the following astronomical event happens.

When a solar month completely covers an amanta month, that is, when there are two new moons, one falling at the beginning and the other at the end of the solar month, the amanta month that begins from the first new moon is treated as a leap month and prefixed with the title *'adhika'* or *'mala'*. We call the leap month *adhika* or *mala* month. The amanta month that runs from the second new moon is considered a regular or normal month and prefixed with the title *'suddha'*. Both amanta months bear the name of the same solar month. A lunar year with an adhika month has 13 amanta months.

An adhika month occurs generally at intervals 2 years 4 months, 2 years 9 months, 2 years 10 months or 2 years 11 months, giving an average interval of about 2 years 8.4 months. The average value can also be obtained by just taking the mean length of a lunar month, which is about 29.5 days, to divide by 11. Since a lunar year is short of the nirayana year by about 11 days, it takes about 29.5/11 = 2 years 8.2 months for the shortfall to accumulate to the average length of a lunar month. Thus an adhika month is added around this time to help the lunisolar calendar keep up with the nirayana year.

We see that the intervals of occurrence of adhika months are quite close. This is because most lunations are shorter than a solar month. See Table 10. See also Table 11 for occurrence of adhika months.

Table 10: Length of lunation versus length of a solar month

Length of solar month				Length of lunation			
d	h	m	s	d	h	m	S

	29	10	48	0	29	5	54	14.4
		to					to	
	31	10	48	0	29	19	36	28.8
Mean Value:	30	10	29	4	29	12	44	3.84

Note: From the mean values, it can be seen that most lunations are shorter than a solar month.

Occurrence of Kshaya Month

Very rarely, an amanta month can completely cover a solar month, that is, there is no new moon falling in that solar month and hence no amanta month naming after it. This 'missing' amanta month is called the *kshaya* or *decayed month*. By astronomical observation, we find that kshaya month may happen at 19 years or 141 years and also at immediate intervals of 4, 65, 76 and 122 years. See Table 12 for occurrence of kshaya months.

A kshaya month can occur because the maximum duration of a lunation is longer than the short solar months Agrahayana, Pausha and Magha since these solar months correspond to the short rasis Vrischika, Dhanus and Makara. Hence a kshaya month is possible only in these three solar months.

When a kshaya month happens in a lunar year, there will always occur two adhika months, one before and the other after, the kshaya month. Let me explain how this can happen. Under normal circumstances, we should have only a new moon falling in a solar month. However, things start to get complicated when new moons and samkrantis fall too close to one another.

When new moons come too close to samkrantis and there are no new moons falling in a particular solar month say A, there will be an extra new moon occurring close before A. This is because the new moon that is supposed to fall in A occurs close to but before A's concerned samkranti. Then there will be a solar month before A that 'captures' a new moon nearby, resulting in two new moons to occur within it. Hence the first adhika month occurs. Since the new moon that falls after A is very close to the new samkranti and knowing that most of the solar months that come after A are longer, then there will be two new moons falling within one of the solar months, giving the second adhika month.

There are also different conventions to handle the adhika months so that the kshaya month can be compensated and the 12 months structure of the lunar year is restored. This will be discussed later.

Table 11: Year and time intervals of occurrence of adhika months in the period Saka 1888 to 1923 (Gregorian 1966 to 2001)

Year		Name of adhika month	Interval of occurrence of adhika months	
Gregorian	Saka		Year	Month
1966	1888	Sravana	2	9
1969	1891	Ashadha	2	11
1972	1894	Vaisakha	2	10
1974	1896	Bhadra	2	4
1977	1899	Sravana @	2	11
1980	1902	Jyaishtha	2	10
(1982)	1904 #	(Asvina)	2	4
(1983)		(Phalguna)	2	9
1985	1907	Sravana	2	10
1988	1910	Jyaishtha	2	10
1991	1913	Vaisakha	2	11
1993	1915	Bhadra	2	4
1996	1918	Ashadha	2	10
1999	1921	Jyaishtha	2	11
2001	1923	Asvina	2	4

@ According to old Surya Siddhantic method of calculation, Ashadha is the adhika month.

In Saka 1904 (AD 1982-1983), Magha is a kshaya month. The interval of occurrences of the two adhika months (Asvina and Phalguna) accompanying Magha is counted from the adhika month (Jyaishtha) in Saka 1902 (AD 1980).

Note: Notice that seven adhika months have occurred in cycles of 19 years as required.

Table 12: Occurrence of kshaya months with the two accompanying adhika months in the period from Saka 692 (AD 770-771) to Saka 1904 (AD 1982-1983)

Year		Year Interval	Kshaya Month	Adhika months before and after Kshaya month
Saka	AD			
692	770-771	-	Pausha	Asvina – Chaitra
814	892-893	122	Agrahayana	Kartika – Chaitra
833	911-912	19	Pausha	Asvina – Chaitra
974	1052-1053	141	Pausha	Asvina – Chaitra
1115	1193-1194	141	Pausha	Asvina – Chaitra
1180	1258-1259	65	Pausha	Kartika – Chaitra

1199	1277-1278	19	Pausha	Kartika – Phalguna
1218	1296-1297	19	Pausha	Agrahayana – Phalguna
1237	1315-1316	19	Agrahayana	Kartika – Phalguna
1256	1334-1335	19	Pausha	Asvina – Phalguna
1302	1380-1381	46	Agrahayana	Kartika – Vaisakha
1321	1399-1400	19	Pausha	Kartika – Chaitra
1397	1475-1476	76	Magha	Asvina – Phalguna
1443	1521-1522	46	Agrahayana	Kartika – Vaisakha
1462	1540-1541	19	Pausha	Asvina – Chaitra
1603	1681-1682	141	Pausha	Asvina – Chaitra
1744	1822-1823	141	Pausha	Asvina – Chaitra
1885	1963-1964	141	Agrahayana	Kartika – Chaitra @
1904	1982-1983	19	Magha	Asvina – Phalguna

@: According to old Surya Siddhantic method of calculation, Pausha is kshaya month and the two adhika months are Asvina and Chaitra.

Tithi

A *tithi* is defined to be the time required for the longitude of the Moon to increase by 12^0 over the longitude of the Sun. Sometimes we call it is a *lunar day*. A lunar month can be divided into 30 tithis, of which 15 are sukla paksha (bright half) counted serially from 1 to 15 with prefix ‘S’ and 15 are krishna paksha (dark half) counted serially from 1 to 14, and the last one 30, with prefix ‘K’ or ‘V’.

Table 13: Names of the 15 tithis with their prefixes and serial numbers

Prefix (es)		Serial number	Name	Prefix (es)		Serial number	Name
S	K	1	Pratipada	S	K	8	Ashtami
S	K	2	Dvitiya	S	K	9	Navami
S	K	3	Tritiya	S	K	10	Dasami
S	K	4	Chaturthi	S	K	11	Ekadasi

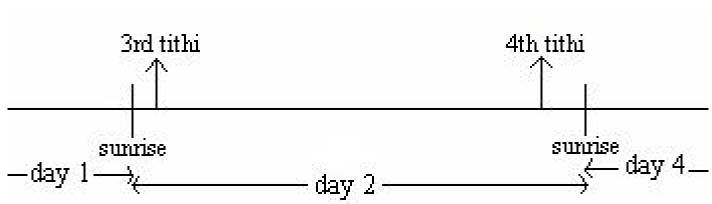
S	K	5	Panchami	S	K	12	Dvadasi
S	K	6	Sashthi	S	K	13	Trayodasi
S	K	7	Saptami	S	K	14	Chaturdasi
				S		15	Purnima
					K	30	Amavasya

When the Moon and the Sun are in conjunction (at new moon), the (K) 30th tithi ends and the (S) 1st tithi begins and continues up to the moment when the Moon gains on the Sun by 12^0 in longitude. Then the (S) 1st tithi ends and the (S) 2nd tithi starts. This process continues and repeats itself at every new moon.

There are 29 or 30 days in an amanta month. Each day is assigned the number of the tithi in effect at sunrise. However, the days are not always counted serially from 1 to 29 or 30. To understand why, we need to look at duration of a tithi. The average duration of a tithi is 23h37m30s (23.625 hours) but the actual value varies from 19h28m48s (19.48 hours) to 26h46m48s (26.78 hours).

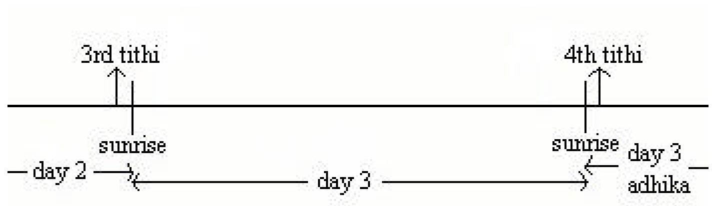
A short tithi may begin after sunrise and end before the next sunrise. In this case, a number is omitted from the day count. For example, when the third tithi begins after sunrise and ends before the next sunrise, the tithi in effect at the next sunrise is the fourth tithi. The sequence of days of the amanta month is 1, 2, 4, 5 and so on. In this case, we have a skipped or *kshaya day*. See Figure 11.

Figure 11: A short tithi that begins after sunrise and ends before the next sunrise, causing a kshaya day



Similarly, a long tithi may span two sunrises, that is, there is no tithi ending in that day. Then a day number is carried over to the second day and is treated as a leap day, suffixed by the term ‘adhika’. For example, the third tithi extends over two days with no tithi ending in the first day. The same tithi number is given to the two days. The sequence of days of the amanta month would be 1, 2, 3, 3 adhika, 4 and so on. Here, we say that a leap or *adhika day* has occurred. See Figure 12.

Figure 12: A long tithi spanning two sunrises, causing an adhika day



Notice that kshaya days occur more frequently than adhika days because most tithis are shorter than a civil day. A civil day is of length very close to 24h. This can be observed by looking at the average length of a tithi. If we compare this with adhika and kshaya months, adhika months happen much more often than kshaya months.

Different conventions in treating the kshaya month

When a kshaya month occurs, there is always a 'lost' amanta month and two adhika months, the first adhika month before and the second one after the kshaya month. In order to replace the 'lost' month and recover the 12 months structure of a lunar year, calendar-makers in different regions set rules to make the compensation. In general, there are three different schools of rules for treating the kshaya month.

1. <u>The Eastern School</u>

   The rule to follow is to treat the first adhika month as the leap month and the second adhika month as a normal or suddha month.

2. <u>The North Western School</u>

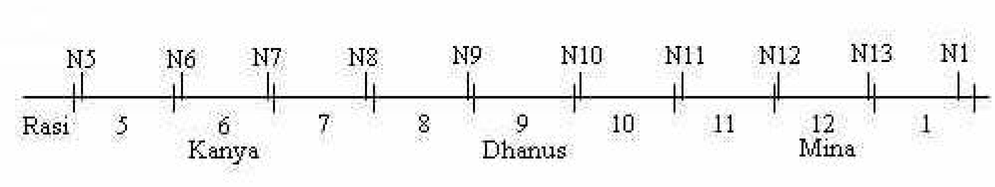
   The procedure is opposite to how rules from the Eastern school handle the adhika months. The first adhika month is treated as a normal or suddha month. The second adhika month is the leap month.

3. <u>The Southern School</u>

   Under this school, both the adhika months are treated as the leap months. The amanta month that contains two *samkramanas* (rasi junctions) that defines a solar month is treated as a *'jugma'* or *dual* (double) *month*, meaning that each tithi of this amanta month is divided into two halves. The first half is a tithi of the concerned amanta month and the second half is the same tithi of the kshaya month. In this way, the amanta month consists of two different months sharing the same tithi and a dual month is obtained. Notice that the amanta month before the dual month gets the same name as if rules of the Eastern school are used and the amanta month after the dual month are named as if procedures from North Western school are used.

The following is an illustration on how rules from the three schools are applied to the lunar year of Saka 1462 (AD 1540-1541) that contains a kshaya month Pausha.

Figure 13: Occurrence of two adhika months of Asvina and Chaitra and the kshaya month of Pausha in the lunar year of Saka 1462 (AD 1540-1541)



Notes:

1. N5, N6, N7, etc are positions of new moons in the solar months of Asvina, Kartika, etc.
2. Amanta months N6-N7 and N12-N13 fall within the solar months of Asvina (Kanya rasi) and Chaitra (Mina rasi). They are adhika Asvina and Chaitra.
3. Amanta month N9-N10 completely overlaps the solar month of Pausha (Dhanus rasi) and hence causes a kshaya month of Pausha.

Table 14: Treatment of kshaya and adhika months in the three different schools, taking the example of the kshaya-month year of Saka 1462, Vikram 1597 or AD 1540-1541, where the kshaya month was Pausha

Amanta month as per notation in Figure 13	Gregorian Calendar dates # from new moon to new moon	Rasi in which initial moment of new moon of amanta month falls	Name of amanta month in different schools		
			Eastern	North Western	Southern
N5-N6	12 Aug to 11 Sep	1540 Simha	Bhadra	Bhadra	Bhadra
N6-N7	11 Sep to 10 Oct	Kanya	(Adhika) Asvina	Asvina	(Adhika) Asvina
N7-N8	10 Oct to 9 Nov	Kanya	Asvina	Kartika	Asvina

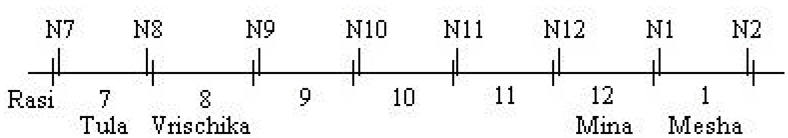
N8-N9	9 Nov to 8 Dec	Tula	Kartika	Agrahayana	Kartika
N9-N10	8 Dec to 7 Jan	1541 Vrischika	Agrahayana	Pausha	Agrahayana & Pausha
N10-N11	7 Jan to 6 Feb	Makara	Pausha	Magha	Magha
N11-N12	6 Feb to 8 Mar	Kumbha	Magha	Phalguna	Phalguna
N12-N13	8 Mar to 6 Apr	Mina	Phalguna	(Adhika) Chaitra	(Adhika) Chaitra
N13-N1	6 Apr to 6 May	Mina	Chaitra	Chaitra	Chaitra

The dates from column 2 are obtained by functions that we have written and they can be found in the Mathematica package IndianCalendar.m.

Observe that from Table 14, it is not always true that a suddha Chaitra amanta month starts the lunar year. It was the adhika Chaitra amanta month that started the (Saka 1462) lunar year if rules from the North Western or the Southern School were used.

From Table 12, we see that for Saka 1443 (AD 1521-1522), the adhika month after the kshaya month is Vaisakha. Under the Eastern school, the amanta month of Chaitra actually falls in the Mesha rasi rather than the Mina rasi. See Figure 14.

Figure 14: Occurrence of two adhika months of Kartika and Vaisakha and the kshaya month of Agrahayana in the lunar year of Saka 1443 (AD 1521-1522)



Notes:

1. N7, N8, N9 etc are positions of new moons in the solar months of adhika Kartika, suddha Kartika, etc.
2. Amanta months N7-N8 and N1-N2 fall within the solar months of Kartika (Tula rasi) and Vaisakha (Mesha rasi). They are adhika Kartika and Vaisakha.
3. Amanta month N8-N9 completely overlaps the solar month of Agrahayana with the linked rasi Vrischika and hence causes a kshaya month of Agrahayana.

4. Under rules from the Eastern school, N7-N8 is adhika Kartika, N8-N9 is suddha Kartika, ……, N11-N12 is Phalguna and N1-N12 is suddha Chaitra.

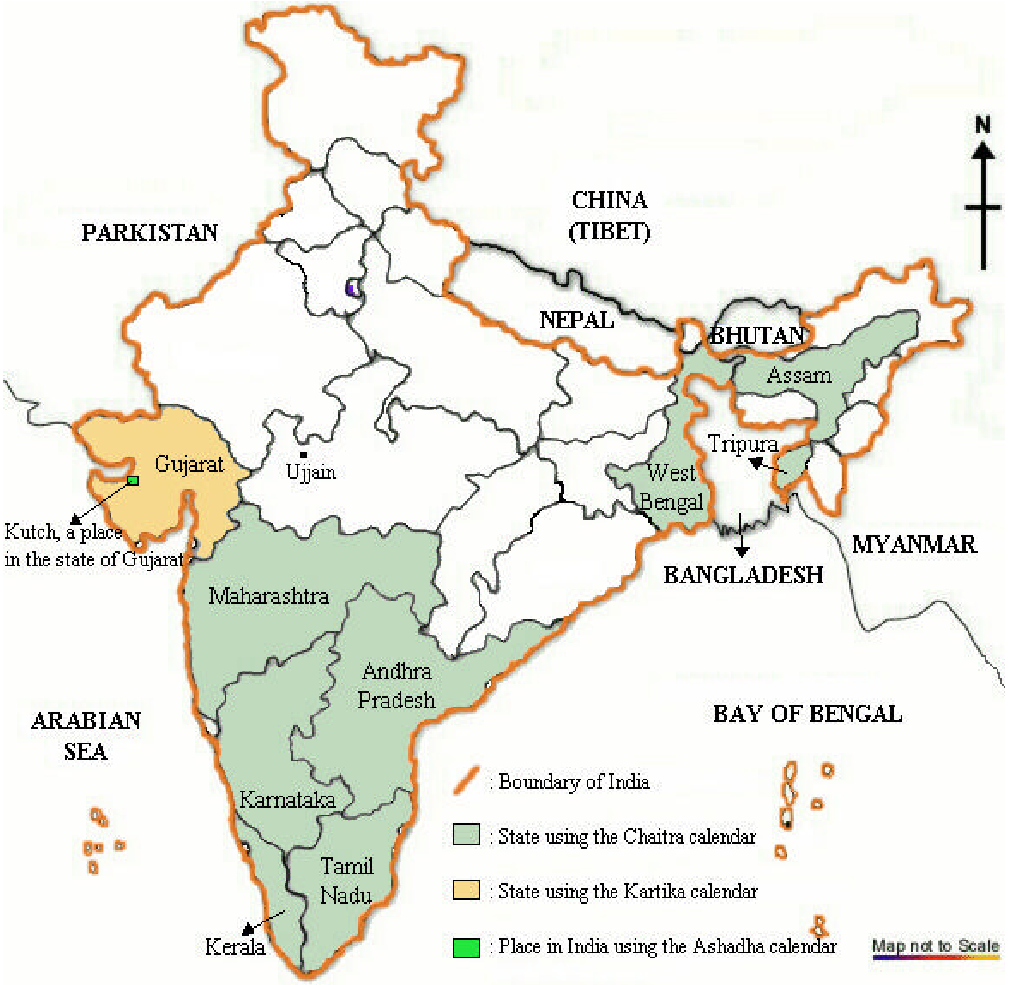
Hence we can conclude that the essential requirement of the Chaitra & other amanta lunisolar calendars is that the 12 months structure of a lunar year must be preserved.

Regions in India where calendars are used

From Table 14, we see that there are actually three variations of the Chaitra calendar and hence, the Kartika and Ashadha calendars, due to the different school rules for treating kshaya month. However, we do not have information on the regions in India that follow these rules. As we have already know the region in India where the amanta lunisolar calendar is used, on Map 3 below, we only indicate the states using the different lunisolar calendars.

The Chaitra calendar is mainly followed in the states of Karnataka, Andhra Pradesh and Maharashtra. The Kartika calendar is used in Gujarat. However, people in Kutch, a place in Gujarat, follow the Ashadha calendar. In these regions, the lunisolar calendars are used mainly for both civil and religious purposes. In the states where solar calendars are used, except in Orissa, Punjab and Haryana, the Chaitra calendar is needed to determine dates of religious and festive events. See Map 3.

Map 3: Areas in India using the different amanta lunisolar calendars



3.4(b) *The Purimanta Lunisolar Calendar*

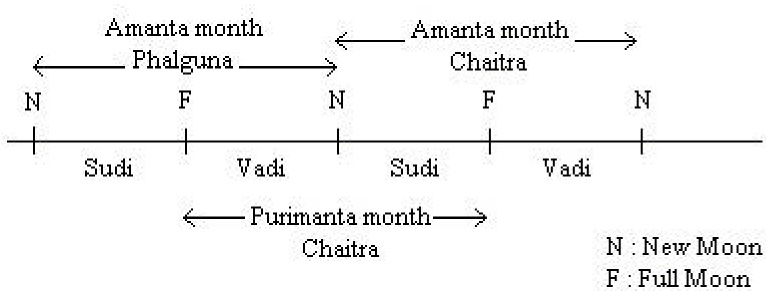
The purimanta lunisolar calendar uses the full moon ending lunar month as its fundamental unit. This calendar is also made to synchronize with the nirayana year in the same way as the amanta lunisolar calendar.

The Purimanta Month

The *purimanta month* is a lunar month that covers the period from full moon to the next full moon. The purimanta month is named after the amanta month in which the moment of its defining subsequent full moon falls. In other words, the purimanta month starts about 15 days earlier and ends in the middle of the concerned amanta month. The purimanta month and consequently the lunar year are expressed in civil days. See Table 8 for the names of 12 purimanta months.

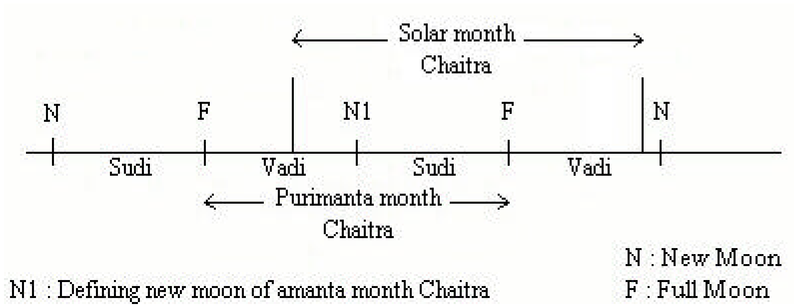
The purimanta month is also divided into two halves. The first is the vadi (krishna paksha) half and the second is the sudi (sukla paksha) half. The definitions are the same as the ones in the amanta month. See Figure 15 for the relationship between the amanta and purimanta months.

Figure 15: Relationship between the amanta and purimanta months



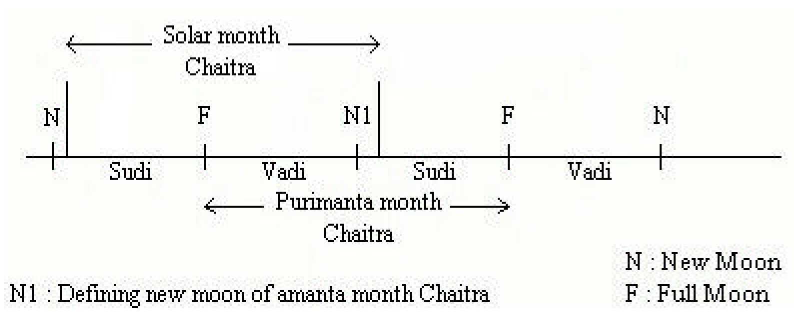
There is a charateristic about the purimanta month. Although an amanta month may fall almost entirely outside its linked solar month, the purimanta month always covers at least half of that solar month. Let us look at two examples to explain this. For the first example, suppose the defining new moon of amanta month Chaitra (N1) falls within the first half of solar month Chaitra, we see that the sudi half of purimanta month Chaitra falls completely within solar month Chaitra. See Figure 16.

Figure 16: First example to show how at least half of the purimanta month falls within its concerned solar month



In the second example, if the defining new moon of the amanta month Chaitra (N1) falls within the second half of solar month Chaitra, we see that the vadi half of purimanta month Chaitra falls completely within solar month Chaitra. See Figure 17.

Figure 17: Second example to show how at least half of the purimanta month falls within its concerned solar month



The Calendar Year

Like the Chaitra calendar, the purimanta lunisolar calendar begins the lunar year with the amanta month of Chaitra, that is, year starts in the middle of the purimanta month of Chaitra. Notice that the vadi half of the purimanta month of Chaitra falls in the old year while its sudi half falls in the new year.

Occurrence and Treatment of Leap or Adhika Month and Kshaya Month

The intervals of occurrence of adhika and kshaya months are generally the same as in the Chaitra calendar. When an adhika amanta month occurs, there will be two purimanta months named after it. The vadi half of the first purimanta month and the sudi half of the second purimanta month are treated as suddha half months while the sudi half and the vadi half of the first and second purimanta months respectively, are treated as a whole adhika month. In this way, the adhika month for both the amanta and purimanta lunisolar calendars are kept at the same month. This can be shown in Table 15.

However, a small number of calendar-makers prefer to have the entire first purimanta month treated as an adhika month and the entire second one as a suddha month. We do not have information on the regions in India where calendar-makers adopt different practices to treat adhika purimanta month.

As for the kshaya month, when it occurs, there will be a kshaya purimanta month and two accompanying adhika purimanta months. However, we are uncleared with the conventions used by calendar-makers to handle kshaya month in the purimanta lunisolar calendar to restore the 12 months structure of the lunar year. See Table 15 for the amanta and purimanta lunar half months in relation to the Gregorian calendar dates for Vikram Samvat year 2050 or Gregorian year 1993-1994.

Table 15: Beginnings of amanta and purimanta months in relations to Gregorian calendar dates for Vikram Samvat year 2050 or Gregorian year 1993-1994

Amanta Month		Corresponding Purimanta month		Beginning of the lunar month in Gregorian calendar date
Phalguna	K	Chaitra	K	AD 1993 9 Mar
Chaitra	S	Chaitra	S	24 Mar @ (Vikram-Chaitradi 2050)
Chaitra	K	Vaisakha	K	7 Apr
Vaisakha	S	Vaisakha	S	23 Apr
Vaisakha	K	Jyaishtha	K	7 May
Jyaishtha	S	Jyaishtha	S	22 May
Jyaishtha	K	Ashadha	K	5 June
Ashadha	S	Ashadha	S	21 June
Ashadha	K	Sravana	K	4 July

Sravana	S	Sravana	S	20 July
Sravana	K	Bhadra	K	3 Aug
(Adhika) Bhadra	S	(Adhika) Bhadra	S	18 Aug
(Adhika) Bhadra	K	(Adhika) Bhadra	K	2 Sept
Bhadra	S	Bhadra	S	17 Sept
Bhadra	K	Asvina	K	1 Oct
Asvina	S	Asvina	S	16 Oct
Asvina	K	Kartika	K	31 Oct
Kartika	S	Kartika	S	14 Nov @@ (Vikram-Kartikadi 2050)
Kartika	K	Agrahayana	K	30 Nov
Agrahayana	S	Agrahayana	S	14 Dec
Agrahayana	K	Pausha	K	29 Dec
Pausha	S	Pausha	S	AD 1994 12 Jan
Pausha	K	Magha	K	28 Jan
Magha	S	Magha	S	11 Feb
Magha	K	Phalguna	K	26 Feb
Phalguna	S	Phalguna	S	13 Mar
Phalguna	K	Chaitra	K	28 Mar

S = Sukla paksha (sudi)
K = Krishna paksha (vadi)

Note:
Lunar year Vikram-Chaitradi 2050 begins from the amanta month of Chaitra for both the amanta and purimanta calendars (marked by @). In Gujarat where Kartika calendar is used, amanta lunar year (Vikram-Kartikadi 2050) starts from the amanta month of Kartika (marked by @@).

Tithi

Like the amanta month, a purimanta month can be divided into 30 tithis or lunar days, of which the first 15 tithis belong to krishna paksha (dark half) and second 15 tithis belong to the sukla paksha (bright half). The names, prefixes and serial numbers of the tithis are the same as what are found in the amanta month. See Table 13.

At full moon, the (S) 15th tithi ends and the (K) 1st tithi begins and continues up to the moment when the Moon gains on the Sun by 12^0 in longitude. Then the (K) 1st tithi ends and the (K) 2nd tithi starts. This process continues and repeats itself at every full moon.

The purimanta month can have 29 or 30 days with occasional occurrence of kshaya days or adhika days due to short and long tithis respectively. Hence, days are not always counted serially from 1 to 29 or 30. See Table 16 for an illustration on how days in the amanta lunar months of Chaitra and Vaisakha corresponding to the purimanta months of Chaitra (S), Vaisakha (K) and (S) and Jyaistha (K) for Saka 1916 or Vikram-Chaitradi 2051 are counted according to the tithi in effect at sunrise.

Table 16: Counting of days in the amanta lunar months of Chaitra and Vaisakha (S) corresponding to the purimanta months of Chaitra (S), Vaisakha (K) and Vaisakha (S) for Saka 1916 or Vikram-Chaitradi 2051 according to the tithi in effect at sunrise.

Civil day	Day Count in Chaitra (S) according to tithi	Grego-rian date AD 1994	Civil day	Day Count in Chaitra (K) according to tithi	Grego-rian date AD 1994	Civil day	Day Count in Vaisakha (S) according to tithi	Grego-rian date AD 1994
1	1	Apr 12	1	1	Apr 26	1	1	May 11
2	2	13	2	2	27	2	2	12
3	3	14	3	3	28	3	3	13
4	4	15	4	4	29	4	4	14
5	5	16	5	5	30	5	4 adhika	15

6	6	17	6	6	May 1	6	5	16
7	7	18	7	7	2	7	6	17
8	8	19	8	8	3	8	7	18
9	9	20	9	9	4	9	→ 9	19
10	10	21	10	10	5	10	10	20
11	11	22	11	11	6	11	11	21
12	12	23	12	12	7	12	12	22
13	13	24	13	13	8	13	13	23
14	→ 15	25	14	14	9	14	14	24
15			15	30	10	15	15	25

S = Sukla paksha (sudi)
K = Krishna paksha (vadi)
→ indicates 'missing' tithi

The Purimanta Lunisolar Eras

Since the amanta and purimanta lunisolar calendars start the lunar year together, they use the same eras. See Table 9 for the lunisolar eras.

Regions in India where calendar is used

In map 4 below, we see that the purimanta lunisolar calendar is mainly used in the states of Uttar Pradesh, Bihar, Madhya Pradesh, Rajasthan, Himachal Pradesh and Jammu and Kashmir for both civil and religious purposes. In the states of Orissa, Punjab and Haryana where the Orissa calendar is followed, the purimanta lunisolar calendar is used to fix dates of religious and festive events.

Map 4: Areas in India using the purimanta lunisolar calendar

Appendix: Computer Codes

(**

The function HinduSolar[date_Integer] from the Mathematica package Calendrica.m follows the convention that solar month begins with the day after the occurrence of its concerned samkranti. We call this convention the DR rule. The function uses old Siddhantic methods to compute HinduSolarLongitude[kyTime_] (true position of the Sun) and hence HinduSunrise[kyTime_] (local time for sunrise at Ujjain), HinduZodiac[kyTime_] (the solar month) and HinduCalendarYear[kyTime_] (Kali Yuga year). The function Samkranti[gYear_, m_], which returns RD moment, is also determined from old Siddhantic methods.

**)

(**

We have written the functions orissaHinduSolar[date_Integer], tamilHinduSolar[date_Integer], malayaliHinduSolar[date_Integer] and bengalHinduSolar[date_Integer] that followed the Orissa rule, the Tamil rule, the Malayali rule and the Bengal rule respectively. They are modifications of the function HinduSolar[date_Integer]. We have also come up with functions ujjainSunrise[kyTime_], ujjainSunset[kyTime_] and ujjainAparahna[kyTime_] to obtain the IST for sunrise, sunset and aparahna at Ujjain. They are used in our written HinduSolar functions when required. However the functions HinduZodiac[kyTime_], HinduCalendarYear[kyTime_] and Samkranti[gYear, m_] are still used. We need to implement a function using modern methods to find true position of the Sun and hence determine the correct solar month, the Kali Yuga year and IST for the samkranti.

**)

(**

ujjainSunrise[kyTime_]

Input: Hindu moment. Output: Hindu moment.

We use the actual longitude for Ujjain, but use the IST meridian to compute the IST for sunrise at Ujjain, i.e., we're computing the IST for sunrise at Ujjain.

**)

```
ujjainSunrise[kyTime_] :=
  Module[{d, latitude, longitude},
  d = Floor[kyTime] + Calendrica`Private`HinduEpoch[];
  latitude = 1389/60;
  longitude = 165/2;
  kyTime + Sunrise[d, latitude, longitude]]
```

(**

ujjain Sunset[kyTime_]

Input: Hindu moment. Output: Hindu moment.

We're computing the IST for sunset at Ujjain using similar methods found in ujjainSunrise[kyTime_].

**)

```
ujjainSunset[kyTime_] :=
   Module[{d, latitude, longitude},
   d = Floor[kyTime] + Calendrica`Private`HinduEpoch[];
   latitude = 1389/60;
   longitude = 165/2;
   kyTime + Sunset[d, latitude, longitude]]
```

(**

ujjainAparahna[kyTime_]

Input : Hindu moment. Output : Hindu moment.

We're computing the IST for aparahna at Ujjain.

**)

```
ujjainAparahna[kyTime_] :=
   ujjainSunrise[kyTime] + 0.6*(ujjainSunset[kyTime] - ujjainSunrise[kyTime])
```

(** The IST for sunrise, sunset and aparahna at Ujjain required by Table3. **)

```
Input:    TimeOfDay[N[ujjainSunrise[HinduDayCount[ToFixed[Gregorian[4, 13, 1989]]]]]]
          TimeOfDay[N[ujjainSunset[HinduDayCount[ToFixed[Gregorian[4, 13, 1989]]]]]]
          TimeOfDay[N[ujjainAparahna[HinduDayCount[ToFixed[Gregorian[4, 13, 1989]]]]]]

Output:   TimeOfDay[5, 41, 48.0244]
          TimeOfDay[18, 19, 55.047]
          TimeOfDay[13, 16, 40.2379]

Input:    TimeOfDay[N[ujjainSunrise[HinduDayCount[ToFixed[Gregorian[4, 14, 1990]]]]]]
          TimeOfDay[N[ujjainSunset[HinduDayCount[ToFixed[Gregorian[4, 14, 1990]]]]]]
          TimeOfDay[N[ujjainAparahna[HinduDayCount[ToFixed[Gregorian[4, 14, 1990]]]]]]

Output:   TimeOfDay[5, 40, 54.1902]
          TimeOfDay[18, 20, 18.3152]
          TimeOfDay[13, 16, 32.6652]

Input:    TimeOfDay[N[ujjainSunrise[HinduDayCount[ToFixed[Gregorian[4, 14, 1991]]]]]]
          TimeOfDay[N[ujjainSunset[HinduDayCount[ToFixed[Gregorian[4, 14, 1991]]]]]]
```

```
         TimeOfDay[N[ujjainAparahna[HinduDayCount[ToFixed[Gregorian[4, 14, 1991]]]]]]

Output:  TimeOfDay[5, 40, 54.1902]
         TimeOfDay[18, 20, 18.3152]
         TimeOfDay[13, 16, 32.6652]

Input:   TimeOfDay[N[ujjainSunrise[HinduDayCount[ToFixed[Gregorian[4, 13, 1992]]]]]]
         TimeOfDay[N[ujjainSunset[HinduDayCount[ToFixed[Gregorian[4, 13, 1992]]]]]]
         TimeOfDay[N[ujjainAparahna[HinduDayCount[ToFixed[Gregorian[4, 13, 1992]]]]]]

Output:  TimeOfDay[5, 40, 54.1902]
         TimeOfDay[18, 20, 18.3152]
         TimeOfDay[13, 16, 32.6652]

Input:   TimeOfDay[N[ujjainSunrise[HinduDayCount[ToFixed[Gregorian[4, 13, 1993]]]]]]
         TimeOfDay[N[ujjainSunset[HinduDayCount[ToFixed[Gregorian[4, 13, 1993]]]]]]
         TimeOfDay[N[ujjainAparahna[HinduDayCount[ToFixed[Gregorian[4, 13, 1993]]]]]]

Output:  TimeOfDay[5, 41, 48.0244]
         TimeOfDay[18, 19, 55.047]
         TimeOfDay[13, 16, 40.2379]

Input:   TimeOfDay[N[ujjainSunrise[HinduDayCount[ToFixed[Gregorian[4, 14, 1994]]]]]]
         TimeOfDay[N[ujjainSunset[HinduDayCount[ToFixed[Gregorian[4, 14, 1994]]]]]]
         TimeOfDay[N[ujjainAparahna[HinduDayCount[ToFixed[Gregorian[4, 14, 1994]]]]]]

Output:  TimeOfDay[5, 40, 54.1902]
         TimeOfDay[18, 20, 18.3152]
         TimeOfDay[13, 16, 32.6652]
```

```
(**
orissaHinduSolar[date_Integer]
Input: Fixed number for RD date. Output: Orissa calendar date of day at sunrise on RD date.
**)
```

orissaHinduSolar[date_Integer] :=

Module[{kyTime, rise, month, year, approx, begin, day},

kyTime = HinduDayCount[date]; (** kyTime gives the number of days of RD date since the start of Kali Yuga or we say kyTime is in Hindu moment. **)

(** We find the rise, month, year, approx, begin and day at kyTime + 1 because the Orissa calendar date is ahead of the HinduSolar calendar date by a day. **)

rise = ujjainSunrise[kyTime + 1];

month = Calendrica`Private`HinduZodiac[rise];

year = Calendrica`Private`HinduCalendarYear[rise] - Calendrica`Private`HinduSolarEra[];

(** year determines the Saka year in which kyTime + 1 falls. **)

approx = kyTime - 2 - Quotient[Mod[Calendrica`Private`HinduSolarLongitude[rise], 1800], 60];

(** approx is a day in Hindu moment that falls in the previous solar month. **)

begin = approx + Calendrica`Private`MSum[(1) &,

approx, (Calendrica`Private`HinduZodiac[ujjainSunrise[#]] =!= month) &];

(** begin returns the starting day of month in Hindu moment. **)

day = kyTime - begin + 2;

(** day gives the day count from the starting day of month to kyTime + 1. **)

orissaHinduSolar[month, day, year]]

(**

tamilHinduSolar[date_Integer]

Input: Fixed number for RD date. Output: Tamil calendar date of day at sunrise on RD date.

**)

tamilHinduSolar[date_Integer] :=

Module[{kyTime, rise1, rise2, month1, month2, year1, year2, approx1,

begin1, day1, samk1, samk2, srise1, srise2, sset1, sset2},

kyTime = HinduDayCount[date];

(** We find rise2 and month2 at kyTime + 1 because under certain criteria, the Tamil calendar date is ahead of the HinduSolar calendar date by a day. **)

rise1 = ujjainSunrise[kyTime];

rise2 = ujjainSunrise[kyTime + 1];

month1 = Calendrica`Private`HinduZodiac[rise1];

month2 = Calendrica`Private`HinduZodiac[rise2];

year1 = Calendrica`Private`HinduCalendarYear[rise1] - Calendrica`Private`HinduSolarEra[];

approx1 = kyTime - 3 - Quotient[Mod[Calendrica`Private`HinduSolarLongitude[rise1], 1800], 60];

begin1 = approx1 + Calendrica`Private`MSum[(1) &,

approx1, (Calendrica`Private`HinduZodiac[ujjainSunrise[#]] =!= month1) &];

day1 = kyTime - begin1 + 1;

samk1 = Samkranti[78 + year1, month1];

srise1 = ujjainSunrise[HinduDayCount[Floor[samk1]]] + Calendrica`Private`HinduEpoch[];

(** srise1 is the IST for sunrise for the day of samk1 if the IST for samk1 falls before midnight. Otherwise srise1 gives the IST for sunrise for the following day. If the latter is true, then samk1 falls after sunset for the same day. **)

sset1 = ujjainSunset[HinduDayCount[Floor[samk1]]] + Calendrica`Private`HinduEpoch[];

(** The explanation for sset1 is similar to that for srise1. **)

If[month1 != month2, (** If month1 != month2, then month2 is the new month. We need the IST for the samkranti, the sunrise and sunset for the day of the samkranti and the Saka year for month2. **)

year2 = Calendrica`Private`HinduCalendarYear[rise2] - Calendrica`Private`HinduSolarEra[];

samk2 = Samkranti[78 + year2, month2];

srise2 = ujjainSunrise[HinduDayCount[Floor[samk2]]] + Calendrica`Private`HinduEpoch[];

sset2 = ujjainSunset[HinduDayCount[Floor[samk2]]] + Calendrica`Private`HinduEpoch[]];

Which[(month1 == month2) && (srise1 <= samk1 < sset1), (** If samk1 falls between srise1 and sset1 on the same day, the Tamil rule and the Orissa rule will both agree. **)

tamilHinduSolar[month1, day1 + 1, year1], month1 == month2,

(** If samk1 is either before sunrise or after sunset the Tamil rule and the DR rule will both agree. **)

tamilHinduSolar[month1, day1, year1],

(** The conditions below are for month1!= month2. **)

srise2 <= samk2 < sset2, tamilHinduSolar[month2, 1, year2],

srise1 <= samk1 < sset1, tamilHinduSolar[month1, day1 + 1, year1],

(** The Tamil rule and DR rules agree unless one or both of the samkrantis fall between sunrise and sunset. **)

True, tamilHinduSolar[month1, day1, year1]]]

(**

malayaliMonth[m_]

Input: (solar)month number. Output: malayali month number.

The Malayali calendar uses the Kollem era, not the Saka traditional era. Starting month of the nirayana year is month 5, the solar month that links with rasi 5. For example, if month = 5, then malayaliMonth = 1.

**)

malayaliMonth[m_] := Calendrica`Private`AdjustedMod[m - 4, 12]

Input: malayaliMonth[5]

Output: 1

(**

malayaliYear[m_, n_]

Input : (solar) month and Saka year. Output : Kollem year. The Malayali calendar uses the Kollem era, not the Saka traditional era. Starting month of the nirayana year is month 5, the solar month that links with rasi 5. Hence malayaliYear number changes to a new year at month 5 and not month 1.

**)

malayaliYear[m_, n_] := If[1 <= m <= 4, n - 747, n - 746]

Input: malayaliYear[1, 1912]

Output: 1165

(**

malayaliHinduSolar[date_Integer]

Input: Fixed number for RD date. Output: Malayali calendar date of day at sunrise on RD date.

**)

```
malayaliHinduSolar[date_Integer] :=
  Module[{kyTime, rise1, rise2, month1, month2, year1, year2, approx1,
   begin1, day1, samk1, samk2, srise1, srise2, aparahna1, aparahna2},
  kyTime = HinduDayCount[date];
```

(** We find rise2 and month2 at kyTime + 1 because under certain criteria, the Malayali calendar date is ahead of the HinduSolar calendar date by a day. Then we make necessary changes to obtain the malayaliMonth and malayaliYear. **)

```
  rise1 = ujjainSunrise[kyTime];
  rise2 = ujjainSunrise[kyTime + 1];
  month1 = Calendrica`Private`HinduZodiac[rise1];
  month2 = Calendrica`Private`HinduZodiac[rise2];
  year1 = Calendrica`Private`HinduCalendarYear[rise1] - Calendrica`Private`HinduSolarEra[];
  approx1 = kyTime - 3 - Quotient[Mod[Calendrica`Private`HinduSolarLongitude[rise1], 1800], 60];
  begin1 = approx1 + Calendrica`Private`MSum[(1) &,
          approx1, (Calendrica`Private`HinduZodiac[ujjainSunrise[#]] =!= month1) &];
  day1 = kyTime - begin1 + 1;
  samk1 = Samkranti[78 + year1, month1];
  srise1 = ujjainSunrise[HinduDayCount[Floor[samk1]]] + Calendrica`Private`HinduEpoch[];
```

(** srise1 is the IST for sunrise for the day of samk1 if the IST for samk1 falls before midnight. Otherwise srise1 gives the IST for sunrise for the following day. If the latter is true, then samk1 falls after sunset for the same day. **)

```
  aparahna1 = ujjainAparahna[HinduDayCount[Floor[samk1]]] +
```

Calendrica`Private`HinduEpoch[];

(** The explanation for aparahna1 is similar to that for srise1. **)

If[month1 != month2,

(** If month1!= month2, then month2 is the new month. We need the IST for the samkranti, the sunrise and the aparahna for the day of the samkranti and the Saka year for month2. **)

year2 = Calendrica`Private`HinduCalendarYear[rise2] - Calendrica`Private`HinduSolarEra[];

samk2 = Samkranti[78 + year2, month2];

srise2 = ujjainSunrise[HinduDayCount[Floor[samk2]]] +

Calendrica`Private`HinduEpoch[];

aparahna2 = ujjainAparahna[HinduDayCount[Floor[samk2]]] +

Calendrica`Private`HinduEpoch[]];

Which[(month1 == month2) && (srise1 <= samk1 < aparahna1), (** If samk1 falls between srise1 and aparahna1 on the same day, the Malayali rule and the Orissa rule will both agree. **)

malayaliHinduSolar[malayaliMonth[month1], day1 + 1, malayaliYear[month1, year1]],

month1 == month2, (** If samk1 is either before sunrise or after aparahna, the Malayali rule and the DR rule will both agree. **)

malayaliHinduSolar[malayaliMonth[month1], day1, malayaliYear[month1, year1]],

(** The conditions below are for month1!= month2. **)

srise2 <= samk2 < aparahna2,

malayaliHinduSolar[malayaliMonth[month2], 1, malayaliYear[month2, year2]],

srise1 <= samk1 < aparahna1,

malayaliHinduSolar[malayaliMonth[month1], day1 + 1, malayaliYear[month1, year1]], (** The Malayali rule and DR rules agree unless one or both of the samkrantis fall between sunrise and aparahna. **) **True,**

malayaliHinduSolar[malayaliMonth[month1], day1, malayaliYear[month1, year1]]]]

(**

BengalHinduSolar[date_Integer]

Input: Fixed number for RD date. Output: Bengal calendar date of day at sunrise on RD date.

**)

bengalHinduSolar[date_Integer] :=

Module[{kyTime, rise1, rise2, month1, month2, year1, year2, approx1,

begin1, day1, samk1, samk2, srise1, srise2, midnight1, midnight2},

kyTime = HinduDayCount[date];

(** We find rise1, month1, year1, approx1, begin1 and day1 at kyTime - 1 because under certain criteria, the Bengal calendar date is behind the HinduSolar calendar date by a day. **)

rise1 = ujjainSunrise[kyTime - 1];

```
rise2 = ujjainSunrise[kyTime];
month1 = Calendrica`Private`HinduZodiac[rise1];
month2 = Calendrica`Private`HinduZodiac[rise2];
year1 = Calendrica`Private`HinduCalendarYear[rise1] - Calendrica`Private`HinduSolarEra[];
approx1 = kyTime - 4 - Quotient[Mod[Calendrica`Private`HinduSolarLongitude[rise1], 1800], 60];
begin1 = approx1 + Calendrica`Private`MSum[(1) &,
        approx1, (Calendrica`Private`HinduZodiac[ujjainSunrise[#]] =!= month1) &];
day1 = kyTime - begin1;
samk1 = Samkranti[78 + year1, month1];
srise1 = ujjainSunrise[HinduDayCount[Floor[samk1]]] + Calendrica`Private`HinduEpoch[];
(** srise1 is the IST for sunrise for the day of samk1 if the IST for samk1 falls before midnight. Otherwise
srise1 gives the IST for sunrise for the following day. If the latter is true, then samk1 < srise1. **)
midnight1 = Floor[samk1] + 1;
(** midnight1 is the midnight following samk1. **)
If[month1 != month2, (** If month1!= month2, then month2 is the new month. We need the IST for the
samkranti, the sunrise for the day of the samkranti and the Saka year for month2. **)
 year2 = Calendrica`Private`HinduCalendarYear[rise2] - Calendrica`Private`HinduSolarEra[];
 samk2 = Samkranti[78 + year2, month2];
 srise2 = ujjainSunrise[HinduDayCount[Floor[samk2]]] + Calendrica`Private`HinduEpoch[];
 midnight2 = Floor[samk2] + 1];
Which[(month1 == month2) && (srise1 <= samk1 < midnight1), (** If samk1 falls between srise1
and midnight1, the Bengal rule and the DR rule will both agree. **)
 bengalHinduSolar[month1, day1 + 1, year1],
 month1 == month2, (** If samk1 is either before sunrise or after midnight, the Bengal calendar date is
behind the HinduSolar calendar date by a day. **)
 bengalHinduSolar[month1, day1, year1],
 (** The conditions below are for month1!= month2. **)
 srise2 <= samk2 < midnight2, bengalHinduSolar[month2, 1, year2],
 srise1 <= samk1 < midnight1,
 bengalHinduSolar[month1, day1 + 1, year1], (** The Bengal rule and DR rules agree unless one or
both of the samkrantis fall between sunrise and midnight. **)
 True, bengalHinduSolar[month1, day1, year1]]]
```

(** Mesha samkranti falls on Gregorian day 14/4/1990 at 3h57m. (between midnight and sunrise) **)

```
Input:  orissaHinduSolar[ToFixed[Gregorian[4, 13, 1990]]]
        tamilHinduSolar[ToFixed[Gregorian[4, 14, 1990]]]
        malayaliHinduSolar[ToFixed[Gregorian[4, 14, 1990]]]
```

bengalHinduSolar[ToFixed[Gregorian[4, 15, 1990]]]

Output: orissaHinduSolar[1, 1, 1912]
tamilHinduSolar[1, 1, 1912]
malayaliHinduSolar[9, 1, 1165]
bengalHinduSolar[1, 1, 1912]

(** Mesha samkranti falls on Gregorian day 14/4/1991 at 10h04m (between sunrise and aparahna). **)
Input: orissaHinduSolar[ToFixed[Gregorian[4, 14, 1991]]]
tamilHinduSolar[ToFixed[Gregorian[4, 14, 1991]]]
malayaliHinduSolar[ToFixed[Gregorian[4, 14, 1991]]]
bengalHinduSolar[ToFixed[Gregorian[4, 15, 1991]]]

Output: orissaHinduSolar[1, 1, 1913]
tamilHinduSolar[1, 1, 1913]
malayaliHinduSolar[9, 1, 1166]
bengalHinduSolar[1, 1, 1913]

(** Mesha samkranti falls on Gregorian day 13/4/1992 at 16h07m (between aparahna and sunset). **)
Input: orissaHinduSolar[ToFixed[Gregorian[4, 13, 1992]]]
tamilHinduSolar[ToFixed[Gregorian[4, 13, 1992]]]
malayaliHinduSolar[ToFixed[Gregorian[4, 14, 1992]]]
bengalHinduSolar[ToFixed[Gregorian[4, 14, 1992]]]

Output: orissaHinduSolar[1, 1, 1914]
tamilHinduSolar[1, 1, 1914]
malayaliHinduSolar[9, 1, 1167]
bengalHinduSolar[1, 1, 1914]

(** Mesha samkranti falls on Gregorian day 13/4/1993 at 22h25m (between sunset and midnight). **)
Input: orissaHinduSolar[ToFixed[Gregorian[4, 13, 1993]]]
tamilHinduSolar[ToFixed[Gregorian[4, 14, 1993]]]
malayaliHinduSolar[ToFixed[Gregorian[4, 14, 1993]]]
bengalHinduSolar[ToFixed[Gregorian[4, 14, 1993]]]

Output: orissaHinduSolar[1, 1, 1915]
tamilHinduSolar[1, 1, 1915]

malayaliHinduSolar[9, 1, 1168]
bengalHinduSolar[1, 1, 1915]

(**
The function HinduLunar[date_Integer] from the Mathematica package Calendrica.m follows the convention that when a kshaya month occurs, the two accompanying adhika months are treated as true leap months. The amanta month that contains two samkramanas is treated as a suddha month instead of a dual month. We call this convention the DR lunar rule. Notice that the DR lunar rule and the Southern school rule both agree except at treating the amanta month containing the whole solar month. The HinduLunar function uses old Siddhantic methods to compute HinduSolarLongitude[kyTime_] (true position of the Sun) and HinduLunarLongitude[kyTime_] (true position of the Moon)and hence HinduSunrise[kyTime_] (local time for sunrise at Ujjain), LunarDay[kyTime_] (tithi), HinduNewMoon[kyTime] (new moon at or before input Hindu moment), HinduZodiac[kyTime_] (the solar month) and HinduCalendarYear[kyTime_] (Kali Yuga year).
**)
(**
We have written the following functions to calculate variations of the amanta lunisolar calendar. They are the amantaEastHinduLunar[date_Integer], amantaNorthWestHinduLunar[date_Integer] and amantaSouthHinduLunar[date_Integer] functions that follow the Eastern school, the North Western school and the Southern school rules respectively. They are modifications of the function HinduLunar[date_Integer]. We have also come up with functions ujjainSunrise[kyTime_], IndianNewMoonAtOrBefore[kyTime_] and IndianFullMoonAtOrBefore[kyTime_] to obtain the IST for sunrise at Ujjain, the new moon and full moon at or before input Hindu moment. They are used in our written HinduLunar functions when required. However the functions HinduZodiac[kyTime_], LunarDay[kyTime_] and HinduCalendarYear[kyTime_] are still used. We need to implement two function using modern methods to find true positions of the Sun and the Moon and hence determine the correct solar month, the Kali Yuga year and the tithi.
**)

(** IndianNewMoonAtOrBefore[kyTime_]
Input: Hindu moment. Output: Hindu moment.
We're computing the IST for new moon at or before kyTime. For this function, we convert kyTime to a julian day number and use NewMoonAtOrBefore[jd_] to find the required new moon (in Greenwich) in julian day number. Then we convert this back to Hindu moment. The fractional part of this Hindu moment gives the IST for the required new moon.
**)
IndianNewMoonAtOrBefore[kyTime_] :=

```
   Module[{JDmoment, newMoon, newMoonInMoment, result},
   JDmoment = JDFromMoment[kyTime + Calendrica`Private`HinduEpoch[]];
   newMoon = NewMoonAtOrBefore[JDmoment];
   newMoonInMoment = MomentFromJD[newMoon];
   result = newMoonInMoment - Calendrica`Private`HinduEpoch[] + 11/48;
```

(** A julian day runs from noon to the next noon. When required new moon (= result) falls on the same julian day as kyTime does, result will be returned even if the time of result is later than that of the kyTime. This is not our desired output. Hence we implement the If condition. If result is later than kyTime, find the last new moon before result. Then let the last new moon be the result and return result. Otherwise, return result straightaway. **)

```
   If[result > kyTime,
        newMoon = NewMoonAtOrBefore[JDmoment - 1];
        newMoonInMoment = MomentFromJD[newMoon];
        newMoonInMoment - Calendrica`Private`HinduEpoch[] + 11/48, result]]
```

(** According to Condensed Ephemeris of Planets' Positions according to 'nirayana' or sidereal system from 1971 to 1981 AD, some of the IST for new moons in AD 1981 fall on 4/5/1981 at 9h49m, on 2/6/1981 at 17h2m and on 2/7/1981 at 0h33m. **)

Input: Gregorian[Floor[IndianNewMoonAtOrBefore[
ujjainSunrise[HinduDayCount[ToFixed[Gregorian[6, 2, 1981]]]]] +
Calendrica`Private`HinduEpoch[]]]

Output: Gregorian[5, 4, 1981]

Input: TimeOfDay[IndianNewMoonAtOrBefore[
ujjainSunrise[HinduDayCount[ToFixed[Gregorian[6, 2, 1981]]]]] +
Calendrica`Private`HinduEpoch[]]

Output: TimeOfDay[9, 49, 20.5944]

Input: Gregorian[Floor[IndianNewMoonAtOrBefore[
ujjainSunrise[HinduDayCount[ToFixed[Gregorian[7, 1, 1981]]]]] +
Calendrica`Private`HinduEpoch[]]]

Output: Gregorian[6, 2, 1981]

Input: TimeOfDay[IndianNewMoonAtOrBefore[

ujjainSunrise[HinduDayCount[ToFixed[Gregorian[7, 1, 1981]]]]] +
Calendrica`Private`HinduEpoch[]]

Output: TimeOfDay[17, 1, 55.2785]

Input: Gregorian[Floor[IndianNewMoonAtOrBefore[
ujjainSunrise[HinduDayCount[ToFixed[Gregorian[7, 2, 1981]]]]] +
Calendrica`Private`HinduEpoch[]]]

Output: Gregorian[7, 2, 1981]

Input: TimeOfDay[IndianNewMoonAtOrBefore[
ujjainSunrise[HinduDayCount[ToFixed[Gregorian[7, 2, 1981]]]]] +
Calendrica`Private`HinduEpoch[]]

Output: TimeOfDay[0, 33, 16.7785]

(**
amantaSouthHinduLunar[date_Integer]
Input: Fixed number for RD date. Output: Chaitra calendar date of day at sunrise on RD date when the Southern school rule is used to handle the kshaya month. Note that this function still follows the DR lunar rule because we need to write a function to find the IST for tithis using true positions of the Sun and the Moon.
**)

amantaSouthHinduLunar[date_Integer] :=
Module[{kyTime, rise, day, leapDay, lastNewMoon, nextNewMoon, solarMonth,
leapMonth, month, year},
kyTime = HinduDayCount[date];
rise = ujjainSunrise[kyTime];
day = Calendrica`Private`LunarDay[rise]; (** day gives the tithi no at sunrise on RD date. 1 to 15 are tithis for the bright half and 16 to 30 are tithis for the dark half. **)
leapDay = day == Calendrica`Private`LunarDay[ujjainSunrise[kyTime - 1]];
(** If tithi number at sunrise on RD (date - 1) = day, then day is a leap day and LeapDay is TRUE. Otherwise, leapDay is FALSE. **)
lastNewMoon = IndianNewMoonAtOrBefore[rise];
nextNewMoon = IndianNewMoonAtOrBefore[Floor[lastNewMoon] + 35];
solarMonth = Calendrica`Private`HinduZodiac[lastNewMoon];

leapMonth = solarMonth == Calendrica`Private`HinduZodiac[nextNewMoon];

(** If solarMonth = solarMonth in which nextNewMoon falls, then lastNewMoon is an adhika month and leapMonth is TRUE. Otherwise leapMonth is FALSE. **)

month = Calendrica`Private`AdjustedMod[solarMonth + 1, 12];

(** month gives the lunar month number. **)

year = Calendrica`Private`HinduCalendarYear[nextNewMoon] – Calendrica`Private`HinduLunarEra[] - If[leapMonth && month == 1, -1, 0];

(** year returns the Vikram year in which kyTime falls. **)

amantaSouthHinduLunar[month, leapMonth, day, leapDay, year]]

(**

checkSkippedRasiForEasternRule[kyTime_]

Input: Hindu moment. Output: List{skippedrasi, leaprasi}.

We're determining whether a kshaya month occurs at or after kyTime in the Vikram year in which kyTime falls. If kshaya month occurs, find the rasis that correspond to the kshaya month and the 2nd accompanying adhika month respectively. In the function, we denote them by skippedrasi and leaprasi.

**)

checkSkippedRasiForEasternRule[kyTime_] :=

Module[{lastNewMoon, solarMonth, nextNewMoon, nextsolarMonth, skippedrasi, leaprasi},

lastNewMoon = IndianNewMoonAtOrBefore[kyTime];

solarMonth = Calendrica`Private`HinduZodiac[lastNewMoon];

nextNewMoon = IndianNewMoonAtOrBefore[Floor[lastNewMoon] + 35];

nextsolarMonth = Calendrica`Private`HinduZodiac[nextNewMoon];

skippedrasi = 0;

leaprasi = 0;

(** Kshaya month possible only for solarMonths 8, 9 and 10. While loop searches for skippedrasi in solarMonths 8, 9 and 10. If solarMonth >= 11, there is no kshaya month at or after kyTime. **)

While[solarMonth < 11,

If[nextsolarMonth == solarMonth + 2, skippedrasi = solarMonth + 1;

(** While loop below searches ahead for leaprasi when skippedrasi != 0. **)

While[nextsolarMonth != solarMonth,

solarMonth = nextsolarMonth;

nextNewMoon = IndianNewMoonAtOrBefore[Floor[nextNewMoon] + 35];

nextsolarMonth = Calendrica`Private`HinduZodiac[nextNewMoon]];

leaprasi = solarMonth;

(** To stop iteration. **)

```
      Break[]];
    solarMonth = nextsolarMonth;
    nextNewMoon = IndianNewMoonAtOrBefore[Floor[nextNewMoon] + 35];
    nextsolarMonth = Calendrica`Private`HinduZodiac[nextNewMoon]];
  (** If kshaya month does not occur, skippedrasi = leaprasi = 0. **)
  {skippedrasi, leaprasi}]
```

(** For Saka 1904 (AD 1982 - 1983), the kshaya month is lunar month 11 because there is no new moon falling in solar month 10. The 2nd adhika month is lunar month 12 that corresponds to solar month 11. **)

Input: checkSkippedRasiForEasternRule[HinduDayCount[ToFixed[Gregorian[9, 17, 1982]]]]

Output: {10, 11}

(**
amantaEastHinduLunar[date_Integer]
Input: Fixed number for RD date. Output: Chaitra calendar date of day at sunrise on RD date when the Eastern school rule is used to handle the kshaya month.
**)

```
amantaEastHinduLunar[date_Integer] :=
  Module[{kyTime, rise, day, leapDay, lastNewMoon, nextNewMoon, solarMonth,
    leapMonth, startkyTime, skippedrasi, leaprasi, month, year},
  kyTime = HinduDayCount[date];
  rise = ujjainSunrise[kyTime];
  day = Calendrica`Private`LunarDay[rise];
  leapDay = day == Calendrica`Private`LunarDay[ujjainSunrise[kyTime - 1]];
  lastNewMoon = IndianNewMoonAtOrBefore[rise];
  nextNewMoon = IndianNewMoonAtOrBefore[Floor[lastNewMoon] + 35];
  solarMonth = Calendrica`Private`HinduZodiac[lastNewMoon];
  nextsolarMonth = Calendrica`Private`HinduZodiac[nextNewMoon];
  leapMonth = solarMonth == nextsolarMonth;
```

(** A kshaya month is possible only in solarMonths 8, 9 and 10. When there is a kshaya month, we assume that the two adhika months that come with it fall between solarMonth 6 to solarMonth 1 of the following nirayana year inclusive. If solarMonth = 1, check whether kshaya month occurred in the previous nirayana year. If 9 <= solarMonth <= 12, check whether kshaya month occurs in the current nirayana year. Otherwise, do not check for occurrence of kshaya month. **)

```
  Which[solarMonth == 1, startkyTime = kyTime - 191, solarMonth >= 9,
    startkyTime = kyTime - (solarMonth - 7)*32];
```

```
   If[2 <= solarMonth <= 8, skippedrasi = leaprasi = 0,
     skippedrasi = First[checkSkippedRasiForEasternRule[startkyTime]];
      leaprasi = Last[checkSkippedRasiForEasternRule[startkyTime]]]; (** skippedrasi determines whether kshaya month occurs. If yes, both the skippedrasi and leaprasi != 0. Otherwise, skippedrasi = leaprasi = 0. **)
    If[skippedrasi != 0, (** The following conditions are for kshaya month and the 2nd adhika month falling in the same nirayana year. **)
     If[skippedrasi < leaprasi, If[skippedrasi < solarMonth < leaprasi, month = solarMonth,
       If[leapMonth && solarMonth == leaprasi,
        leapMonth = ! (solarMonth == nextsolarMonth); month = solarMonth,
        month = Calendrica`Private`AdjustedMod[solarMonth + 1, 12]]],
      (** The following conditions are for the 2nd adhika month falling in solarMonth 1 of the following nirayana year. **)
      If[(solarMonth > skippedrasi) || (solarMonth < leaprasi),
       month = solarMonth, If[leapMonth && solarMonth == leaprasi,
        leapMonth = ! (solarMonth == nextsolarMonth); month = solarMonth,
        month = Calendrica`Private`AdjustedMod[solarMonth + 1, 12]]]],
     month = Calendrica`Private`AdjustedMod[solarMonth + 1, 12]];
   If[month == solarMonth, year = Calendrica`Private`HinduCalendarYear[lastNewMoon] -
       Calendrica`Private`HinduLunarEra[] - If[leapMonth && month == 1, -1, 0],
     year = Calendrica`Private`HinduCalendarYear[nextNewMoon] -
           Calendrica`Private`HinduLunarEra[] - If[leapMonth && month == 1, -1, 0]];
   amantaEastHinduLunar[month, leapMonth, day, leapDay, year]]

(** checkSkippedRasiForNorthWesternRule[kyTime_]
Input: Hindu moment. Output: List{skippedrasi, leaprasi}.
We're determining whether a kshaya month occurs at or after kyTime in the Vikram year in which kyTime falls. If kshaya month occurs, find the rasis that correspond to the kshaya month and the 1st accompanying adhika month respectively. In the function, we denote them by skippedrasi and leaprasi.
**)
checkSkippedRasiForNorthWesternRule[kyTime_] :=
   Module[{lastNewMoon, solarMonth, nextNewMoon, nextsolarMonth, skippedrasi,
     leaprasi, newMoon, lastsolarMonth, startsolarMonth},
   lastNewMoon = IndianNewMoonAtOrBefore[kyTime];
   solarMonth = Calendrica`Private`HinduZodiac[lastNewMoon];
   nextNewMoon = IndianNewMoonAtOrBefore[Floor[lastNewMoon] + 35];
   nextsolarMonth = Calendrica`Private`HinduZodiac[nextNewMoon];
```

```
   skippedrasi = 0;
   leaprasi = 0;
   newMoon = lastNewMoon;
(** Kshaya month possible only for solarMonths 8, 9 and 10. While loop searches for skippedrasi in solarMonths 8, 9 and 10. If solarMonth >= 11, there is no kshaya month at or after kyTime. **)
   While[solarMonth < 11,
      If[nextsolarMonth == solarMonth + 2, skippedrasi = solarMonth + 1;
      lastNewMoon = IndianNewMoonAtOrBefore[Floor[newMoon] - 28];
      lastsolarMonth = Calendrica`Private`HinduZodiac[lastNewMoon];
      (** While loop below searches behind for leaprasi when skippedrasi != 0. **)
      While[lastsolarMonth != solarMonth,
        solarMonth = lastsolarMonth;
        lastNewMoon = IndianNewMoonAtOrBefore[Floor[lastNewMoon] - 28];
        lastsolarMonth = Calendrica`Private`HinduZodiac[lastNewMoon]];
      leaprasi = solarMonth;
      (** To stop iteration. **)
      Break[]];
     newMoon = nextNewMoon;
     solarMonth = nextsolarMonth;
     nextNewMoon = IndianNewMoonAtOrBefore[Floor[nextNewMoon] + 35];
     nextsolarMonth = Calendrica`Private`HinduZodiac[nextNewMoon]];
   (** If kshaya month does not occur, skippedrasi = leaprasi = 0. **)
   {skippedrasi, leaprasi}]
```

(** For Saka 1904 (AD 1982 - 1983), the kshaya month is lunar month 11 because there is no new moon falling in solar month 10. The 1st adhika month is lunar month 7 that corresponds to solar month 6. **)

Input: checkSkippedRasiForNorthWesternRule[HinduDayCount[ToFixed[Gregorian[9, 17, 1982]]]]

Output: {10, 6}

(**
amantaNorthWestHinduLunar[date_Integer]
Input: Fixed number for RD date. Output: Chaitra calendar date of day at sunrise on RD date when the North Western school rule is used to handle the kshaya month.
**)

```
amantaNorthWestHinduLunar[date_Integer] :=
   Module[{kyTime, rise, day, leapDay, lastNewMoon, nextNewMoon, solarMonth,
```

```
    leapMonth, startkyTime, skippedrasi, leaprasi, month, year},
   kyTime = HinduDayCount[date];
   rise = ujjainSunrise[kyTime];
   day = Calendrica`Private`LunarDay[rise];
   leapDay = day == Calendrica`Private`LunarDay[ujjainSunrise[kyTime - 1]];
   lastNewMoon = IndianNewMoonAtOrBefore[rise];
   nextNewMoon = IndianNewMoonAtOrBefore[Floor[lastNewMoon] + 35];
   solarMonth = Calendrica`Private`HinduZodiac[lastNewMoon];
   nextsolarMonth = Calendrica`Private`HinduZodiac[nextNewMoon];
   leapMonth = solarMonth == nextsolarMonth;
(** A kshaya month is possible only in solarMonths 8, 9 and 10. When there is a kshaya month, we assume
that the two adhika months that come with it fall between solarMonth 6 to solarMonth 1 of the following
nirayana year inclusive. If 6 <= solarMonth <= 10, check whether kshaya month occurred in the current
nirayana year. Otherwise, do not check for occurrence of kshaya month. **)
   If[6 <= solarMonth <= 10, startkyTime = kyTime + (7 - solarMonth)*29;
    skippedrasi = First[checkSkippedRasiForNorthWesternRule[startkyTime]];
    leaprasi = Last[checkSkippedRasiForNorthWesternRule[startkyTime]],  skippedrasi = leaprasi =
0];
(** skippedrasi determines whether kshaya month occurs. If yes, both the skippedrasi and leaprasi != 0.
Otherwise, skippedrasi = leaprasi = 0. **)
   If[skippedrasi != 0,
(** leaprasi is always < skippedrasi and both must fall in the same nirayana year. **)
    If[(leaprasi <= solarMonth < skippedrasi) && (! (leapMonth = solarMonth == nextsolarMonth)),
     month = solarMonth + 2, If[solarMonth == leaprasi,
      leapMonth = ! (solarMonth == nextsolarMonth)];
     month = Calendrica`Private`AdjustedMod[solarMonth + 1, 12]],
    month = Calendrica`Private`AdjustedMod[solarMonth + 1, 12]];
   year = Calendrica`Private`HinduCalendarYear[nextNewMoon] -
         Calendrica`Private`HinduLunarEra[] - If[leapMonth && month == 1, -1, 0];
   amantaNorthWestHinduLunar[month, leapMonth, day, leapDay, year]]
```

(** For Saka 1904 (AD 1982 - 1983), the kshaya month is lunar month 11. The 1st adhika month is lunar month 7. The 2nd adhika month is lunar month 12. **)

(** This is under the Southern school rule. **)

```
Input:    amantaSouthHinduLunar[ToFixed[Gregorian[9, 18, 1982]]]
          amantaSouthHinduLunar[ToFixed[Gregorian[10, 17, 1982]]]
          amantaSouthHinduLunar[ToFixed[Gregorian[11, 16, 1982]]]
```

amantaSouthHinduLunar[ToFixed[Gregorian[12, 16, 1982]]]
amantaSouthHinduLunar[ToFixed[Gregorian[1, 15, 1983]]]
amantaSouthHinduLunar[ToFixed[Gregorian[2, 13, 1983]]]
amantaSouthHinduLunar[ToFixed[Gregorian[3, 15, 1983]]]
amantaSouthHinduLunar[ToFixed[Gregorian[4, 14, 1983]]]

Output: amantaSouthHinduLunar[7, True, 1, False, 2039]
amantaSouthHinduLunar[7, False, 1, False, 2039]
amantaSouthHinduLunar[8, False, 1, False, 2039]
amantaSouthHinduLunar[9, False, 1, False, 2039]
amantaSouthHinduLunar[10, False, 1, False, 2039]
amantaSouthHinduLunar[12, True, 1, False, 2039]
amantaSouthHinduLunar[12, False, 1, False, 2039]
amantaSouthHinduLunar[1, False, 1, False, 2040]

(** This is under the Eastern school rule. **)
Input: amantaEastHinduLunar[ToFixed[Gregorian[9, 18, 1982]]]
amantaEastHinduLunar[ToFixed[Gregorian[10, 17, 1982]]]
amantaEastHinduLunar[ToFixed[Gregorian[11, 16, 1982]]]
amantaEastHinduLunar[ToFixed[Gregorian[12, 16, 1982]]]
amantaEastHinduLunar[ToFixed[Gregorian[1, 15, 1983]]]
amantaEastHinduLunar[ToFixed[Gregorian[2, 13, 1983]]]
amantaEastHinduLunar[ToFixed[Gregorian[3, 15, 1983]]]
amantaEastHinduLunar[ToFixed[Gregorian[4, 14, 1983]]]

Output: amantaEastHinduLunar[7, True, 1, False, 2039]
amantaEastHinduLunar[7, False, 1, False, 2039]
amantaEastHinduLunar[8, False, 1, False, 2039]
amantaEastHinduLunar[9, False, 1, False, 2039]
amantaEastHinduLunar[10, False, 1, False, 2039]
amantaEastHinduLunar[11, False, 1, False, 2039]
amantaEastHinduLunar[12, False, 1, False, 2039]
amantaEastHinduLunar[1, False, 1, False, 2040]

(** This is under the North Western school rule. **)
Input: amantaNorthWestHinduLunar[ToFixed[Gregorian[9, 18, 1982]]]
amantaNorthWestHinduLunar[ToFixed[Gregorian[10, 17, 1982]]]

amantaNorthWestHinduLunar[ToFixed[Gregorian[11, 16, 1982]]]
amantaNorthWestHinduLunar[ToFixed[Gregorian[12, 16, 1982]]]
amantaNorthWestHinduLunar[ToFixed[Gregorian[1, 15, 1983]]]
amantaNorthWestHinduLunar[ToFixed[Gregorian[2, 13, 1983]]]
amantaNorthWestHinduLunar[ToFixed[Gregorian[3, 15, 1983]]]
amantaNorthWestHinduLunar[ToFixed[Gregorian[4, 14, 1983]]]

Output: amantaNorthWestHinduLunar[7, False, 1, False, 2039]
amantaNorthWestHinduLunar[8, False, 1, False, 2039]
amantaNorthWestHinduLunar[9, False, 1, False, 2039]
amantaNorthWestHinduLunar[10, False, 1, False, 2039]
amantaNorthWestHinduLunar[11, False, 1, False, 2039]
amantaNorthWestHinduLunar[12, True, 1, False, 2039]
amantaNorthWestHinduLunar[12, False, 1, False, 2039]
amantaNorthWestHinduLunar[1, False, 1, False, 2040]

(**
IndianFullMoonAtOrBefore[kyTime_]
Input: Hindu moment. Output: Hindu moment.
We're computing the IST for full moon at or before kyTime. For this function, we convert kyTime to a julian day number and use FullMoonAtOrBefore[jd_] to find the required full moon (in Greenwich) in julian day number. Then we convert this back to Hindu moment. The fractional part of this Hindu moment gives the IST for the required full moon.
**)

IndianFullMoonAtOrBefore[kyTime_] :=
Module[{JDmoment, fullMoon, fullMoonInMoment, result},
JDmoment = JDFromMoment[kyTime + Calendrica`Private`HinduEpoch[]];
fullMoon = FullMoonAtOrBefore[JDmoment];
fullMoonInMoment = MomentFromJD[fullMoon];
result = fullMoonInMoment - Calendrica`Private`HinduEpoch[] + 11/48;

(** A julian day runs from noon to the next noon. When required full moon (= result) falls on the same julian day as kyTime does, result will be returned even if the time of result is later than that of the kyTime. This is not our desired output. Hence we implement the If condition. If result is later than kyTime, find the last full moon before result. Then let the last full moon be the result and return result. Otherwise, return result straightaway. **)

If[result > kyTime,
fullMoon = FullMoonAtOrBefore[JDmoment - 1];

```
        fullMoonInMoment = MomentFromJD[fullMoon];
        fullMoonInMoment - Calendrica`Private`HinduEpoch[] + 11/48,
    result]]
```

(** According to Condensed Ephemeris of Planets' Positions according to 'nirayana' or sidereal system from 1971 to 1981 AD, some of the IST for full moons in AD 1981 fall on 14/9/1981 at 8h39m, on 13/10/1981 at 18h19m and on 12/11/1981 at 3h56m. **)

```
Input:   Gregorian[Floor[IndianFullMoonAtOrBefore[
         ujjainSunrise[HinduDayCount[ToFixed[Gregorian[10, 13, 1981]]]]] +
         Calendrica`Private`HinduEpoch[]]]

Output:  Gregorian[9, 14, 1981]

Input:   TimeOfDay[IndianFullMoonAtOrBefore[
         ujjainSunrise[HinduDayCount[ToFixed[Gregorian[10, 13, 1981]]]]] +
         Calendrica`Private`HinduEpoch[]]

Output:  TimeOfDay[8, 38, 47.451]

Input:   Gregorian[Floor[IndianFullMoonAtOrBefore[
         ujjainSunrise[HinduDayCount[ToFixed[Gregorian[11, 11, 1981]]]]] +
         Calendrica`Private`HinduEpoch[]]]

Output:  Gregorian[10, 13, 1981]

Input:   TimeOfDay[IndianFullMoonAtOrBefore[
         ujjainSunrise[HinduDayCount[ToFixed[Gregorian[11, 11, 1981]]]]] +
         Calendrica`Private`HinduEpoch[]]

Output:  TimeOfDay[18, 19, 42.6265]

Input:   Gregorian[Floor[IndianFullMoonAtOrBefore[
         ujjainSunrise[HinduDayCount[ToFixed[Gregorian[11, 12, 1981]]]]] +
         Calendrica`Private`HinduEpoch[]]]

Output:  Gregorian[11, 12, 1981]
```

Input: TimeOfDay[IndianFullMoonAtOrBefore[
ujjainSunrise[HinduDayCount[ToFixed[Gregorian[11, 12, 1981]]]]] +
Calendrica`Private`HinduEpoch[]]

Output: TimeOfDay[3, 56, 59.4279]

Bibliography

Main references for Chapter 1 and 2

1. Helmer Aslaksen, *The Mathematics of the Chinese Calendar*, available at http://www.math.nus.edu.sg/aslaksen/.

2. L.E. Doggett, *Calendars*, available at http://charon.nmsu.edu/~lhuber/leaphist.html.

3. James Evans, *The History and Practice of Ancient Astronomy*, Oxford University Press, 1998.

4. *The 1996 Grolier™ Multimedia Encyclopedia*, © Grolier Incorporated, 1995.

5. *DK Multimedia: Encyclopedia of Science 2.0*, © Dorling Kindersley, 1996.

6. Lee Tang Hwee, *Calendars in Singapore*, available at http://www.math.nus.edu.sg/aslaksen/.

Main references for Chapter 3

7. S.K. Chatterjee, *Indian Calendric System*, Publications Division, Ministry of Information and Broadcasting, Government of India, 1998.

8. M.H. Saha and N.C. Lahiri, *History of the Calendar*, Council of Scientific and Industrial Research, New Delhi, 1992.

9. S.N. Sen and K.S. Shukla, *History of Astronomy in India*, Indian National Science Academy, New Delhi, 1985.

Main references for Appendix: Computer Codes

10. Nachum Dershowitz and Edward M. Reingold, *Calendrical Calculations*, Cambridge University Press, 1997.

11. N.C. Lahiri, *Condensed Ephemeris of Planets' Positions according to 'nirayana' or sidereal system from 1971 to 1981 AD*, Astro-Research Bureau, Calcutta, 1990.