On Taxation and the Control of Externalities

By WILLIAM J. BAUMOL*

It is ironic that just at the moment when the Pigouvian tradition has some hope of acceptance in application it should find itself under a cloud in the theoretical literature. James Buchanan has argued that its recommended taxes and subsidies may even increase resource misallocation in the presence of monopoly. Otto Davis and Andrew Whinston (1962) have, in effect, raised doubts about its applicability in the presence of oligopoly. And Ronald Coase has asserted that the tradition has not selected the correct taxation principle for the elimination of externalities, and may not even have chosen the right individuals to tax or to subsidize. In this paper I will suggest that these authors have led the discussion in our profession to focus on the wrong difficulties. In doing so they have, albeit inadvertently, drawn attention away from some of the most important limitations of the Pigouvian prescription as an instrument of policy and from consideration of the means that might prove effective in practice.

The main purpose of the paper is to show that, taken on its own grounds, the conclusions of the Pigouvian tradition are, in fact, impeccable. Despite the various criticisms that have been raised against it in the large numbers case, which is of primary importance in reality and to which Pigou's analysis directs itself, his tax-subsidy programs are generally those required for an optimal allocation of resources. Moreover, I will attempt to show that where an externality is (like the usual pollution problem) of the public goods variety, neither compensation to nor taxation of those who are affected by it is compatible with optimal resource allocation. Pigouvian taxes (subsidies) upon the generator of the externality are all that is required.

However, as is well known, the Pigouvian proposals suffer from a number of serious shortcomings as operational criteria when one seeks to implement them precisely as they emerge from the theory. I therefore discuss a modified approach that recommends itself more for its promise of effectiveness, than its theoretical nicety. It consists of two basic steps: the setting of standards, more or less arbitrarily, of levels of pollution, congestion and the like, that are considered to be tolerable, and the design of taxes and effluent charges whose rates are shown by experience to be sufficient to achieve the selected standards of acceptability. Such a system of charges will, at least in principle, effect any preselected reduction in,

* Professor of economics, Princeton University and New York University. I would like to express my gratitude to the National Science Foundation whose assistance helped materially in the completion of the paper and to my colleagues James Litvack, Wallace Oates, and David Bradford, to my students Mark Gaudry and Bryan Boulier, and to Peter Bohm, James Buchanan, Ronald Coase, Karl-Göran Mäler, Herbert Mohring, and Ralph Turvey who have given me many very helpful suggestions, and saved me from a number of serious errors. Mohring and J. Hayden Boyd have written an extremely illuminating paper dealing, among other relevant matters, with the portions of the Coase-Buchanan-Turvey arguments in the case where the polluters and their victims "can and do negotiate." Since the present paper concerns itself only with the "relevant" large numbers case where there is no negotiation, it deliberately makes no attempt to consider the interesting negotiation case examined so helpfully by Mohring and Boyd.

say, the pollution content of our rivers, at minimum cost to society. It automatically achieves an efficient allocation of the required reduction in emissions among the offending firms *even if they are neither pure competitors nor profit maximizers*. Thus, a persuasive case can be made for the use of taxes and subsidies to control externalities, even if they will not produce an optimal allocation of resources in the complex world of reality.

I. The Coase Argument in the Case Without Negotiation

Recommendations designed for the competitive case can clearly run into difficulties in the presence of monopolistic elements. Buchanan reminds us that, if a polluting monopolistic industry already restricts the outputs of its products below their competitive levels, the imposition of an effluent charge to restrict output still further is hardly likely to be appropriate. And Davis and Whinston (1962) show for the case of externalities under oligopoly that it is rather difficult to come up with an ideal set of taxes since in the small numbers case just about anything is possible by way of pricing and output levels. However, these arguments have little direct bearing on the Pigouvian analysis because it is couched entirely in terms of pure competition (on this see Stanislaw Wellicz' illuminating discussion), which, in view of the large numbers involved in virtually all of the externalities problems that worry us today, is entirely apropos.

Coase's arguments, buttressed by impressive legal erudition, are less easily dealt with. He offers us a number of illuminating observations, among them the interesting point (see his Section IV) that (in the relatively unimportant cases) where only a small number of decision makers is involved, a process of voluntary bargaining and side payments among those concerned by an externality may produce an optimal allocation of resources, even in the absence of liability for damage. This implies that where small numbers are involved, the imposition of a "corrective" Pigouvian tax may be too much of a good thing—it can produce a misallocation rather than eliminating it.

Coase suggests, however, that even in cases where there is no negotiation among the parties affected by an externality the Pigouvian taxes and subsidies may be the wrong remedy—that they may only modify the character of the misallocation of resources. Coase's central argument appears to be the following: Every social cost is inherently reciprocal in nature. The nearby residents who breathe smoke spewn by a factory must share with the management of the factory the responsibility for the resulting social cost. True, if the factory were closed up the social cost would disappear. But the same holds for its neighbors—were they to move away no one would suffer smoke nuisance. Put another way, just as the smoke emitted by the factory imposes at least a psychic cost on its neighbors, the latter's insistence on the installation of purification devices or a reduction in the pollution-producing activity imposes a cost on the factory.

This position, though at first glance very odd (the murder victim too, is then always an accessory to the crime), grows more persuasive as one considers it further. Coase does not raise the issue as a matter of distributive justice. Rather, he suggests, because of the reciprocal structure of the externality, the traditional taxes and subsidies are likely to lead to a misallocation of resources.[1] If it is socially less costly to

[1] Thus Coase starts out with

> . . . the case of a confectioner, the noise and vibrations from whose machinery disturbed a doctor in his work. To avoid harming the doctor would inflict harm on [be costly to] the confectioner. The problem posed by this case was essentially whether it was worthwhile, as a

remove the neighbors from the vicinity of the factory than to reduce the quantity of pollutants emitted by the plant (taking into account the location preferences of the current residents), surely the former is the course of action which is more desirable socially.

In that case, should not a tax sometimes be levied, at least in part on those who choose to live near the factory rather than upon the factory owners?[2] Otherwise might not too many persons be induced to move near the factory thus, incidentally, increasing the magnitude of the Pigouvian tax since the social damage caused by the smoke must then rise correspondingly?

A simple model shows readily that, properly stated, the prescription of the Pigouvian tradition is (at least formally) correct. An appropriately chosen tax, levied only on the factory (without payment of compensation to local residents) is precisely what is needed for optimal resource allocation under pure competition. No tax on nearby residents is required or, taken in real terms, is even compatible with optimal resource allocation. Thus the obvious and apparently common interpretation of the Coase position is simply invalid. We will see, however, that the issue Coase himself intended to raise was rather more subtle and his conclusions are not necessarily at variance with the Pigouvian prescription as I interpret it.

II. Analysis: Should the Victims of Externalities be Taxed or Compensated?

To formalize the argument we construct an elementary general equilibrium model designed to represent in most explicit form the conditions envisioned in the Coase argument, departing from it only by an assumption of universal perfect competition, including thereby the critical stipulation that costs of negotiated and voluntary control of externalities are prohibitive. In addition, we adopt the simplifying premises that there is only one scarce resource, labor, and that the externality (smoke) only affects the cost of production of neighboring laundries, rather than causing disutility for consumers. It is easy to show (see for example, fn. 5) that neither of these simplifications, nor the assumption that there are only four activities, affects the substance of the discussion. We utilize the following notation: Let

x_1, x_2, x_3, and x_4 be the outputs of the economy's four activities, I, II, III, and IV

R be the total supply of the labor resource available

x_5 be the unused quantity of labor (which is assumed to be utilized as leisure)

---

result of restricting the methods of production which could be used by the confectioner, to secure more doctoring at the cost of a reduced supply of confectionery products. [Section II, p. 2]

[2] If the factory owner is to be made to pay a tax equal to the damage caused, it would clearly be desirable to institute a double tax system and to make residents of the district pay an amount equal to the additional cost incurred by the factory owner (or the consumers of his products) in order to avoid the damage. [Coase, Section IX, p. 41] An even stronger statement on this subject occurs in Buchanan and Stubblebine (Section III):

> . . . full Pareto equilibrium can *never* be attained via the imposition of unilaterally imposed taxes and subsidies until all marginal externalities are eliminated. If a tax subsidy method, rather than 'trade,' is to be introduced, it should involve bi-lateral taxes (subsidies). Not only must B's behavior be modified so as to insure that he will take the costs externally imposed on A into account, but A's behavior must be modified so as to insure that he will take the costs 'internally' imposed on B into account. [italics added]

However, in a recent letter Buchanan commented:

> In my own thinking . . . I did not ever think of this sort of [double] tax at all, and it would have surely seemed bizarre to me to suggest that taxes be levied on both the factory and the laundries. What we were proposing was the Wicksellian public-goods approach. Suppose that existing property rights allow the factory to put out the smoke . . . There is a public goods problem here; the residents get together, impose a tax on *themselves* to subsidize the factory to install the smoke prevention device.

x_{ij} be the quantity of x_i consumed by individual j $(i=1, \ldots, 5)$ $(j=1, \ldots, m)$

p_1, p_2, p_3, p_4, and p_5 be the prices of the four outputs and leisure

$u_j(x_{1j}, \ldots, x_{5j})$ be the utility function of individual j, and

$c_1(x_1)$, $c_2(x_1, x_2)$, $c_3(x_3)$ and $c_4(x_4)$ be the respective total labor cost functions for our four outputs

Here x_1 is an output whose production imposes external costs on the manufacture of x_2 (say, industry II is the oft-cited laundry industry whose costs are increased by I's smoke). To permit the full range of Coase's alternatives (moving of the factory's neighbors and elimination of smoke by the factory), each of these two products is taken to have a perfect substitute. The substitute for x_1 is x_3 whose production yields no externalities, but whose cost is different (presumably higher) than that of x_1. We may think of commodity III as identical with I, but produced in a factory equipped with smoke elimination equipment. Similarly, industry IV is taken to offer the same output as II but its operations have been relocated (at a cost) in order to avoid the effects of the externalities.[3] Thus, by changing the ratio between x_2 and x_4 the model can relocate as much of the laundry output as is desired.

All prices are expressed in terms of hours of labor so that, identically,

$$p_5 = 1 \tag{1}$$

Pareto optimality then requires maximization of the utility of any arbitrarily chosen individual, say m, subject to the requirement that there be no loss in utility to any of the $m-1$ other persons, i.e., given any feasible level for these other persons' utility. Thus the problem is[4] to maximize

$$u_m(x_{1m}, \ldots, x_{5m})$$

subject to

$$u_j(x_{1j}, \ldots, x_{5j}) = k_j \text{ (constant)} \qquad (j = 1, 2, \ldots, m-1)$$

$$\sum_{j=1}^{m} x_{ij} = x_i \qquad (i = 1, \ldots, 5)$$

and the labor requirement (production function) constraint

$$c_1(x_1) + c_2(x_1, x_2) + c_3(x_3) + c_4(x_4) + x_5 = R$$

We immediately obtain our Lagrangian

$$\begin{aligned} L = {} & \sum_{j=1}^{m} \lambda_j[u_j(x_{1j}, \ldots, x_{5j}) - k_j] \\ & + \sum_i \nu_i(x_i - \sum_j x_{ij}) \\ & + \mu[R - c_1(x_1) - c_2(x_1, x_2) \\ & - c_3(x_3) - c_4(x_4) - x_5] \end{aligned} \tag{2}$$

where we may take $\lambda_m = 1$, $k_m = 0$.

We use the notation u_{ji} to represent $\partial u_j/\partial x_{ij}$ and c_{ik} to represent $\partial c_i/\partial x_k$ (or dc_i/dx_k, where appropriate).

Then, differentiating in turn with respect to the x_{ij} and the x_i we obtain the first-order conditions

$$\partial L/\partial x_{ij} = \lambda_j u_{ji} - \nu_i = 0 \quad (i = 1, \ldots, 5) \quad (j = 1, \ldots, m)$$

$$\partial L/\partial x_1 = -\mu(c_{11} + c_{21}) + \nu_1 = 0$$

$$\partial L/\partial x_i = -\mu c_{ii} + \nu_i = 0 \quad (i = 2, 3, 4)$$

$$\partial L/\partial x_5 = -\mu + \nu_5 = 0$$

[3] Since product III is a perfect substitute for product I and product IV is a perfect substitute for product II, the utility function for individual j can be written as $u_j(x_{1j}+x_{3j}, x_{2j}+x_{4j}, x_{5j})$. This is, of course, a special case of the more general utility function utilized in the text, and as the reader can verify, the conclusions are totally unaffected by the use of the particular form of the utility function just described.

[4] For a more sophisticated variant of this model, using the techniques of non-linear programming, see Robert Meyer.

Now, from consumer equilibrium analysis, we know that for any two commodities, a and b, and any two prices, p_a and p_b, we have $p_a/p_b = u_{ja}/u_{jb}$ $(j=1, \cdots, m)$ or $\omega_j p_i = u_{ji}$ for all i and some ω_j.

Hence, $\lambda_j u_{ji} = \lambda_j \omega_j p_i$, so that writing $s_j = \lambda_j \omega_j$ the first of our first-order conditions becomes $\nu_i = s_j p_i$ for all individuals, j. Consequently the value of s_j must equal the same number, $s = \nu_i / p_i$ for every individual, and that first equation of the first-order conditions now becomes simply $\nu_i = s p_i$ for all i. Substituting this expression for ν_i into the other first-order conditions, we obtain

$$\begin{aligned} sp_1 &= \mu(c_{11} + c_{21}) \\ sp_i &= \mu c_{ii} \qquad (i = 2, 3, 4) \\ sp_5 &= s = \mu \quad \text{since } p_5 = 1 \text{ [by (1)]} \end{aligned} \tag{3}$$

By (3) we may then divide through the preceding conditions by $s=\mu$, and they therefore reduce just to[5]

$$\begin{aligned} p_1 &= c_{11} + c_{21} \\ p_2 &= c_{22} \\ p_3 &= c_{33} \\ p_4 &= c_{44} \\ p_5 &= 1 \end{aligned} \tag{4}$$

In other words, the optimal price for the externality-generating product is equal to the (Pareto optimal) level of its entire social[6] marginal cost, $c_{11}+c_{21}$, while the optimal price for any item, i, which generates no externalities is simply its marginal private cost, c_{ii}. To obtain these prices in our world of pure competition, one need merely levy an excise tax on item I equal to c_{21} (labor hours) dollars per unit, just as the Pigouvian tradition requires. Assuming the appropriate concavity-convexity conditions hold, this will automatically satisfy the necessary and sufficient conditions for the Pareto optimal output levels.[7] In the competitive case, where negotiation is impractical, that is all there is to the matter. The generalization to the case of n outputs, each of them imposing externalities on a number of the others, is immediate.

It is important to observe that, *the solution calls for neither taxes upon x_2, the neighboring laundry output, nor compensation to that industry* for the damage it suffers.

One way to look at the reason is that our model (and the pollution model in general) refers to the important case of *public* externalities. The laundry whose output is

[5] The analysis can also take account of constraints on the availability of land at the relevant locations, which give rise to rents that equalize costs at all locations actually utilized. If S_a and S_b represent the availability of land near and away from the factory, respectively, presumably we would add to the labor constraint in the model the two additional land-use constraints $g_a(x_1, x_2, x_3) + s_a = S_a$ and $g_b(x_4) + s_b = S_b$, with the quantities of unused land, s_a and s_b perhaps entering the utility functions. It then follows, just as before, that the equilibrium conditions are now $p_1 = c_{11} + c_{21} + p_a g_{a1}$; $p_2 = c_{22} + p_a g_{a2}$; $p_3 = c_{33} + p_a g_{a3}$; $p_4 = c_{44} + p_b g_{b4}$; $p_5 = 1$; $p_a = \rho_a/\mu$; $p_b = \rho_b/\mu$; where ρ_a and ρ_b are the Lagrange multipliers for the new constraints and p_a and p_b are the (labor) prices of land at the two locations. Our previous conclusions are, thus, totally unaffected. Only the smoke producer's product sells for more than its marginal private cost of labor plus land.

[6] The social cost is not c_{21} alone but is the sum of the private and the external costs together (see the illuminating terminological discussion by D. W. Pearce and Stanley Sturmey). Note that the tax, implicitly, is a tax on *smoke* not a tax on x_1, the output of the smoke producing industry. For if s is the quantity of smoke and t the unit tax we may write $t = c_{21} = (\partial c_2/\partial s)(ds/dx_1)$ and obviously the firm can reduce its tax rate by decreasing the second of these terms, the smokiness of its product. This point has been emphasized by Charles Plott, who showed that a fixed tax per unit of x_1 might even conceivably increase s, if s were an inferior input.

[7] Moreover, measured in real terms this is the only tax arrangement that satisfies the optimality requirements, neglecting the possibility of a lump sum tax or subsidy which does not affect the marginal conditions. F. Trenery Dolbear has shown that it is generally not possible to find an optimal tax rate that compensates fully those who suffer the effects of the externality. Since no compensation is paid to industry II, the solution that is derived here does not run into Dolbear's problem. We also do not run into the problem of a multiplicity of solutions corresponding to the various points on Dolbear's contract curve because we are dealing with a world of pure competition with a given initial distribution.

damaged by smoky air does not, by an increase in its own output, make the air cleaner or dirtier for others. As with all public goods, an increase in one user's consumption does not reduce the available supply to others.[8] Hence, the appropriate price (compensation) to a user of a public good (victim of a public externality) is *zero* except, of course, for lump sum payments. Thus, perhaps, rather than saying there is no price that will yield an optimal quantity of a public good (externality), it may be more illuminating to say that a double price is required: a nonzero price (tax) to the supplier of the good, and a zero price to the consumer. Of course, no ordinary price can do this job, but a Pigouvian tax, without compensation to those affected by an externality, can indeed do the trick.

III. What Prevents an Excessive Influx of Neighbors?

When only smoke emission is taxed, with the tax level based on the magnitude of x_2, nearby laundry output, what will prevent too many laundries from moving near the smoky factory? The answer is that, when the tax on the externality producer is set properly, the externalities themselves keep down the size of the nearby population. Moreover, the level of the tax will control both the magnitude of smoke emission and thereby (indirectly), the size of the nearby population. A high tax rate will discourage smoke and hence encourage migration into the neighborhood. A low tax rate will encourage smoke and, hence, drive residents away. A tax on smoke alone is all that is needed to control the magnitudes of *both* variables. That is why, as shown by the mathematics of the preceding section, just a tax on the smoke producer is sufficient to produce an optimal allocation of resources among all the activities in our model.[9]

A diagram may help to make the point clearer. Figure 1 shows the response of our two industries' outputs to a change in the tax rate on the polluting industry, I. We see that as the tax rate varies, industry I's output response follows the curve RR'. Thus, if the tax level is t, the output of industry I will be x_{1t}. But, because of the externalities, the output of industry II, in turn, reacts to the output of I. This relationship is described by reaction curve PP'. With $x_1 = x_{1t}$ we see that $x_2 = x_{2t}$.

The tax rate on II can vary all the way from $t=0$, yielding output combination (x_{10}, x_{20}), to a prohibitive tax rate, t_p, that drives I out of business altogether, so that $x_1 = 0$ and $x_2 = x_{2p}$. Obviously, the ratio x_1/x_2 then decreases monotonically as the tax rate increases and, assuming continuity, there will be some intermediate tax rate at which the two activities will be in balance. The tax will keep x_1 in check while the external cost imposed by x_1 on industry II will keep x_2 to the right relative level. There is no need for a separate tax on II to achieve this goal.

[8] In his discussion of these matters Coase seems at one point to skate awfully close to an error analogous to the confusion between pecuniary and technological externalities. He writes (section IX):

> The tax that would be imposed would . . . increase with an increase in the number of those in the vicinity . . . But people deciding to establish themselves in the vicinity of the factory will not take into account [the resulting] fall in the value of production which results from their presence. This failure to take into account costs imposed on others is comparable to the action of a factory-owner in not taking account the harm resulting from his emission of smoke. [p. 42]

This is analogous to the argument that where the supply curve of labor is rising an increase in output by firm A must produce externalities, by raising B's labor costs. But, of course, this merely represents a transfer from B to his workers and is not a real net cost to society. For that reason, as is well known, pecuniary externalities do *not* lead to resource misallocation. Like a price change, the variation in taxes constitutes a pecuniary externality. Both have real consequences but they are merely "movements along" the production and utility functions, i.e., any given vector of inputs will be able to produce the same outputs as before the change in tax rates, and any vector of output levels will still be able to yield the same utility levels.

[9] See the Appendix for a discussion of an argument by Buchanan and Stubblebine which is related to Coase's.

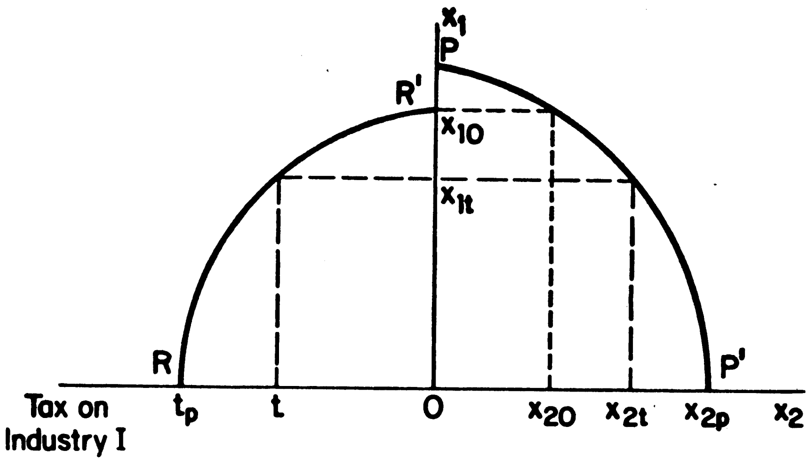

FIGURE 1

In order for this arrangement to work it is clearly necessary that the laundries *not* be compensated (at the margin) for the smoke damage they suffer. If they received in compensation an amount which varied with the magnitude of the smoke damage, that externality would not restrict the level of laundry activity near the factory. If the laundry operators' smoke costs were offset by damage compensation payments, obviously they would lose the economic incentive to eschew the vicinity of the smoky factory[10] and then Coase's tax on laundries would indeed be required to keep them away. But then the tax would be needed only to sop up the compensation payments which should never have been given in the first place.

IV. Multiple Local Maxima in the Coase Model

Coase's discussion is, however, right in pointing out the possibility that the economy may make the wrong choice between smoke elimination and laundry relocation: however the source of the problem, a multiplicity of local maxima, does not emerge clearly. Coase writes:

> Assume that a factory which emits smoke is set up in a district previously free from smoke pollution, causing damage valued at $100 per annum. Assume that the taxation solution is adopted and that the factory owner is taxed $100 per annum as long as the factory emits the smoke. Assume further that a smoke-preventing device costing $90 per annum to run is available. In these circumstances, the smoke-preventing device would be installed.
>
> . . . Yet the position achieved may not be optimal. Suppose that those who suffer the damage could avoid it by moving to other locations or by taking various precautions which would cost them, or be equivalent to a loss in income of, $40 per annum. Then there would be a gain in the value of production of $50 if the factory continued to emit its smoke and those now in the district moved elsewhere or made other adjustments to avoid the damage. [Section IX]

One curious feature of this example is its assumption that while smoke damage is $100, the cost of moving to other locations is only $40. Under these circumstances one

[10] Of course, as smoke cost increases in the neighborhood of the factory, rents will fall to some extent and serve as partial compensation to the laundries. However, this does not change the analysis fundamentally. It is analogous to the case of rise in the price of an input which, as is well known, will tend to reduce the output of competitive firms, even though prices of other complementary inputs fall as a result. As the discussion of footnote 5 shows, explicit consideration of the price of land does not change the character of the solution.

may well wonder why people living near the factory do not just move elsewhere on their own initiative. Moreover, this may not simply be a matter of the numbers he happens to have chosen. The problem arises whenever the cost of moving away from the factory is less than the cost of elimination of the smoke, which in turn is less than the cost of the smoke damage, as the logic of Coase's example requires.

It is perhaps more important to recognize that the example presents us with a choice between (at least) two local optima. As will be argued later, a multiplicity of maxima is generally rendered more likely by the presence of externalities so that this issue is not a pecularity of Coase's illustrations. The first of the two local optima in Coase's example (call it solution A) involves zero smoke emission and a full complement of residents near the factory. In the second optimum (solution B) no one remains in residence next to the factory and there is no restriction in smoke emission by the plant. Assuming that the (undesirable) initial position is the only other possibility, as Coase seems to suggest, which of these two will in fact be the global optimum depends on the cost of moving everyone away (m dollars) and the cost of elimination of the smoke (s dollars).

Assume with Coase that the initial cost of smoke damage is \$100, that $s<100$, but that $s<m$ so that it is cheaper to eliminate the smoke than to move the factory's neighbors. In this case, A is obviously the optimal solution. Since inhabitants surround the plant, and smoke emission, by assumption, cannot be changed by small amounts, the incremental social damage of an increase in smoke emission is \$100. Thus the correct Pigouvian tax is \$100 and, since $s<100$, with such a tax it will pay the factory to do the right thing by society—to install the smoke eliminator.

Now assume instead that $m<s<100$ (it is cheaper to move people than to stop the smoke). This time B is the optimal solution, and since under B no one lives near the factory, the incremental cost of smoke is clearly zero. Therefore the proper Pigouvian tax is zero, a value that induces the factory to continue smoking, and its neighbors will find it advantageous (since $100>m$) to exit (coughing) from the area. Thus the zero Pigouvian tax value automatically satisfies the requirements of solution B when B is optimal just as the \$100 Pigouvian tax leads to solution A when A is optimal.

Of course, if B happens to be the true global optimum and society mistakenly imposes the \$100 Pigouvian tax appropriate for (local) optimum A, the economy may well end up with the inferior equilibrium A. This is the usual difficulty one encounters whenever there is a multiplicity of maxima, a problem that Pigou so clearly recognized (pp. 140, 224).

V. Departures from the Optimum and Adjustments in the Tax

If there is a departure from the optimal solution, for whatever reason, the value of the Pigouvian tax need not change. If, for example, B is the global optimum so that the optimal tax is zero, that tax need not be increased if a few (misguided) individuals choose to move back near the factory so that additional smoke now incurs (say) \$50 in damage. *At the optimal solution* the marginal cost of smoke is zero, and the equilibrium Pigouvian tax remains zero—it does not increase to \$50.

Here we have arrived at the issue which, I now understand, was really Coase's main point in the portion of his article we are considering. He writes in a letter:

> . . . Let us assume your optimum tax is imposed. Now suppose that A establishes himself near the plant which produces the damaging emissions and thus

> increases the amount of damage. Would your tax increase? My guess is that it would not (certainly if your tax system is right it should not). The tax system I was attacking was one which would in these circumstances, automatically lead to an increase in the tax as the damage increased.

This point is, surely, quite different from the issue he is usually interpreted to have raised (see the quotations in fn. 1, above, which suggest how the "usual interpretation" arose). It is, however, not inconsistent with the optimal solution derived in the previous section nor is it inconsistent with what I take to be the Pigouvian tradition.

But even on this issue Coase's strictures are not necessarily valid. Suppose that a regulator, having no way of calculating the *optimal* values of the Pigouvian tax is, however, able to determine the value of any marginal social damage at any point in time. *Faut de mieux* he therefore sets a tax rate equal to *current* marginal social damage on the smoke producer. This causes him to reduce his smoke, and so brings more laundries into the neighborhood. The tax is then readjusted to equal the new (higher) value of damage per puff of smoke, more laundries move in, and so on. Will this process of trial and error adjustments of the tax level, always setting it equal to current marginal smoke damage, converge to the optimum of Section II? That is, will the sequence of tax values converge to the optimal Pigouvian tax level, and will resource allocation approach optimality? That now seems to be Coase's main question.

Obviously, such a learning process always involves wastes and irreversabilities, just like the process of convergence of competitive prices to their equilibrium values in the absence of externalities. But if we follow the usual practice of assuming away these costs, one can show that the process may be expected to converge to the optimum, provided the equilibrium is unique and stable. That is, there is then nothing inherently different about gradually moving taxes and prices towards their equilibrium here, and the process of adjustment toward competitive equilibrium when there are no externalities.

Specifically, letting s_t represent the tax per unit on commodity 1 at time t, and G_i be the ith adjustment function we may set

$$\begin{aligned} dx_{1t}/dt &= G_1[p_{1t} - s_t - c_{11}(x_{1t})] \\ (5)\quad dx_{2t}/dt &= G_2[p_{2t} - c_{22}(x_{1t}, x_{2t})] \\ dx_{it}/dt &= G_i[p_{it} - c_{ii}(x_{it})] \quad (i = 3, 4) \end{aligned}$$

$$(6)\, s_t = c_{21}(x_{1t}, x_{2t}) \quad p_{it} = f_i(x_{1t}, \ldots, x_{5t})$$

and where, as usual, we take

$$(7)\qquad G_i(0) = 0$$

$$(8)\qquad G_i' > 0$$

Going back to Section II, when optimality conditions (4) hold, we see by substituting them into (5) that all $dx_{it}/dt = 0$, i.e., (4) is indeed an equilibrium position for the dynamic system (5)–(8). Furthermore, any solution that does not satisfy (4) must involve at least one nonzero argument in the adjustment functions (5), and so no solution that fails to satisfy (4) can be an equilibrium.

It follows that if the dynamic system (5)–(8) is stable, and the solution to (4) is unique, the process with taxes set equal to *current* marginal damage and imposed *only on the polluter* will converge toward the optimum. One does not need to have calculated the optimal tax values from the beginning and stick to them.

The reason this process of simultaneous learning and adjustment does not work in Coase's example is that it involves (at least) two local maxima, as we have already noted. And in such a case, obviously, the adjustment mechanism may well take us to the wrong maximum. Unfortunately, as we will see presently, in the presence of externalities, a multiplicity of maxima is all too likely to be with us.

VI. Implementation Problems

Despite the validity in principle of the tax-subsidy approach of the Pigouvian tradition, in practice it suffers from serious difficulties. For we do not know how to estimate the magnitudes of the social costs, the data needed to implement the Pigouvian tax-subsidy proposals. For example, a very substantial portion of the cost of pollution is psychic; and even if we knew how to evaluate the psychic cost to some one individual we seem to have little hope of dealing with effects so widely diffused through the population.[11]

This would not necessarily be very serious if one could hope to learn by experience. One might try any plausible set of taxes and subsidies and then attempt, by a set of trial and error steps, to approach the desired magnitudes. Unfortunately, convergence toward the desired solution by an iterative procedure of this sort requires some sort of measure of the improvement (if any) that has been achieved at each step so that the next trial step can be adjusted accordingly. But we do not know the socially optimal composition of outputs, so we simply have no way of judging whether a given change in the trial tax values will even have moved matters in the right direction.

These difficulties are compounded by another characteristic of externalities which has already been mentioned—the likelihood that in the presence of externalities there will be a multiplicity of local maxima (see Richard Portes, D. A. Starrett, and Baumol). Consequently, even if an iterative process were possible it might only drive us toward a local maximum, and may thus fail to take advantage of the really significant opportunities to improve economic welfare.

A simple model in the spirit of that of Section II can be used to show that the presence of "strong" externalities can be expected to produce a violation of the convexity conditions in whose absence one normally finds a multiplicity of local optima.

Let us assume (to permit the use of a two-dimensional diagram) that there exist only the first two of our four activities (the smoky output, x_1, and nearby laundry, x_2), and that their respective cost functions are, as before, $c_1(x_1)$ and $c_2(x_1, x_2)$. As a result, the equation of the production possibility locus is

$$c_1(x_1) + c_2(x_1, x_2) = R$$

For convenience let us use k as a parameter measuring the strength of the (marginal) externality.[12] Assume first that there are diminishing returns (increasing costs) in the production of the two outputs, and that there are no marginal external effects so that $k=0$. (At the margin industry I's output produces no smoke or smoke is harmless to industry II.) In that case it is easy to show that the production possibility locus must satisfy $dx_2^2/dx_1^2 < 0$, i.e., that the locus must assume the general shape AC_0B in Figure 2 with the concavity property required by the second-order conditions.

Now, suppose that the activity of in-

[11] For an excellent discussion of some of the work done in trying to implement Pigouvian taxes in practice, see Allen Kneese and Blair Bower, esp. ch. 6 and 8. The difficulty of determining the magnitude of the Pigouvian tax-subsidy level is one of Coase's major points, one that seems often to be overlooked in discussions of his paper. Thus Coase writes in a letter, "The view I expressed in my article was not that such an optimum tax system (levied solely on the damage producing firm) was inconceivable but that I could not see how the data on which it would have to be based could be assembled." An interesting approach to application for the small numbers case that is based on the decomposition principle of mathematical programming is presented by Davis and Whinston (1966).

[12] E.g., k may be interpreted as $\partial^2 c_2/\partial x_1 \partial x_2$, i.e., the additional marginal resources cost of output 2 resulting from a unit increase in output 1.

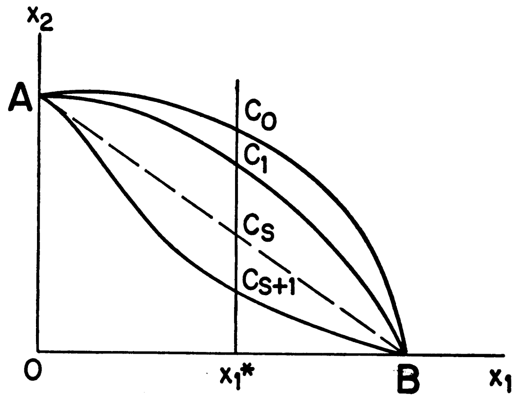

FIGURE 2

dustry I does produce some external damage ($k>0$). What happens to the production possibility locus? First I will argue that neither of its end points, A or B, will normally be affected. At point B, laundry output, x_2, is zero. Hence, no matter how much smoke is produced, there is no laundry output to be damaged. Point B is therefore invariant with the magnitude of k. Similarly, at A, the smoke creating output is zero. Consequently, no matter how smoky the process of producing output II may be (no matter how large the value of k) the total smoke emitted will be (output x_1) $\cdot$ (smoke per unit of output) $=0$, since the first of these factors is zero. Thus the position of point A remains invariant with the magnitude of k.

The effect on intermediate points such as C_0 on the locus is quite different. As k increases it takes increasing quantities of resources to produce a given volume of laundry. Thus, with any fixed value of x_1, say x_1^*, as k increases, the quantity of laundry that can be turned out with a given quantity of resources, R, must decline. Point C_0 will be pushed down to some lower point, C_1. With a still greater value of k it will be lowered still further. As smoke damage increases without limit it will take larger and larger quantities of resources to turn out a given quantity of laundry and eventually we approach a limit point γ on the horizontal axis, at which it is no longer possible to produce clean clothes with any finite quantity of resources.

Now draw in straight line segment AB whose position does not vary with k since neither A nor B does. It is clear that as k increases we will eventually come to some point C_s beyond which all remaining points in the sequence C_{s+1}, C_{s+2}, . . . lie below AB. Beyond this point, obviously, the second-order conditions must be violated, as the production possibility curve approaches the axes, AOB.

Thus we see that the presence of sufficiently strong detrimental externalities will generally produce a violation of the second-order conditions. Only in the presence of insignificant externalities can one have any degree of confidence that the convexity conditions will hold.[13]

It is easy to offer an intuitive reason indicating how the presence of externalities increases the likelihood of a multiplicity of maxima, a reason that suggests that the problem is very real and potentially very serious in practice. Where a particular activity reduces the efficiency of another it becomes plausible that the optimal level of that activity, at least at some particular locations, is zero. If there are one hundred possible locations for the plants of a smoke-producing industry the worst possible solution might be to place some plants in each candidate location. *Any* solution leaving at least some combination of smoke-free areas may be preferable, and may well constitute a local maximum.

To make the point more concretely, suppose we are dealing with an island separated by a ridge of mountains that pre-

[13] The analysis can be extended to the case of n activities and externalities that enter utility as well as production functions. The analysis here confines itself to externalities producing inefficiencies on the production side following a suggestion of Jacob Marschak that the argument is more persuasive if framed in these terms.

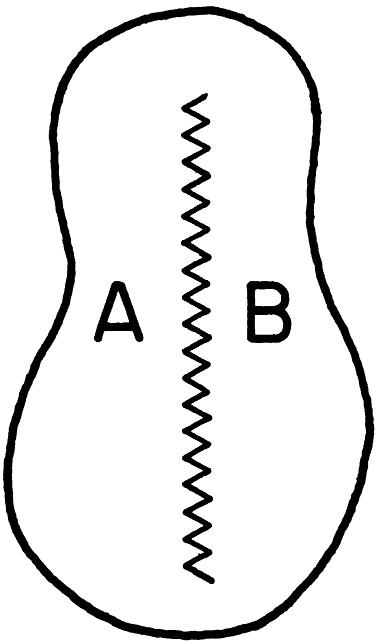

FIGURE 3

vent smoke from going from one side to the other (Figure 3). Let S_a and S_b be the volume of smoke-producing activity located on the two respective sides of the island, and let P_a and P_b be the corresponding number of residents living there. Let $S_a+S_b=S$ and $P_a+P_b=P$. Then, if the social cost of the smoke is great enough, there will obviously be at least two local optima: $(P_a=P,\ P_b=0,\ S_a=0,\ S_b=S)$ and $(P_a=0,\ P_b=P,\ S_a=S,\ S_b=0)$. For either of these arrangements keeps the smoke and the people apart. This does not mean, of course, that the two solutions are equally desirable. If A offers great scenic attractions while B is closer to raw materials we may expect the former of the two local maxima to be preferable. We cannot preclude the possibility of a third (interior) maximum, for once there is some industrial activity on each of the two sides of the island there may be some least cost distribution of people and industrial activity. But we see that we may well expect to encounter *at least* two local maxima. With more separated locations and more sources of externalities the number of combinations of zero-valued variables that constitute local maxima may well grow astronomically.

The presence of a number of local maxima clearly means that an "improvement" may merely represent a move toward some minor peak in the social welfare function and it can, therefore, impose serious opportunity losses on society. All in all, we are left with little reason for confidence in the applicability of the Pigouvian approach, literally interpreted. We do not know how to calculate the required taxes and subsidies and we do not know how to approximate them by trial and error.

VII. An Alternative Approach—Adjustment of Taxes to Achieve Acceptable Externality Levels

There is an alternative approach to the matter that seems perfectly natural. On issues as important as those we are discussing, given the limited information at our disposal, it is perfectly reasonable to act on the basis of a set of minimum standards of acceptability. If, say, we treat the sulphur content of the atmosphere as one of the outputs of the economic system, it is not unreasonable to select some maximal level of this pollutant that is considered satisfactory and to seek to determine a tax on the offending inputs or outputs capable of achieving the chosen standard. This is precisely the approach employed in the formulation of stabilization policy, where it is decided that an employment rate exceeding w percent and a rate of inflation exceeding ν percent per year are simply unacceptable, and fiscal and monetary measures are then designed accordingly.[14]

[14] As this discussion indicates, I join Wellicz in refusing to abandon externalities policy entirely to Little's "administrative decisions" (p. 184) or to Ralph Turvey's "applied economist" (p. 313). For further discus-

The advantages (as well as the limitations) of this approach are clear—unlike the Pigouvian procedure, it promises to be operational because it requires far less information for its implementation. Moreover, it utilizes global measures and avoids direct controls with all of their heavy administrative costs and their distortions of consumer choice and inefficiencies. It does not use the police and the courts as the prime instrument to achieve the desired modification of the outputs of the economy. Its effects are long lasting, not depending on the vigor of an enforcement agency, which all too often proves to be highly transitory. Unlike most other measures that have been proposed in the area it need not add to the mounting financial burdens of the state and local governments. Finally, it can be shown that, unlike any system of direct controls, it promises, at least in principle, to achieve decreases in pollution or other types of damage to the environment at minimum cost to society.[15]

sion see Baumol and Wallace Oates. For an earlier proposal that is very similar in spirit, see John H. Dales, ch. 6.

[15] This proposition has been suggested elsewhere (see, for example, Kneese and Bower, chs. 5 and 7; Larry Ruff, p. 79), and will be fairly obvious to anyone familiar with the analysis of the allocative effects of price changes and their efficiency properties. Specifically, suppose it is desired to reduce the pollution content of a river by k percent. Obviously a k percent reduction in the number of gallons emitted by each of the plants discharging wastes into the river will generally not be the desired solution. The theorem in question then asserts the following:

Given the production of any desired vector of final outputs by the plants along the river, a tax per gallon of effluent sufficient to reduce the overall pollution content of the river to the desired level will automatically achieve this decrease at minimum total cost to all plants combined.

The proof of the theorem is a straightforward exercise in constrained maximization (see Baumol and Oates). It works, of course, because the lower the marginal cost of reduction in pollution outflows of a particular plant, the larger the reductions it will pay it to undertake to avoid the corresponding tax payment.

What is surprising about the proposition, if anything, is that, unlike many results in welfare analysis, it does not require the firms along the river, or any other firms, to be perfect competitors, nor does it have to assume that they maximize profits rather than share of market or growth or some other target variable. All it requires is that the firms wish to produce whatever output they select at minimum cost to themselves.

One can expect an acceptability criterion procedure to be operational because policy makers think quite naturally in terms of minimum acceptability standards, and while it is no doubt an exaggeration to say that they can arrive at them easily, there are all sorts of precedents indicating that such standards can be decided upon in practice.

Though we are unlikely to be able to determine in advance precisely a set of tax values that will achieve the desired output standards, the output level achieved by a given tax arrangement is readily observed and, at least in principle, it is possible to learn by trial and error, continuing the direction of change of any tax modifications that turn out to bring outputs closer to their target levels. Since the procedure is a satisficing rather than a maximizing approach the possibility of a multiplicity of maxima is not relevant.

That is to say, one generally expects a considerable number of solutions to satisfy a particular set of acceptability conditions (various resource allocation patterns may be able to achieve a given set of reductions in pollution levels) *whether or not the second-order conditions are satisfied.* If several of these do so, then the essence of the satisficing approach is that one simply utilizes the first of the acceptable solutions that is discovered. One gives up any attempt to achieve any standard of optimality (other than minimization of cost[16] for a given degree of protection of the environment) and rests content with *any* solution that happens to satisfy the standards that have been selected.

[16] Of course it is conceivable that there may be more than one local *cost* minimum. In that case an effluent charge that yields an acceptable pollution level may not yield the global cost minimum. This may be something that practical policy simply has no way of avoiding.

Thus, the acceptability criterion approach does not dispose of the difficulties involved in finding a true optimum—rather it sweeps those difficulties under the rug. Even with pollution reduced to acceptable levels, there will remain the possibility that the (undiscovered) global optimum offers us a world far better than what we have managed to achieve—if only we knew how to attain it. But if we permit ourselves to be paralyzed by councils of perfection we may have still greater cause for regret.

It may be that with time we can learn to improve the workings of a set of standards of acceptability. If, say, it turns out to be unexpectedly cheap to attain the initial pollution standards, it may be reasonable to tighten the standards on the presumption that marginal costs will not yet have equalled the marginal social benefits. Successive modifications in the criteria based on experience and revaluation may produce results that on the whole are not too bad.

If firms are put on notice that the acceptability standards may well be modified in the future this may lead them to construct what George Stigler describes as more flexible plants,—plants which are designed to keep down the cost of response to changes in standards. Of course, flexibility itself is not costless. However, it may be precisely what is appropriate for a society which is only beginning to learn how to grapple with its environmental problems.

Appendix

Buchanan, Stubblebine and Taxation of Both Parties to an Externality

Buchanan and Stubblebine have raised objections to the Pigouvian solution similar to those offered by Coase (see fn. 2, above). Much of their discussion deals with the case where voluntary negotiation in the presence of externalities will lead automatically to a Pareto optimum. As already admitted, in this case a Pigouvian tax will only cause trouble. However, the authors also appear to offer an argument against the Pigouvian tax for the case in which negotiation is absent.

Their argument, if I understand it correctly, is that after industry I adjusts to a Pigouvian tax on its output, for that industry the marginal yield of an increase in x_1 is zero. However, for industry II, at the point γ the marginal yield of x_1 is $c_{21}<0$. There must, consequently, be potential gains from trade between the two industries. They state:

> So long as $[(\partial c_2/\partial x_1)/(\partial c_2/\partial x_2)]$ remains nonzero, a Pareto-relevant marginal externality remains, despite the fact that the full 'Pigouvian solution' is attained. The apparent paradox here is not difficult to explain. Since, as postulated, [II] is not incurring any cost in securing the change in [I's] behavior, and since there remains, by hypothesis, a marginal diseconomy, further 'trade' can be worked out between the two parties. . . . The important implication to be drawn is that full Pareto equilibrium can never be attained via the imposition of unilaterally imposed taxes and subsidies . . .
> [Section III, pp. 382–83]

No doubt this is true—in a competitive situation two interrelated industries can generally increase their joint profits ("gain from trade") by collusion at the expense of the general public. In the case under discussion, if the output of x_1 is reduced it is true that industry I will lose nothing and industry II will gain c_{21}. However, society as a whole will experience no net gain.

Since the analysis deals exclusively with resource *allocation* we must assume that the labor released by the reduced value of x_1 will be employed elsewhere to produce more of some other output or more leisure. Consequently, the goods or services represented by the t units in taxes must be redistributed to the general public either by remission of another tax, increased provision of government services or some other means.

We may now evaluate the consequences of a unit increase in the output of x_1 on the entire society by summing up the direct effects on each of the three groups immediately

	Industry I	Industry II	Consumers	General Public
Incremental gain or revenue	p		$u_1 = c_{11} + t$	$t = c_{21}$
Incremental cost	$(c_{11} + t) = (c_{11} + c_{21})$	c_{21}	p	

concerned: industry I, industry II, consumers, and the consequences of the tax receipts for the general public (which encompasses all consumers and producers, including those already mentioned). These are shown in the table above Adding up the incremental gains and revenues we see that the net social gain is zero, precisely as optimality requires. There is only a redistribution from industry II to the general public.

In a recent letter Buchanan comments:

> As for the nonoptimality of a unilaterally imposed tax, the problem here is that income effects enter to make the benefit-receiving side change behavior so that still further adjustments would be necessary . . . Our point was that this new position would not be one of full equilibrium if income effects enter. The laundries would now find that they secure the benefits of cleaner air without cost to themselves. Presumably this would make them do more laundry. This change in behavior would in turn change the apparent optimal solution. Admittedly, the imposed solution qualifies as Pareto-optimal if further trading is prohibited. And here Pareto-equilibrium does take on a different meaning from Pareto-optimal. Gains-from-trade exist, as you agree and, once these take place, we are not in an optimal solution.

In this paper I deal with the case where trading fails to take place not because it is prohibited, but because (as seems characteristic of our most important externalities problems in reality) large numbers make trading virtually impossible to arrange (where have we seen automobile drivers pay one another to cut down their exhaust?). Moreover, one must distinguish between the role of Buchanan's income effect and that of "further trading." Of course, further trading can destroy the optimality of the results achieved by a Pigouvian tax. For, as just argued, in that case the two affected groups gain by exploiting the community. On the other hand, the "income effect"—the influx of laundries near the factory as clean air becomes cheaper is precisely the reason a tax on the smoke producer alone can lead *everyone* to behave Pareto optimally (see Section III).

REFERENCES

W. J. Baumol, "External Economies and Second-order Optimality Conditions," *Amer. Econ. Rev.*, June 1964, *54*, 358–72.

——— **and W. E. Oates**, "The Use of Standards and Pricing for Protection of the Environment," *Swedish J. Econ.*, Mar. 1971, *73*, 42–54.

J. M. Buchanan, "External Diseconomies, Corrective Taxes and Market Structure," *Amer. Econ. Rev.*, Mar. 1969, *59*, 174–7.

——— **and W. C. Stubblebine**, "Externality," *Economica*, Nov. 1962, *29*, 371–84.

R. H. Coase, "The Problem of Social Cost," *J. Law Econ.*, Oct. 1960, *3*, 1–44.

J. H. Dales, *Pollution, Property, and Prices*, Toronto 1968.

O. A. Davis and A. Whinston, "Externalities, Welfare and the Theory of Games," *J. Polit. Econ.*, June 1962, *70*, 241–62.

——— **and** ———, "On Externalities, Information, and the Government-Assisted Invisible Hand," *Economica*, Aug. 1966, *33*, 303–18.

F. T. Dolbear, Jr. "On the Theory of Optimum Externality," *Amer. Econ. Rev.*, Mar. 1967, *57*, 90–103.

A. V. Kneese and B. T. Bower, *Managing Water Quality: Economics, Technology, Institutions*, Baltimore 1968.

I. M. D. Little, *A Critique of Welfare Economics*, 2d ed., New York 1957.

H. Mohring and J. H. Boyd, "Analyzing 'Externalities': 'Direct Interaction' vs. 'Asset

Utilization' Frameworks," *Economica,* forthcoming.

R. A. **Meyer, Jr.**, "Externalities as Commodities," *Amer. Econ. Rev.,* Sept. 1971, *61,* 736–40.

D. W. Pearce and G. S. Sturmey, "Private and Social Costs and Benefits: A New Terminology," *Econ. J.,* Mar. 1966, *76,* 152–57.

A. C. Pigou, *The Economics of Welfare,* 4th ed., London, 1932.

C. R. Plott, "Externalities and Corrective Taxes," *Economica,* Feb. 1966, *33,* 84–7.

R. D. Portes, "The Search for Efficiency in the Presence of Externalities," forthcoming.

L. E. Ruff, "The Economic Common Sense of Pollution," *Publ. Interest,* spring 1970, *19,* 69–85.

D. A. Starrett, "Fundamental Non-Convexities in the Theory of Externalities," Harvard 1971, unpublished.

G. J. Stigler, "Production and Distribution in the Short Run," *J. Polit. Econ.,* June 1939, *47,* 305–27.

R. Turvey, "On Divergences Between Social Cost and Private Cost," *Economica,* Aug. 1963, *30,* 309–13.

S. Wellicz, "On External Economies and the Government-Assisted Invisible Hand," *Economica,* Nov. 1964, *31,* 345–62.