Electrical Properties of the Fair-weather Atmosphere and the Possibility of Observable Discharge on Moving Objects

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Fair-weather Electric Field

For the remote sensing of aircraft and rockets traversing the atmosphere, the interaction of these objects with ambient electromagnetic fields should be considered, and whether observable effects exist. A fair-weather electric field is observed to exist at the earth's surface, downward pointing and with a global mean average of around 100 V/m.

$$\mathbf{E}(r = R_{\oplus}) \equiv \mathbf{E}_0 = -100\ \hat{r}$$

In the previous century Kelvin suggested a model of the earth's atmosphere as a giant capacitor, with the ionosphere and ground as its plates. The ionosphere is held at positive charge, the ground negative. This simple picture remains useful for organizing observations.

To understand the variation with height of the electrical properties between ground and ionosphere, consider the conductivity of the intervening atmosphere. The atmospheric density decreases exponentially with height h: $n \propto e^{-h/H_0}$, where H_0 is about 8 km. Because the atmosphere is chiefly neutral at the surface and for 10s of kms above, the conductivity in this region will be dominated by collisions with neutral molecules. Since scattering cross section scales inversely with density, and the conductivity is proportional to scattering cross section, conductivity σ will increase exponentially with height:

$$\sigma \propto e^{h/H_1}$$

where H_1 is about 5 km. $H_1 < H_0$ because cosmic rays contribute to the ionization of the upper atmosphere, thereby increasing the conductivity in the upper regions, and shortening the conductive scale height. In magnitude the conductivity near the ground is around 10^{-14} mks, but increases to around 10^{-2} by 100 km height, an increase of 12 orders of magnitude. The conductivity in the ground is also around 10^{-2}. Thus the Kelvin picture of the atmosphere as a capacitor bound by highly conductive plates is an appropriate one.

With a finite conductivity and an electric field, current must flow. In steady state, this current must be constant with height; otherwise charge would build up to catastrophic levels at one height or another, contradicting the assumption of steady state. Invoking Ohm's law for constant current density $J = \sigma E$ (ignore vector behavior of E, and consider only vertical currrents and fields):

$$E \propto \mathrm{e}^{-h/H_1}$$

The electric field *decreases* exponentially with height, and the strongest field magnitude occurs right at the surface.

Now, since the electric field is varying exponentially with height, it will have a finite divergence. And Gauss' law tells us that a divergence in the electric field implies a charge density ρ: $dE/dh = \rho/\epsilon_0$. Therefore we also have a net space charge density existing in the atmosphere which vanishes approaching the ionosphere, and increases exponentially toward the ground. At the ground, $\rho_0 \equiv E_0\epsilon_0/H_1 \simeq 2 \times 10^{-13}$ C/m^3. The sign of the charge density works out to be positive; the upper plate of our capacitor is effectively spread through the atmosphere. The highly conducting regions near the ionosphere 'boundary' carry little charge. It is instead concentrated in the dielectric of the atmosphere near the negative plate of the earth surface.

Let $Q_\oplus$ be the total charge on the earth's surface, and $Q_\oplus/4\pi R_\oplus^2$ be the average surface charge density. Since the height of the ionosphere, $\sim$ 100 km, is much less than the earth radius of curvature, we can use the flat plate expression to give the surface field strength in terms of the surface charge density: $E_0 = Q_\oplus/(4\pi R_\oplus^2 \epsilon_0)$, and $E = E_0 \mathrm{e}^{-h/H_1}$. The potential drop $V_\oplus$ is:

$$V_\oplus \equiv \frac{Q_\oplus}{C_\oplus} = \int_0^{100\ \mathrm{km}} E dh \simeq H_1 E_0 \simeq 5 \times 10^5\ \mathrm{V}$$

The upper limit of the integral is arbitrarily picked to represent where the electric field drops to zero. There is a voltage drop of half a million volts between the ionosphere and ground. The global capacitance $C_\oplus \sim$1 farad. The surface charge density is about 9×10^{-10} C/m^2, or 6×10^5 electrons per square cm.

The current $I_\oplus = \sigma_0 E_0 4\pi R_\oplus^2$ flowing through this global capacitor is about 10^3 amps; with current density around 10^{-12} amps/m^2. The power dissipation $P_\oplus$ by the global circuit $\equiv I_\oplus V_\oplus \sim$ 500 MW.

A cylinder of clear air, 20 km in radius, from sea level to 10 km has about 400 MΩ of resistance. Between 10 km and the ionosphere, the resistance of the same column is perhaps 40 MΩ. A cylinder of cloudy air, by comparison, would comprise 1 GΩ of resistance between sea level and 10 km. Yet with these large values, the global average resistance $R_\oplus = V_\oplus / I_\oplus$ is only about 500 Ω. Its large deviation from the clear air values may be attributed to orographic features, variations in conductivity due to ocean salinity, etc. Therefore the time constant for the discharge of this potential is $2\pi R_\oplus C_\oplus \simeq 1200$ s. With such a short discharge time, some entity must constantly maintain the potential. It is thought that thunderstorm activity is the primary mechanism, although magnetospheric activity and atmospheric tides can also contribute. So thunderstorms do not discharge the global charge separation, they drive it.

The primary evidence that this is so is the Carnegie Curve, a plot of variously the electric field or current measured at the earth surface as a function of time of day. The surface fields show global diurnal variations of about 15%. These diurnal variations apparently mirror the activity of thunderstorms; both the thunderstorms and the fields peak at the same absolute time of day, when thunderstorm activity peaks over the Amazon. Even though an observer might be in a time zone with little thunderstorm activity, the high conductivities of the ionosphere and ground ensure that the effects of the sourcing thunderstorms will be communicated instantaneously to all parts of the globe. Individual thunderstorms exhibit a charge separation which mirrors the global capacitor, with positive high and negative low. However the cloud bottom can be much lower in potential than the ground, and thus the strike. The excess charge then travels away over the highly conducting ground. Potential differences between cloud bottom and ground can be 10^7 V. One lightning strike may release 20 coulombs of charge, and the cloud can recharge fast enough to strike every 5 seconds, thus delivering 4 amps of current. Since the charge carriers are negative electrons flowing out of the cloud, the current is measured as flowing upward. This global current then flows out over the ionosphere, and back to the ground through fair weather.

Typical Parameters of the Global Atmospheric Electric Environment

surface electric field E_0	100 V
global current $I_\oplus$	10^3 A
global power dissipated $P_\oplus$	500 MW
global resistance $R_\oplus$	500 Ω
global capacitance $C_\oplus$	1 f
global time constant	1200 s
atmospheric conductivity at surface σ_0	10^{-14} mks
conductivity at 100 km	10^{-2} mks
conductivity of ground $\sigma_\oplus$	10^{-2} mks
conductive scale height H_1	5 km
atmospheric charge density at surface ρ_0	10^{-13} C/m^3
charge density of ground	10^{-9} C/m^2

Possible Observable Consequences for Moving Objects

So can you hold out a pair of electrodes and light a bulb? Or would you be electrocuted jumping out of a tree? No to both questions. Trees and human bodies are highly conductive, and we are electrically part of the ground plane. We represent bumps in the surface of equipotential, so that the potential at our feet is the same as at our head. The potential at the top of a tree is the same as at the roots.

For an object to come to equilibrium at a certain height, not connected to the ground, would require transportation of charge either onto or off the object, depending on whether it was raised or lowered in height. If the object is losing mass, the transfer is much more rapid than just conduction through ambient air. An aircraft or rocket will conveniently transfer charge through its exhaust, and gradually change its potential as it climbs. A reentering object can transfer charge through any ablation associated with its reentry. For a truly isolated object moving through the atmosphere, charging and catastrophic discharge may occur depending on the relative timescales of its motion and conduction through the air. It is conceivable a large charge may be accumulated by such an object, leading to an observable discharge. Such discharges will be most noticable in the lower atmosphere, where the conductivity is relatively small, the charge density large, and larger charges can accumulate on an object.

We saw there is roughly 10^{-9} C $= Q_{1m}$ of charge in a column of 1 square meter. As a body moves through the atmosphere and between regions of differing charge density, it will sweep up charge. The efficiency of this sweeping process is unknown, but for bodies of interest an upper limit would be the total charge in a 1 square meter column; bodies larger and smaller than 1 m would sweep up more or less charge; respectively. If this charge were distributed over a body of size 1 meter, the associated electrostatic potential energy would be $= {Q_{1m}}^2/(4\pi\epsilon_0) \simeq 10^{-9}$ J $= 10^{10}$ eV. This amount of energy could in principle be observable; it would be similar to a miniature lightning flash.

But for this charge to accumulate to catastrophic levels requires that the timescale for the atmosphere to conduct the charge away must be shorter than the timescale for motion through the charging levels of the atmosphere. The speeds of ballistic missiles approach several km per second, implying such an object could traverse the conductive scale height in a matter of seconds. Would the charge leak away during such a fast traversal?

The conductivity of a gas σ can be related to the mean scattering time τ by:

$$\tau = \frac{m\sigma}{\rho q} = \tau_0 e^{2h/H_1}$$

where m and q are the mass and charge of the charge carriers. For the atmosphere, we can use atomic mass 28 (for N_2), and a single quantum of charge. For these values, τ_0 is about 10^{-8} seconds. At 100 km, $\tau = 10^9$ s. The charge carriers evidently go only a short distance near the surface, but travel unimpeded in the ionosphere.

One estimate of the discharge time through the atmosphere would be the time for charge carriers to diffuse out a distance from the object equal to the characteristic size of the object D, taken to be 1 meter. For a random walk, the mean square distance is proportional to the number of collisions N and the mean free path $\lambda = v\tau$, where v may be taken to be the thermal speed $\sqrt{3kT/m}$. The time required for an N-step walk is $t = N\tau$. Thus the time to diffuse a distance D is:

$$t = \frac{D^2 \rho q}{3kT\sigma} = \frac{D^2 \rho_0 q}{3kT\sigma_0} e^{-2h/H_1}$$

At 300 K at the surface, this time is about 10^3 seconds. At 100 km, this time is 10^{-15} s. This is to be compared with the timescale for traversal at speed s: H_1/s. One may then expect catastrophic discharge only for objects travelling faster than about 500 m/s, or about 1000 miles/hr.

Evidently, objects moving as fast as a km/s can accumulate charge at heights below 10 km or so. But objects moving through the upper atmosphere at speeds of kms/s have their charge leaked away as fast as it's accumulated. Unfortunately, the observational effects will manifest best only for objects traversing the lower parts of the atmosphere, where remote sensing may be of least interest.