

A Review of Flight Dynamic Simulation Model of Missiles

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Abstract: - A full six degrees of freedom (6-DOF) simulation flight dynamics model is applied for the accurate prediction of short and long range trajectories of high and low spin-stabilized projectiles and small bullets via atmospheric flight to final impact point. The projectile is assumed to be both non-flexible and rotationally symmetric about its spin axis launched at low and high pitch angles. The projectile maneuvering motion depends on the most significant forces and moments variations, in addition to wind and gravity. The computational flight analysis takes into consideration the Mach number and total angle of attack effects by means of the variable aerodynamic coefficients. The present paper describes the results of the comparison i) data of verified experiments and ii) computational codes on atmospheric dynamics model flight analysis for projectiles (M105 mm) and small bullets (7,62 mm) which stated in [8], [9].

Key-Words: - projectiles; trajectory; high and low pitch angles.

1 Introduction

Ballistics is the science that deals with the motion of projectiles. The word ballistics was derived from the Latin 'ballista,' which was an ancient machine designed to hurl a javelin. The modern science of exterior ballistics [12] has evolved as a specialized branch of the dynamics of rigid bodies, moving under the influence of gravitational and aerodynamic forces and moments.

The first rigid six-degree-of-freedom projectile

exterior ballistics model constructed by Fowler, Gallop, Lock, and Richmond [7]. Many authors have extended this projectile model in various direction i.e. [2, 3, 5, 10].

The present work address a full six degrees of freedom (6-DOF) projectile flight dynamics analysis for accurate prediction of short and long range trajectories of high spin-stabilized projectiles and small bullets. The proposed flight dynamic model takes into consideration the influence of the most

significant forces and moments variations, in addition to wind, gravity forces and Magnus effects.

2 Projectile model

The Cartridge 105mm HE M1 projectile is used with various 105mm Howitzers such as US M49 with M52, M52A1 cannons, M2A1 & M2A2 with M101, M101A1 cannons, M103 with M108 cannon, M137 with M102 cannon as well as NATO L14 MOD56 and L5. For technical characteristics and operational data of 105mm HE M1 see table 1.

Technical Description	Technical Data
105mm HE M1 consists Projectile: Hollow steel forging	Total Length: 790 mm
Explosive charge: 2,09kg TNT	Total weight: 18,15 kg
Supplementary charge: 0,14kg TNT	Temperature Limits: -40 °C to +52 °C for operation
Cartridge case: M14	
Percussion primer: M28B2	-62 °C to 71 °C for storage

Table 1. Technical Characteristics of 105mm HE M1 Howitzers Projectile.

Physical and geometric characteristics data of the 7.62 mm bullet are illustrated in Table 2.

Characteristics	Technical Data
Diameter	7,62mm
Total length(mm)	71,88
Weight (kg)	0,385
Axial moment of inertia	$7,2282 \cdot 10^{-8}$
Transverse moment of inertia	$5,3787 \cdot 10^{-7}$
Center of gravity from the base	12,03

Table 2. Characteristics of 7,62 small bullet.

3 Trajectory Flight Simulation Model

A six degree of freedom rigid-projectile model [1, 4, 6, 11] has three rotations and three translations. The three translations components (x,y,z) describing the position of the projectile's center of mass and the three Euler angles (φ, θ, ψ) describing the orientation of the projectile with respect to translation from the body frame (no-roll-frame, NRF, $\varphi=0$) to the plane fixed (inertial frame, if). For such flight bodies, X_{NRF} axis of the projectile in the no-roll-frame coordinate system usually lies along the projectile

axis of symmetry and Y_{NRF}, Z_{NRF} axes are then oriented so as to complete a right-hand orthogonal system.

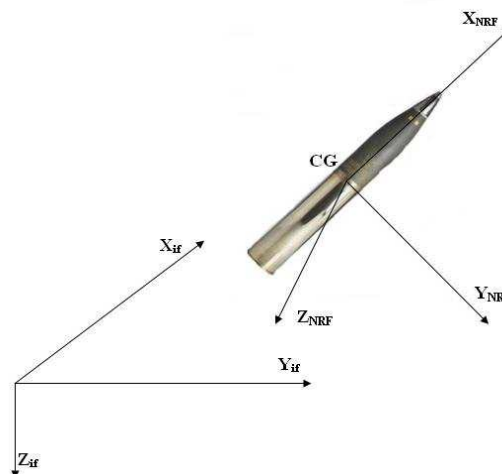


Fig. 1. Coordinate System

Newton's laws of the motion state that rate of change of linear momentum must equal the sum of all the externally applied forces and the rate of change of angular momentum must equal the sum of all the externally applied moments, respectively. The force acting on the projectile comprises the weight, the aerodynamic force and the Magnus force. The moment acting on the projectile comprises the moment due to the standard aerodynamic force, the Magnus aerodynamic moment and the unsteady aerodynamic moment.

Therefore, the twelve state variables $x, y, z, \varphi, \theta, \psi, u, v, w, p, q$ and r are necessary to describe position, flight direction and velocity at every point of projectile's atmospheric flight trajectory. Introducing the components of the acting forces and moments with regards to the no-roll-frame (~) rotating coordinate system with the arc length as an independent variable, we derive the following equations of motion for six-dimensional flight:

$$x' = D \theta - \beta D \psi + \alpha \theta D \quad (1)$$

$$y' = D \psi + \beta D + \alpha \theta \psi D \quad (2)$$

$$z' = -D \theta + \beta D \quad (3)$$

$$\phi' = \frac{D}{V} \tilde{p} + \frac{D}{V} \tan|\theta| \tilde{r} \quad (4)$$

$$\theta' = \frac{D}{V} \tilde{q} \quad (5)$$

$$\psi' = \frac{D}{V \cos \theta} \tilde{r} \quad (6)$$

$$\begin{aligned} \tilde{u}' = & -\frac{D}{V} g \theta - \frac{\pi}{8m} \rho V D^3 C_{D0} \\ & - D^3 \frac{\pi}{8m} \rho V C_{D2} \alpha^2 - \\ & - D^3 \frac{\pi}{8m} \rho V C_{D2} \beta^2 + \\ & + \beta D \tilde{r} - \tilde{q} \alpha D \end{aligned} \quad (7)$$

$$\begin{aligned} \tilde{v}' = & -D^3 \frac{\pi}{8m} \rho \beta V C_{NA} + \\ & + D^4 \frac{\pi}{16m} \tilde{p} \rho \alpha C_{NPA} + \\ & + \alpha D \tilde{p} \tan|\theta| - D \tilde{r} \end{aligned} \quad (8)$$

$$\begin{aligned} \tilde{w}' = & \frac{D}{V} g - D^3 \frac{\pi}{8m} \rho V C_{NA} \frac{\tilde{w}}{V} - \\ & - D^4 \frac{\pi}{16m} \tilde{p} \rho C_{NPA} \frac{\tilde{v}}{V} + \\ & + D \tilde{q} - \frac{D}{V} \tilde{p} \tilde{v} \end{aligned} \quad (9)$$

$$\tilde{p}' = D^5 \frac{\pi}{16I_{XX}} \tilde{p} \rho C_{LP} \quad (10)$$

$$\begin{aligned} \tilde{q}' = & D^3 \frac{\pi}{8I_{YY}} \rho V C_{NA} \alpha L E_{MCP} + \\ & + D^4 \frac{\pi}{16I_{YY}} \rho C_{YPA} \tilde{p} \beta L E_{MCM} + \\ & + D^5 \frac{\pi}{16I_{YY}} \rho C_{MQ} \tilde{q} + D^4 \frac{\pi}{8I_{YY}} \rho C_{MA} - \\ & - \frac{D}{V} \tilde{r} \frac{I_{XX}}{I_{YY}} \tilde{p} - \frac{D}{V} \tilde{r}^2 \tan|\theta| \end{aligned} \quad (11)$$

$$\begin{aligned} \tilde{r}' = & -D^3 \frac{\pi}{8I_{YY}} \rho V C_{NA} \alpha L E_{MCP} + \\ & + D^4 \frac{\pi}{16I_{YY}} \tilde{p} \rho C_{YPA} \beta L E_{MCM} + \\ & + D^5 \frac{\pi}{16I_{YY}} \rho C_{MQ} \tilde{r} - D^4 \frac{\pi}{8I_{YY}} \rho C_{MA} + \\ & + \frac{D}{V} \tilde{p} \tilde{q} \frac{I_{XX}}{I_{YY}} + \frac{D}{V} \tilde{q} \tilde{r} \tan|\theta| \end{aligned} \quad (12)$$

where

C_{D0} = zero-yaw drag aerodynamic coefficient

C_{D2} = yaw drag aerodynamic coefficient

C_{LP} = roll damping aerodynamic coefficient

C_{MQ} = pitch damping aerodynamic coefficient

C_{MA} = pitch moment coefficient

C_{YPA} = Magnus moment coefficient

C_{NPA} = Magnus force aerodynamic coefficient

x, y, z = projectile position, in inertial space

$\tilde{u}, \tilde{v}, \tilde{w}$ = projectile velocity components,

expressed in no-roll-frame

\tilde{p} = projectile roll rate, expressed in no-roll-frame

\tilde{q}, \tilde{r} = projectiles pitch and yaw rates

expressed in no-roll-frame

θ, ψ = projectiles pitch and yaw angles

ϕ = projectile roll angle

I_{XX}, I_{YY}, I_{ZZ} =diagonal components of the inertia matrix

I_{XY}, I_{YZ}, I_{XZ} =off-diagonal components of the inertia matrix

V = total aerodynamic velocity

ρ = atmospheric density

m = mass of projectile

t = time

α, β = aerodynamic angles of attack and sideslip

D = projectile reference diameter

g = gravity

The projectile dynamic model consists of 12 highly first order non-linear ordinary differential equations, which are solved simultaneously by resorting to numerical integration using 4th order Runge-Kutta method.

4 Results

The flight path trajectories of the present dynamic model with initial firing velocity of 490 m/sec and rifling twist rate 1 turn in 18 calibers (1/18) of the

M103 Howitzer at initial pitch angles of 45° and 70° are indicated for two cases: constant and variable aerodynamic coefficients. The impact points of the above trajectories with the present variable coefficients method compared with accurately estimations of McCoy's flight atmospheric model [12] provide satisfactory agreement for the same conditions.

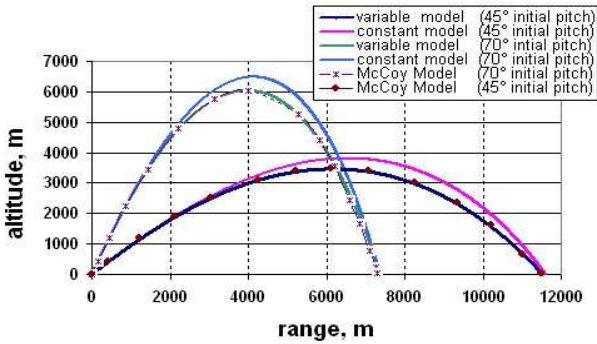


Fig. 2. Impact points and flight path trajectories with constant and variable aerodynamic coefficients for 105mm projectile compared with McCoy's trajectory data.

The diagram shows that for 105mm M1 projectile, fired at sea-level (without wind) with an angle of 45° (dark blue solid line) the predicted range to impact is approximately 11520 m, the time of flight is slight greater than 53 sec, and the maximum height is 3465 m. At 70° (green solid line), the predicted level-ground range is 7340 m, the time of flight to impact is about 72,5 sec, and the maximum height is slight over 6030 m. In the same diagram, the basic differences from McCoy's flight data are remarked in trajectory models with constant aerodynamic coefficients.

A small bullet of 7.62 mm diameter is examined for its atmospheric constant flight trajectory predictions at low and high pitch angles 0.84° and 32° , respectively, with initial firing velocity of 793 m/s, initial yaw angle 2 degrees, yaw rate 25 rad/s and rifling twist 12 inches per turn. The impact points of the above trajectories are compared with a corresponding 5.0 m/s mean crosswind blowing motion and a accurately flight path prediction with Nennstiel's trajectory analysis [11], for cartridge 7.62 mm ball M80 bullet type with initial firing velocity of 838 m/s. The main differences are presented at high altitudes from the firing site sea-level.

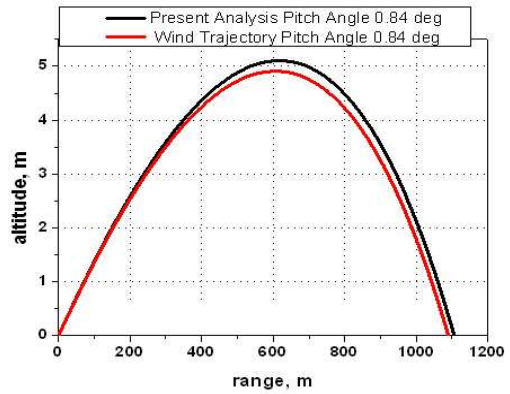


Fig. 3. Constant aerodynamic coefficients impact points for 7.62 mm bullet.

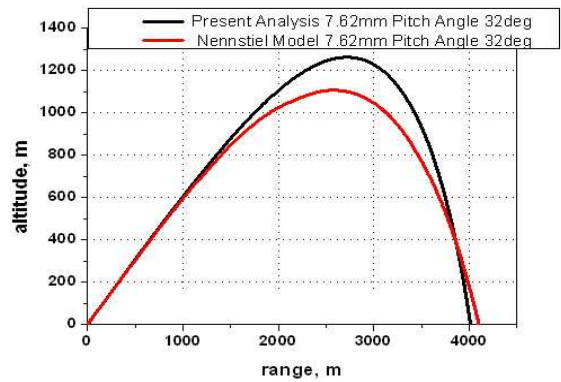


Fig. 4. Constant flight atmospheric trajectory analysis for small bullet compared to Nennstiel prediction computational algorithm

At 0.84 degrees the 7.62 mm bullet fired at sea-level has a range with the wind model of almost 1,090 m (red solid line). The presented method for the same initial pitch angle gives a corresponding value of 1,105 m, and the height is slight over 5.1 m. At 32 degrees initial pitch angle, the predicted range of 7.62 mm bullet to impact point of Nennstiel analysis^[11] is approximately 4,100 m, and the maximum height is 1,170 m, respectively. On the other hand, the proposed computational no-wind flight analysis gives an impact point at almost 4 km and a maximum height at about 1,3 km.

5 Conclusions

The six degrees of freedom (6-DOF) simulation flight dynamics model is applied for the accurate prediction of short and long range trajectories of high and low spin-stabilized projectiles and small bullets. The computational analysis takes into consideration effects of constant aerodynamic coefficients, Mach number and total angle of attack variation effects. The computational results of the

dynamic flight trajectories with constant aerodynamic force and moment coefficients are in good agreement compared with other technical data and recognized projectile flight dynamic models.

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