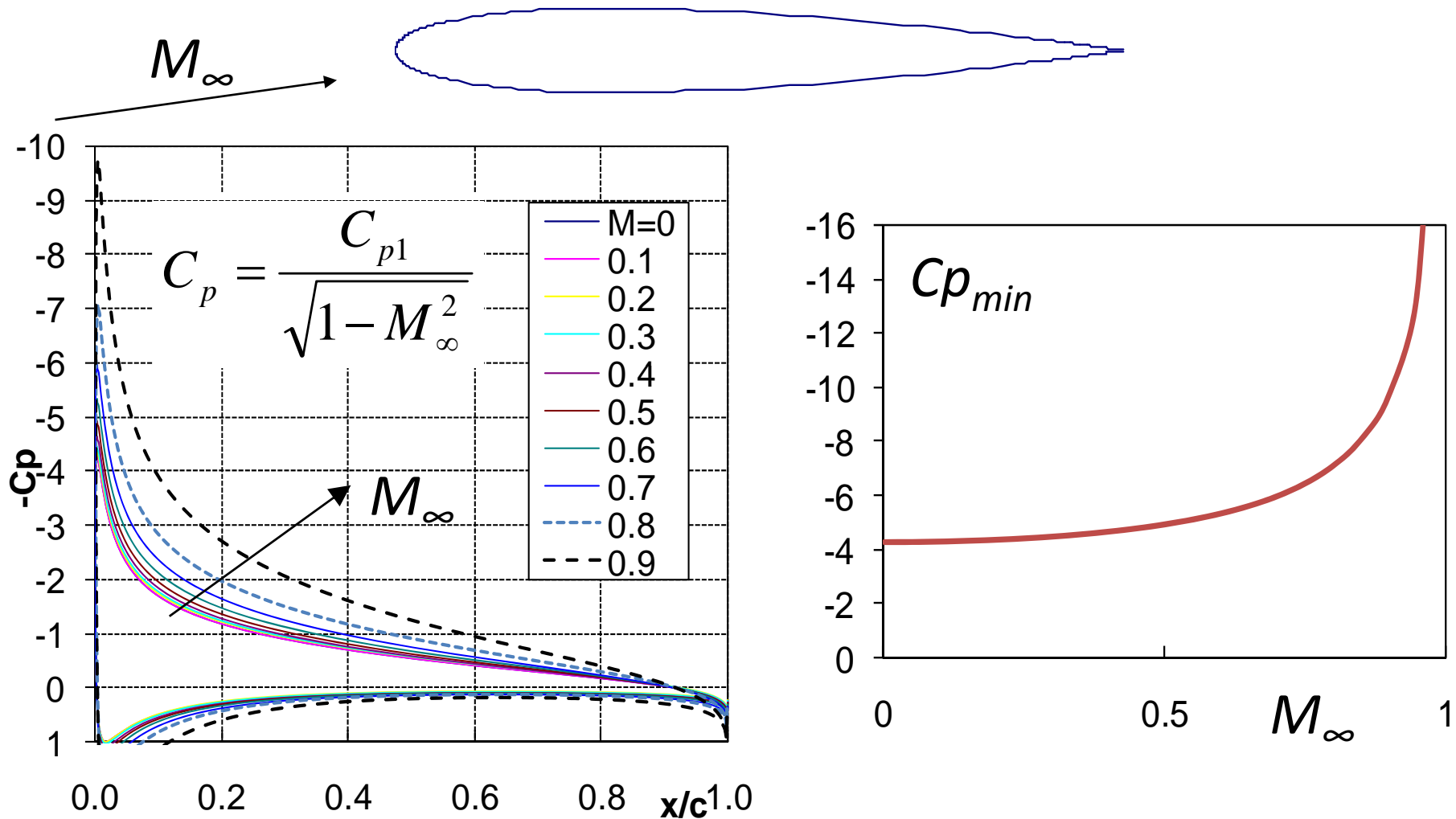


# Transonic Aerodynamics

- Critical Pressure Coefficient and Critical Mach Number
- Drag Divergence
- Mitigating the Drag Problem
  - The supercritical airfoil
  - Sweep
  - The Area Rule

# Limit of the Compressibility Correction

NACA 0012 AT 8 DEGREES



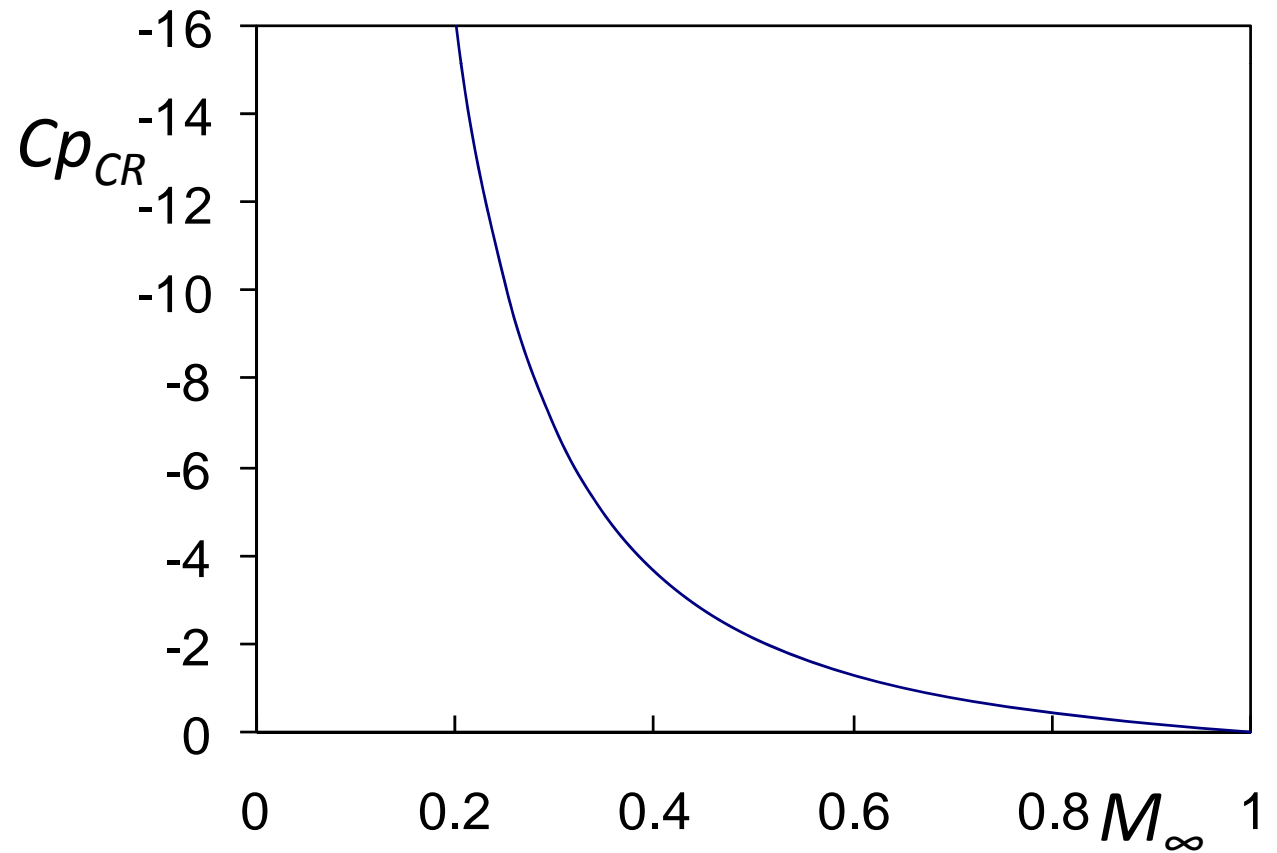
# Critical Pressure Coefficient

*PRESSURE COEFFICIENT WHERE THE LOCAL MACH NUMBER IS 1*

# Critical Pressure Coefficient

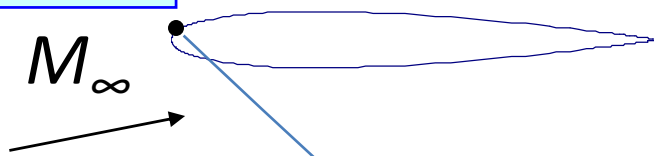
*PRESSURE COEFFICIENT WHERE THE LOCAL MACH NUMBER IS 1*

$$Cp_{CR} = \frac{1}{\frac{1}{2} \gamma M_{\infty}^2} \left[ \left( \frac{1 + \frac{1}{2} (\gamma - 1) M_{\infty}^2}{\frac{1}{2} (\gamma + 1)} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right]$$

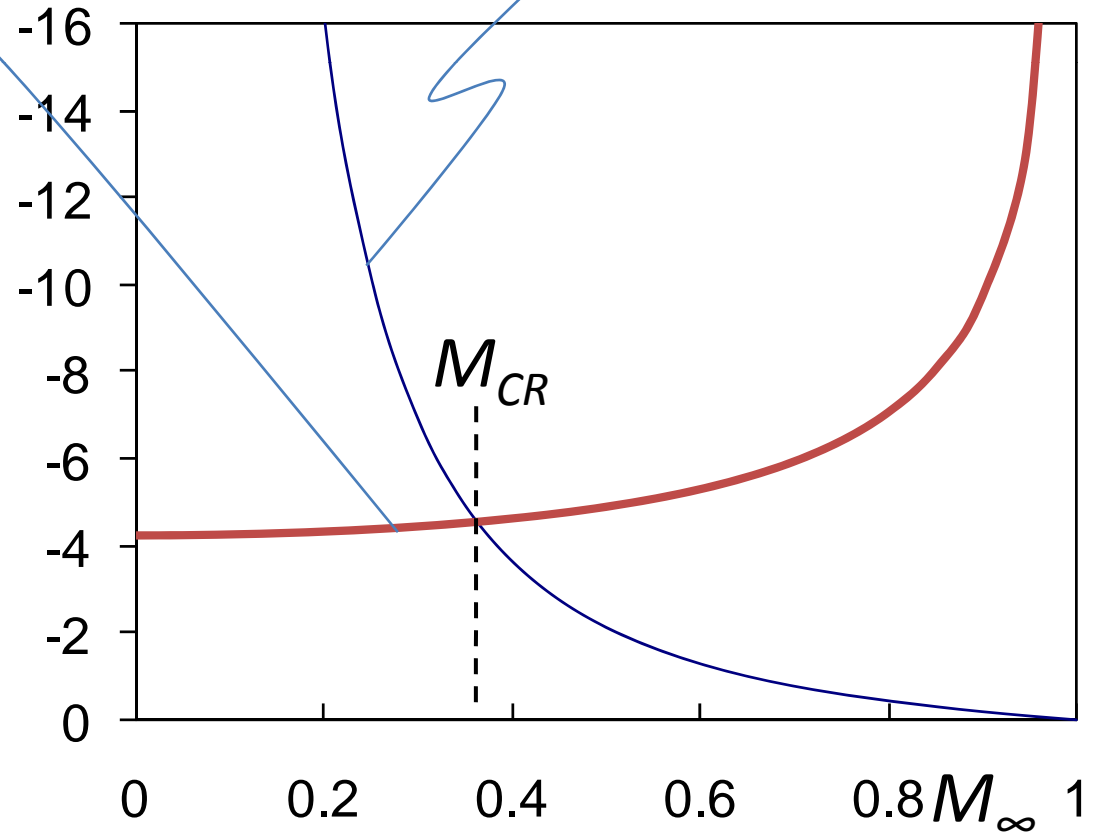


# Critical Mach Number

$$Cp_{\min} = \frac{Cp_{1_{\min}}}{\sqrt{1 - M_{\infty}^2}}$$



$$Cp_{CR} = \frac{1}{\frac{1}{2}\gamma M_{\infty}^2} \left[ \left( \frac{1 + \frac{1}{2}(\gamma - 1)M_{\infty}^2}{\frac{1}{2}(\gamma + 1)} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right]$$



# Critical Mach Number $M_{CR}$

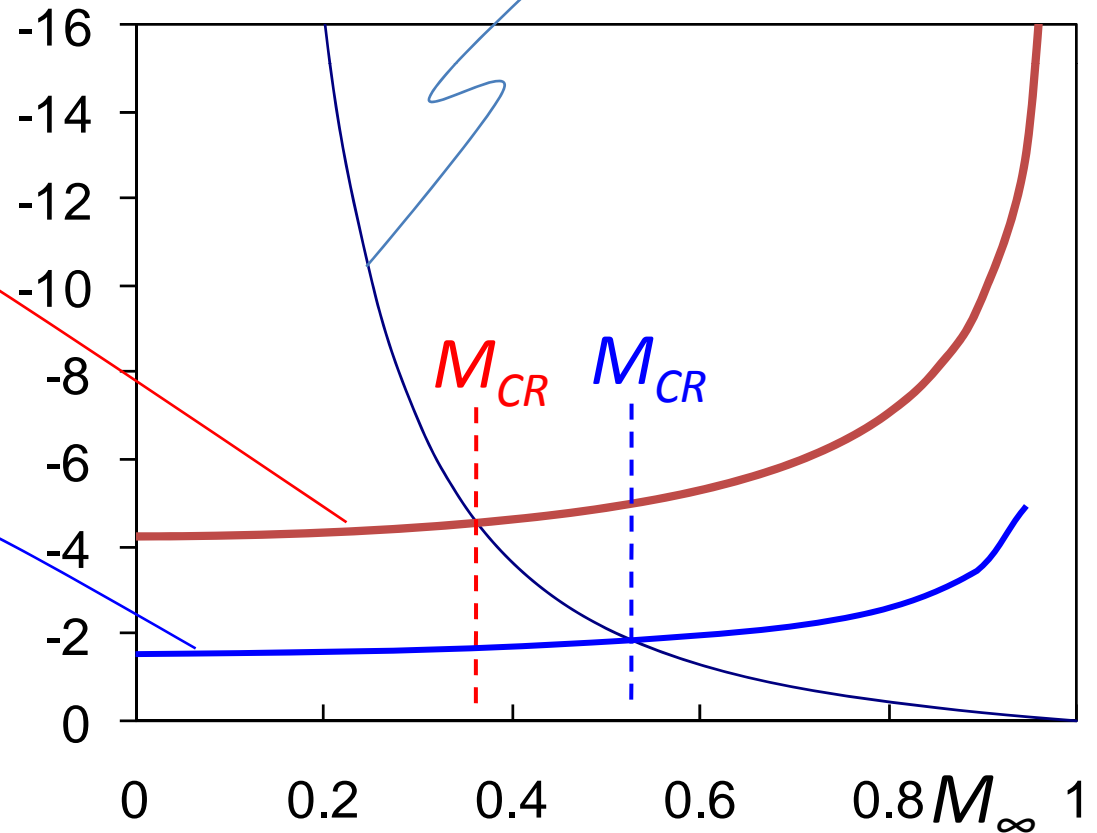
$$Cp_{\min} = \frac{Cp_{1_{\min}}}{\sqrt{1 - M_{\infty}^2}}$$



$$Cp_{CR} = \frac{1}{\frac{1}{2}\gamma M_{\infty}^2} \left[ \left( \frac{1 + \frac{1}{2}(\gamma - 1)M_{\infty}^2}{\frac{1}{2}(\gamma + 1)} \right)^{\frac{\gamma}{\gamma - 1}} - 1 \right]$$

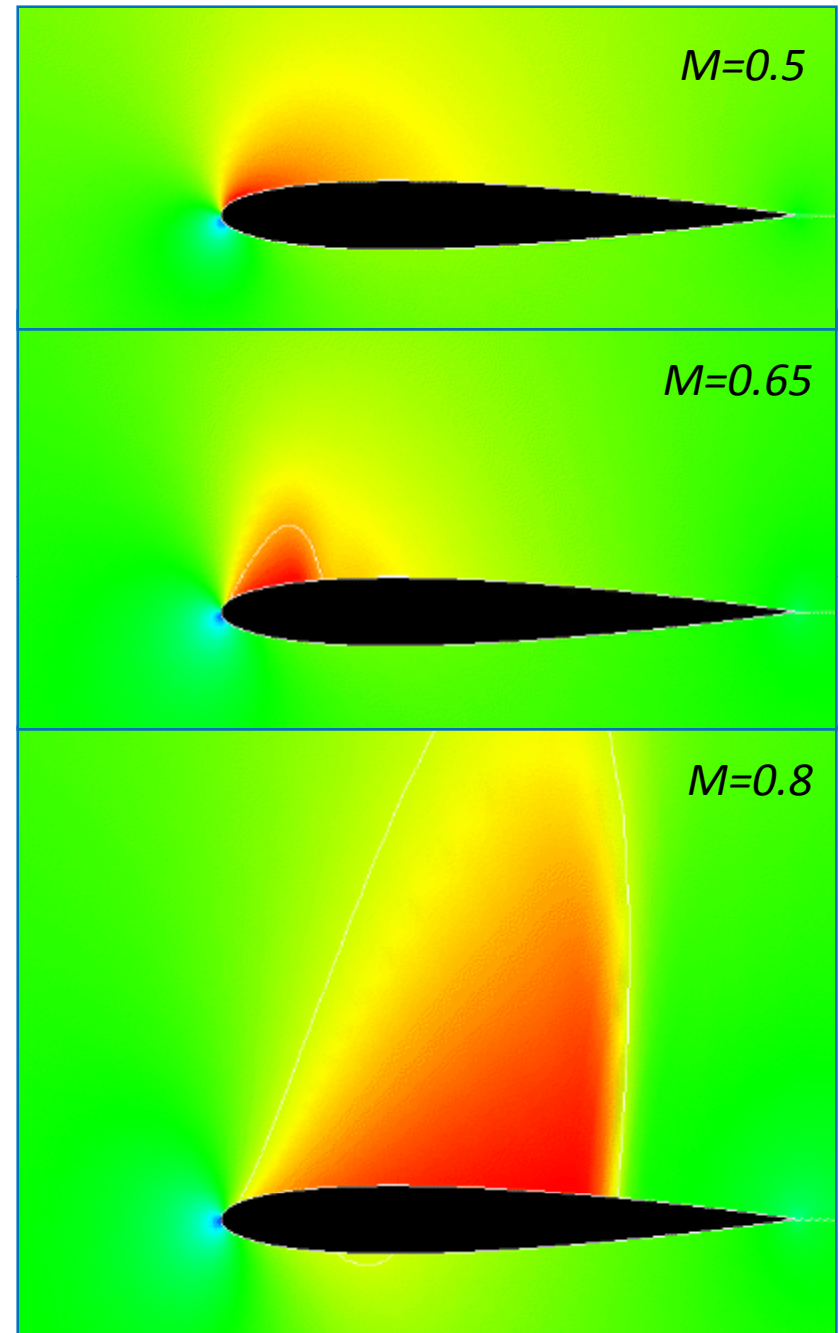
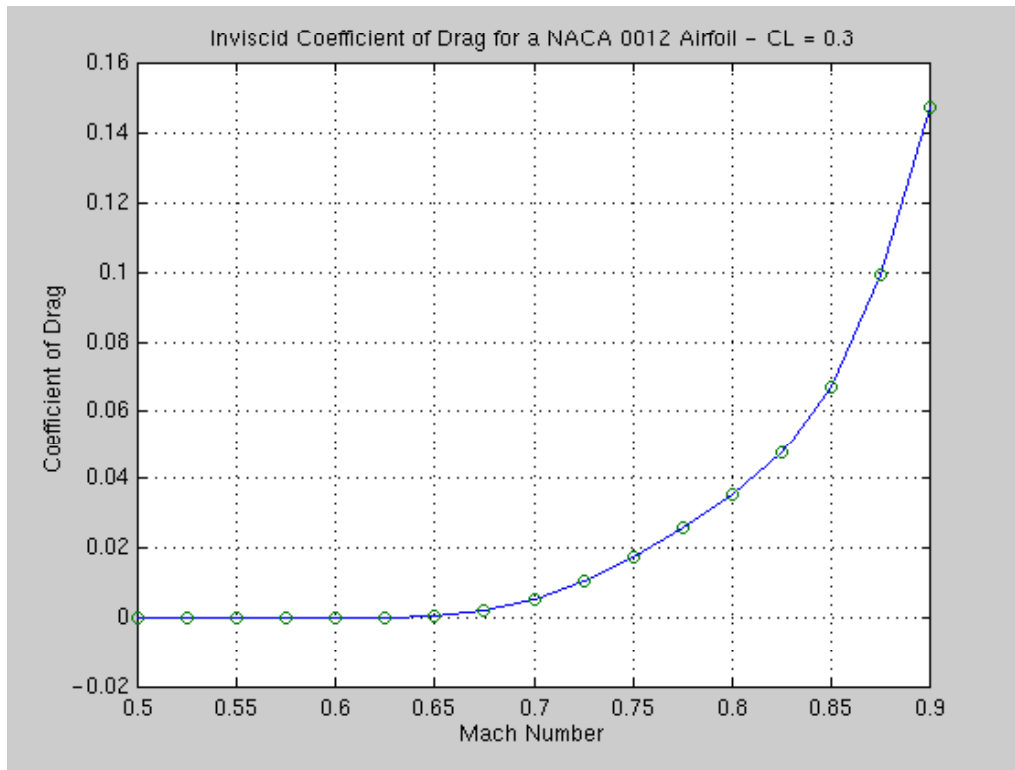
*0012 at 8 deg.*

*0012 at 4 deg.*



# Drag Divergence

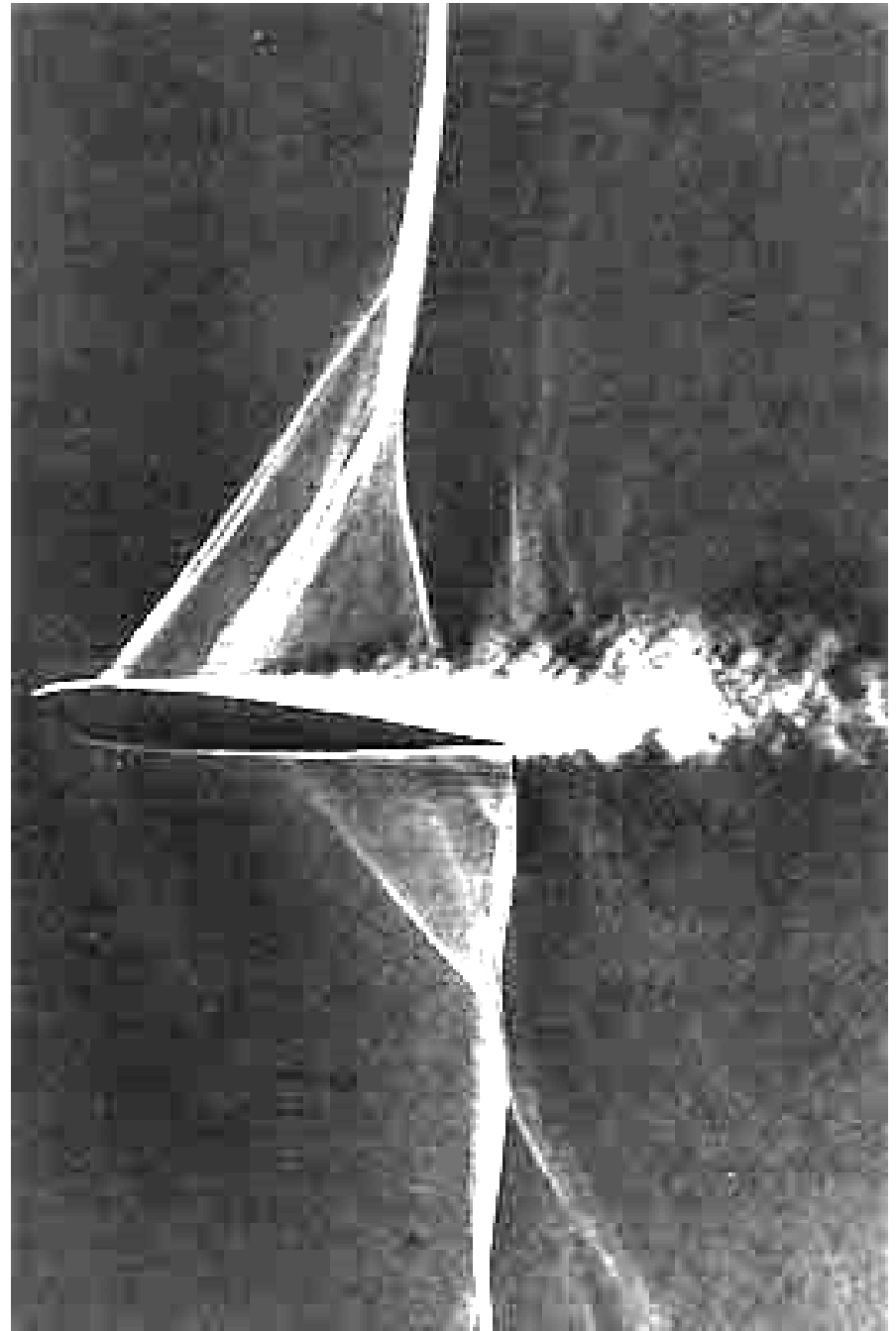
*INVISCID CALCULATION*



Ilan Kroo and J Alonso, Stanford University  
<http://adg.stanford.edu/aa241/drag/dragrise.html>

# Shock Induced Separation

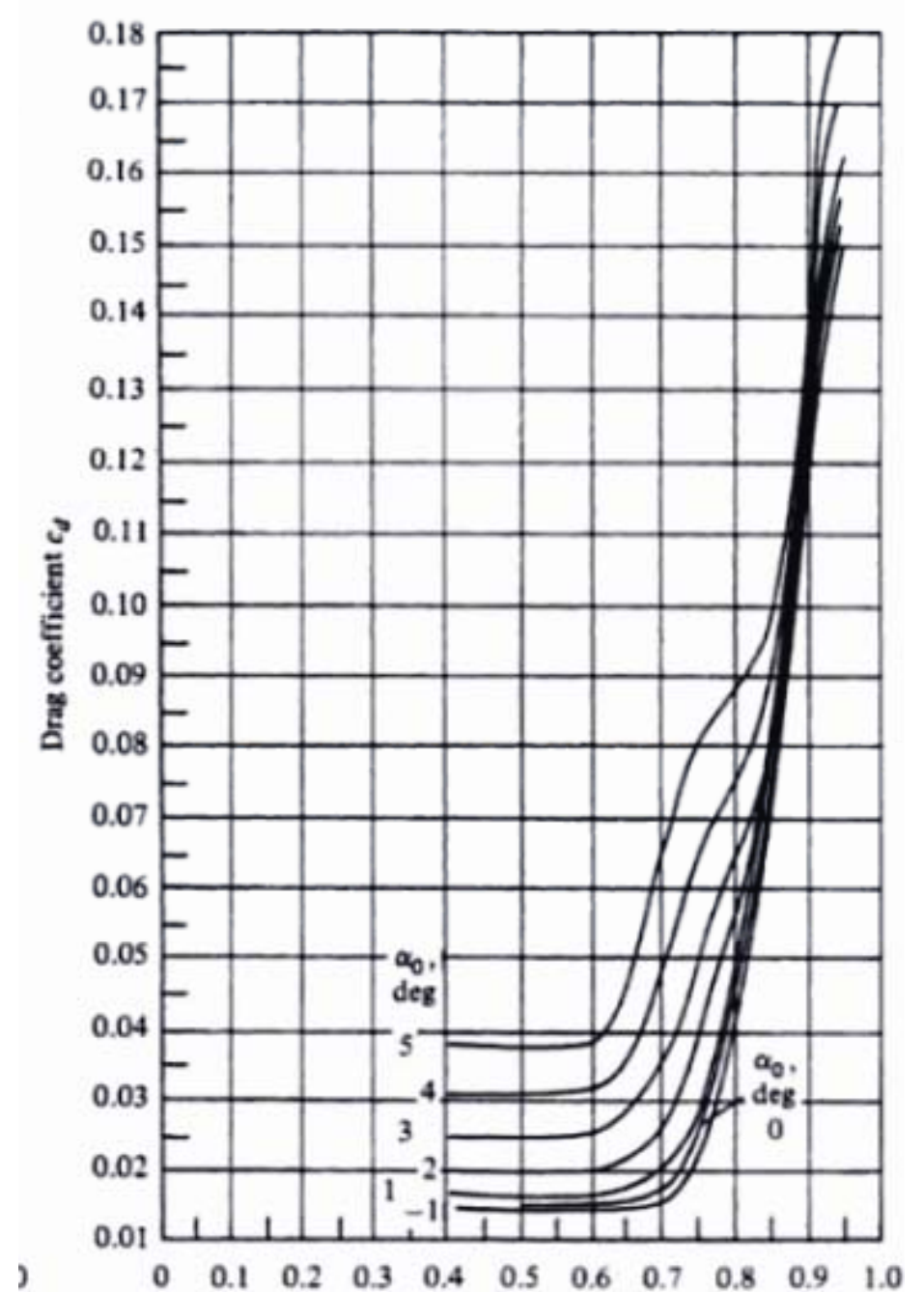
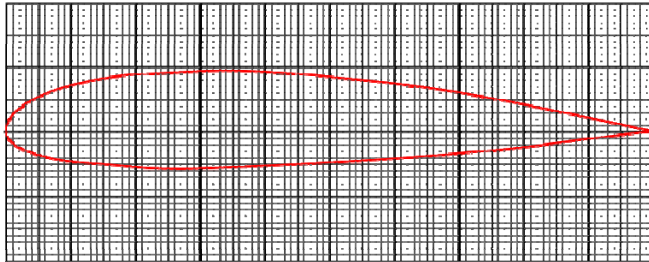
[www.hq.nasa.gov/pao/History/SP-440/ch7-2.htm](http://www.hq.nasa.gov/pao/History/SP-440/ch7-2.htm)





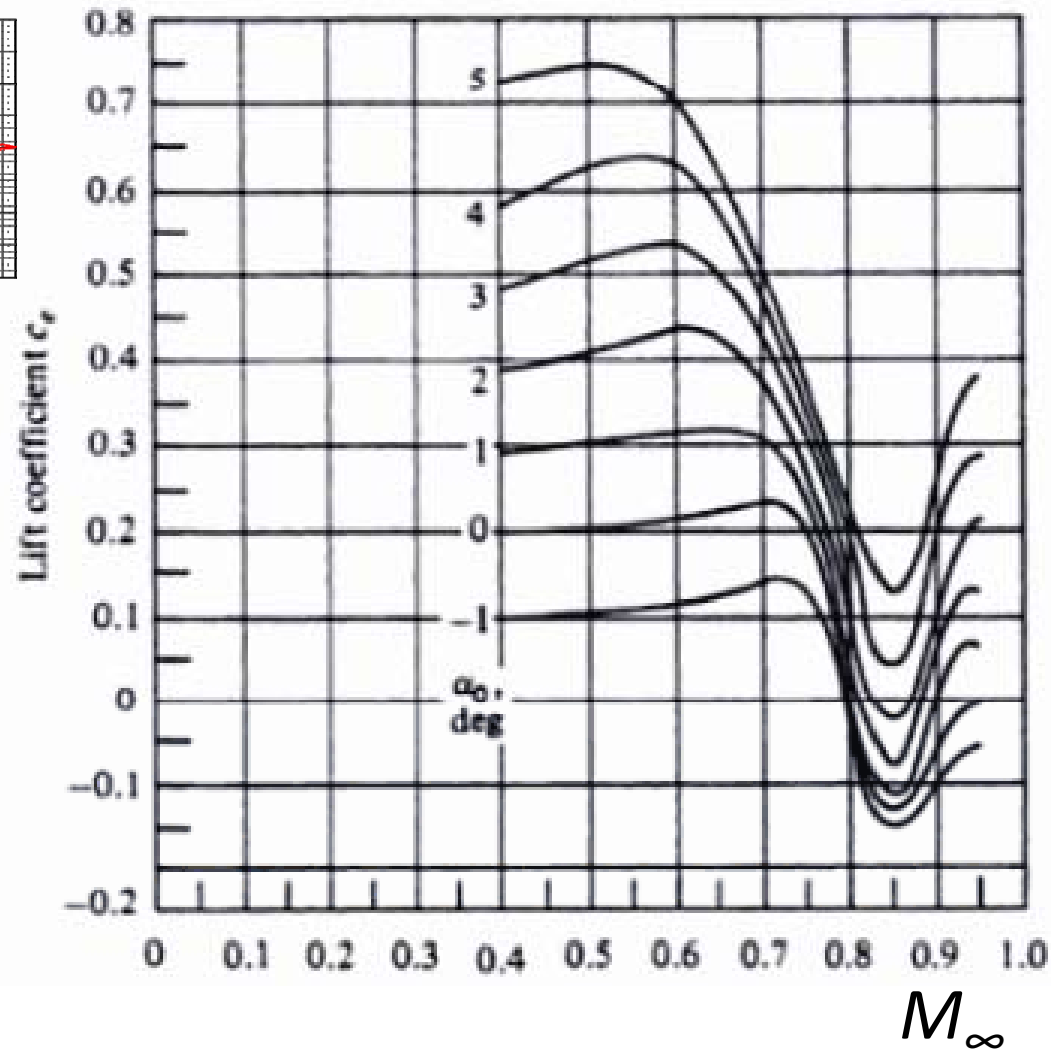
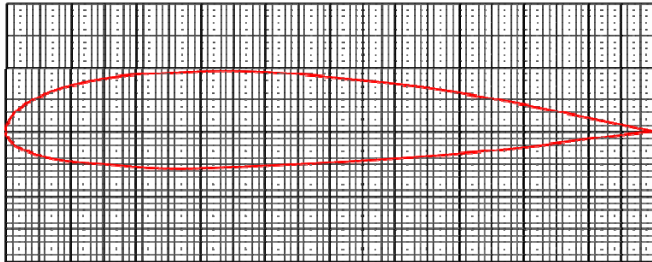
# Drag Divergence

NACA 2315



# Effects on Lift

NACA 2315

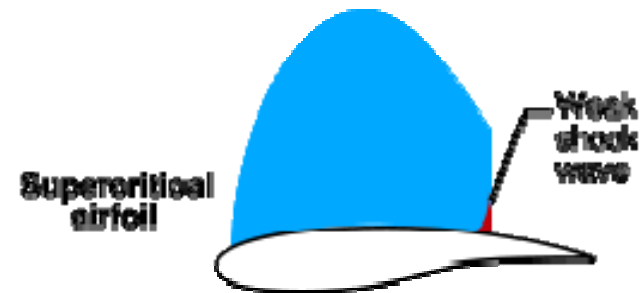
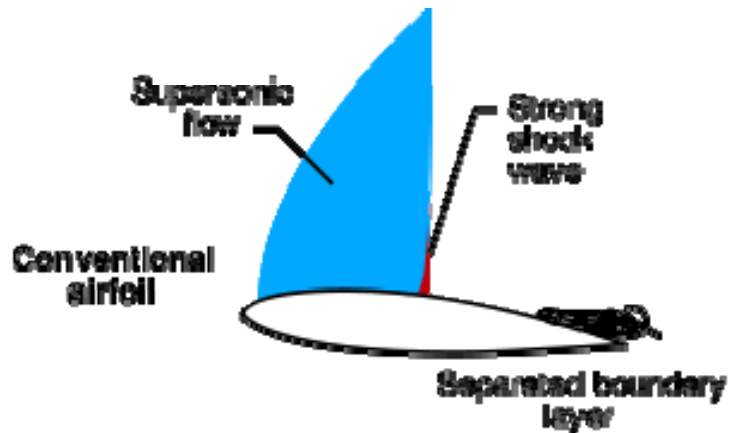


# Mitigating the Drag Problem

- The Supercritical Airfoil
- Sweep
- The Area Rule

# The Supercritical Airfoil

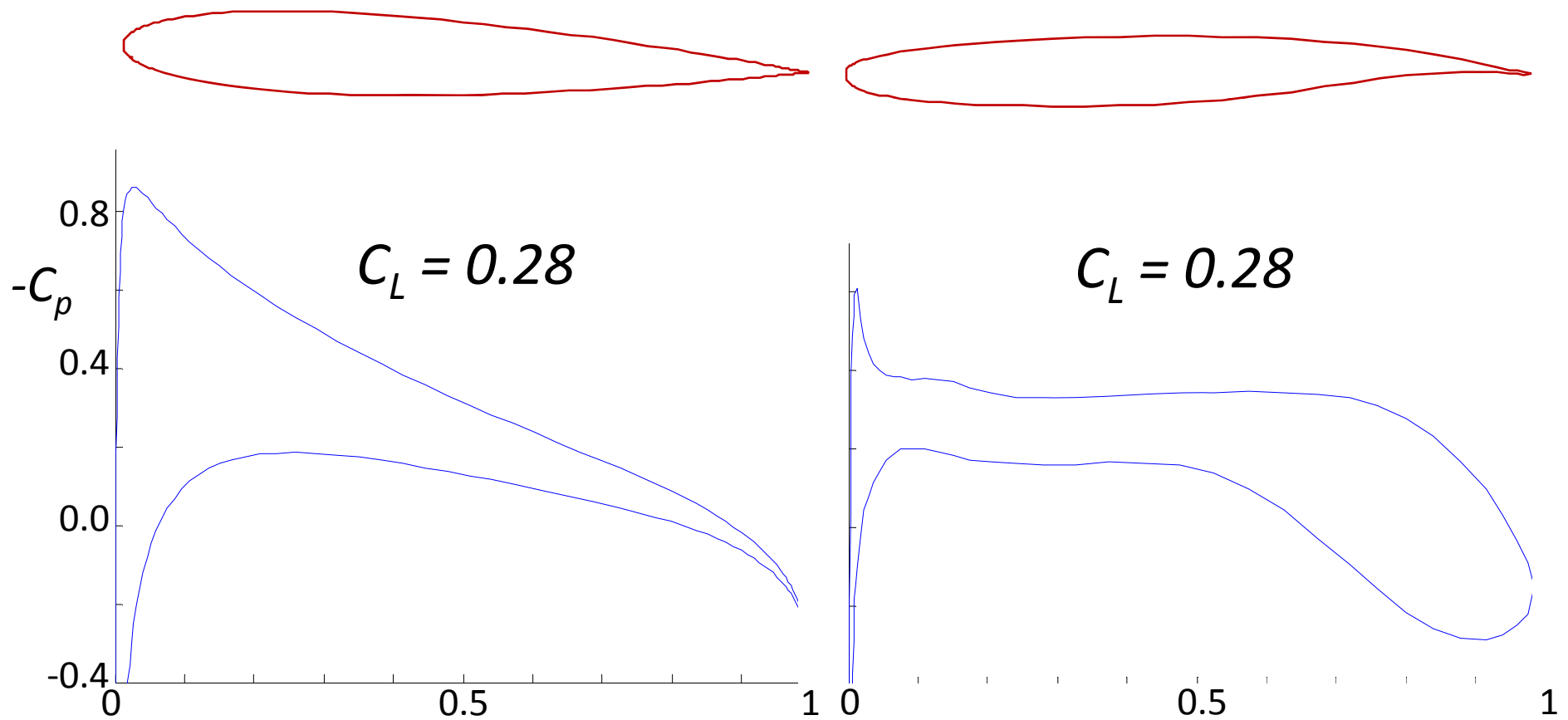
Designed to maximize  $M_{DIV}$

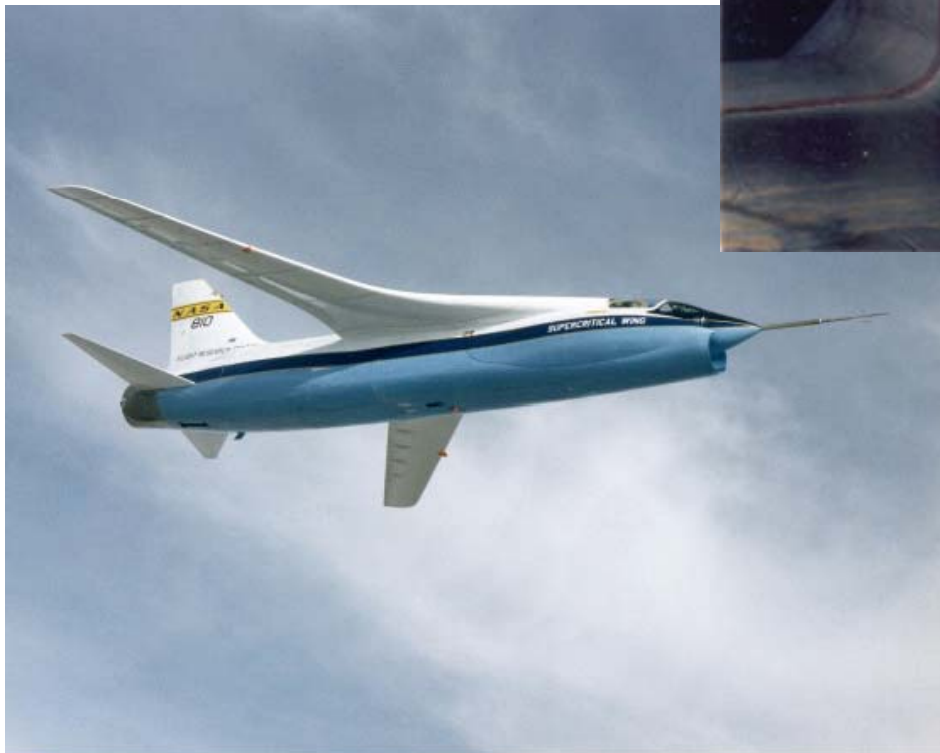
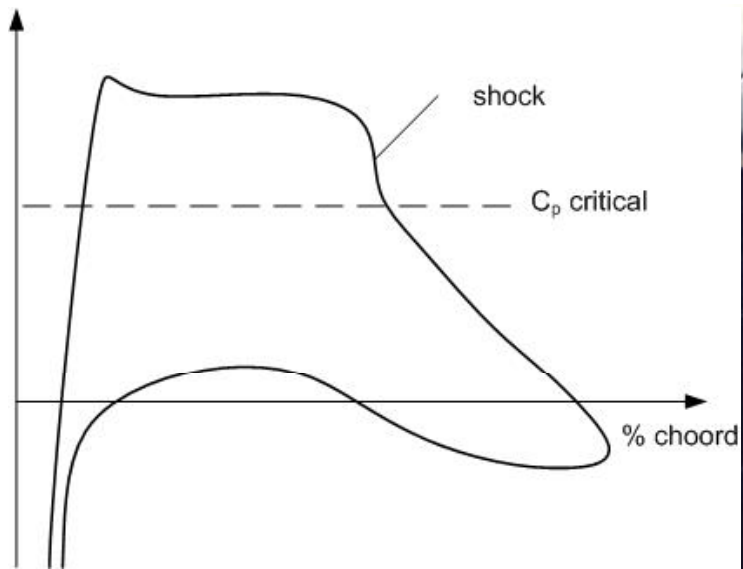


# Conventional vs. Supercritical

*NACA 0012 at 2.3 deg.*

*Boeing BACJ at 0 deg.*





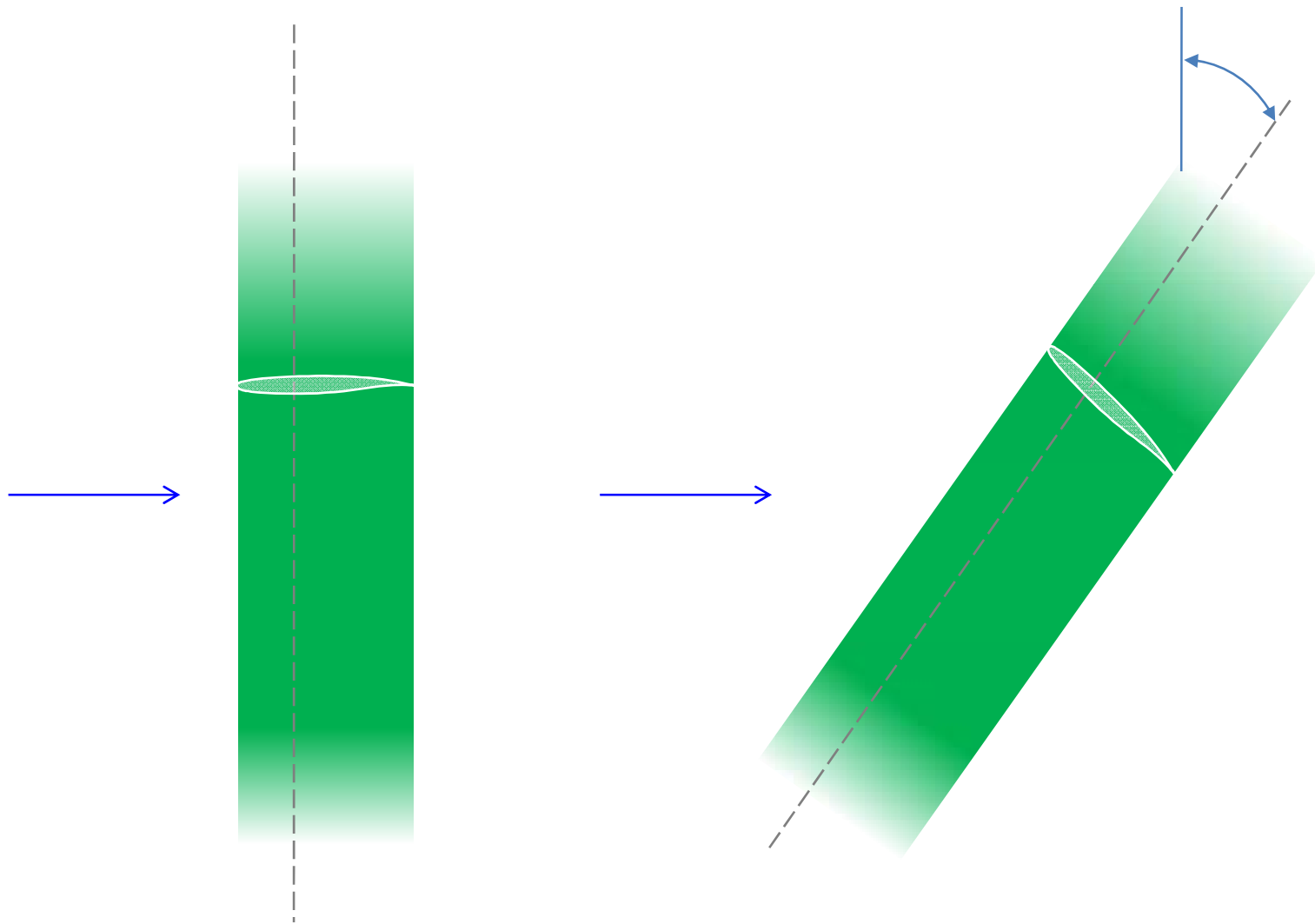
<http://oea.larc.nasa.gov/PAIS/Concept2Reality/supercritical.html>



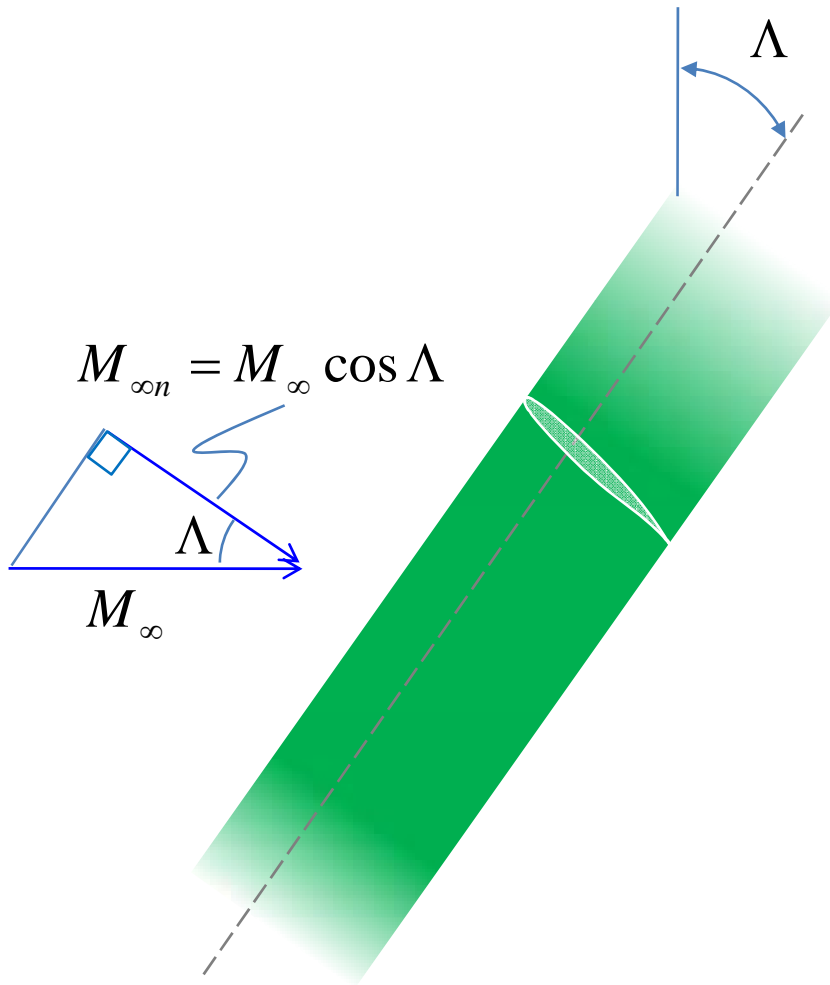
Dryden Flight Research Center E73-3468 Photographed 1973  
F-8 modified with Supercritical Wing. NASA photo



# Sweep - The Independence Principle



# Necessary Sweep Angle





# Effects on Drag Divergence

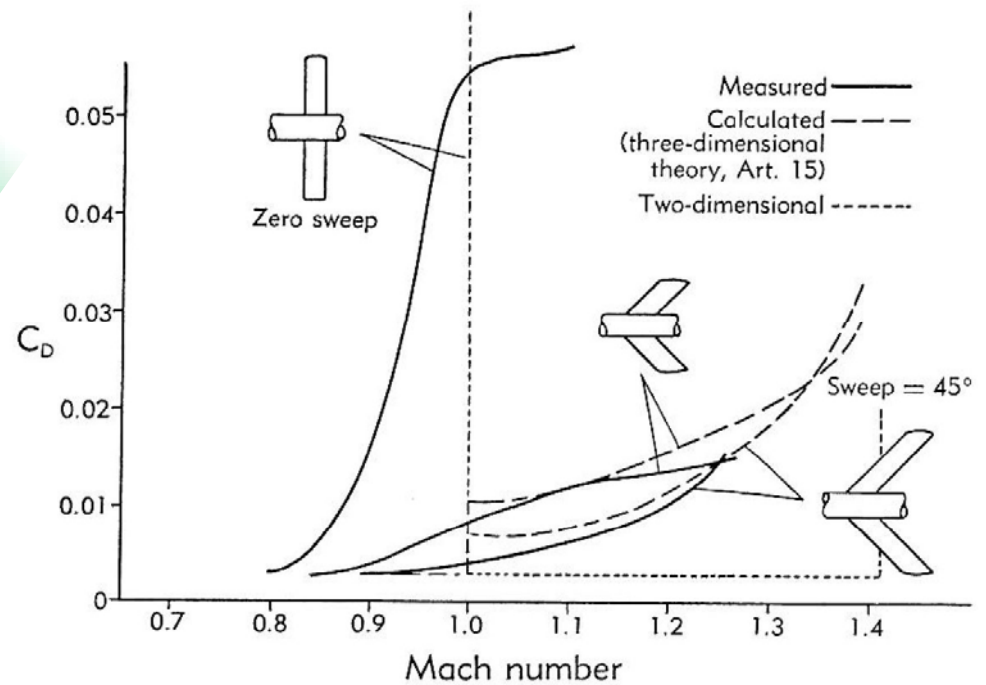
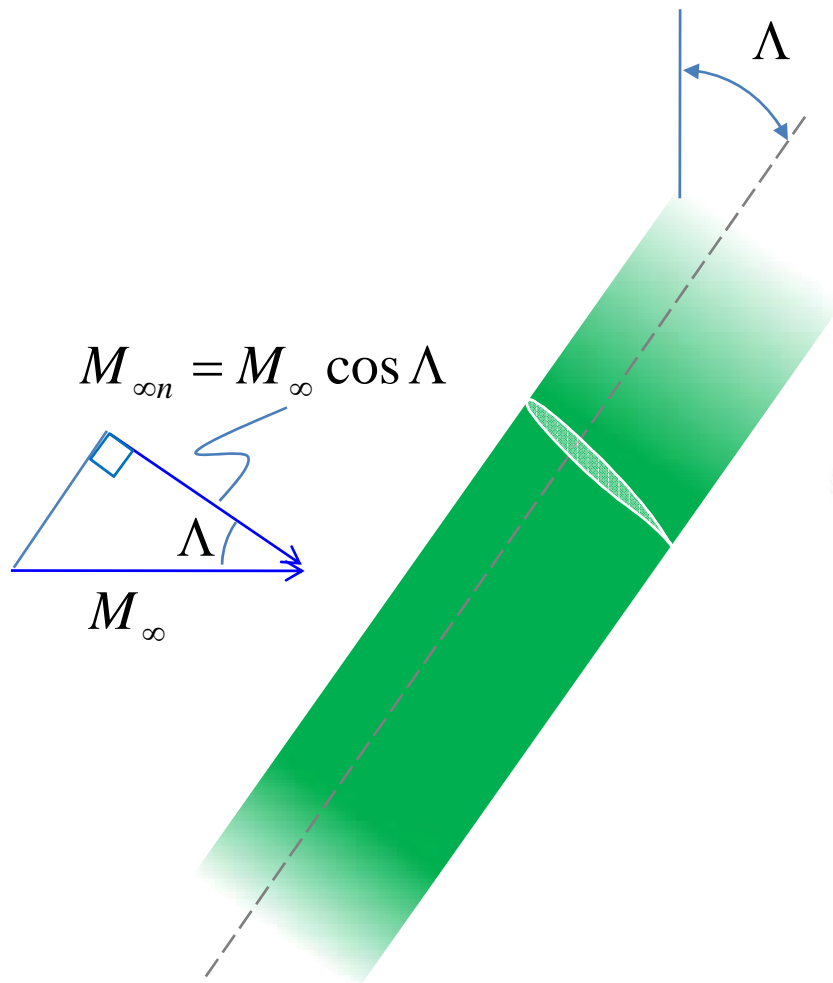
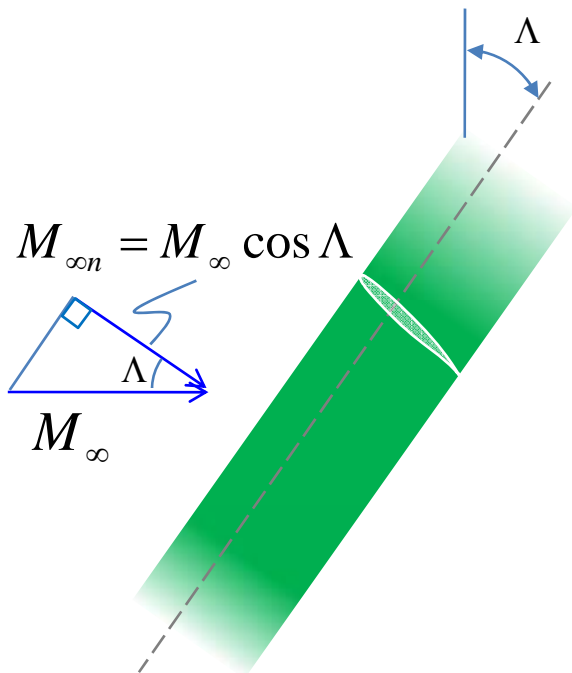
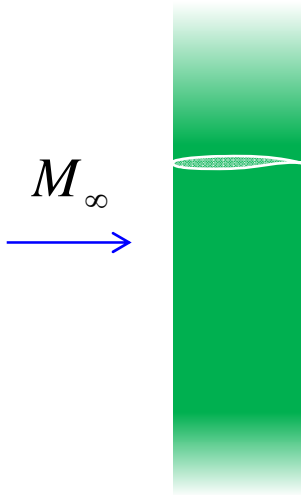


Fig. A,4d. Drag coefficients of straight and swept wings, determined in free-fall experiments.

R. T. Jones, *High Speed Wing Theory*,  
Princeton University Press, 1960

# Effects on Lift



# Effects on Lift

$$C_L = \frac{2\pi\alpha \cos^2 \Lambda}{\sqrt{1 - M_\infty^2 \cos^2 \Lambda}}$$

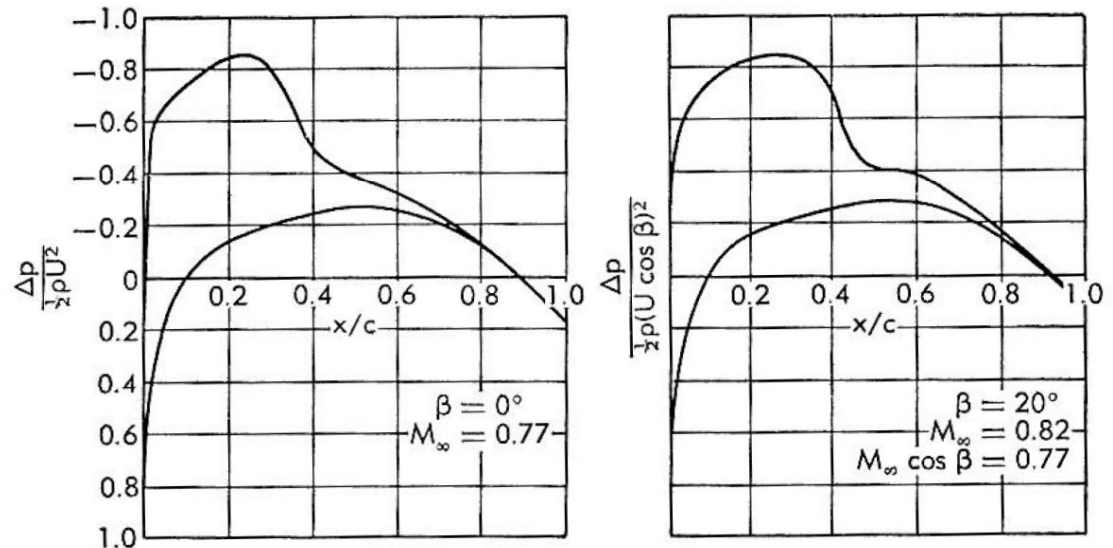
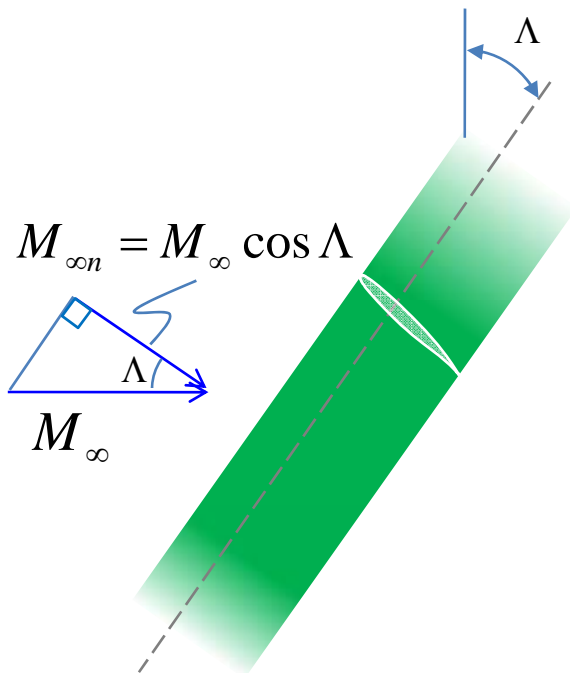
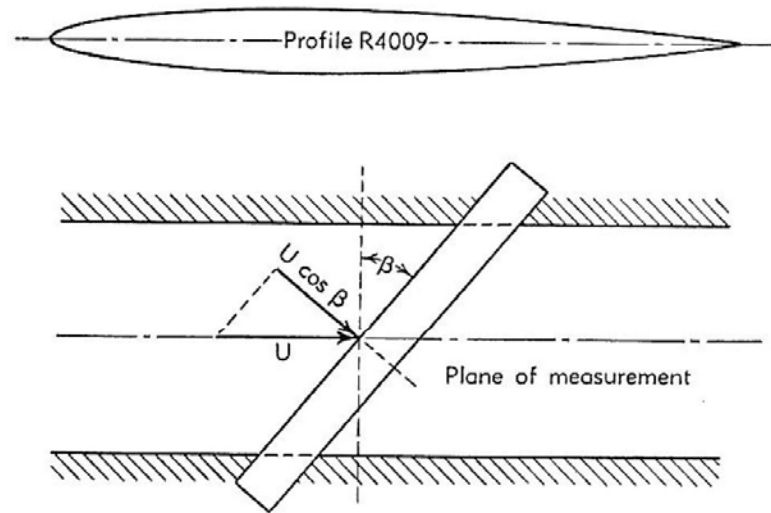


Fig. A,4g. Pressure distributions on oblique airfoil at  $M_\infty \cos \beta = 0.77$ .

R. T. Jones, *High Speed Wing Theory*,  
Princeton University Press, 1960

# Effects on Lift

$$C_L = \frac{2\pi\alpha \cos^2 \Lambda}{\sqrt{1 - M_\infty^2 \cos^2 \Lambda}}$$

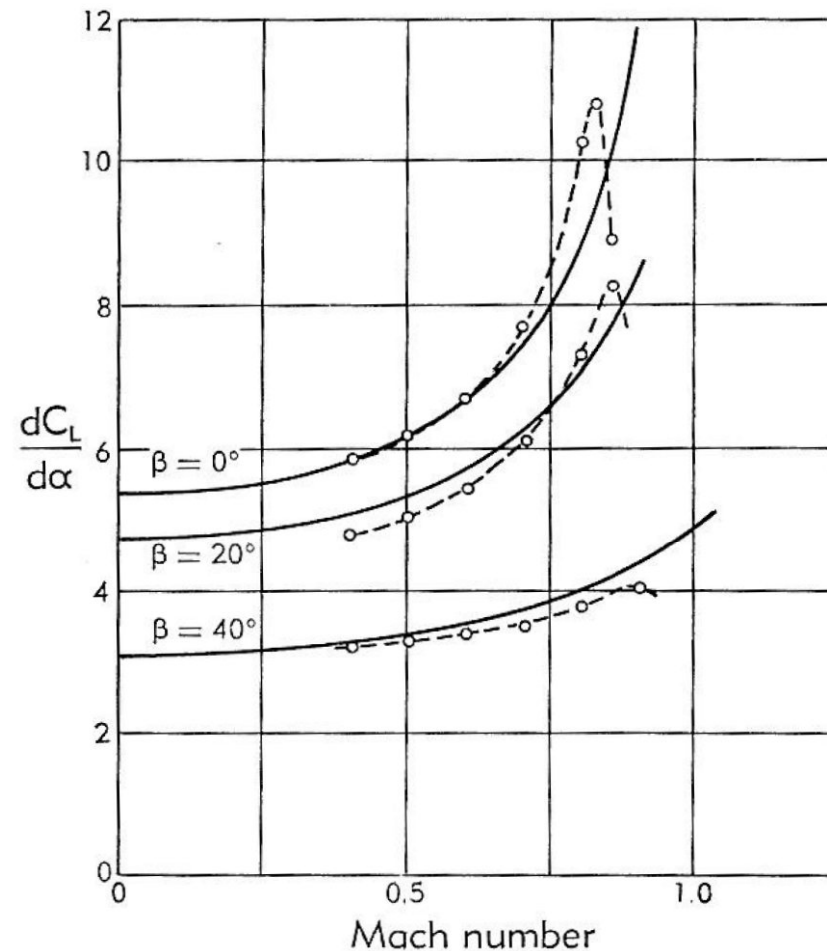
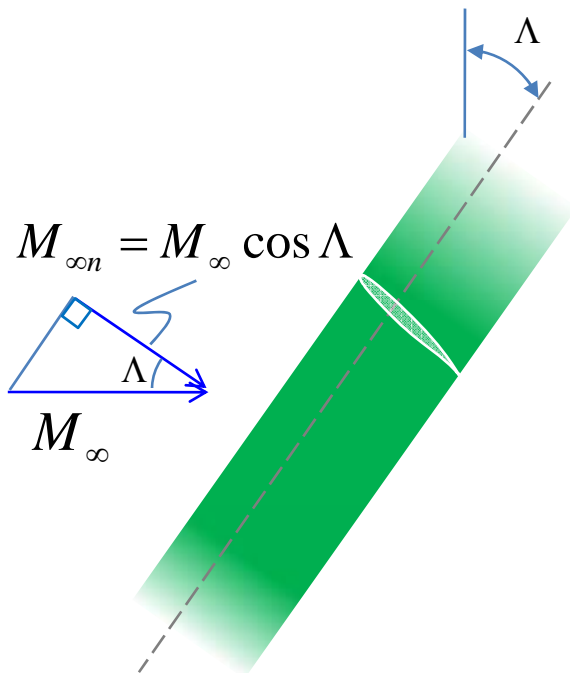
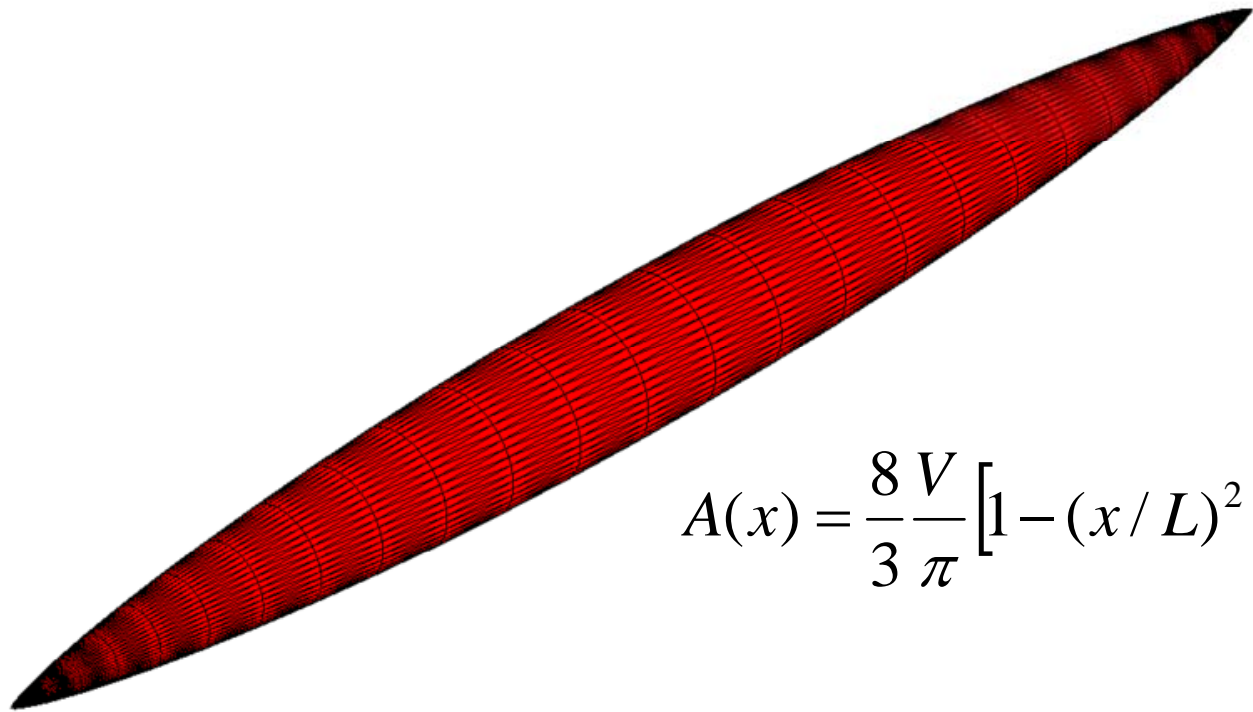


Fig. A,4h. Variation of lift curve slope with Mach number for oblique airfoils.

R. T. Jones, *High Speed Wing Theory*,  
Princeton University Press, 1960

# The Area Rule

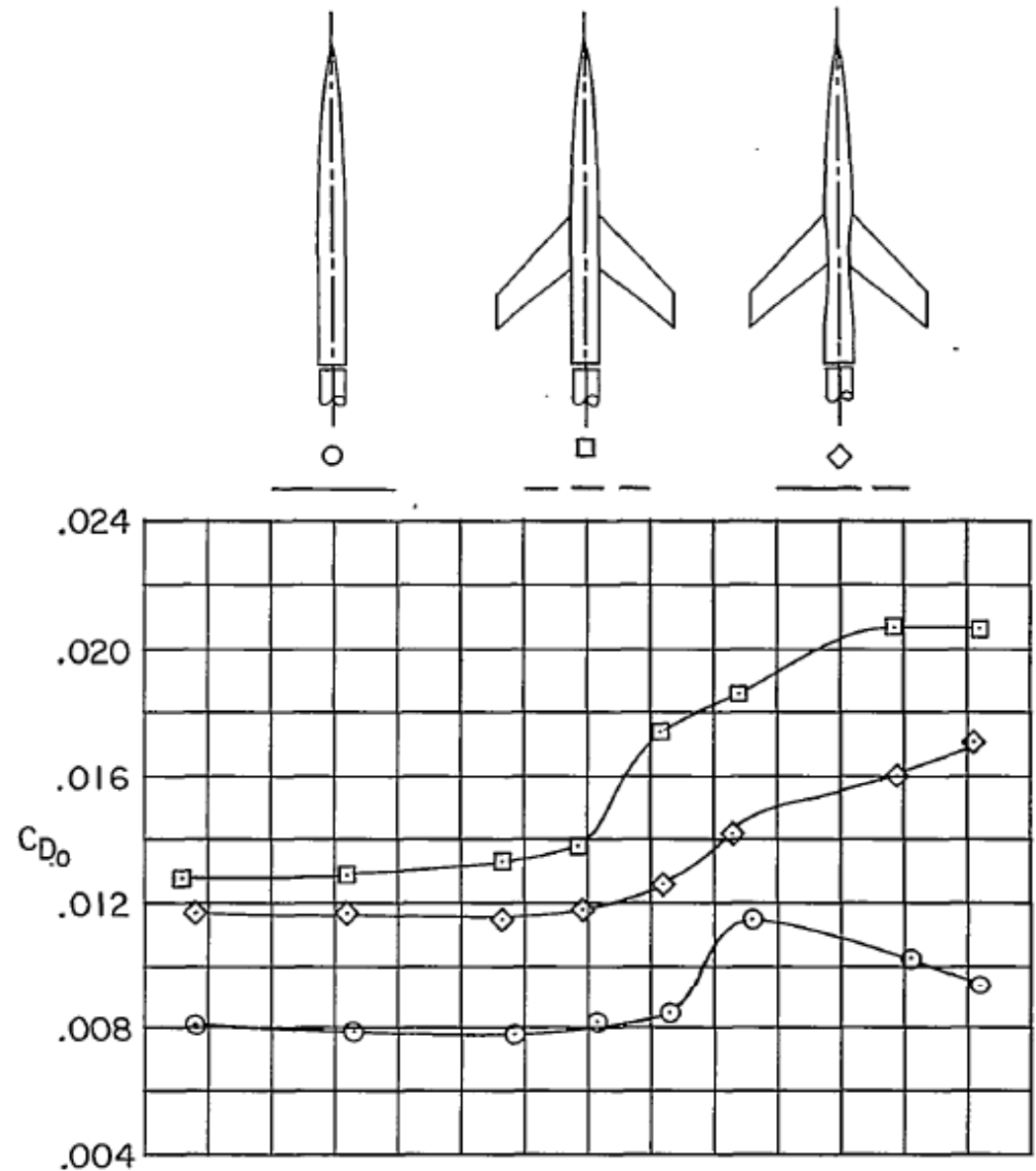
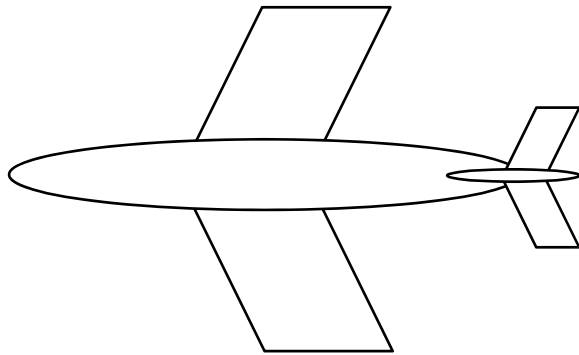
*THE SEARS HAACK BODY*



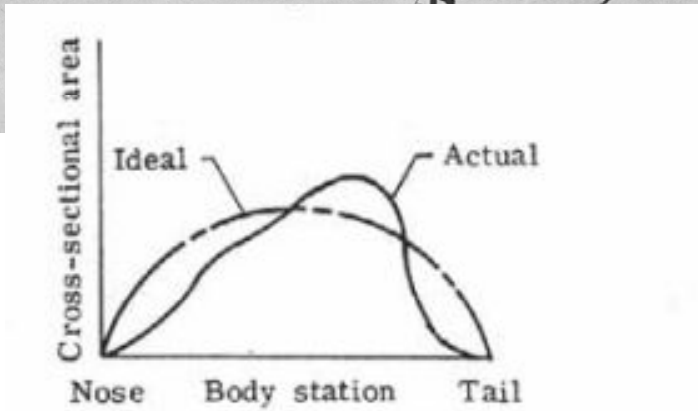
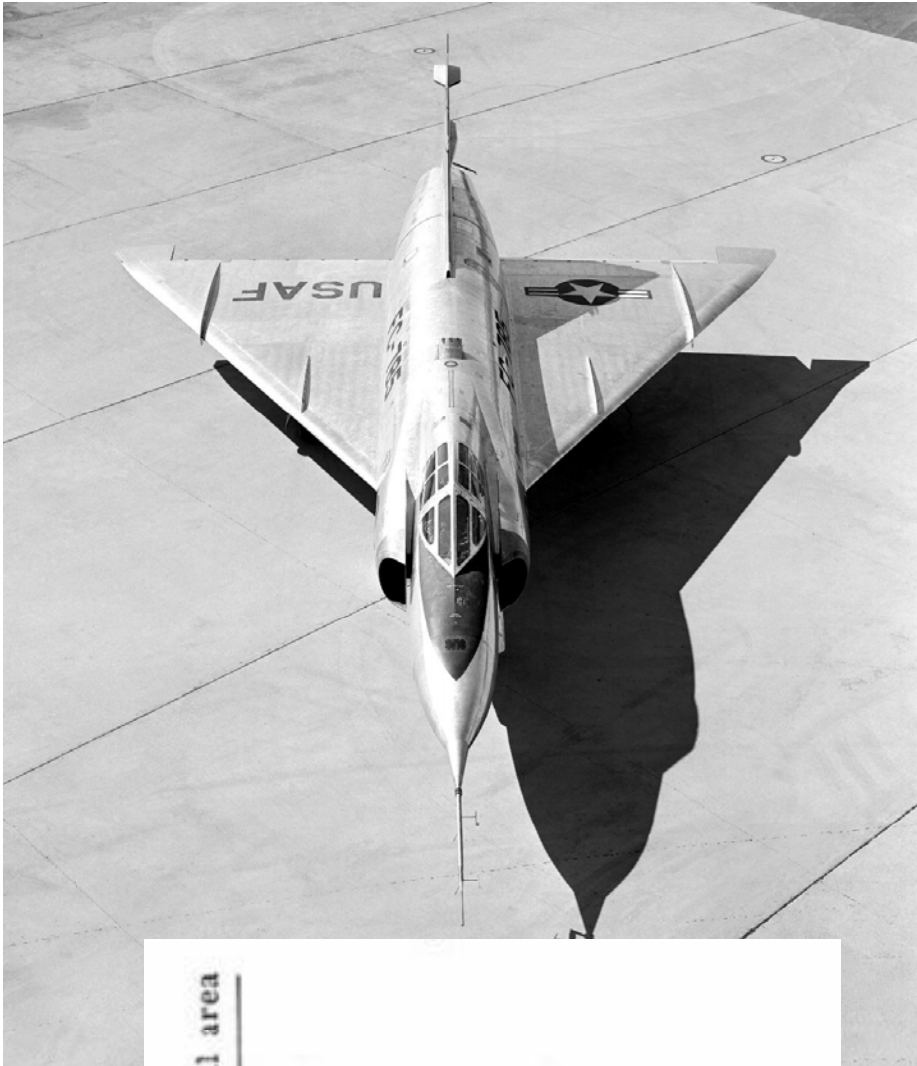
$$A(x) = \frac{8V}{3\pi} \left[ 1 - (x/L)^2 \right]^{3/2}$$

# The Area Rule

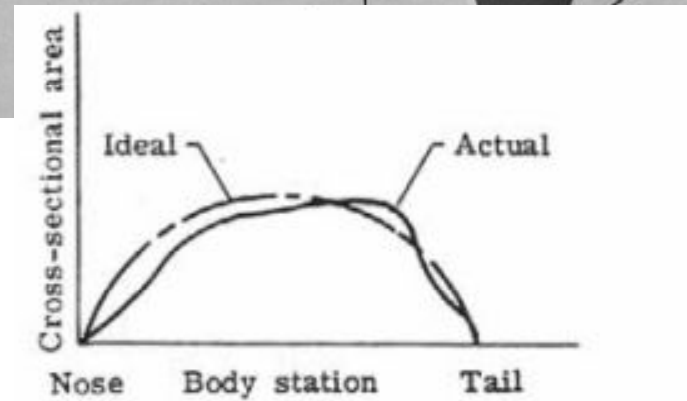
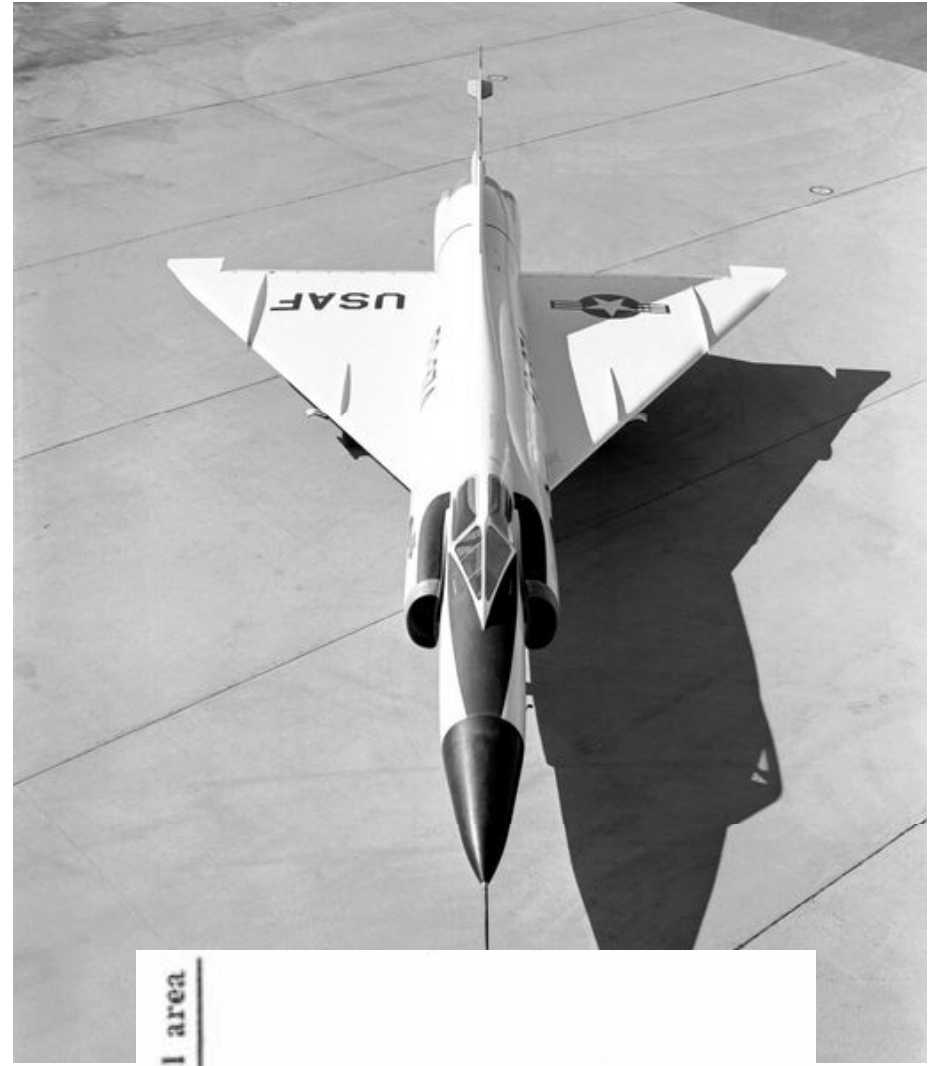
WHITCOMB'S OBSERVATIONS



Whitcomb, R., 1952 *A Study of the Zero Lift Drag-Rise Characteristics of Wing Body Combinations Near the Speed of Sound*, NACA RM L52H08



(a) YF-102A before area ruling.



(b) F-102A after area ruling.

# Boeing 747

747 200B (Air Force 1), Max Mach 0.89



747 400, Max Mach 0.92



# Generalizing the Area Rule

- At supersonic Mach numbers the Area is measured along the Mach plane instead of perpendicular to the flight direction
- Optimal shapes for more general requirements (like finite base areas, area requirements at particular cross sections) are given by W.T. Lord and E. Eminton in “Slender Bodies of Minimum Wave Drag,” *Journal of the Aeronautical Sciences*, August, 1954, pages 569-570. These have been programmed by Prof. Mason, see [http://www.aoe.vt.edu/~mason/Mason\\_f/MRsoft.html#MINDRAG](http://www.aoe.vt.edu/~mason/Mason_f/MRsoft.html#MINDRAG)