New Insights into Harappan Town-Planning, Proportions and Units, with Special Reference to Dholavira*

Michel Danino
80 Swarnambika Layout, Ramnagar
Coimbatore 641 009 Tamil Nadu
Email: michel_danino@yahoo.com

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Abstract

Dholavira's elaborate town-planning rests on the conscious use of specific proportions for its successive enclosures. Those proportions combined with the city's dimensions allow us to calculate precisely the unit of length used for the fortifications, to relate it to the Lothal ivory scale, and to work out potential subunits. Both proportions and units receive overwhelming confirmation from structures of Dholavira and other Harappan sites. Units are finally refined to a *dhanus* of 190.1 cm and an *angula* of 1.76 cm, the former being 108 times the latter. The Dholavirian scheme of units is then shown to be related to historical unit systems in several ways; in particular, the *Arthashastra*'s scheme of linear measures conclusively has Harappan roots. Finally, the paper attempts to outline some of the abstract concepts underlying Dholavira's geometry, taking a peep at a hitherto neglected component of the Harappan mind.

Dholavira's Ratios

Unlike most Harappan cities, Dholavira in the Rann of Kachchh (23°53′10″ N, 70°13′ E), excavated by R. S. Bisht in the 1990s, presents us with a largely undisturbed plan and clearly delineated multiple enclosures covering about 48 hectares. This fascinating site displays two marked specificities. While Harappan town-planning is usually based on a duality acropolis / lower town, Dholavira's plan (**Fig. 1**) is triple: an acropolis or upper town consisting of a massive "castle" and an adjacent "bailey," a middle town (including a huge ceremonial ground), and a lower town, a large part of which was occupied by a series of reservoirs. (Throughout this paper, terms such as "castle," "bailey," "granary," "college," etc., have been used with quotation marks to remind the reader that they are conjectural in nature; similarly, the neutral term "acropolis" has been preferred to the misleading "citadel," as it is now largely accepted that the upper town in Harappan settlements had no inherent military purpose; in the same spirit, "fortifications" has been preferred to "defences.")

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* This paper is dedicated to the memory of the late Dr. S. P. Gupta, whom I was lucky to know and to learn from over a decade.

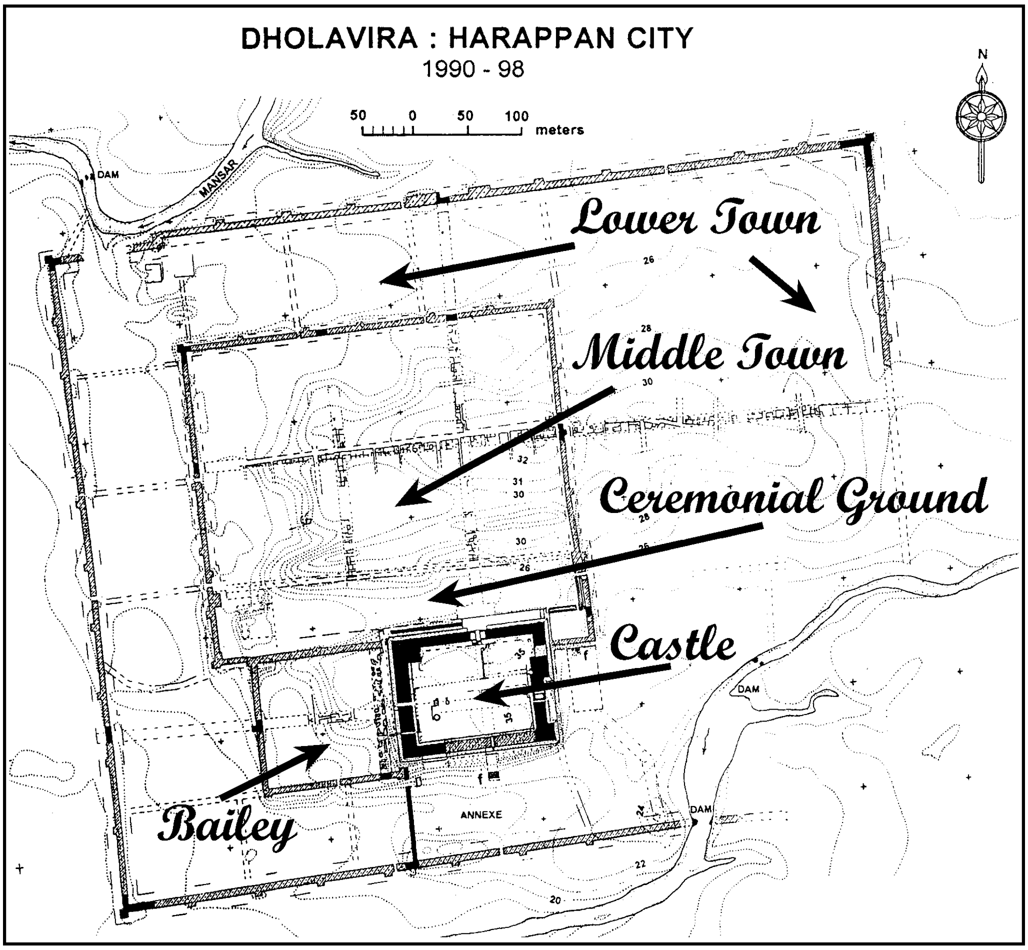

Fig. 1. Plan of Dholavira (adapted from Bisht 1999)

Table 1 summarizes the city's dimensions, as supplied by the excavator (Bisht 1997, 1999, 2000), with a maximum margin of error of 0.5% (Bisht 2000: 18). Because these figures are our primary data on which all further calculations of margins of error will depend, they have not been rounded off and are quoted as published.

Dimension	Measurement (in metres)	
	Length	Width
Lower town (entire city)	771.1	616.85
Middle town	340.5	290.45
Ceremonial ground	283	47.5
"Castle" (inner)	114	92
"Castle" (outer)	151	118
"Bailey"	120	120

Table 1: Dholavira's dimensions

Dholavira's second specificity is the conspicuous use of precise ratios or proportions in the various enclosures. Bisht highlighted some of them as follows (added in parentheses are margins of error calculated on the basis of Table 1 and rounded off to the first decimal):

1. The "**castle**" also reflects the city's ratio of **5 : 4** (0.9% inner, 2.4% outer);
2. the "**bailey**" is square (ratio **1 : 1**);
3. the **middle town**'s length and breadth are in a ratio of **7 : 6** (0.5%);
4. the **ceremonial ground**'s proportions are **6 : 1** (0.7%);
5. the **city**'s length (east-west axis) and width (north-south) are in a ratio of **5 : 4** (0.0%, a perfect match).

All but one ratios are verified within 1%, an excellent agreement considering the irregularities of the terrain and possible erosion and tectonic movements in the course of millennia. The exception, the outer dimensions of the "castle," can be explained by the fact that this monumental stone structure, which formed the earliest part of the city, was altered when the middle town was added shortly before the mature phase, so as to bring its dimensions in line with the desired proportions 5 : 4, while other enclosures were directly built to plan.

In two earlier papers (Danino, 2005 & in press), a few other important ratios at work in Dholavira were worked out, some as "axioms" (i.e., basic proportions that the town-planners would have chosen in order to define the whole city geometrically), others as consequences of those basic choices, so that all the ratios are interrelated in a mathematically consistent system:

1. The castle's outer and inner lengths are in the ratio of **4 : 3** (verified within 0.7%).
2. The width of the castle's eastern and western fortification walls (half of the difference between the outer and inner lengths) is **1/8**th of the outer length and **1/6**th of the inner length (the last fraction identical to the proportions of the ceremonial ground). Margins of error are 2% and 2.7%, a little high but acceptable in view of the irregularities of the castle's fortifications: their calculated dimensions are only averages.
3. The middle town's length and the castle's internal length are in the ratio of **3 : 1** (0.4%).
4. The middle town's length and the castle's outer length are precisely in the ratio of **9 : 4** (0.2%).
5. That the above ratio is a conscious choice is made clear by its repetition: the city's length and the middle town's length are in the same ratio of **9 : 4** (0.6%). In other words, the length of the city is to that of the middle town what the

length of the middle town is to that of the "castle" — clearly no accident. We will return to this fundamental principle of recursion.

6. The middle town's length and the ceremonial ground's length are in the ratio **6 : 5** (precisely verified within 0.3%).
7. A few more ratios play a role in the complete picture, notably **3 : 2**.

Here again, margins of error are so small that such ratios cannot by any means be accidental. The principal ones are summarized in **Fig. 2** and **Table 2**.

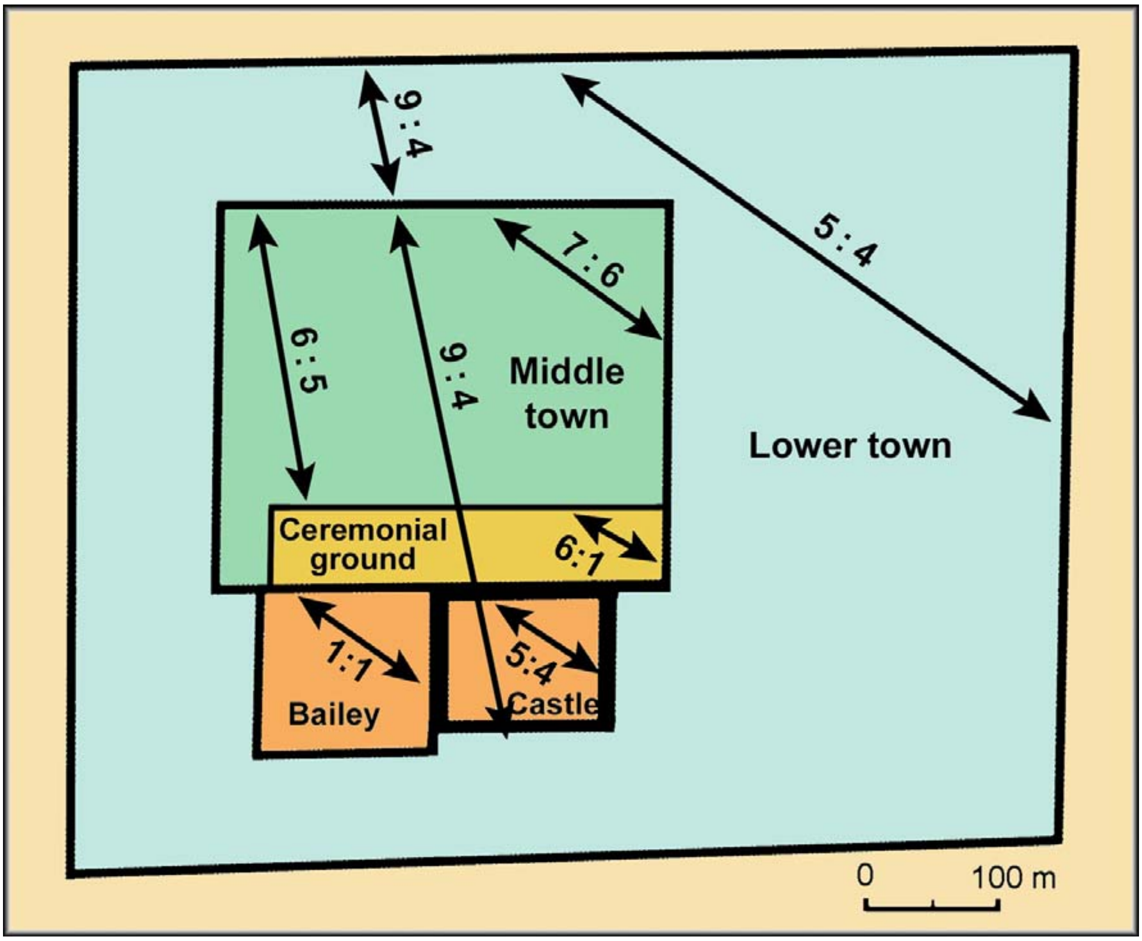

Fig. 2. Main ratios at work in Dholavira

Dimensions	Ratio	Margin of error (%)
"Castle," inner*	**5 : 4**	0.9
"Castle," outer*	**5 : 4**	2.4
"Bailey"*	**1 : 1**	0.0
Middle town*	**7 : 6**	0.5
Ceremonial ground*	**6 : 1**	0.7
Lower town (entire city)*	**5 : 4**	0.0
Castle's outer to inner lengths**	**4 : 3**	0.7
Middle town's length to castle's internal length**	**3 : 1**	0.4
Middle town's length to castle's outer length**	**9 : 4**	0.2
City's length to middle town's length**	**9 : 4**	0.6
Middle town's length to ceremonial ground's length**	**6 : 5**	0.3
* = proposed by R. S. Bisht ** = proposed by Michel Danino		

Table 2. Dholavira's ratios and margins of errors

Dholavira's Master Unit of Length

With such a set of precise ratios and dimensions, we can work out the unit of length that was used to measure out the city's enclosures. Let us call it "D" for Dholavira.

In a previous paper (Danino, in press), a simple procedure was used to calculate, with no *a priori* assumption, the largest possible value of D that will result in most of the city's dimensions being expressed as integral (or whole) multiples of D. The procedure, briefly put, consists in algebraically expressing the smallest dimension in our scheme (i.e., the average width of the castle's western and eastern fortifications) as a multiple of the unknown unit D (or "n times D," n being an integer); then, using the precise ratios noted above, to work out the larger dimensions and express all of them in terms of "n times D." For instance, the castle's outer length, which, as we saw, is 8 times the width of its fortifications, becomes 8 nD. But while most dimensions now become integral multiples of "n times D," a few are fractional expressions. To make those fractions disappear, all we have to do is choose "n" as the least common multiple of their denominators. It turns out that with n = 10, all fractional results disappear, except one.

Going back to our initial formula, the width of the castle's western and eastern fortifications, which we expressed as "n times D," is now 10 D. Bringing into play the proportions listed above, the castle's inner dimensions become 60 D x 48 D, outer dimensions 80 D x 64 D, the bailey 63 D x 63 D, the middle town's length 180 D, the ceremonial ground 150 D x 25 D, and the lower town 405 D x 324 D. (The only non-integral multiple of D in this scheme is the middle town's width, equal to 6/7th of the length and therefore about 154.3 D.) A few additional dimensions were worked out in terms of D, which need not be detailed here. **Fig. 3** summarizes Dholavira's dimensions in terms of D.

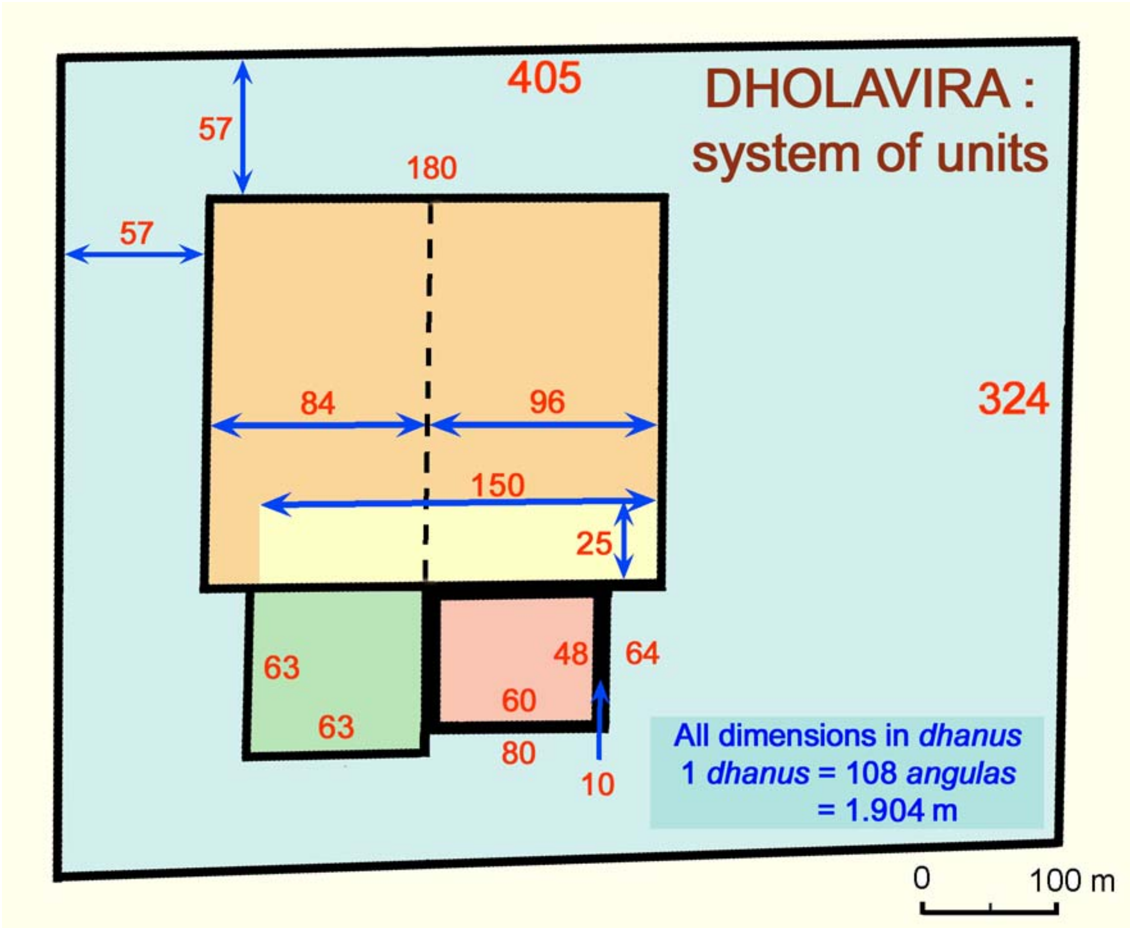

Fig. 3: Dholavira's main dimensions expressed in terms of *dhanus*, Dholavira's master unit of length

It only remains to determine the value of D, which is done with the greatest precision by deriving it from the city's largest dimension, the length of the lower town: if 771.1 m = 405 D, then **D = 1.904 m** or 190.4 cm.

To remove any doubt regarding the soundness of our scheme of units and ratios, all we have to do is to start afresh from D = 1.904 m, calculate the theoretical dimensions using **Fig. 3**, and compare with the actual dimensions supplied by Bisht. **Table 3** summarizes the results, as well as the margin of error between theoretical and actual dimensions:

Dimension	Length (in terms of unit D)	Theoretical measurement (in metres)	Actual measurement (in metres)	Margin of error (%)
Lower town's length	**405**	771.1	771.1	0.0
Lower town's width	**324**	616.9	616.85	0.0
Middle town's length	**180**	342.7	340.5	+0.6
Middle town's width	**154.3**	293.8	290.45	+1.1
Ceremonial ground's length	**150**	285.6	283	+0.9
Ceremonial ground's width	**25**	47.6	47.5	+0.2
Inner castle's length	**60**	114.2	114	+0.2
Inner castle's width	**48**	91.4	92	– 0.7
Outer castle's length	**80**	152.3	151	+0.9
Outer castle's width	**64**	121.9	118	+3.2
Bailey's length & width	**63**	120.0	120	0.0

Table 3: Comparison between theoretical and actual dimensions

Margins of error are remarkably modest, 0.6% on average, the highest being, again, in the outer dimensions of the "castle"; if we leave them aside, the average margin of error drops below 0.4%. These almost perfect matches evidenced by large structures rule out the play of chance.

It is important to note that from a mathematical perspective, what we have done so far is merely *to calculate the largest possible unit in terms of which all dimensions will be expressed as integers* (except one, the middle town's width, for reasons explained in Danino, in press). Our sole assumption is that Dholavira's architects would have planned the city's dimensions as integral multiples of their unit (48, 60 ...) rather than as non-integral ones (e.g. 48.4, 60.3 ...), a natural expectation for aesthetical as well as pragmatic reasons. It will be amply confirmed in the next sections when we find integral multiples of D at work in other Dholavirian structures and further afield in other Harappan cities.

Ratios in Harappan Settlements

Let us first examine other Harappan cities and structures in the light of our Dholavirian scheme of ratios. (In the rest of this paper, whenever published dimensions are stated or implied to be approximate, no margin of error has been added as mathematically such a margin would not be significant.)

Dholavira's nested enclosures appear to be in a class of their own. Wherever fortifications have been traced in other sites, an overall ratio of **1 : 2** is the most common: **Kalibangan**'s acropolis is 120 x 240 m (Lal 1997: 122), while **Surkotada**'s overall dimensions are 130 x 65 m (Lal 1997: 135). We find the same ratio in **Mohenjo-daro**'s acropolis, which rests on a huge brick platform measuring 400 x 200 m (Jansen 1988: 134), although whether the acropolis was ever fortified remains unclear. The division of Kalibangan's acropolis and of Surkotada into two equal halves does broadly recall the complex formed by Dholavira's "bailey" and "castle," but that is as far as we can get.

However, when we study a variety of structures from other Mature Harappan sites, we find many of Dholavira's chief ratios unmistakably in use:

- *Ratio 3 : 1* is found at Mohenjo-daro's "college" whose average dimensions are 70.3 x 23.9 m (Mackay 1938: 10).
- *Ratio 3 : 2* is the overall ratio of Kalibangan's lower town (approximate dimensions 360 x 240 m) (Lal 1998: 119), as well as of a sacrificial pit (1.50 x 1 m) (Lal 1998: 96). It is also the ratio (within 1.7 %) of a reservoir in Dholavira's "castle" measuring 4.35 x 2.95 m (Lal 1998: 43). We find it again (within 1%) at Mohenjo-daro in the massive platform of the "granary" (also called "warehouse"), which measures 50 x 33 m (Jansen 1979: 420).
- *Ratio 4 : 3* is visible in Mohenjo-daro's "granary": this structure is composed of 27 brick platforms (in 3 rows of 9); while all platforms are 4.5 m wide (in an east-west direction), their length (in a north-west direction) is 8 m for the first row, 4.5 m for the central row, and 6 m for the third row (Jansen 1979: 420). It is singular that both pairs (8, 6) and (6, 4.5) precisely reflect the ratio 4 : 3.
- *Ratio 6 : 1* is reflected in Lothal's dockyard (average dimensions 216.6 x 36.6 m) (Rao 1979: 1:123).
- *Ratio 5 : 4*, Dholavira's prime ratio, is found at two settlements in Gujarat: Lothal, whose overall dimensions are 280 x 225 m (Lal 1997: 129); and Juni Kuran (just forty kilometres away from Dholavira in Kachchh), whose acropolis measures 92 x 72 m (Chakrabarti 2006: 166), which approximates 5 : 4 by 2.2%. Ratio 5 : 4 is also found in structures elsewhere: in Harappa's "granary" of 50 x 40 m (Kenoyer 1998: 64); and in a major building of Mohenjo-daro's HR area measuring 18.9 x 15.2 m (Dhavalikar & Atre 1989: 195-97), thus with a precision of 0.5%.
- *Ratio 5 : 4* is repeated in other ways. At Dholavira, for instance, there are 5 salients on the northern side of the middle town's fortification, against 4 on its eastern and western sides, a clear reminder, should one be needed, of this ratio's pre-eminence. (If we include the corner salients, their numbers grow to 7 and 6, which reflect the middle town's ratio.) Returning to Mohenjo-daro, the "assembly hall," also called "pillared hall," located on the southern part of the acropolis, had four rows of five pillars each (Jansen 1988: 137).

- *Ratio 7 : 6*, the ratio of Dholavira's middle town, is evidenced in the dimensions of the very same "pillared hall," which measures "approximately 23 by 27 metres" (Possehl 2002: 194). It is quite intriguing that this hall, in its dimensions (7 : 6) as well as rows of pillars (5 : 4), should reflect Dholavira's two key ratios!
- *Ratio 9 : 4* is found at Mohenjo-daro's long building located just north of the Great Bath, called "block 6" and measuring approximately 56.4 x 25 m (Mackay 1938: 17), thus within 0.3%.

The above examples are no more than a first sampling and call for a more systematic study, but they do show that Dholavira's ratios are not exclusive to it and are part of a broader Harappan tradition of town planning and architecture. So far, in fact, we have found 7 of Dholavira's 10 ratios at other sites.

In addition, more ratios emerge from other sites, for instance:

- *At Harappa*, the "granary" has 12 rooms measuring 15.2 x 6.1 m each (Kenoyer 1998: 64), i.e., in a ratio of **5 : 2** (0.3%). Near mound AB, "14 symmetrically arranged small houses" were found, each measuring 17.06 x 7.31 m (Chakrabarti 2006: 156), i.e. in a perfect ratio of **7 : 3**.
- *At Gola Dhoro,* a small fortified settlement near Bagasra on the Gulf of Kachchh in Gujarat, has a shell workshop measuring 5.6 x 3.2 m (Bhan *et al.* 2005), exactly in a ratio of **7 : 4**.

Until hundreds of such cases are studied and clear patterns brought out through statistical comparisons, what can safely be said at this stage is that Harappan architects and masons did not believe in haphazard constructions, but followed precise canons of aesthetics based on specific proportions.

Dimensions in Harappan Settlements Expressed in Terms of D

Ratios apart, we should expect Dholavira's unit D = 1.904 m reflected elsewhere: indeed examples abound, beginning with dimensions quoted in the previous section. A few more are proposed below, but it should be kept in mind that most of the following dimensions have clearly been rounded off by the authors referred to; therefore any agreement within 1 or 2% may be considered satisfactory. (A more problematic situation occurs when different authors publish different measurements for the same dimensions; it is hoped that the most reliable ones have been selected here.)

- *Mohenjo-daro*: We saw above a major building in the HR area measuring 18.9 x 15.2 m, which is neatly expressed as **10 x 8 D** (0.7%, 0.2%); the "college" is 70.3 m long, precisely **37 D** (0.2%); "block 6" is about 56.4 m long, i.e. **30 D**; and the "pillared hall" (23 x 27 m) is **14 x 12 D**. According to Possehl (2002: 101), Mohenjo-daro's "First Street" is 7.6 m wide, which is exactly **4 D** (0.2%); the "Central

Street," 5.5 m wide, is nearly **3 D** (3.8%); a smaller street jointing "First Street" is 3.8 m wide, or exactly **2 D** (0.2%).

- *Kalibangan:* As we saw, the acropolis measures about 120 x 240 m, which is equivalent to **63 x 126 D**. Note also that the first dimension is identical to the inner dimensions of Dholavira's "bailey," while the second is also the width of Kalibangan's lower town.
- *Harappa:* We mentioned that the "granary" measures 50 x 40 m, the smaller side being exactly **21 D** (0%); each of its 12 rooms is 15.2 x 6.1 m, or **8 D** lengthwise (0.2%). We also mentioned 14 small houses each 17.06 m long, which is precisely **9 D** (0.4%).
- *Gola Dhoro:* The excavators give its inner dimensions as "approximately 50 x 50 m" (Bhan *et al.* 2005); a more precise reading of its plan yields 52.7 x 45 m, and outer dimensions of 64.6 x 56.4 m (averaging opposite sides, as the plan is not perfectly square); the latter dimensions are very close to **34 x 30 D**. The shell workshop, mentioned earlier, is 5.60 m long, which translates as **3 D** (2%).
- *Chanhu-daro:* This important Harappan town in Sind has a street 5.68 m wide (Chakrabarti 2006: 154), which is precisely **3 D** (0.6%).
- *Lothal:* The above-mentioned dockyard (average dimensions 216.6 x 36.6 m) can be precisely expressed as **114 x 19 D**.
- *Dholavira:* Finally, a few compelling cases come from **Dholavira** itself: a large rock-cut reservoir, south of the "castle," measures 95 m x 11.42 m (Bisht 1999: 28) or very precisely **50 x 6 D** (0.2%, 0%); while the length is a minimum, the width, measured at the top (it is a bit narrower at the bottom), gives us a remarkably perfect match. At the eastern end of a broad street traversing the "castle" stand two pillars 3.8 m apart (Lal 1998: 44); that is exactly **2 D**. The middle town's major north-south street is 5.75 m wide (Lal 1998: 44); that is almost exactly the same width as Chanhu-daro's street, and therefore **3 D** (0.7%).

Naturally, every single dimension cannot be expected to be a whole multiple of D; it is therefore striking enough that so many should turn out to be. This makes a strong case for Dholavira's unit to have been one of the standards in the Harappan world, at least as far as town-planning and architecture are concerned.

The Case of the Great Bath

The difficulty in obtaining reliable measurements for important structures finds an apt illustration in the case of Mohenjo-daro's famous Great Bath. Its discoverer, John Marshall, gave the dimensions of the central bath as 39 x 23 feet (1931: 24), which appear suspiciously rounded off; V. B. Mainkar (1984: 147), translating feet into metres, has 11.89 x 7.01 m. Kenoyer (1998: 63) offers approximately 12 x 7 m, probably from the same dimensions supplied by Jansen (1978/1997: 227); elsewhere, however (Jansen 1988: 134), the latter proposes 11.7 x 6.9 m. A careful measurement by this author of a digital scan of a precise isometric plan of the complex made after Marshall (Possehl 2002: 189) yielded averages of 12.1 x 7.1 m. Such substantial differences rule out a precise study, but temporary dimensions of $6\frac{1}{4}$ or $6\frac{1}{3}$ x $3\frac{2}{3}$ D (equivalent to 11.9 or 12 x 7 m) may be proposed.

Of greater interest is the colonnade surrounding the bath, consisting of 10 columns along the length and 7 along the width. A digital study by this author of the above-mentioned isometric plan yielded averages of 26.7 x 18.9 m, which happen to be precisely **14 x 10 D**. This introduces a ratio not found so far in our studies, **7 : 5**. **Fig. 4** illustrates most of our findings at Mohenjo-daro's acropolis.

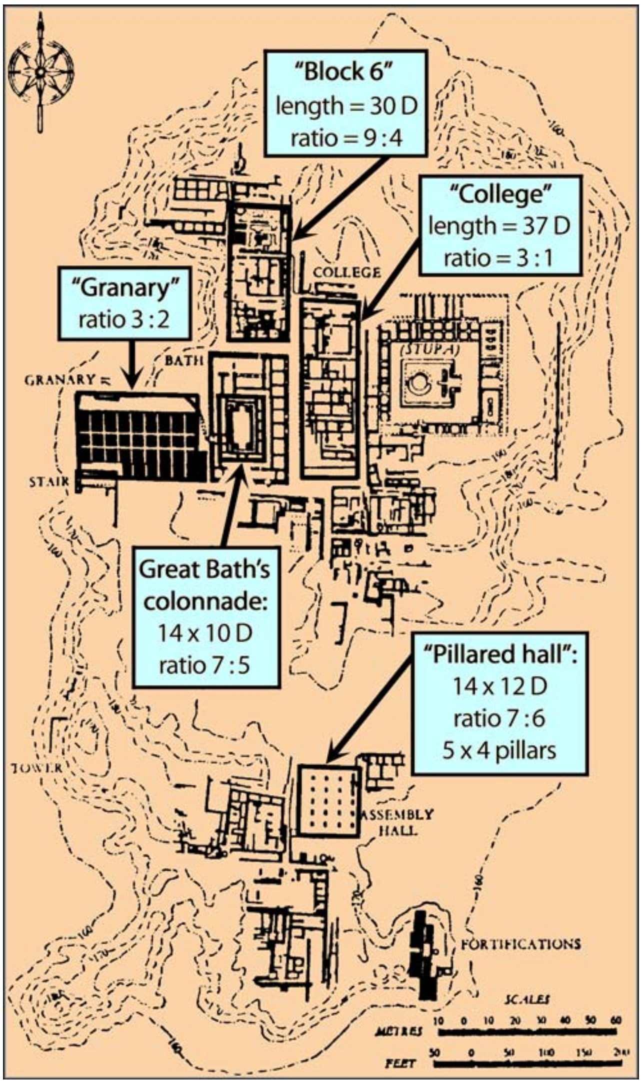

Fig. 4: Mohenjo-daro's acropolis: a few ratios and dimensions
Expressed in terms of Dholavira's unit D = 1.9 m

Dholavira's *Dhanus* and *Angula*

A unit never exists singly, however; it is always part of a system. D = 1.904 m would plainly be inconvenient to measure out bricks, walls or even small rooms. In order to determine its subunits, let us turn to divisions on the three known Harappan scales: those of Mohenjo-daro (6.7056 mm), Harappa (9.34 mm) (Mainkar 1984: 146), and Lothal (1.77 mm). The last is evidenced on an ivory scale found at Lothal, which has 27 graduations covering 46 mm. (Both S. R. Rao (Rao 1979: (2)626) and V. B. Mainkar erred in dividing 46 mm by 27, when the length must of course be divided by the 26 divisions formed by the 27 graduations.)

Dividing D by the first two units yields no clear result. Dividing it by the Lothal unit, we get 1075.7, or, with an approximation of 0.4%, 1080. This last number can be written 108 x 10. So expressed, D begins to make sense as 108 times 1.77 cm. But what is so special about 1.77 cm? First, let us remember that the values of the traditional digit in the ancient world, be it in Egypt, Mesopotamia, China, Greece, Japan, or the Roman Empire, fluctuated between 1.6 and 1.9 cm (Rottländer 1983: 205); 10 times the Lothal unit falls precisely in that range.

Then, the *Arthashastra* defines a digit (*angula* in Sanskrit) as eight widths of barley grain (2.20.6) or "the maximum width of the middle part of the middle finger of a middling man" (2.20.7) (Kangle 1986: 138). Some eight centuries later, Varahamihira's *Brihat Samhita* (LVIII.2) repeats the first definition; that is the "standard" *angula* of classical India — there are indeed variations in regional traditions of iconometry, but they need not detain us here. Most scholars from J. F. Fleet down took the angula to be "roughly equating ... ¾th of an inch" (Chattopadhyaya 1986: 231), that is, 1.9 cm. K. S. Shukla (1976: 19), Ajay Mitra Shastri (1996: 327) or A. K. Bag (1997: 667), to quote just a few, endorsed this approximate value.

Mainkar (1984: 147) traced the "development of length and area measures in India" and narrowed the value of the *angula* to 17.78 mm. He was perhaps the first to suggest that 10 times the Lothal unit, i.e. 17.7 mm, was thus almost identical to the traditional *angula*. Let us build on Mainkar's suggestion and define a "Lothal *angula*" as $\mathbf{A_L}$ being ten times the Lothal unit. With this definition, we can now write $\mathbf{D = 108\, A_L}$: Dholavira's unit is equal to 108 Lothal *angulas*. An organic relationship between the Dholavira unit and the Lothal scale is not surprising, considering the geographical proximity of the two cities.

The above result is arresting, since the concept of "108 *angulas*" is well attested in classical India. For instance, one of the systems of units described in Kautilya's *Arthashastra* (2.20.19) fits very well in the Dholavirian scheme: "108 *angulas* make a *dhanus* (meaning a bow), a measure [used] for roads and city-walls...." (Kangle 1986: 139).

We should note that S. R. Rao provides no margin of error in the reading of the Lothal scale; our above approximation of 0.4% corresponds to less than 0.2 mm from the 46 mm read on the scale, and we can safely assume that such a minute difference would have been rounded off. With our unit D = 1.904 m firmly established by Dholavira's geometry and structures in other settlements, we can propose a more precise value for the Lothal *angula* $\mathbf{A_L}$: 190.4 / 108 = **1.763 cm** (instead of 1.77 cm).

More on Subunits

The Harappan brick provides us with a serendipitous confirmation of the Lothal *angula*. In the Mature phase (and occasionally in the Early phase), most bricks follow ratios of 1 : 2 : 4 in terms of height-width-length; among several different sizes in this ratio, one dominates by far: 7 x 14 x 28 cm, measured and averaged over numerous samples (Kenoyer 1998: 57 and Jansen quoted by Rottländer 1983: 202); the first dimension, 7 cm, is almost exactly 4 Lothal *angulas* (the difference being just 0.5 mm or 0.7%). This is an important confirmation of the *angula*, independent of the Lothal scale. So the humble brick's dimensions can be elegantly expressed as **4 x 8 x 16** $\mathbf{A_L}$.

Between the *angula* and the *dhanus*, there must have been several important subunits. Across the ancient world, units for the digit, the palm, the span, the foot and the cubit were common, together with multiple variations. The Egyptian royal cubit, for instance, had no fewer than six subunits between itself (52.4 cm) and the digit (1.87 cm), its 28th part, in multiples of 4, 5, 12, 14, 16, 24 digits. Below are a few case studies of the use of possible multiples of the Lothal *angula*, or subunits of the *dhanus*.

A useful analysis derives from 13 long dimensions of Mohenjo-daro houses; they were measured by R.C.A. Rottländer from precise plans, not on the ground (Rottländer 1983: 201). Let us now express those dimensions in terms of *dhanus* and *angulas*. For the purpose, however, let us first notice that one of the dimensions, 380.2 cm, is twice 190.1 cm, which is virtually identical to our *dhanus* D = 190.4 cm; but since, according to Rottländer, this dimension appears "five times as the inside diameter of inner rooms of houses," the difference, however small (0.2%), may be significant. Till a discussion further below, let us adopt here a "Mohenjo-daro *dhanus*" or $\mathbf{D_M}$ **= 190.1 cm**, with a corresponding "Mohenjo-daro *angula*" or $\mathbf{A_M}$ equal to 190.1 / 108 = **1.76 cm**. **Table 4** translates Rottländer's measurements into those two units for further study (because dimensions are long ones, all results are within 0.1%).

No.	Dimension (in cm)	In terms of D_M & A_M
1	345.2	1 D_M 88 A_M
2	369.9	1 D_M 102 A_M
3	380.2	2 D_M
4	506.8	2 D_M 72 A_M
5	567.2	3 D_M
6	647.6	3 D_M 44 A_M
7	682.2	3 D_M 64 A_M
8	702.7	3 D_M 75 A_M
9	796.2	4 D_M 20 A_M
10	845.2	4 D_M 48 A_M
11	873.2	4 D_M 64 A_M
12	1106.1	5 D_M 88 A_M
13	1440.2	7 D_M 62 A_M

Table 4: Rottländer's dimensions of houses at Mohenjo-daro, and their expressions in the proposed scheme of *dhanus* and *angulas*.

The third column suggests the following conclusions:

1. Nos. 3 and 5 are perfect multiples of D_M.
2. No. 9, with a remainder of 20 A_M, suggests twice **10 A_M**.
3. No. 8, with a remainder of 75 A_M, suggests five times **15 A_M**.
4. Nos. 7, 10 and 11, which have a remainder of 48 or 64 A_M, suggest a multiple of **16 A_M**. This subunit is also the length of the most common brick, as we saw.
5. No. 4, with a remainder of 72 A_M, suggests twice **36 A_M** (this subunit of the *dhanus* makes sense, since the latter is 108 *angulas* = 36 x 3, but it could also be formed out of a subunit of **12 A_M**).
6. With the above proposed subunits, No. 1's and No. 12's remainder of 88 A_M can be expressed as 36 x 2 + 16 A_M. No. 2's remainder of 102 A_M is 36 x 2 + 15 x 2 A_M, and No. 13's remainder of 62 A_M is 15 x 2 + 16 x 2 A_M.
7. Finally, No. 6's remainder of 44 A_M can be explained with an additional subunit of **8 A_M** as 36 + 8 A_M. This new subunit is not unnatural, since it is the width of the most common brick.

It should be stressed that the above subunits are, at this stage, tentative suggestions based on a few measurements; many precise dimensions, especially smaller ones (in the range of 50-200 cm) would be needed to confirm the proposed system, and add to or subtract from it. (A discussion of Rottländer's conclusions drawn from the above 13 measurements is beyond the scope of this paper; let it simply be said here that they are based on a few arbitrary choices and do not appear to result in an internally consistent system.)

For the present, let us examine a few such smaller dimensions suggestive of specific subunits:

1. **Kalibangan**'s streets have widths in an arithmetic progression: 1.8, 3.6, 5.4 and 7.2 m (Lal 1997: 127). Such widths are found at other sites: **Banawali**'s bigger streets measure 5.4 m (Lal 1997: 127). Moreover, 1.8 m occurs in other contexts: it is the height of the corbelled drain forming the outlet of Mohenjo-daro's Great Bath (Jansen 1988: 136). This dimension is nearly 102 A_M (0.3%), which is the same as the remainder in dimension No. 2 of Table 4.

2. Mackay reports at **Mohenjo-daro** a lane and a doorway having both a width of 1.42 m (Mackay 1938: 9, 11). This dimension, equal to 81 A_M (0.4%), is interesting because it is exactly 3/4 of 108 A_M or 1 D_M, i.e. three quarters of a *dhanus*. This suggest that one quarter, **27 A_M,** could have been another subunit.

3. Mainkar noticed a connection between the three known Harappan scales: Mohenjo-daro's (a broken piece of shell with divisions of 6.706 mm), Harappa's (a piece of bronze rod with divisions of 9.34 mm), and Lothal's ivory scale (Mainkar 1984: 146). He suggested that **10 Mohenjo-daro units + 15 Lothal units = 10 Harappa units**; Mainkar's margin of error was too high (0.8%), because of his wrong value for the Lothal unit. With our value of A_M (1.76 cm), this works out to 67.06 + 26.4 = 93.46 mm, correct to less than 0.1%. This remarkably low margin of error makes an accidental relationship between the three scales extremely unlikely. Mathematically, it can also be expressed as: **1 Harappa unit is equal to 1 Mohenjo-daro unit plus 3/2 or 1.5 times the Lothal unit**. The advantage of the first expression is that it again brings out subunits in multiples of 10 and 15.

Adding 4 *angulas*, the height of the common brick, the above dimensions suggest the play of 7 subunits of the *dhanus*: 4, 8, 10, 15, 16, 27 and 36 *angulas*. The first, 4 A_M or 7.0 cm, and the last, 36 A_M or 63.4 cm, are respectively comparable to the palm unit and the "double foot" of other ancient cultures (Rottländer 1983: 205). If Harappan builders did see it as a "double foot," then they may have used its half, 18 *angulas* or 31.7 cm, as a "foot," which would add another subunit to the above list.

That is as far as we can go regarding subunits until a number of precise dimensions can be examined.

Which *Dhanus-Angula* pair?

We derived one pair of units from Lothal and Dholavira (1.763 / 190.4 cm), and another from Mohenjo-daro (1.76 / 190.1 cm). Although the two pairs are almost identical (differing by less than 0.2%), is there any ground for preferring one to the other? Returning to Mainkar's relationship between the three Harappan units, the margin of error rises to 0.11% if we use the Lothal *angula*. Similarly, if we recalculate margins of error between theoretical and real values in the case of a few fairly precise dimensions examined earlier, we find that the Mohenjo-daro pair comes closer than the Dholavira pair in three cases out of four, as shown in **Table 5**.

Structure	Expression in *dhanus*	Margin of error with Dholavira units (%)	Margin of error with Mohenjo-daro units (%)
Rock-cut reservoir at Dholavira	50 x 6	0.2, 0.0	0.0, 0.1
Rooms of Harappa's "granary"	8 (length)	0.2	0.0
Houses at Harappa (row of 14)	9 (length)	0.4	0.3

Table 5: Comparison of two pairs of units

Finally, if we revisit Table 3 of Dholavira's dimensions and apply to it the Mohenjo-daro rather than the Lothal pair, we find all margins of error reduced by 0.2%; four margins (concerning the lower town's and the bailey's dimensions) now become negative: we have 7 positive margins and 5 negative ones, instead of a single negative margin earlier. Statistically speaking, this is a better distribution.

All these considerations suggest that the Mohenjo-daro pair gives consistently closer results. We may therefore remove subscripts "L" and "M" and adopt, subject to further, more detailed studies, a general Harappan *angula* **A = 1.76 cm**, and a general *dhanus* **D = 190.1 cm** (which may be rounded off to **1.9 m** when dimensions are not extremely precise).

Continuity of the Dholavira Scheme of Ratios and Units

The scheme of ratio and units found at Dholavira finds unmistakable echoes in historical times.

Earlier (Danino, in press), it was recalled how the *Arthashastra,* the *Natyashastra* and Varahamihira's *Brihat Samhita* used various multiples of the *angula*. For instance, the *danda* ("staff") could be defined as 96, 108 or more *angulas*; classical treatises of Hindu architecture such as the *Manasara* recommend measurements with a rod of 108 *angulas*; and iconometry prescribes heights of 84, 86, 108 or 120 *angulas* for statues of deities, although "many [early texts] concentrate on the description of an image of 108 *angulas* in length" (Nardi 2006: 260).

The origin of the concept behind the sacred number 108 is probably multiple. It could be simply based on the human body: 108 *angulas* (1.9 m) is the height of a tall man, as specifically mentioned by Varahamihira in his *Brihat Samhita* (68.105) (Bhat 1981: 642). Also if, as our above analysis suggests, 27 A and 36 A were standard subunits, the least common multiple of those two numbers happens to be 108 (= 27 x 4 or 36 x 3). From a different perspective, simple but compelling astronomical considerations behind 108 have been demonstrated by Subhash Kak (2000: 101-02 & 124).

Dholavira's ratios must have been perceived as specially auspicious, otherwise every enclosure might as well have been square. Some of those ratios are still in use in various traditions of Vastu Shilpa. In the sixth century A.D., Varahamihira, for instance, wrote in his *Brihat Samhita* (53.4 & 5): "The length of a king's palace is greater than the breadth by a quarter.... The length of the house of a commander-in-chief exceeds the width by a sixth...." (Bhat 1981: 451-52). These two ratios, 1 + 1/4 and 1 + 1/6, are identical to 5/4 and 7/6 — very precisely Dholavira's ratios (5/4 for the "castle" and the lower town, 7/6 for the middle town). Such a perfect double match is beyond the realm of coincidence.

A recent work by Mohan Pant and Shuji Funo (2005) compared the grid dimensions of building clusters and quarter blocks of three cities: Mohenjo-daro, Sirkap (Taxila, early historical), and Thimi (in Kathmandu Valley, a contemporary town of historical origins). The results are striking: the authors, after a careful superimpositions of grids on published plans of all three cities (their own in the case of Thimi), find that block dimensions measure 9.6 m, 19.2 m (= 9.6 m x 2), or multiples of such dimensions. This, they argue, evokes the *Arthashastra*'s unit called *rajju*, equal to 10 *dandas*. As regards the *danda*, which has four possible traditional values, the authors choose that of 108 *angulas* as prescribed in the *Arthashastra* (2.20.18-19); it is the same passage which this paper quoted earlier to define the *dhanus*, and the *danda* is mentioned in it as another name of the *dhanus*: for our purpose, the two terms are identical. Pant and Funo conclude from their grids that the unit of length common to the planning of those three cities of very different epochs was the *rajju* = 19.2 m, based on the *danda* = 1.92 m.

If Pant's and Funo's work (of which this author was unaware in his first studies of Dholavira's geometry) finds acceptance among scholars well versed in Harappan and historical town-planning, it will have two important implications. First, it will provide a dramatic confirmation of the scheme worked out for Dholavira, since, proceeding from completely independent methods and different structures (blocks and clusters), it yields a unit of 1.92 m consisting of 108 *angulas* (the difference with our value of 1.901 m is 2 cm or 1%, probably well within the margin of error of the two authors' grid-based method). Secondly, it will lend support to our own conclusion that such concepts survived the collapse of Harappan urbanism, and that Kautilya's canons of urbanism had Harappan roots. Is it so surprising when we already know that the weight system, metallurgical, agricultural and craft techniques did live on, apart from numerous religious symbols and practices? (Danino, 2003)

Indeed, preliminary surveys by this author of dimensions at historical sites have been rewarded by a high proportion of multiples of Dholavira's *dhanus*. However, this line demands further research before it can be presented systematically.

In the meantime, having established a basic continuity of linear measures between India's two urbanizations, we may ask whether the *angula-dhanus* system, as well as some important ratios found in the Mature phase, are in evidence at Late Harappan sites. Unfortunately, a full answer to this question may have to wait, as very few sites of that phase have so far been subjected to substantial horizontal excavations.

Archaeological Considerations

One might object that if Harappans made such a rigorous and systematic use of units of length, we should expect to find many more scales in their cities and towns. But we already have four Harappan scales (from Mohenjo-daro, Harappa, Lothal and Kalibangan) against none at all anywhere from the historical period, when units of lengths were certainly in common use as the *Arthashastra* and later texts testify; the number of extant scales is thus not a reliable indicator. Also, scales made of terracotta, such as Kalibangan's, were necessarily fragile and smaller fragments could have escaped notice. Finally — and this would have applied to historical times too — linear measures were conceivably produced for daily use in the form of sticks or even knotted ropes or strings copied from one master standard of length. In fact, the making of measuring rods from the wood of specific trees and the weaving of measuring ropes from specific fibres or types of grass form an integral part of traditional iconometry and architecture in India, especially in the South (Ganapathi Sthapati & Ananth 2002: 238-39).

Indeed, the very word *rajju* means "rope." If we try to picture the physical act of laying out a city like Dholavira, with dimensions running into hundreds of metres, the combined use of the *rajju* and the *dhanus* makes eminent sense: with one rope carefully measured out as a *rajju* (19.01 m) and another measured out as a *dhanus* (1.901 m), the whole city's layout can be quickly and securely translated on the ground: taking the case of lengths alone, that of the inner "castle" will be 6 ropes, of the outer "castle" 8 ropes, of the ceremonial ground 15 ropes, of the middle town 18 ropes, and of the entire city 40½ ropes. It would be hard to conceive of a more felicitous scheme.

Another objection might be in the form of this question: If Mohenjo-daro's and Harappa's units are related to Lothal's, where and how do the former two come into play? Indeed, for decades, archaeologists and other scholars have referred to an "Indus inch" (defined as 5 times the Mohenjo-daro "unit" of 6.7056 mm, therefore 3.3528 cm) and an "Indus foot" 10 times longer, but those hypothetical units, based on the Mohenjo-daro scale alone, never resulted in any significant analysis of available dimensions, as Mackay readily conceded: "Few of the widths of the doorways are actual multiples of the unit marked on the scale that has been found [at Mohenjo-daro]" (Mackay 1938: 405).

By comparison, the *dhanus-angula* system appears to work better. Apart from the many examples provided in earlier sections, a study of frequent dimensions of doorways at Mohenjo-daro is eloquent. **Table 6** translates Mackay's six most common dimensions (1938: 167) into centimetres and expresses them in terms of the nearest "Indus inch" and the nearest *angula*.

Width of doorway (in cm)	In "Indus inches"	Margin of error (%)	In *dhanus-angulas*	Margin of error (%)
71.1	21	1	40 A	1
101.6	30	1	58 A	0.5
111.8	33	1	64 A	0.8
147.3	44	0.1	84 A	0.4
180.3	54	0.4	102 A	0.5
243.8	73	0.4	1 D 30 A	0.4

Table 6: Comparison of widths of doorways at Mohenjo-daro, expressed in two different systems of units

Margins of error are slightly smaller with the *dhanus-angula* system, but more importantly, it yields multiples that are far more natural (all of them as even numbers) and wholly compatible with the system of subunits tentatively outlined above, while, as Mackay observed, the multiples of Indus inches cannot be combined into a coherent system. Working from a different angle, Rottländer doubted whether Mohenjo-daro's shell scale was a ruler at all: "There is a high chance that it was part of an ornament or finger-board of a stringed instrument" (1983: 202). It is clearly necessary to revisit the whole field of Harappan metrology from a fresh perspective.

In this respect, R. Balasubramaniam and Jagat Pati Joshi (unpublished), recently submitting Kalibangan's "crude terracotta scale" to careful scrutiny, established that it is based on a unit of 1.75 cm. This is almost identical to our *angula* of 1.76 cm; given that the scale would have slightly contracted during the firing process, the difference of one tenth of a millimetre (0.6%) actually points to a high precision. The Kalibangan scale therefore provides a powerful confirmation of the Harappan *angula*.

Harappan and Classical Concepts

On a cultural level, the presence of carefully proportioned fortifications as at Dholavira might be as much a specific cultural trait as pyramids are to Egypt or ziggurats to Mesopotamia. Here, instead of erecting colossal buildings, enormous energy was spent on defining spaces: the space of the rulers and administrators (the acropolis) and the spaces for other classes of citizens. Demarcating was a vital need not for defence, but for self-definition.

But there may also be deeper motives at work. Ratios and units apart, we can discern a few important principles underlying Dholavira's fascinating harmony, in an almost Pythagorean sense of the term.

In a recent study of the origins of geometry in various civilizations, Olivier Keller, turning to the *Sulbasutras*, is struck by "the very frequent addition, in Vedic numerology, of one unit. ... The addition of a unit is a way to envelop the various parts in a unity; thus 7 can be the result of the addition of the four cardinal directions, above, below, *and space itself*. Or again 21, which is 3 times 7, can also represent man because of his 20 fingers *and his body*... We can discern, as in the analogies related to the parts of the [Vedic] altars, the profound thought of a totality reflecting itself in every part, and conversely of the union of various parts in a totality" (Keller 2006: 138, italics in the original). Similarly, when Varahamihira specifies the length of a king's palace, rather than express it as 5/4 or 1.25 times the width, he asks us to add a quarter to the width, that is, to the unit: 1 + 1/4. Adding one more unit, we get 1 + 1 + 1/4 = 9/4. We can now understand that these two fundamental ratios of Dholavira emerge from the same principle of *addition of a unit*; 9 : 4 is, in reality, nothing but 5 : 4 plus one unit.

This addition to the unit of a fraction of itself can also be seen as a process of *expansion*, of *auspicious increase* symbolizing or inviting prosperity. The *Manasara*, a treatise of Hindu architecture, applies this process when it specifies (35.18-20) that "the length of the mansion [to be built] should be ascertained by commencing with its breadth, or increasing it by one-fourth, one-half, three-fourth, or making it twice, or greater than twice by one-fourth, one-half or three-fourths, or making it three times" (Acharya 1934/1994: 374). The outcome is a series of auspicious ratios: 5/4, 3/2, 7/4, 2/1, 9/4, 5/2, 11/4, 3/1. It is significant that we have found all but one ratios (11/4) at Dholavira or other Harappan settlements, and reasonable to assume that the concept behind such ratios is the same in Harappan and in historical times.

Also found at Dholavira is the principle of *recursion*, or repetition of a motif. In our case, the ratio 9 : 4, between the lengths of the "castle" and of the middle town, is repeated between the lengths of the middle and the lower towns. This principle is visible in classical architecture; in temples, for instance, *shikharas* of increasing height build up towards the towering last one. This is another way of repeating the initial unity and grow from it.

The Harappans' use of a *decimal system* is already evidenced, as is well known, in the weight system. (We must stress that contrary to a common misconception, an *empirical* decimal system in no way requires numerals with a decimal place-value notation and the zero: that development occurred in India in the fourth or fifth century AD.) The Harappan decimal system is conspicuous at Dholavira: in the Lothal *angula* equal to 10 units of the Lothal scale, and, at the other end of our range, in Pant's and Funo's *rajju* equal to 10 *dandas* (*dhanus*).

As an aside, we can now express the theoretical width of the castle's eastern and western fortifications as 10 *dhanus* = 1 *rajju*. Is it not to be marvelled at that members of the elite that occupied this high point of the city chose to embed this concept of **one unit** in their massive walls? Had they merely wished to have effective defences, a few metres would have sufficed, even if inner rooms had to be integrated; a width of up to 19 m might appear as a mindless waste of material and labour, but it is a strong clue to an underlying cultural or sacred concept.

The common thread connecting those principles was anticipated by astrophysicist J. McKim Malville, who saw in Dholavira's features "the apparent intent ... to interweave, by means of geometry, the microcosm and the macrocosm" (Malville & Gujral 2000: 3). To the ancient mind, the concept of sacred space is inseparable from the practice of town-planning and architecture.

There would be more to say on Dholavirian numbers, especially number 3 which is the key to the whole city, and its omnipresent square 9, but this must wait till a body of evidence can be built from other Harappan sites. In the meantime, we have covered enough ground to strengthen Jim Shaffer's thesis of a strong connection between Harappan urbanism and the urbanism of historical times, in which he finds "a unique cultural tradition traceable for millennia" (Shaffer 1993: 54), or Dilip Chakrabarti's recent observation: "The ideals of ancient Indian town planning seem to run deep through the concepts embedded in the Harappan cities like Mohenjodaro and Dholavira" (Chakrabarti 2006: 166).

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