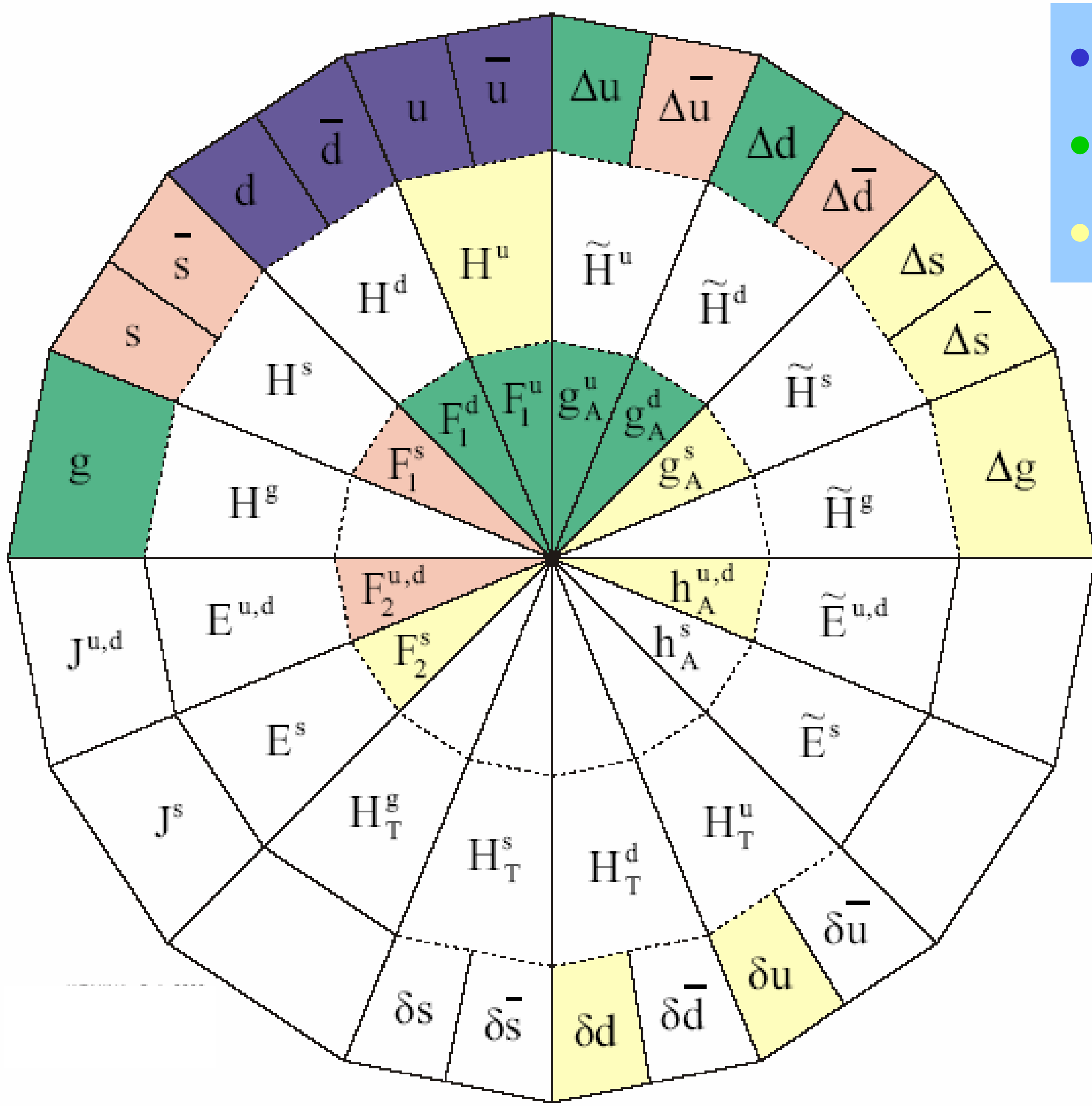
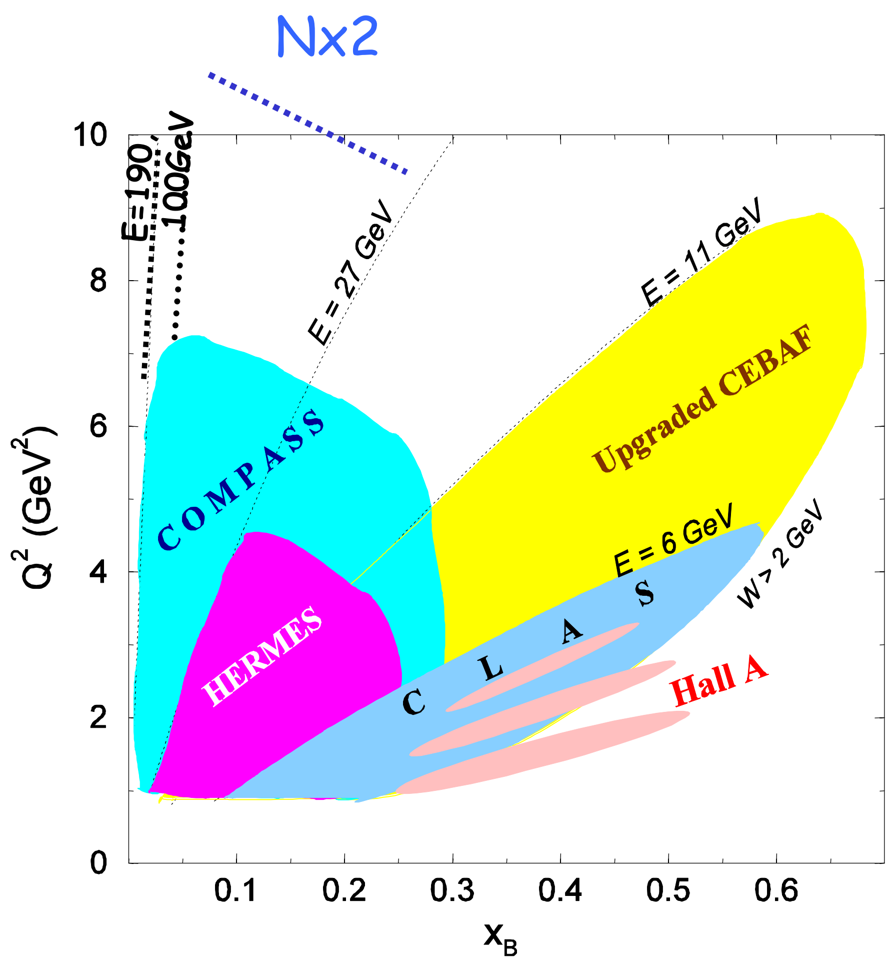


- well known
- known
- uncertain

Kinematical domain



Collider :

H1 & ZEUS 0.0001<x<0.01

Fixed target :

JLAB	6-11GeV	SSA,BCA?
HERMES	27 GeV	SSA,BCA

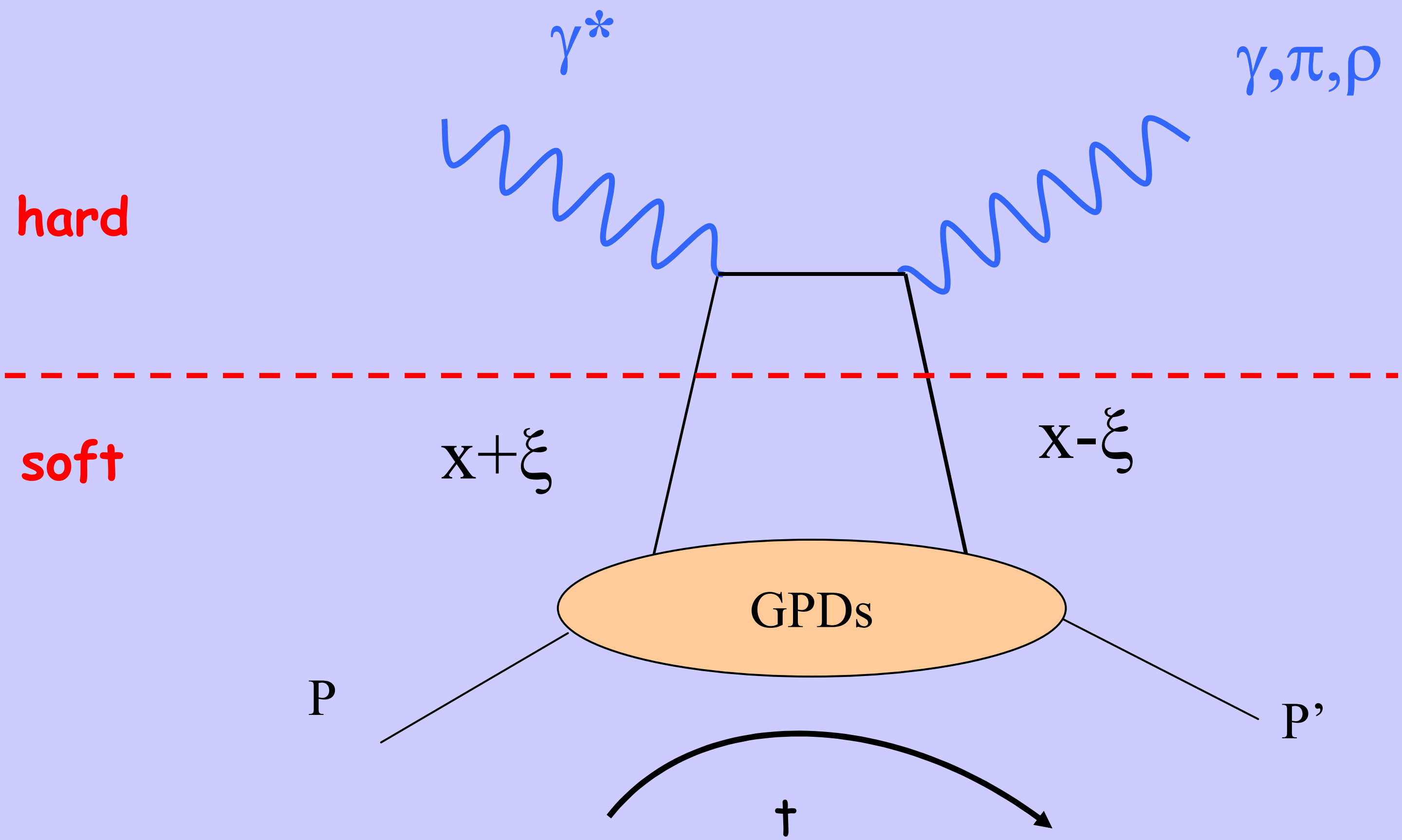
COMPASS could provide data on :

- Cross section (190 GeV)
- BCA (100 GeV)
- Wide Q^2 and x_{bj} ranges

Limitation due to luminosity

Generalized Parton Distributions
Recent Progress

Pervez Hoodbhoy
Quaid-e-Azam University
Islamabad



Factorisation:

Q^2 large, $-t<1$ GeV^2

What is a GPD?

- It is a proton matrix element which is a hybrid of elastic form factors and Feynman distributions
- GPDs depend upon:

  x : *fraction of the longitudinal momentum carried by struck parton*

  t: *t-channel momentum transfer squared*

  ξ: *skewness parameter (a new variable coming from selection of a light-cone direction)*

  Q^2 : probing scale

X. Ji (1997) A. Radyushkin (1997)

Formal definition of GPDs:

$$\int \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x} \langle p' \big| \bar{q}(-\tfrac{1}{2}\lambda n)\gamma^{+} q(\tfrac{1}{2}\lambda n) \big| p \rangle = H(x,\xi,t)\bar{u}\gamma^{+}u + E(x,\xi,t)\bar{u}\frac{i\sigma^{+\nu}q_{\nu}}{2M}u$$

- x_i and x_f are the momentum fractions of the struck quark, and $x = \frac{1}{2}(x_i + x_f)$.
- $\xi = (x_f - x_i)/2$ is skewness. Depends on lightcone direction.
- $\int dx H(x,\xi,t) = \mathrm{F}_1(\mathrm{t})$
- $\int dx E(x,\xi,t) = \mathrm{F}_2(\mathrm{t})$

Relation of GPDs to Angular Momentum

Generalized form factor and quark angular momentum:

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\left[A^{q,g}_{20}(t)\,\gamma^{(\mu}P^{\nu)} + B^{q,g}_{20}(t)\,\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M}\right]U(P)$$

Total quark angular momentum:

$$J^{u+d} = \frac{1}{2}\left[A^{u+d}_{20}(0) + B^{u+d}_{20}(0)\right] = \frac{1}{2}\left[\langle x\rangle^{u+d} + B^{u+d}_{20}(0)\right]$$

Quark angular momentum (Ji's sum rule)

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x dx\left[H^q(x,\xi,0) + E^q(x,\xi,0)\right]$$

X. Ji, Phy.Rev.Lett.78,610(1997)

GPDs And Orbital Angular Momentum Distribution:

$$O^{\beta\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\gamma^{(\beta} iD^{\mu_1} iD^{\mu_2} \cdots iD^{\mu_n)}\psi$$

Define generalized angular momentum tensor:

$$M^{\alpha\beta\mu_1\mu_2\cdots\mu_n} = \xi^{\alpha} O^{\beta\mu_1\mu_2\cdots\mu_n} - \xi^{\beta} O^{\alpha\mu_1\mu_2\cdots\mu_n} \text{ (minus traces)}$$

$$\int d^4\xi \langle p | M^{\alpha\beta\mu_1\mu_2\cdots\mu_n}(\xi) | p\rangle = J_n \times \text{tensor structures} \times (2\pi)^4 \delta^4(0)$$

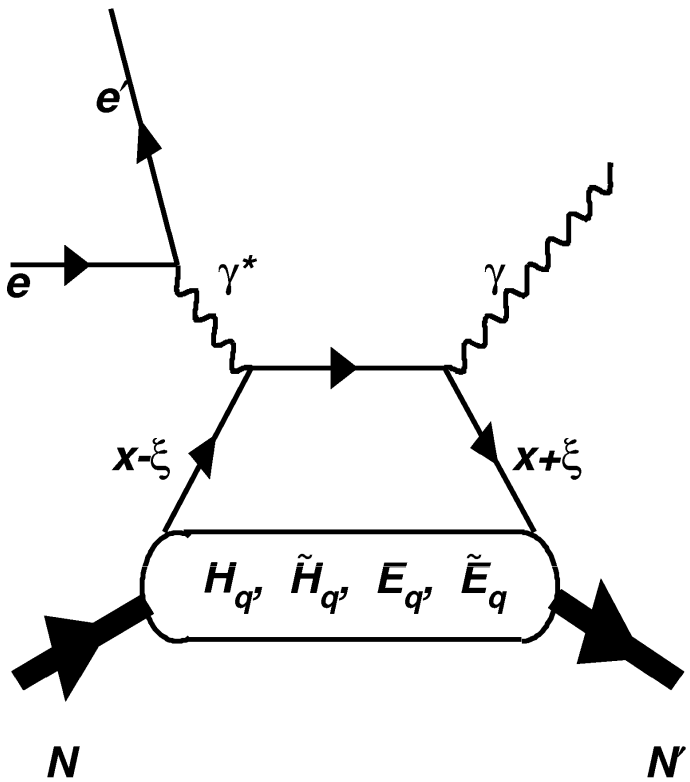
reduced matrix element

$$\int d^3\xi \langle p | M^{12\cdots+++}(\xi) | p\rangle = S^{+\cdots+} + \tilde{L}^{+\cdots+} + \Delta\tilde{L}^{+\cdots+}$$

$$\boxed{L(x) = \frac{1}{2}\left[xq(x) + xE(x) - \Delta q(x)\right]}$$

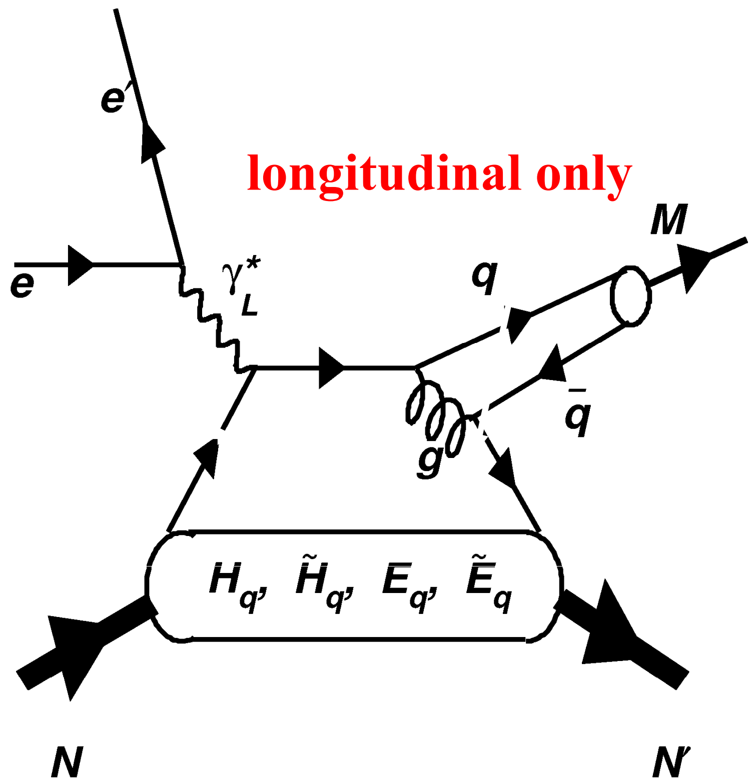
Ji+Hoodbhoy

DVCS



a)

DVMP

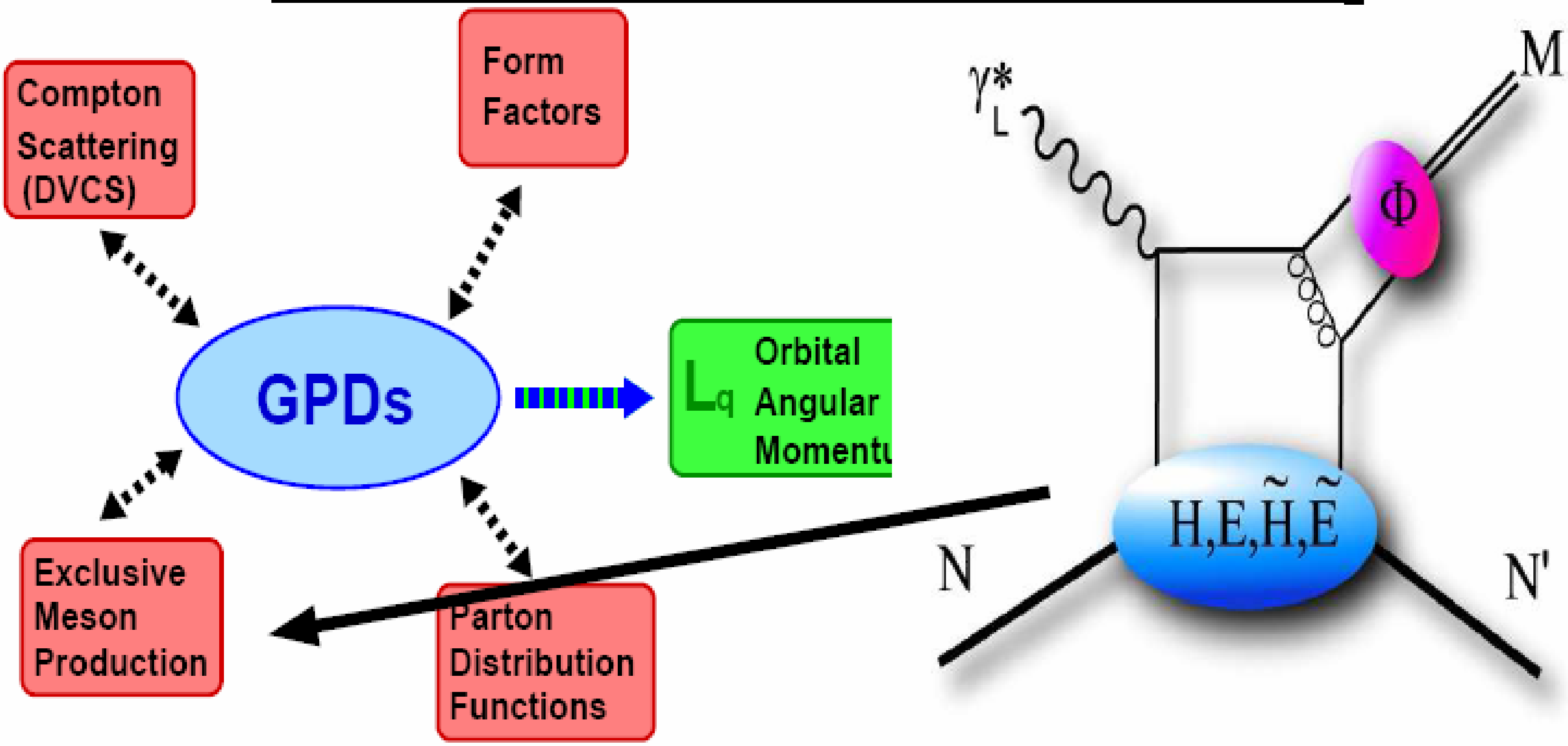


b)

DVCS cannot separate u/d quark contributions.

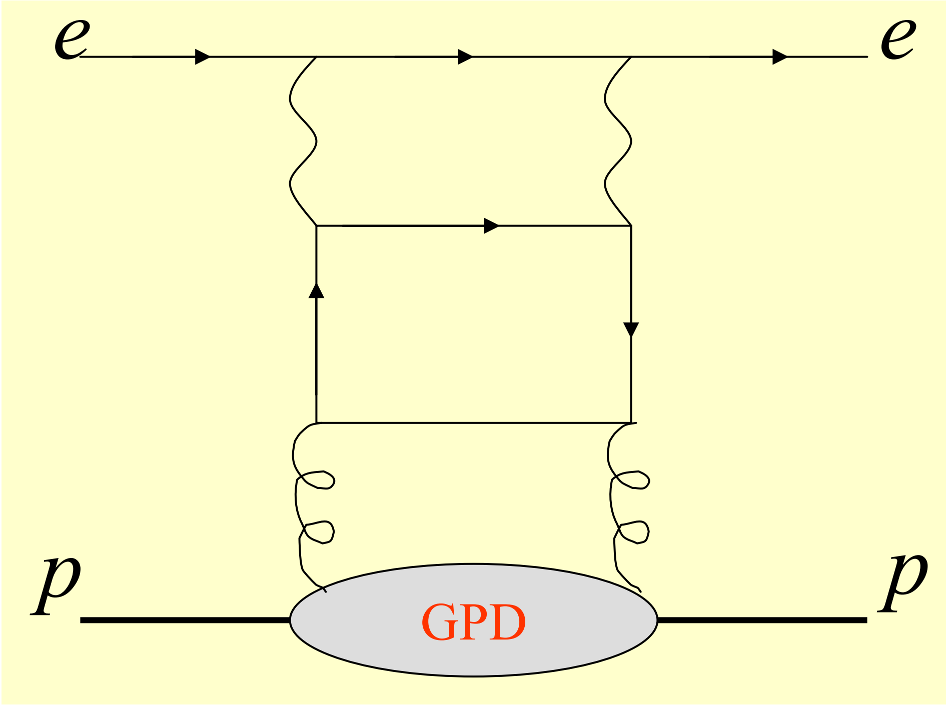
M = ρ/ω select H, E, for u/d flavors
M = π, η, K select H, E

Exclusive Reactions & GPDs



☞ Quantum numbers of final meson state select different $GPDs$

- Pseudoscalar mesons ($\pi, \eta ...$): $\tilde{H}, \tilde{E}$
- Vector mesons ($\rho, \omega, \phi ...$): H, E (flavour singlet)
- f-meson family ($f_0, f_2, ...$): H, E (flavour non-singlet)



P. Hoodbhoy and X. Ji, PRD

Helicity-flip GPDs

$$\frac{1}{x}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P'S'|F^{(\mu\alpha}(-\frac{\lambda}{2}n)F^{\nu\beta)}(\frac{\lambda}{2}n)|PS\rangle$$
$$= H_{Tg}(x,\xi)\bar{U}(P'S')\frac{\bar{P}^{([\mu}i\Delta^{\alpha]}\sigma^{\nu\beta)}}{M}U(PS)$$
$$+ \quad E_{Tg}(x,\xi)\bar{U}(P'S')\frac{P^{([\mu}\Delta^{\alpha]}}{M}\frac{\gamma^{[\nu}\Delta^{\beta])}}{M}U(PS)+\ldots .$$

TMD Parton Distributions

- These appear in the processes in which hadron transverse-momentum is measured, often together with TMD fragmentation functions.
- The leading-twist ones are classified by Boer, Mulders, and Tangerman (1996,1998)
  - There are 8 of them

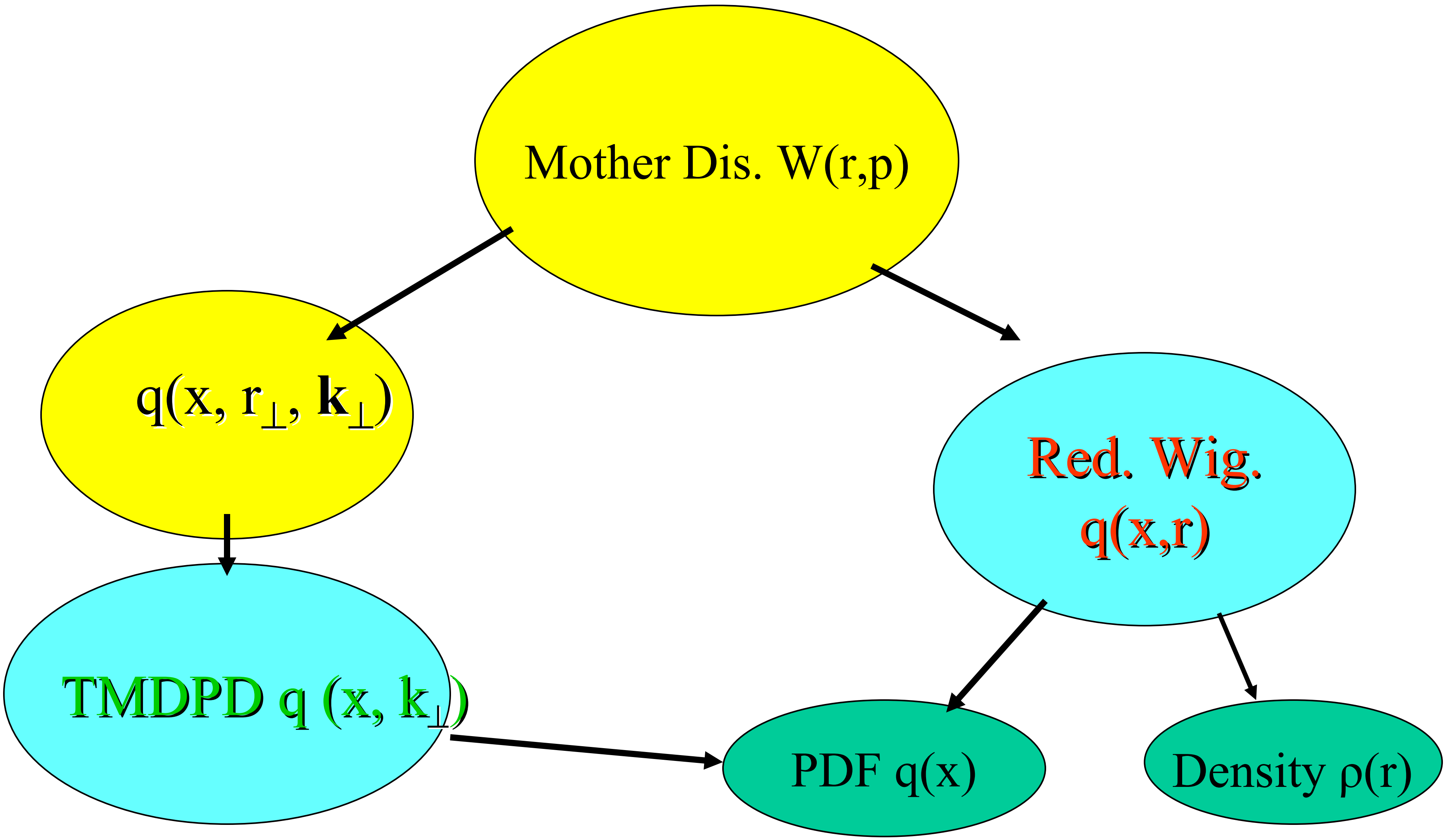
$q(x, k_\perp)$, $q_T(x, k_\perp)$,

$\Delta q_L(x, k_\perp)$, $\Delta q_T(x, k_\perp)$,

$\delta q(x, k_\perp)$, $\delta_L q(x, k_\perp)$,

$\delta_T q(x, k_\perp)$, $\delta_T q(x, k_\perp)$

Wigner parton distributions & offsprings (Ji)



Wigner parton distributions (WPD)

$$W(x,p) = \int \psi^*(x-\eta/2)\psi(x+\eta/2)e^{ip\eta}d\eta \ ,$$

$$\langle O(x,p)\rangle = \int dx dp O(x,p) W(x,p)$$

- When integrated over p, one gets the coordinate space density $\rho(x)=|\psi(x)|^2$
- When integrated over x, one gets the coordinate space density $n(p)=|\psi(p)|^2$

Wigner distributions for quarks in proton

- Wigner operator (X. Ji,PRL91:062001,2003)

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int \overline{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2)e^{ik\cdot\eta}d^4\eta \ ,$$

- Wigner distribution: "**density**" for quarks having *position r and 4-momentum k^μ* (off-shell)

$$\begin{aligned} W_\Gamma(\vec{r},k) &= \frac{1}{2M}\int\frac{d^3\vec{q}}{(2\pi)^3}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}(\vec{r},k)\right|-\vec{q}/2\right\rangle \\ &= \frac{1}{2M}\int\frac{d^3\vec{q}}{(2\pi)^3}e^{-i\vec{q}\cdot\vec{r}}\left\langle \vec{q}/2\left|\hat{\mathcal{W}}(0,k)\right|-\vec{q}/2\right\rangle \end{aligned}$$

Reduced Wigner Distributions and GPDs

- The 4D reduced Wigner distribution f(**r**,x) is related to Generalized parton distributions (GPD) H and E through simple FT,

$$f_\Gamma(\vec{r},x) = \frac{1}{2M}\int \frac{d^3\vec{q}}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{r}} F_\Gamma(x,\xi,t) \ .$$

$$\frac{1}{2M}F_{\gamma^+}(x,\xi,t) = [H(x,\xi,t) - \tau E(x,\xi,t)] + i(\vec{s}\times\vec{q})^z \frac{1}{2M}[H(x,\xi,t) + E(x,\xi,t)]$$

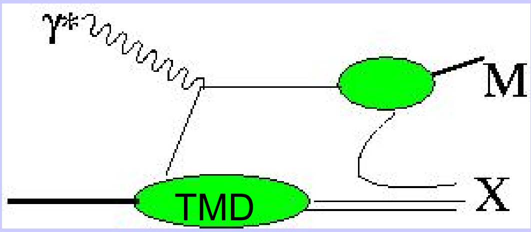
$t = -q^2$

$\xi \sim q_z$

H,E depend only on 3 variables. There is a rotational symmetry in the transverse plane..

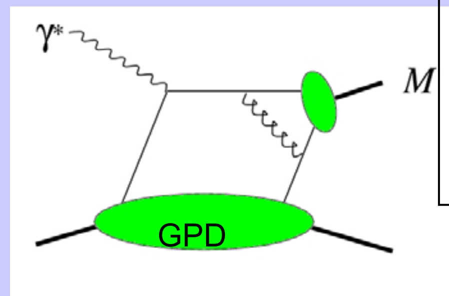
$W_p^u(x,\mathbf{k},\mathbf{r})$ "Parent" Wigner distributions

Probability to find a quark **u** in a nucleon **P** with a certain polarization in a position **r** and momentum **k**



d^3r

d^2k_T (FT)



TMD PDFs: $f_p^u(x,k_T), g_1, f^{\perp}_{1T}, h_1, h^{\perp}_{1L}$

GPDs: $H_p^u(x,\xi,t)$, $E_p^u(x,\xi,t)$, $\tilde{H}, \tilde{E}, \ldots$

Measure momentum transfer to quark

k_T distributions also important for exclusive studies

d^2k_T

$\xi=0, t=0$

dx

Measure momentum transfer to target

Exclusive meson data important in understanding of SIDIS measurements

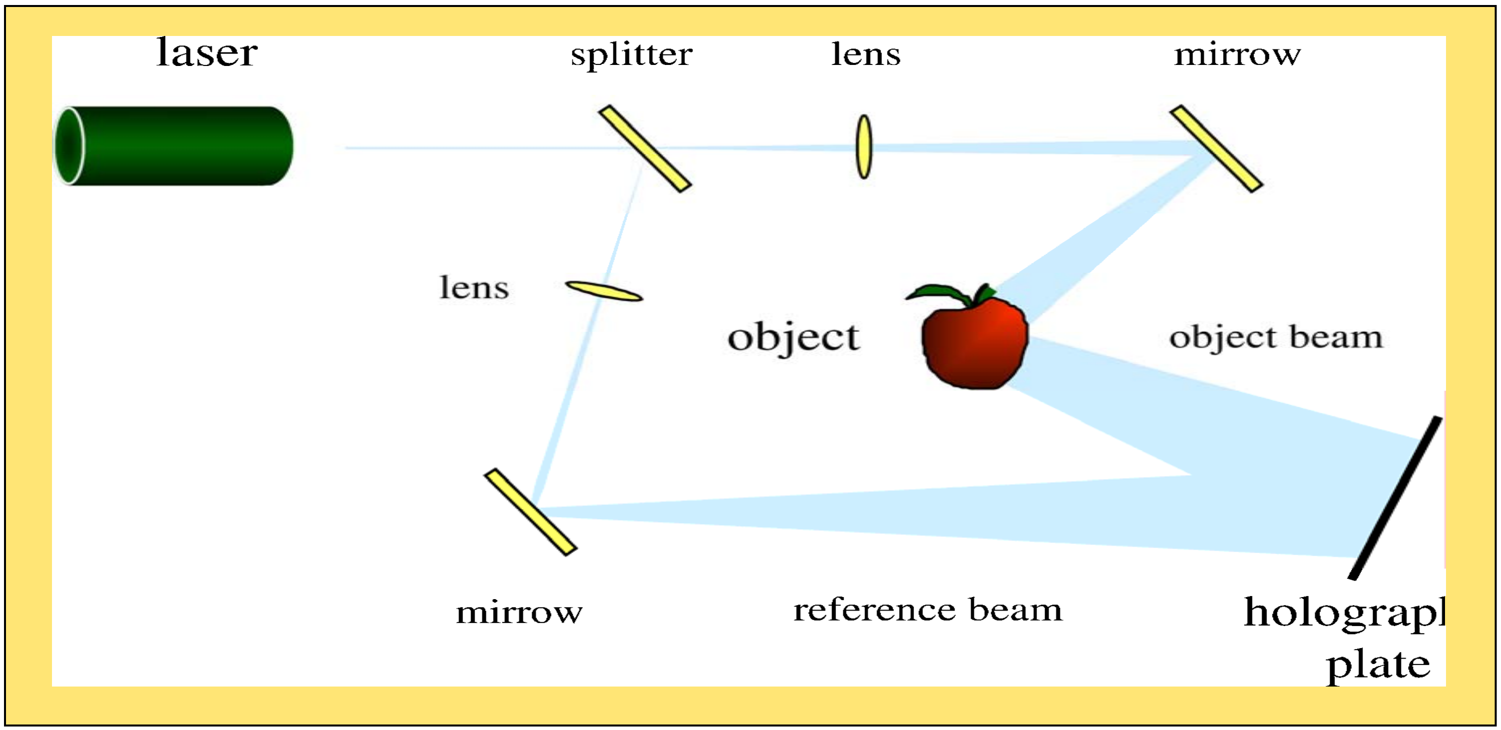
PDFs $f_p^u(x,k_T)$, g_1, h_1

FFs $F_{1p}^u(t), F_{2p}^u(t)$..

Some PDFs same in exclusive and semi-inclusive analysis

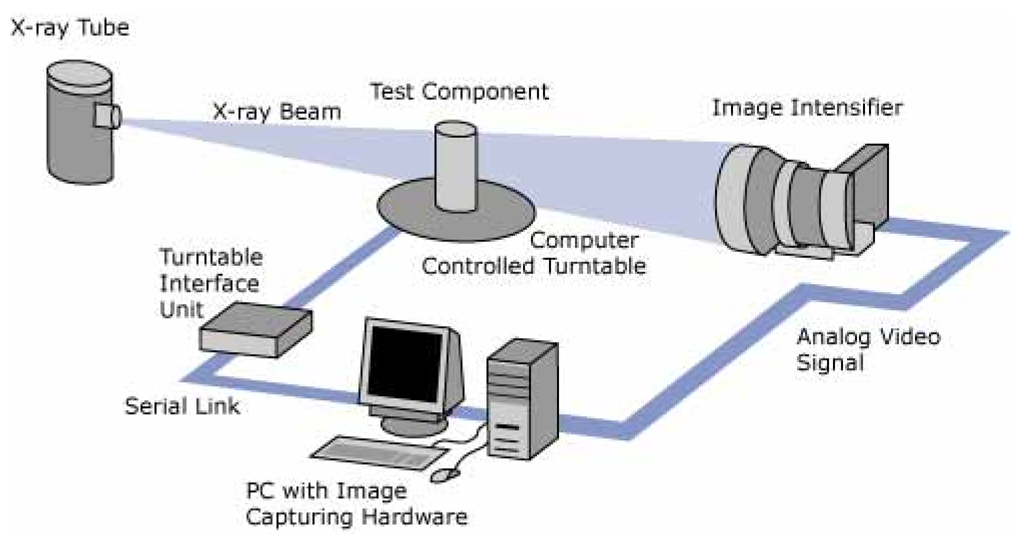
Analysis of SIDIS and DVMP are complementary

Holography is "lensless photography" in which an image is captured not as an image focused on film, but as an interference pattern at the film. Typically, coherent light from a laser is reflected from an object and combined at the film with light from a reference beam. This recorded interference pattern actually contains much more information that a focused image, and enables the viewer to view a true three-dimensional image which exhibits parallax.



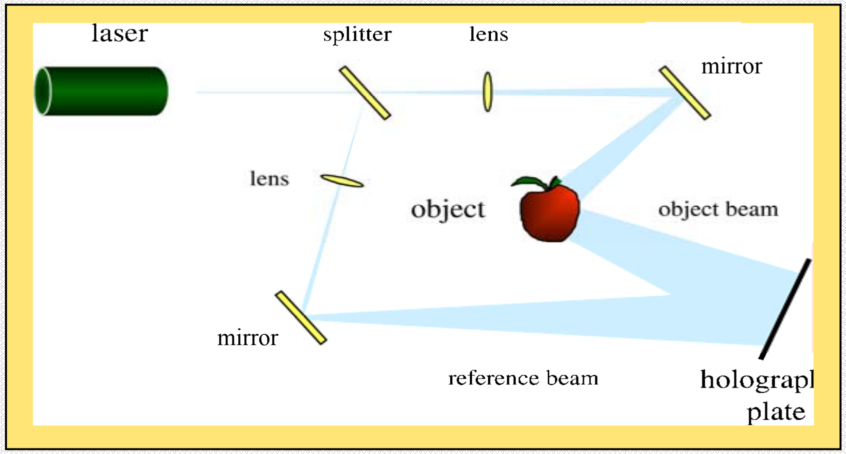
Computed Tomography

Computed Tomography (CT) is a powerful nondestructive evaluation (NDE) technique for producing 2-D and 3-D cross-sectional images of an object from flat X-ray images. Characteristics of the internal structure of an object such as dimensions, shape, internal defects, and density are readily available from CT images.



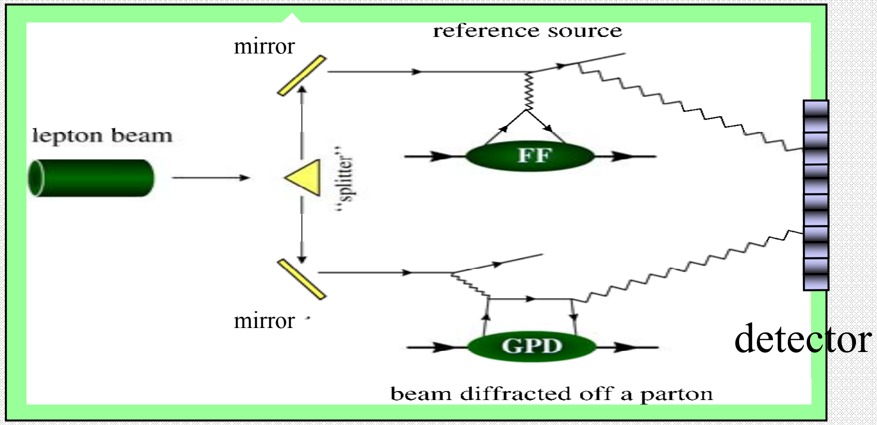
From Holography to Tomography

A. Belitsky, B. Mueller, NPA711 (2002) 118

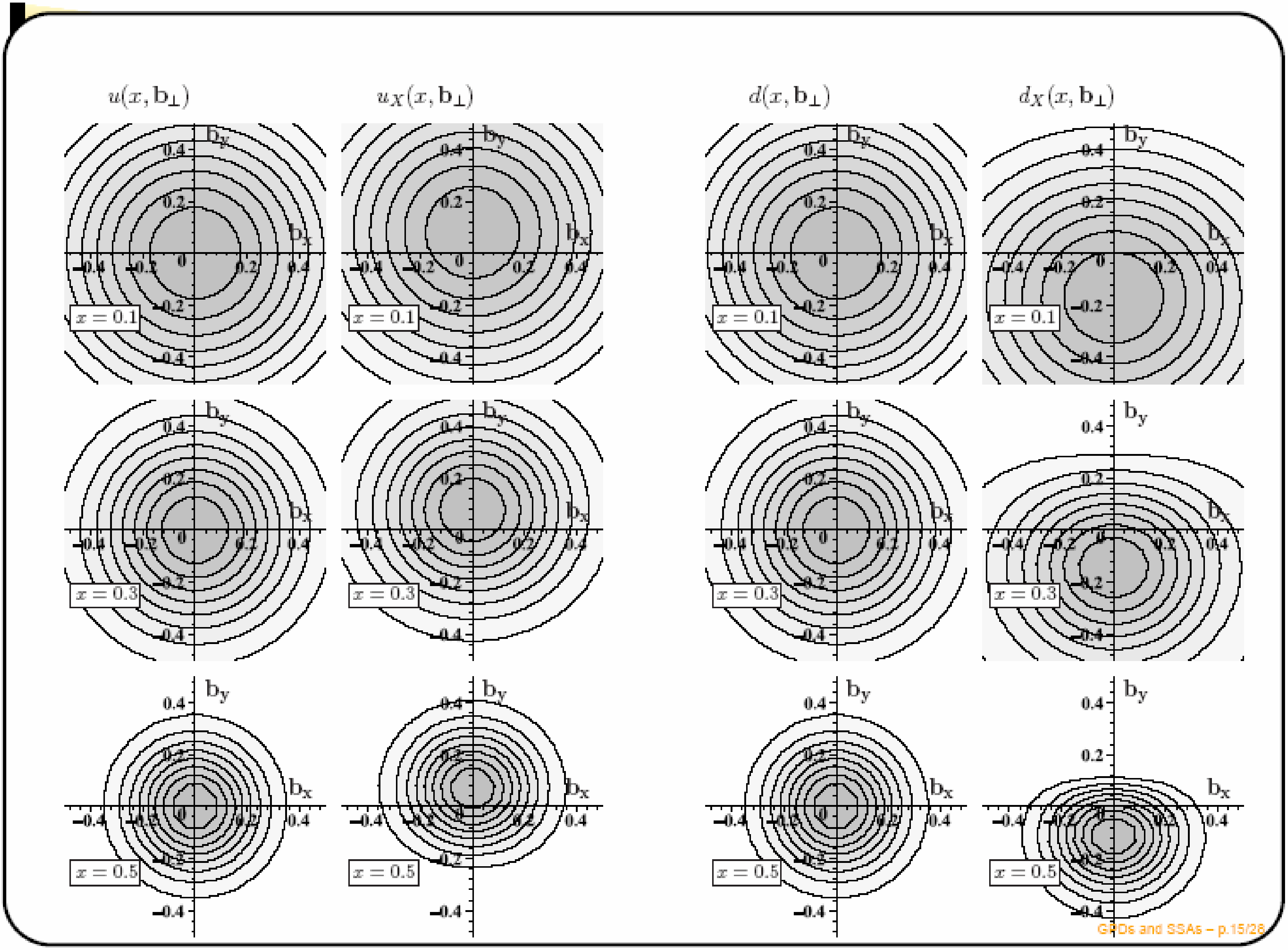


An Apple

A Proton



By varying the energy and momentum transfer to the proton we probe its interior and generate tomographic images of the proton ("femto tomography").

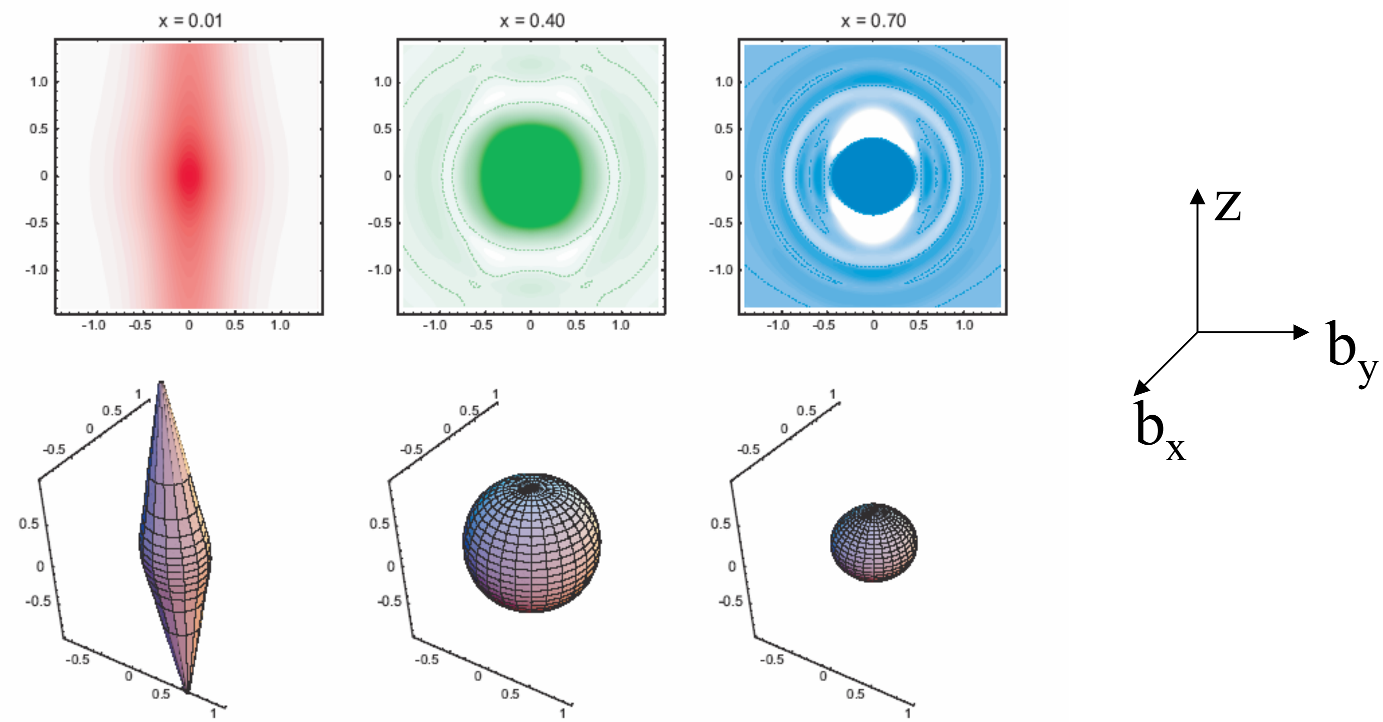


Burkardt

$$\lim_{x\to 1} q(x, \vec{b}_\perp) \propto \delta^2(\vec{b}_\perp)$$

Imaging quarks at fixed Feynman-x

- For every choice of x, one can use the Wigner distributions to picture the nucleon in 3-space; **quantum phase-space tomography!**



GPDs ON A LATTICE

$$\mathcal{O}_q^{\{\mu_1\cdots\mu_n\}} = \overline{q}\,\gamma^{\{\mu_1}\overleftrightarrow{D}^{\mu_2}\ldots\overleftrightarrow{D}^{\mu_n\}}\,q$$

$\longrightarrow$ Generalised Form Factors

$$\langle p', s'|\mathcal{O}^{\{\mu_1\cdots\mu_n\}}(\Delta)|p, s\rangle =$$

$$\bar{u}(p', s')\gamma^{\{\mu_1}u(p, s)\sum_{i=0}^{\frac{n-1}{2}} A_{qn,2i}(t)\Delta^{\mu_2}\ldots\Delta^{\mu_{2i+1}}\overline{p}^{\mu_{2i+2}}\ldots\overline{p}^{\mu_n\}}$$

$$+\bar{u}(p', s')\frac{i\sigma^{\{\mu_1\nu}\Delta_\nu}{2m}u(p, s)\sum_{i=0}^{\frac{n-1}{2}} B_{qn,2i}(t)\Delta^{\mu_2}\ldots\Delta^{\mu_{2i+1}}\overline{p}^{\mu_{2i+2}}\ldots\overline{p}^{\mu_n\}}$$

$$+C_{qn}(t)\,\frac{1}{m}\bar{u}(p', s')u(p, s)\Delta^{\mu_1}\ldots\Delta^{\mu_n}|_{\text{n even}}$$

$$A_{10}^{q}(Q^{2}) = F_{1}^{q}(Q^{2})$$

$$B_{10}^{q}(Q^{2}) = F_{2}^{q}(Q^{2})$$

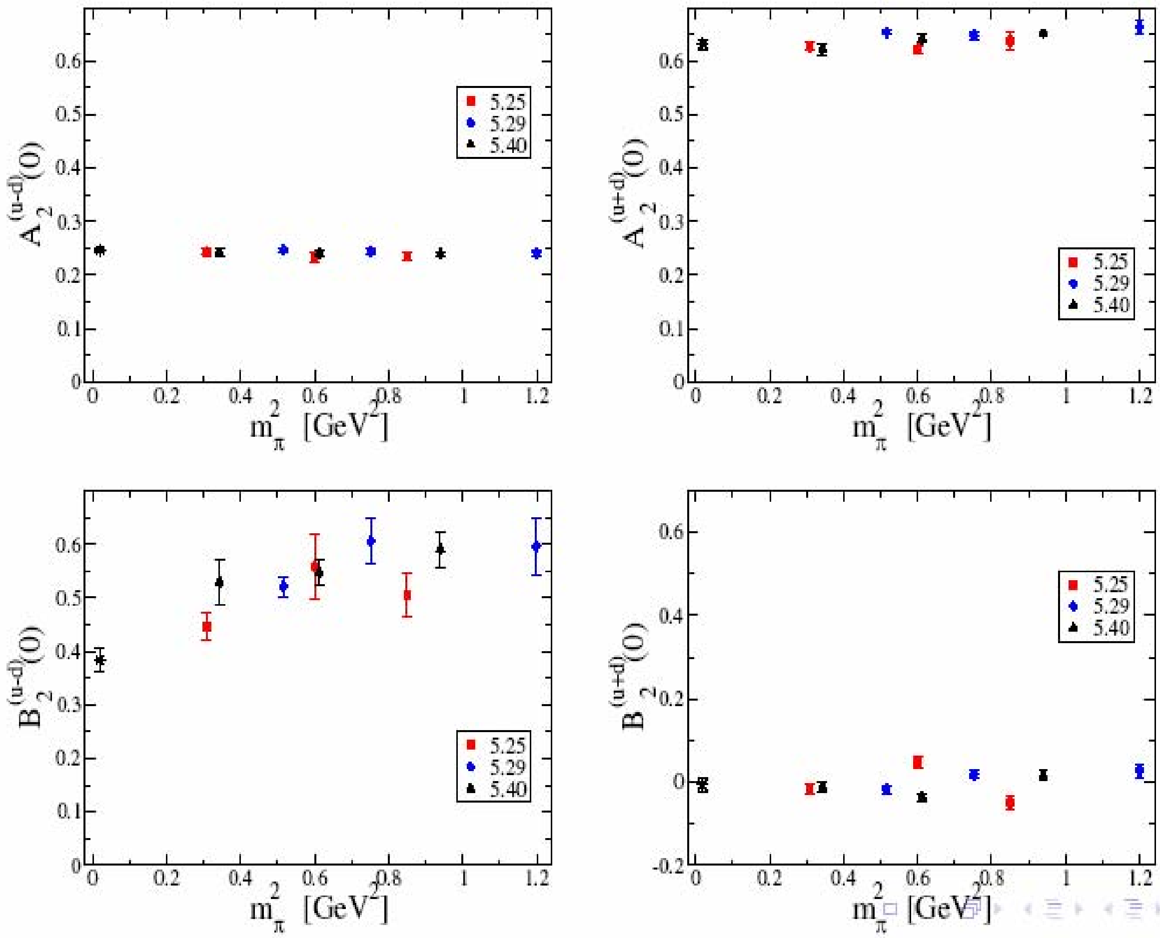
$$\tilde{A}_{10}^{q}(Q^{2}) = G_{A}^{q}(Q^{2})$$

$$\tilde{B}_{10}^{q}(Q^{2}) = G_{P}^{q}(Q^{2})$$

$$J^{q} = \tfrac{1}{2}\left(A_{20}^{q}(0) + B_{20}^{q}(0)\right)$$

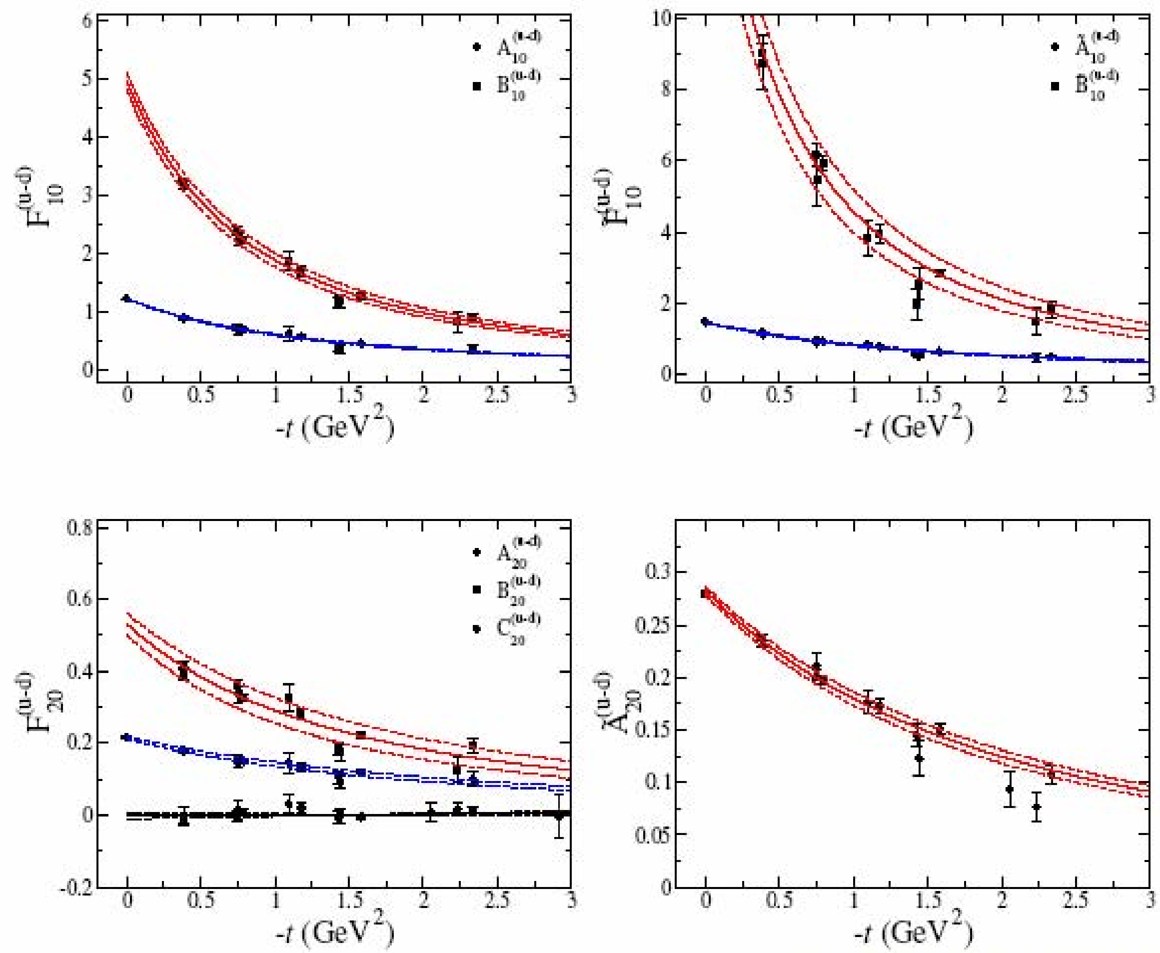
$$\tfrac{1}{2}\Sigma^{q} = \tilde{A}_{10}^{q}(0)$$

Angular Momentum $J^q = L^q + S^q = \frac{1}{2}(A_2^q + B_2^q)$, ($\overline{\text{MS}}$ 4 GeV^2)



Zanotti

Generalised Form Factors, ($m_\pi \approx 950\text{MeV}$)

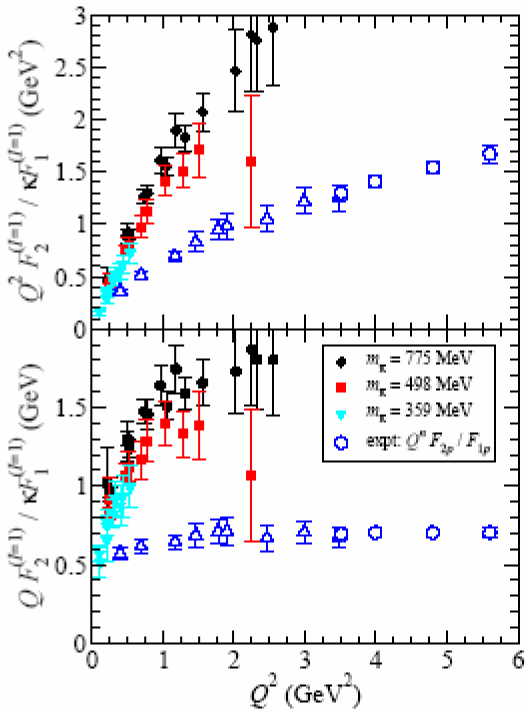


Zanotti

Nucleon F_2/F_1 on the Lattice (I)

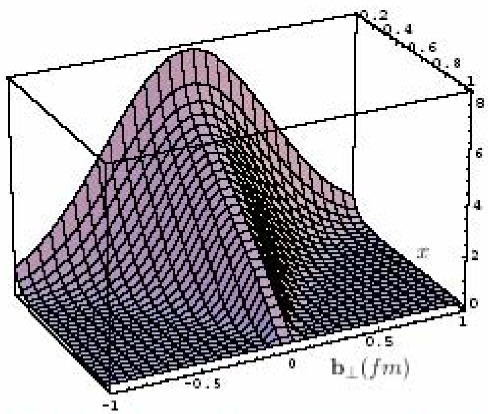
PRELIMINARY

- Only $I = 1$ form factors computed so far to avoid disconnected diagrams. $F_1^{I=1} = F_{1p} - F_{1n}$ but F_{1n}, F_{2n} not known accurately for $Q^2 \gtrsim 1$ GeV^2.
- Our normalization is $F_2(Q^2) \to \kappa$ as $Q^2 \to 0$.



Fleming

Transverse quark distributions



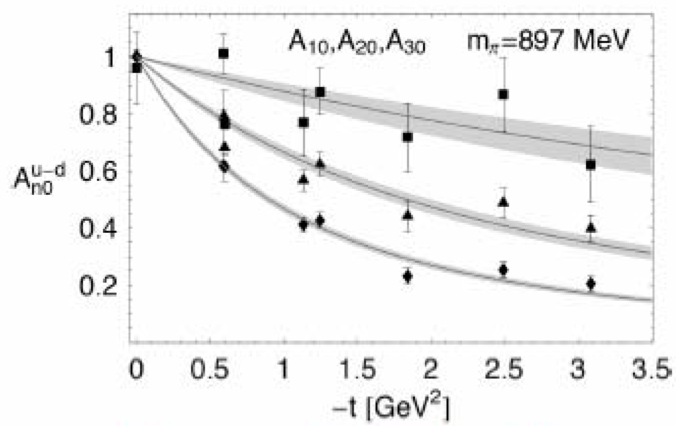
M. Burkardt hep-ph/0207047

$$A^q_{n0}(-\Delta_\perp^2) = \int d^2 b_\perp e^{i\mathbf{\Delta}_\perp \cdot \mathbf{b}_\perp} \int_{-1}^{1} x^{n-1} q(x, \mathbf{b}_\perp)$$

$$\langle b_\perp^2 \rangle^q_{(n)} = -4 \frac{A^{q\prime}_{n0}(0)}{A^q_{n0}(0)}$$

$$\lim_{x \to 1} q(x, \mathbf{b}_\perp) \propto \delta(b_\perp^2)$$

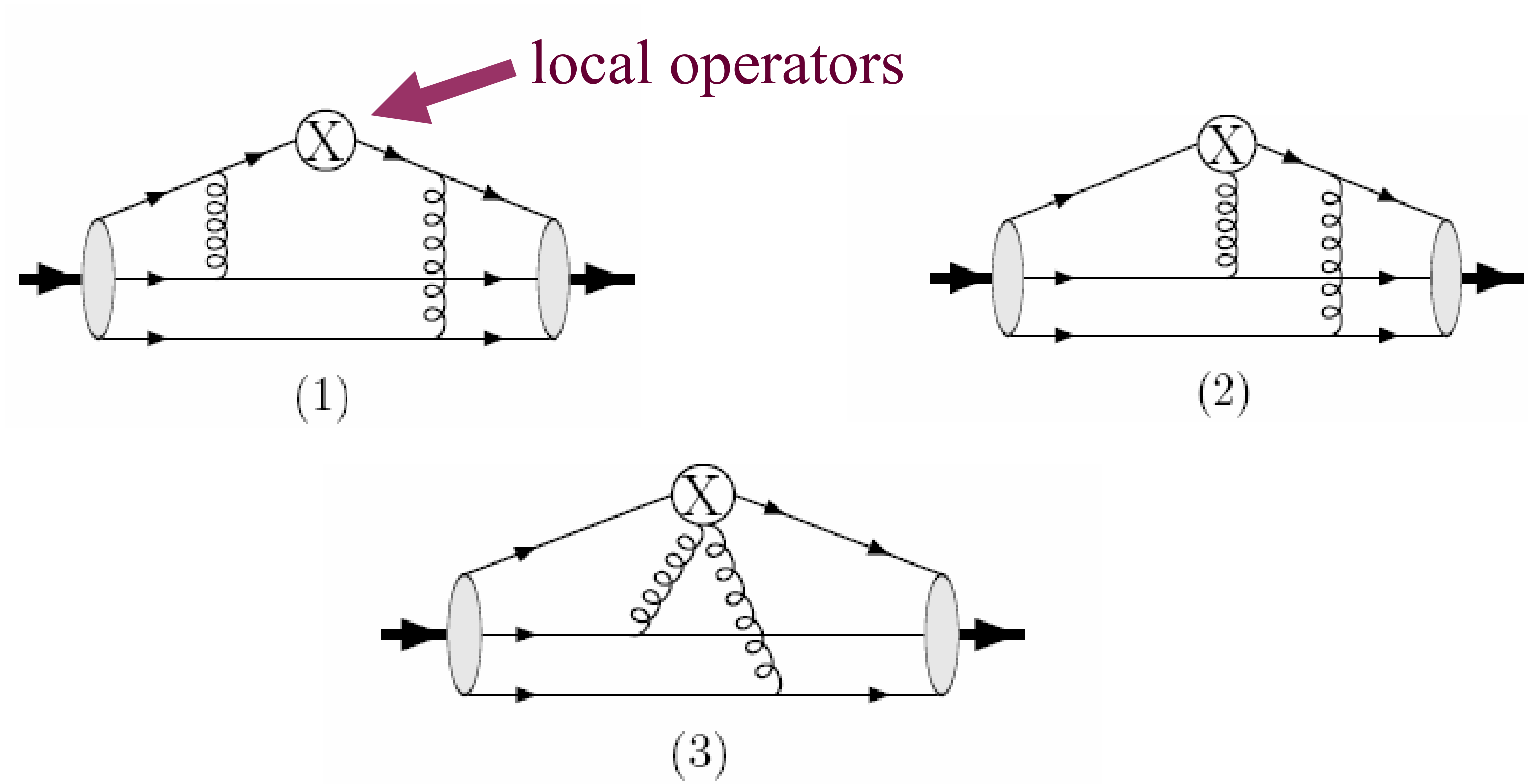
- Higher moments A_{n0} weight $x \sim 1$.
- Slope of A^q_{n0} decreases as n increases.
- Slope of $A^{u-d}_{10}(0) =$ -0.93(4) (GeV)^2.
- Slope of $A^{u-d}_{30}(0) =$ -0.13(3) (GeV)^2.
- Will this continue at light pion masses?



D. Renner (LHPC/SESAM)

Fleming

The lattice gives no intuition. Can we do build models in some sensible limit ?



$$H_q(x,\xi,t) \;=\; \int [dx][dy]\Phi_3^*(y_1,y_2,y_3)\Phi_3(x_1,x_2,x_3)T_{Hq}(x_i,y_i,x,\xi,t)\;,$$

GPDs - Experimental Aspects

- DVCS measured at HERA (at H1 and Zeus)
- DVCS measured at JLab (fixed target,CLAS)
- DVCS planned at COMPASS, CERN

- DVMP measured at HERA
- DVMP measured at JLab
- DVMP measured (old data, 2002) at COMPASS

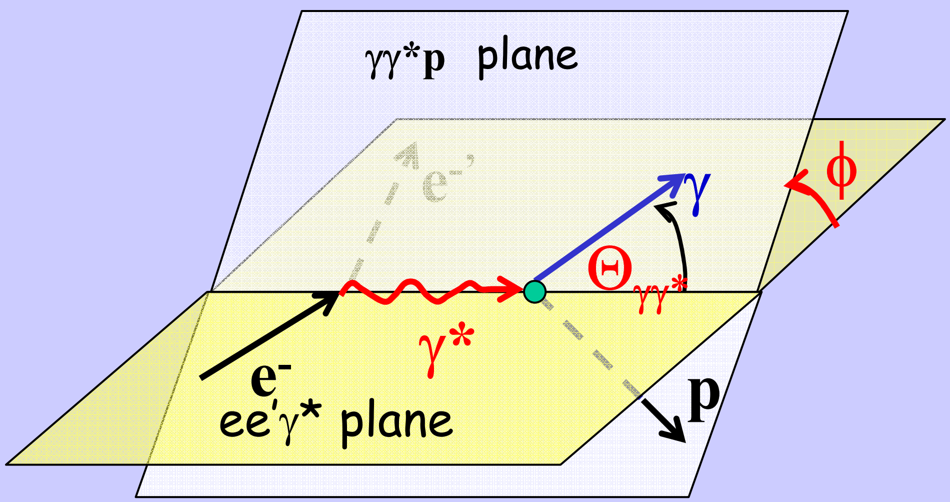
- DDVCS planned at JLab

Some Generalities

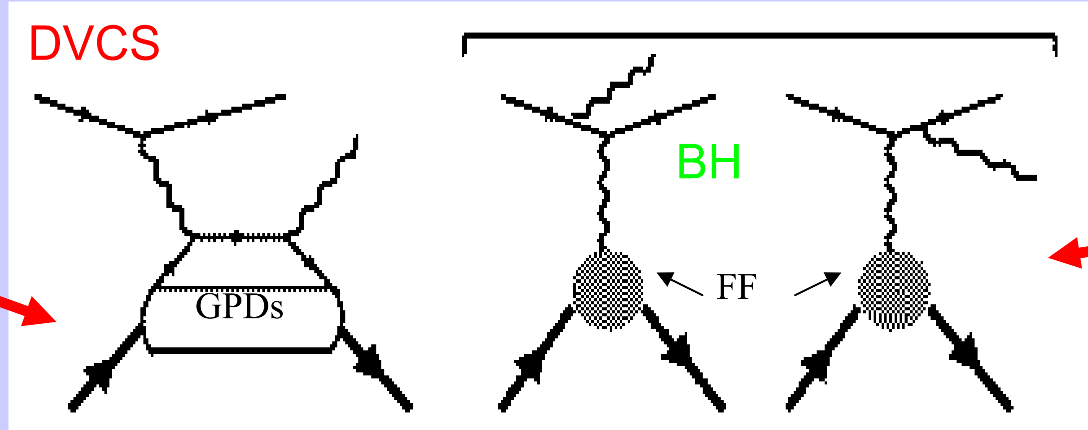
$$\frac{1}{x-\xi+i\varepsilon} = P\left(\frac{1}{x-\xi}\right) - i\pi\delta(x-\xi)$$

$$\Rightarrow \mathrm{Im}\{F\} = \pi\sum e_q^2 \left\{F^q(\xi,\xi,t,Q^2)\,\mathrm{m}F^q(-\xi,\xi,t,Q^2)\right\}$$

$$\mathrm{Re}\{F\} = -\sum e_q^2 P\int_{-1}^{+1} dx F^q(x,\xi,t,Q^2)\left\{\frac{1}{x-\xi} \pm \frac{1}{x+\xi}\right\}$$



small signal



big noise

$$A_{LU}(\phi) = \frac{d\overrightarrow{\sigma}(\phi) - d\overleftarrow{\sigma}(\phi)}{d\overrightarrow{\sigma}(\phi) + d\overleftarrow{\sigma}(\phi)}$$ (Beam Spin Asymmetry, BSA)

$$A_{C}(\phi) = \frac{d\sigma^{+}(\phi) - d\sigma^{-}(\phi)}{d\sigma^{+}(\phi) + d\sigma^{-}(\phi)}$$ (Beam Charge Asymmetry, BCA)

find that:

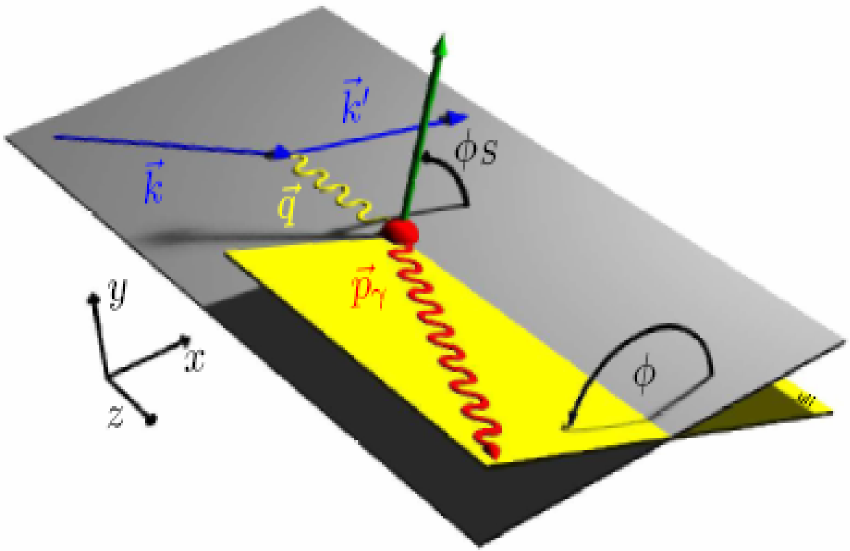
$A_{LU}(\phi) \propto \mathrm{Im}(\tilde{M})\sin\phi$ and $A_{C}(\phi) \propto \mathrm{Re}(\tilde{M})\cos\phi$

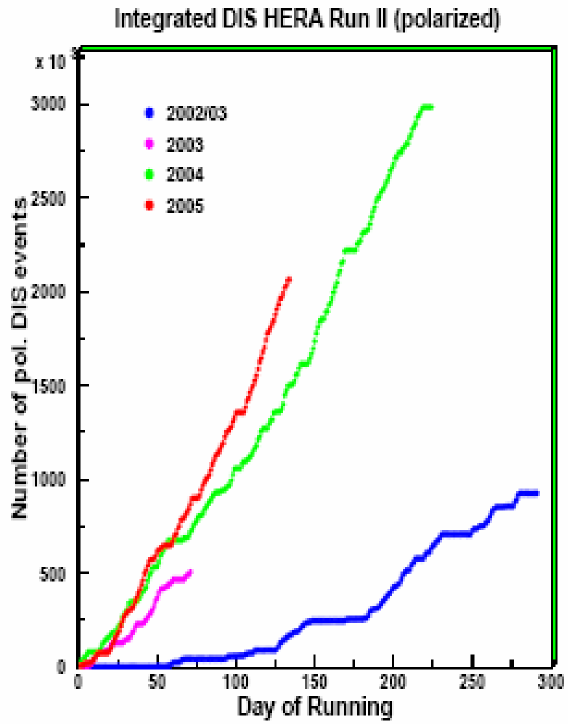
where: $$\tilde{M} = \frac{\sqrt{t_0 - t}}{2m}\left[F_1 \mathrm{H} + \xi(F_1 + F_2)\tilde{\mathrm{H}} - \frac{t}{4m^2}\mathrm{E}\right]$$

What about the GDP E ?

A_{UT}: unpolarized beam,
transversely pol. target

Data taking with transverse Hydrogen target in progress ...
$\approx$ 6 million on tape





$$A_{UT}^{\sin(\phi-\phi_s)\cos\phi} \sim \frac{-t}{4M_p}(F_2 H_1 - F_1 E_1)$$

$$A_{UT}^{\cos(\phi-\phi_s)\sin\phi} \rightarrow \frac{-t}{4M_p}(F_2 \widetilde{H}_1 - \xi F_1 \widetilde{E}_1)$$

$\approx$ 8 Mio expected in total (November 2005)

Ellinghaus

JLAB progress

GPD	Reaction	Obs.	Expt		Status	
$H(\pm\xi,\xi,t)$	ep→epγ (DVCS)	BSA	CLAS	4.2 GeV	Published PRL	*From*
			CLAS	4.8 GeV	Preliminary	*$ep \to epX$*
			CLAS	5.75 GeV	Preliminary	
		(+ σ)	Hall A	5.75 GeV	Fall 04	*Dedicated set-up*
			CLAS	5.75 GeV	Spring 05	
$\widetilde{H}(\pm\xi,\xi,t)$	**ep→epγ (DVCS)**	TSA	CLAS	5.65 GeV	Preliminary	
$E(\pm\xi,\xi,t)$	**e(n)→enγ (DVCS)**	BSA	Hall A	5.75 GeV	Published	
$(u+d)$	**ed→edγ (DVCS)**	BSA	CLAS	5.4 GeV	under analysis	
$H(\|x\|<\xi,\xi,t)$	**ep→epe$^+$e$^-$ (DDVCS)**	BSA	CLAS	5.75 GeV	under analysis	
$\int_x H,E \quad (u+d)$	**ep→epρ**	σ_L	CLAS	4.2 GeV	Published PLB	
			CLAS	5.75 GeV	under analysis	
$\int_x H,E \quad (2u-d)$	**ep→epω**	(σ_L)	CLAS	5.75 GeV	Accepted EPJA	

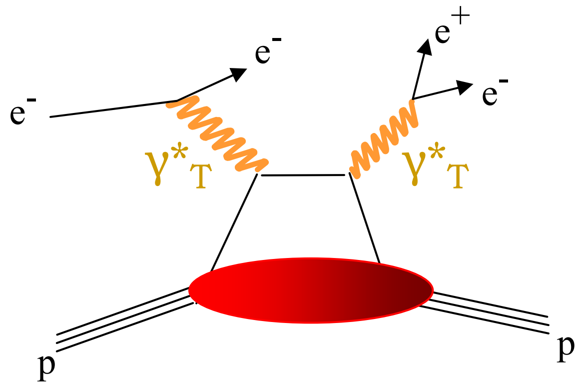
+ other meson production channels π, η, Φ under analyses in the three Halls.

New Ideas For Investigating GPDs

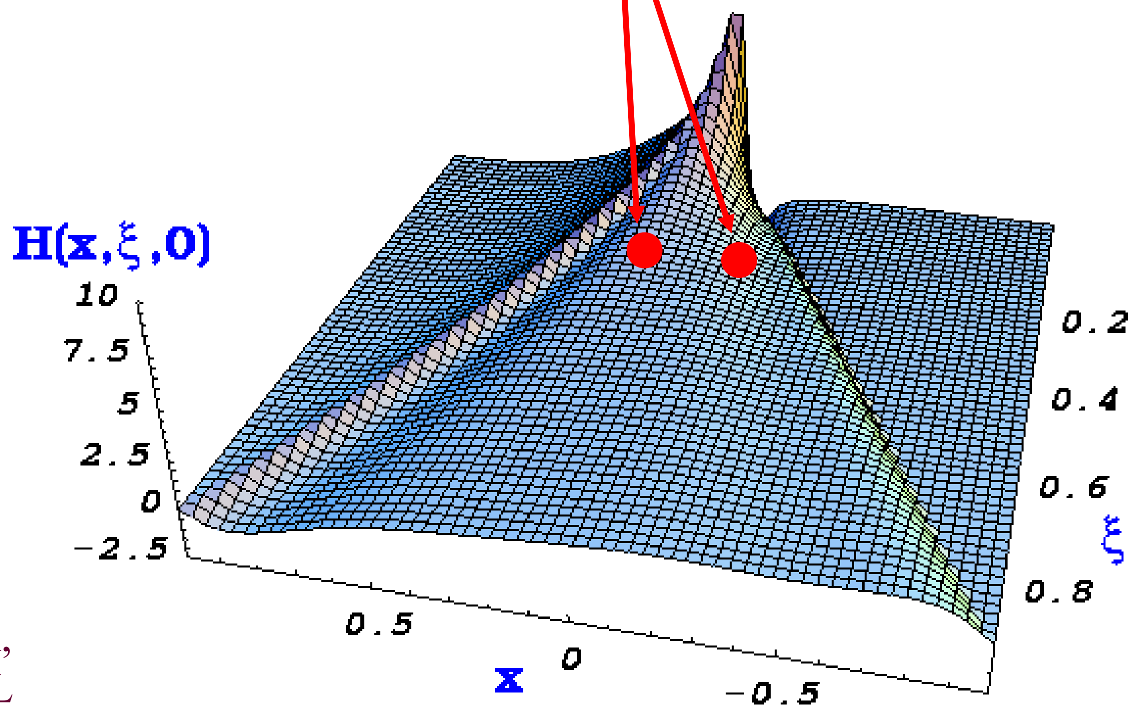
DDVCS
(Double Deeply Virtual Compton Scattering)

DDVCS-BH interference generates a **beam spin asymmetry** sensitive to

$$\mathrm{Im}\,T^{DDVCS} \sim H(\pm x(\xi, q'), \xi, t) + \mathrm{K}$$

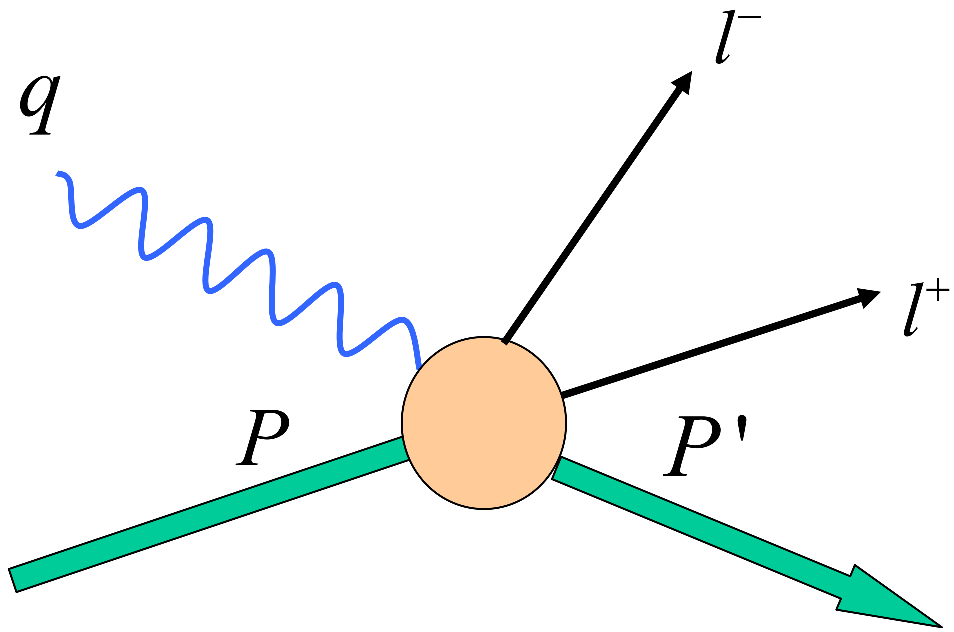


The (continuously varying) virtuality of the outgoing photon allows to "tune" the kinematical point (x,ξ,t) at which the GPDs are sampled (with |**x** | < ξ).



M. Guidal & M. Vanderhaeghen,
A. V. Belitsky & D. Müller, PRL

Using real photons to probe proton structure.



Large scale provided by $t = (P - P')^2$

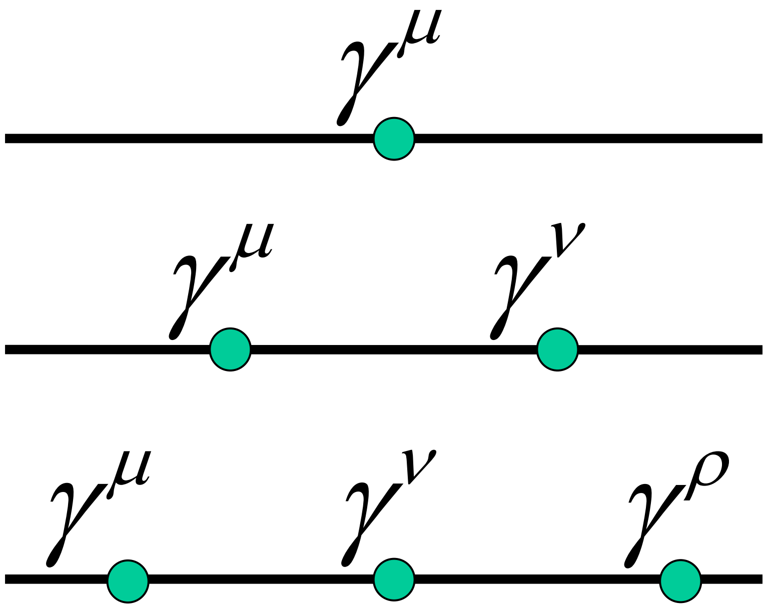
PH – to be published in PRD, April 2006

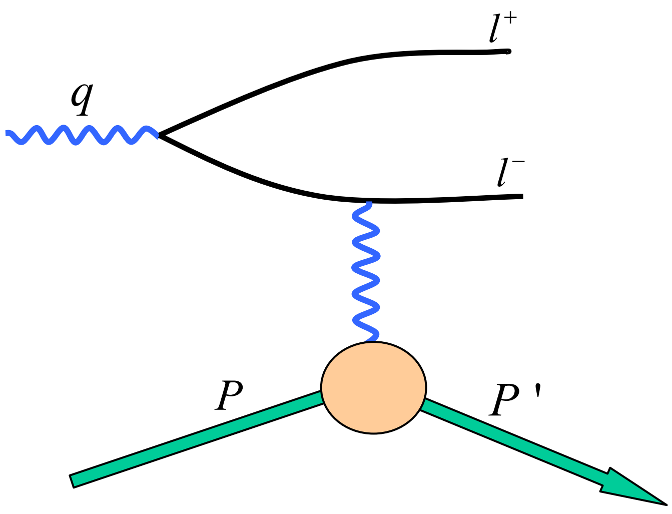
$$C\gamma^\mu C^{-1} = -\gamma^\mu$$

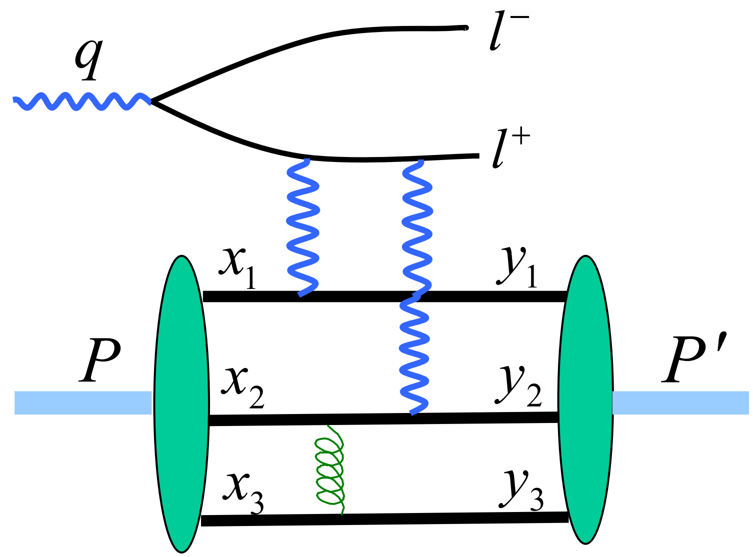
$$C\gamma^\mu\gamma^\nu C^{-1} = +\gamma^\mu\gamma^\nu$$

$$C\gamma^\mu\gamma^\nu\gamma^\rho C^{-1} = -\gamma^\mu\gamma^\nu\gamma^\rho$$

etc.







Large and calculable.
Gives zero asymmetry.

Calculable (72 diagrams) but
small. Also gives zero asymmetry.

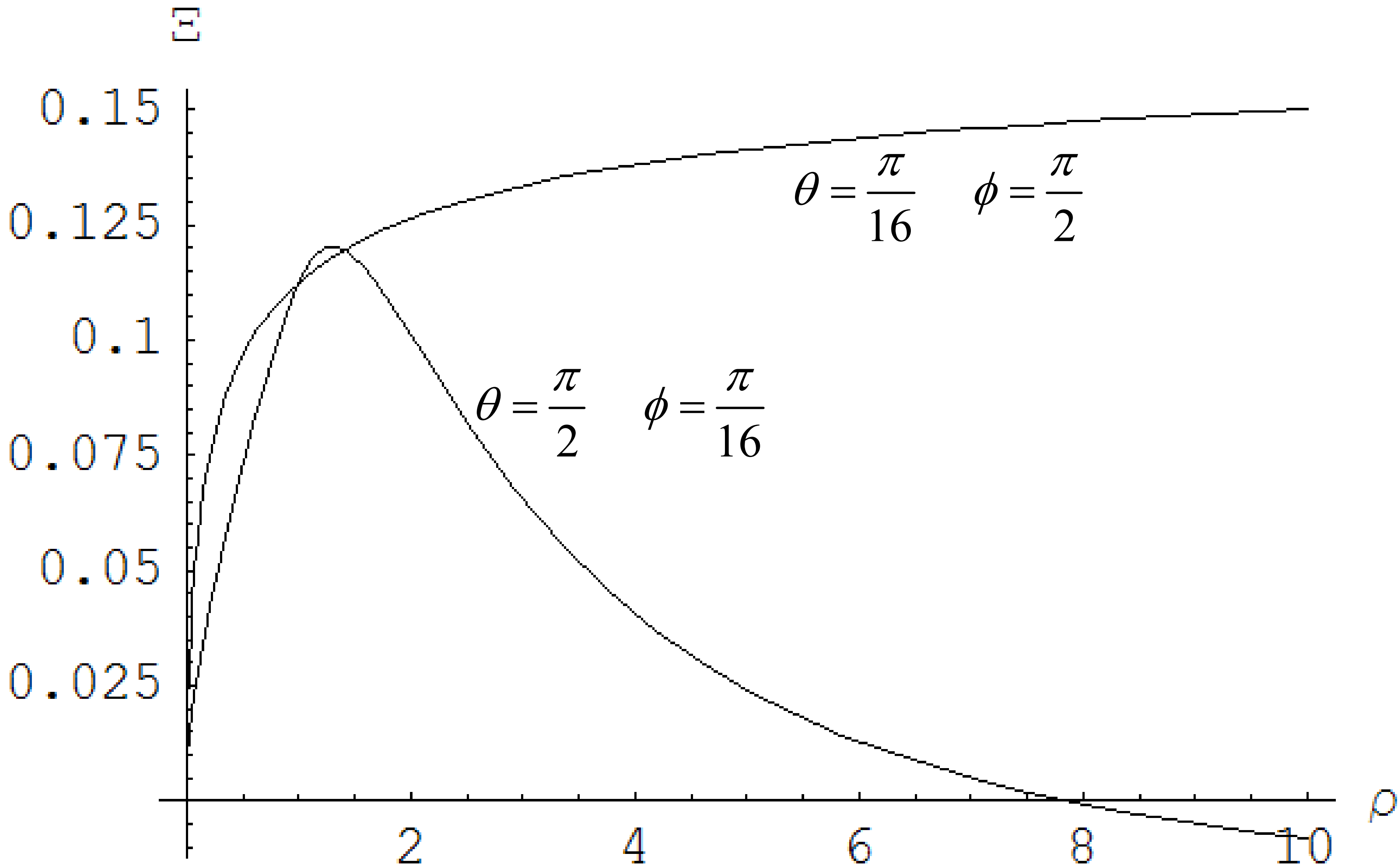
Interference of these two is the key !

$$\Theta_N(\theta,\phi) = \frac{\sum_{\gamma,s}(\mathrm{A}_1^{\gamma,s}\mathrm{A}_2^{*\gamma,s} + \mathrm{A}_1^{*\gamma,s}\mathrm{A}_2^{\gamma,s})}{\sum_{\gamma,s}\left|A_1^{\gamma,s}\right|^2}$$

$$= \frac{64\pi^2\alpha_s e_1}{(-t)^2 G_M(t)}\left(\cos\theta + e^{-i\phi}\frac{M}{\sqrt{-t}}\sin\theta\right)\times$$

$$\int[dx][dy]\frac{\left(e_2x_2y_2 + e_3x_3y_3\right)\Phi(x_1,x_2,x_3)\Phi(y_1,y_2,y_3)x_1\delta(x_1 - y_1)}{x_1x_2x_3y_1y_2y_3(y_1\bar{y}_1 + \Lambda)\bar{y}_1{}^2}$$

- $\mathrm{G}_M(t) \to (-t^{-2})$ at large t.



Ξ
0.15
0.125
0.1
0.075
0.05
0.025
$\theta = \frac{\pi}{16} \quad \phi = \frac{\pi}{2}$
$\theta = \frac{\pi}{2} \quad \phi = \frac{\pi}{16}$
2
4
6
8
10
ρ

- Real photons are used to probe nucleon structure.
- Real photons are easily available at many labs.
- At large-t the proton structure is much simpler.
- The expression for the asymmetry is very compact.
- The size of the signal is large at modest –t.
- Only F_1 form-factor considered here: F_2 involves spin-flip which is zero for massless, collinear quarks.

OPEN QUESTIONS

- How big will Sudakov effects be?

- Will the next order calculation (few thousand diagrams!) change the angular structure?

- Will it dominate the present calculation?

GPD CHALLENGES

- Goal: map out the full dependence on x, ξ, t, Q^2
- Develop models consistent with known forward distributions, form factors, polynomiality constraints, positivity,…
- More lattice moments, smaller pion masses, towards unquenched QCD,…
- Launch a world-wide program for analyzing GPDs perhaps along the lines of CTEQ for PDFs.
- High energy, high luminosity is needed to map out GPDs in deeply virtual exclusive processes such as DDVCS (JLab with 12GeV unique).

HOW TO TEACH SCIENCE

by

Arvind Gupta (IUCAA, India)

on

Monday, April 3, 2006

- Physics Auditorium, Physics Department, QAU.
- Vice-Chancellor's introduction: 1:25pm-1:30pm
- Presentation by Arvind Gupta: 1:30pm-2:15pm
- Questions and Answers: 2:15pm-2:45pm