

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

# APOLLO

GUIDANCE AND NAVIGATION

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R-500

SPACE NAVIGATION  
GUIDANCE AND CONTROL

Volume 1 of 2

JUNE 1965

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**INSTRUMENTATION  
LABORATORY**

CAMBRIDGE 39, MASSACHUSETTS

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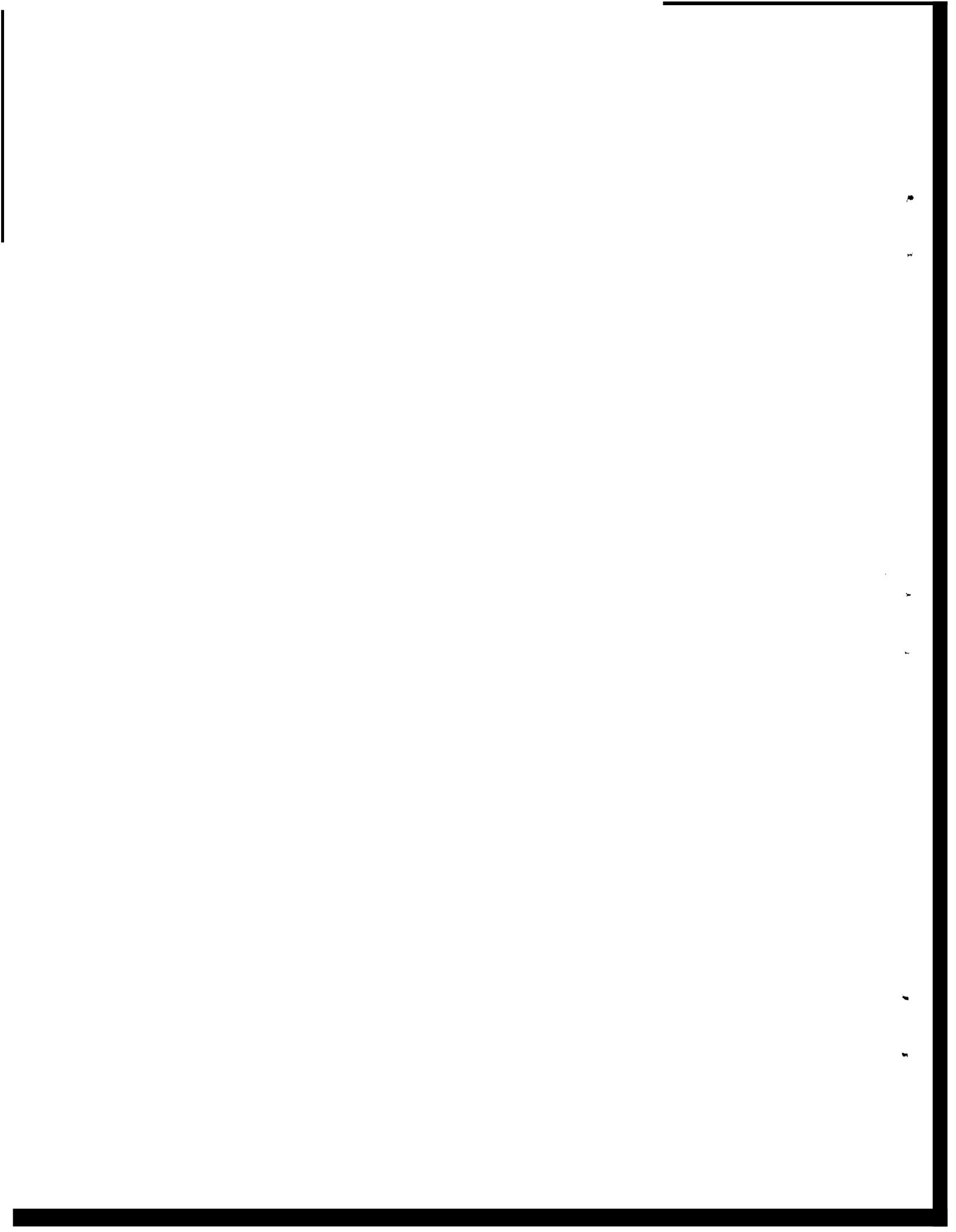
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## PREFACE

The material in this book was assembled to support a series of lectures to be given by the authors in Europe in June 1965, under the sponsorship of the Advisory Group for Aerospace Research and Development, an agency of NATO.

The general subject of Space Vehicle Control Systems is the subject of discussion with particular application to the present Manned Lunar Landing Program. The man-machine interaction along with requirements of the mission are first described. These mission requirements in terms of specific hardware along with the performance requirements and underlying reasons for choice are next explained. Lastly, the theoretical background, the system analysis and the derivation of the control functions to integrate the hardware into a precision guidance, navigation and control system are discussed. The book is organized into seven sections following the pattern of the lectures.

Section I provides historical background to the fundamental problems of guidance and navigation. The basic physical phenomenon and associated instrument techniques are discussed.

Section II continues with background information going more specifically into the problems and approach of the guidance, navigation and control of the Apollo manned lunar landing mission. This section illustrates some of the basic philosophy and approaches to the Apollo tasks, such as the success enhancing decision to provide equipment that will perform all necessary operations on-board and using all ground based help when available.

Section III concerns in detail the analytic foundation for performing on-board calculations for navigation and guidance. The achievement of a unified and universal set of equations provides an economy in on-board computer program to perform all the various mission tasks.

Section IV covers in detail the mechanization of the inertial sensor equipment of the Apollo guidance and control system.

Section V provides the same visibility into the optical navigation sensors and measurement techniques.

Section VI provides background and specific techniques in the mechanization of on-board digital computers. Application to the Apollo mission illustrates several problems of interest such as the method for providing reasonable and straightforward astronaut data input and readout.

Section VII concerns the specific problems and solutions of vehicle attitude control under conditions both of rocket powered flight and the free-fall coast conditions. The Apollo mission provides a diversity of examples of this area of technology in the control schemes of the command and service module, the lunar landing vehicle, and the earth entry return configuration.

The general problems of Space Navigation, Guidance, and Control requires a great variety of discipline from the engineering and scientific fields. The successful completion of any one space mission or phase of a space mission requires a team effort with a unified approach. Of equal importance are the software deliveries and performance with the hardware. This lecture series is ~~an attempt to integrate~~ many of the disciplines involved in creating successful and accurate space vehicle control systems,

These lectures represent, on every ones part, an interplay between equipment and theory. While in each case emphasis may be on one or the other, in the whole equal emphasis is applied.

All sections may be treated as separate entities however in the case of Section III through VII it is helpful to have the background of Section II. There is cross reference between sections to avoid unnecessary duplication.

It is observed that the authors have emphasized the Apollo mission and hardware as examples in their treatment of the subjects. This is partially because of their intimate familiarity with Apollo in the development work at the Instrumentation Laboratory of MIT and partially because Apollo provides, in an existing program, an excellent example in its multiple requirements and diversity of problems. Because Apollo is currently under development, no particular attempt has been made to make reference only to the latest configuration details. Indeed the authors have utilized various stages of the Apollo development cycle without specific identification in every case as they provide the guidance, navigation, or control technique example desired.

The authors wish to express their appreciation to NASA for the opportunity to participate in the lecture series and for permission to use material from the research and development contracts NAS 9-153 and NAS 9-4065. They also recognize that this does not constitute approval by NASA of this material. In addition, they wish to thank the many members of M. I. T. 's Instrumentation Laboratory; who are working on the Apollo system, for their inspiration and generation of material.

PART I

GUIDANCE - BASIC PRINCIPLES

by

Dr. C. Stark Draper  
Dr. Walter Wrigley

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Dr. Draper was born in Windsor, Missouri on October 2, 1901. In 1922 he received a Bachelor of Arts degree in psychology from Stanford University. He then entered M. I. T. where he earned a B. S. in electrochemical engineering in 1926; an S.M. without specification of department and an Sc. D in physics in 1938.

Dr. Draper is President of the International Academy of Astronautics and his many memberships include the American Academy of Arts and Sciences, the National Academy of Sciences, and the American Institute of Aeronautics and Astronautics. In 1965 President Johnson personally awarded the National Medal of Science to Dr. Draper for his "many engineering achievements for the defense of the country"!

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Walter Wrigley is Professor of Instrumentation and Astronautics, Educational Director of the Instrumentation Laboratory, and Acting Director of the Experimental Astronomy Laboratory at the Massachusetts Institute of Technology. He has been active in the development of basic concepts for fire control and navigation systems and has written extensively in these and related fields.

Dr. Wrigley was born in Brockton, Mass., on March 26, 1913. He received his Bachelor of Science degree in 1934 and an Sc. D. in Physics in 1941, both from M. I. T.

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## Part I

### GUIDANCE - BASIC PRINCIPLES

#### INTRODUCTION

Guidance is the process of collecting and applying information for the purpose of generating maneuver commands to control vehicle movements. In effect, this process represents closure of the essential control feedback branch that has to be associated with structure and propulsion in order for any vehicle system to operate successfully. Strong airframes and powerful engines can provide the capability for flight, but without control to give stability, guidance to determine proper paths and to generate maneuver commands for realizing these paths, the most sophisticated and expensive craft are of no practical value. The finest airplane will not begin to serve as a means of transportation until the pilot takes his seat and assumes the functions of control and guidance. It is true that in recent times the demands of these functions have often gone beyond the abilities of human senses, human muscles and the speed of human thought processes. Man must now retain his hold on the command of many situations through his invention, production and application of inanimate devices that aid or replace his own limited powers for direct action. The collective brains of mankind are again demonstrating the ages-old truth that thought is very powerful among the factors that determine progress. Advances in the technology of control and guidance have been substantial during the past two decades, but these advances have generated much misunderstanding, controversy and often strong opposition as the demands for attention and funding support have increased. However, the spectacular results that have been achieved, particularly in the fields of ballistic missiles, submarines, satellites and space vehicles, have tended to reduce this resistance and to encourage the development of a technology essential to the advancement and perhaps the survival of our country.

A wide spectrum of possibilities for the future have already been revealed by results now in the records, but essential decisions associated with a continuation of work toward pioneering improvements in performance remain to be made in the near future. Matters of national policy, strategy, tactics, economics, politics, company profits and human emotions are *so* inter-mixed with basic physical laws

and technological developments that any significant clarifications of basic problems associated with guidance and control are certainly helpful in forming plans for constructive action. The authors of this paper hope to provide some assistance by a discussion of basic principles, requirements, mechanization features and natural performance limitations of components and of systems to meet the needs of military operations. Representative numerical values for typical cases are cited, but specific results from particular equipments are not presented.

## CHAPTER I-1

### PROBLEMS OF GUIDANCE

The traditional method of directing the motion of a vehicle from a port of departure to a port of destination is based on the position and direction information generated by navigation. This situation is suggested by Fig. I-1. Because the terminal phases of many missions are made to depend upon direct contacts with facilities at the destination, the accuracy required of navigation is in general not very great. If navigation can bring a vehicle into an area extending a few miles around the destination, its function has been accomplished. For flights covering not more than a few hours it follows that performance inaccuracies not greater than one to three miles for each hour of flight are often considered satisfactory for navigation systems.

Attacks on area targets with weapons able to cause destruction over areas several miles in radius is suggested in Fig. I-2a. The purposes of such attacks can be served by control and guidance systems giving CEP's - Circular Error Probability (the radius in which half of a significant number of flights would terminate) with the order of one nautical mile. This inaccuracy should be substantially independent of range and time of flight.

Any scheme of navigation or guidance that does not use direct contacts based on either natural or artificial electromagnetic (or acoustic) radiation, must depend upon the identification of points on the earth's surface in terms of measured distances from established bench marks or by means of angles between local gravity directions with respect to coordinate axes fixed in the earth. The association of these directions with points on a theoretical mapping surface makes it possible to identify well-surveyed positions on the earth with inaccuracies somewhat less than one-tenth of a nautical mile. This means that a CEP of one-tenth nautical mile as the performance goal for the instrumentation of navigation and guidance systems is consistent with the map grids now available. Effective attacks on many military targets, such as bridges and hardened missile sites, which can be given map locations within one-tenth nautical mile require inaccuracies around the aim point of approximately the same one-tenth mile order of magnitude. Figure I-2b suggests the situation associated with a ballistic attack on a hardened missile site.

Control, navigation and guidance always involve a reference coordinate system with axes having known working relationships with directions in the space used for defining the essential path of motion. This definition involves directions with respect to the reference coordinates, and also distances and motion between the guided entity and the destination or target. In practice, the reference coordinates may be established in several ways and the essential distances may also be indicated by various methods.

Figure 1-3 illustrates one of the simplest situations for guidance with one airplane making an attack on another craft with guns. The problem is for the attacker to fly a path which causes projectiles from his armament to strike the target. His problem centers around the line of sight to the target with reference coordinates for maneuvers fixed in the attacking plane. Usually roll, pitch and yaw axes fixed to the aircraft would be instinctively chosen for judging direction and magnitude of maneuvers. Success is achieved when the attacker flies so that his gunfire destroys the target.

Navigation and guidance present situations that are more complex than the circumstances of an air-to-air duel because direct visual line-of-sight contact with the destination is not generally possible, so that a reference space outside the moving vehicle is necessary in order to describe positions and motions. When flight paths are between points associated with the earth's surface it is natural to use earth's coordinates established by north and the vertical for reference purposes. It is also reasonable to use radiation such as radio and radar for determining distance and direction.

Situations of many kinds appear, depending upon circumstances, but it is possible to illustrate the principles involved in terms of the diagrams of Figs. I-4, I-5 and I-6.

Figure 1-4 suggests the situation in which ground-based equipment having a known orientation with respect to earth coordinates has artificial radiation contacts with the moving vehicle. When these contacts are by pulse tracking, radar distance measurements are direct and give position when combined with indications of direction from tracking signals,

Figure 1-5 illustrates the relationship of distances between points on the earth's surface to angles between gravitational vectors at the points in question. Measurements of directions of gravity are generally associated with positions by the means of celestial navigation procedures. In these methods the gravity vector angles from lines of sight to selected stars are corrected for earth's rotation and then related to map information. It is noted on the figure that an inaccuracy of 60 seconds of arc (one

minute of arc) gives a position inaccuracy of 6000 feet (one nautical mile), while inaccuracies of 6 seconds of arc and 1 second of arc correspond to 600 feet and 100 feet respectively.

Figure I-6 illustrates the indication of distance moved by a vehicle over the surface of the earth by integration of signals from an accelerometer with its input axis stabilized along the direction of vehicle motion. One integration of these signals from a given initial instant gives changes in velocity. A second integration gives changes in position. Assuming perfect orientation of the input axis during a one-hour time of flight, an average accelerometer inaccuracy of  $30 \times 10^{-6}$  earth gravity leads to approximately 6000 feet inaccuracy in the indication of distance traveled. Under similar circumstances accelerometer inaccuracies of  $3 \times 10^{-6}$  and  $0.5 \times 10^{-6}$  earth gravity produce approximately 600 feet and 100 feet respectively.

The use of a vehicle-borne accelerometer implies that the means to stabilize the member on which it is mounted be aligned with the direction of the earth gravity vectors which identify positions on the earth's surface. On the basis of numbers given in Fig. I-6, initial alignment inaccuracies of 6 arc seconds, 0.6 arc seconds and 0.1 arc second will mean position inaccuracies of 6000 feet, 600 feet and 100 feet respectively. Additional inaccuracies of similar magnitude will accumulate if the orientational reference keeping the accelerometer input axis at right angles to the local gravity directions drifts at average rates that accumulate the given angular inaccuracies,

It is convenient to express these drift rates in terms of earth's rate for the purposes of describing system performance. Earth's rate called an "eru" unit is 15 degrees per hour or 900 minutes per hour. One-thousandth of earth's rate (called one milli-earth-rate-unit, one meru) is thus one minute per hour (0.015 degree per hour) which corresponds to a position inaccuracy of about one nautical mile per hour. This means that one-tenth nautical mile (600 feet) corresponds to one-tenth meru (0.0015 degree per hour, or 6 seconds per hour) while one-hundred feet corresponds to 0.0167 meru (0.00025 degree per hour, 1 arc-second per hour).

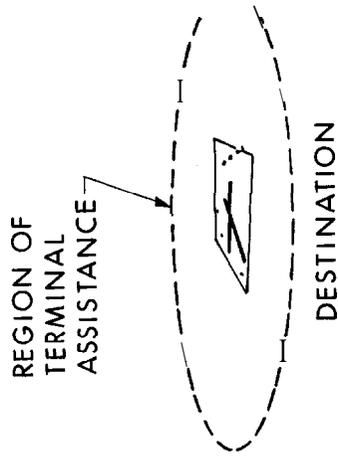
It is to be noted that the numbers mentioned in the last paragraph are only rough approximations. Any mechanization would require higher performance from its individual components in order to account for the interactions that inevitably exist in complete systems.

In summary, it appears that radiation link inaccuracy is directly that of the instrumentation used. When gravitational directions are used to indicate positions, a 60 arc-second error gives one nautical mile error, with the corresponding error for 100 feet being one arc-second. If one-hour flight time is allowed to accumulate

these errors the stabilization drift error must be less than 1 meru (0.015 degree per hour) for one mile error in an hour while a drift of 0.0167 meru (0.00025 degree per hour, 1 arc-second per hour) is required if not more than 100 feet position error is to be developed in one hour.

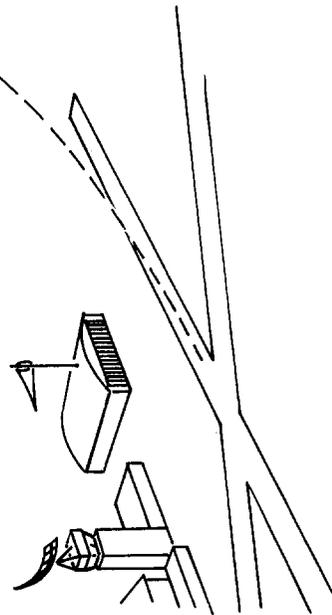
When signals from an accelerometer with its input along the vehicle flight path are used to generate position change data, one mile error in one hour needs an accelerometer inaccuracy of about  $30 \times 10^{-6}$  earth gravity, while 100 feet error in one hour needs accelerometer performance in the range of  $0.5 \times 10^{-6}$  earth gravity.

FUNCTION OF NAVIGATION IS TO DIRECT FLIGHT SO THAT THE MOVING VEHICLE ARRIVES WITHIN THE REGION OF TERMINAL ASSISTANCE AT THE DESTINATION



a

DEPARTURE

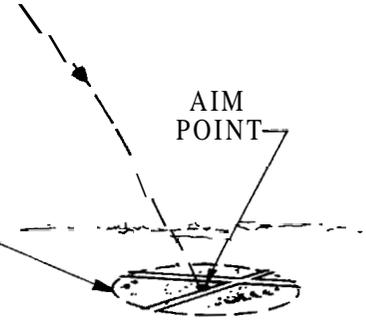


GROUND-BASED FACILITIES COOPERATING WITH AIR-BORNE EQUIPMENT PROVIDE A RADIUS OF SEVERAL MILES FOR THE REGION OF TERMINAL ASSISTANCE

Fig. I-1 Navigation

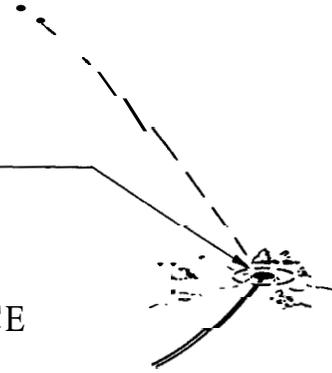


CEP OF ABOUT 1 MILE  
( APPROXIMATELY ONE ORDER  
OF MAGNITUDE -LESS THAN  
THE TOLERABLE ERRORS FOR  
NAVIGATION



a. AREA TARGET GUIDANCE

CEP OF ABOUT 0.1 MILE (600FT)  
FOR EFFECTIVE RESULTS  
FROM AVAILABLE WARHEADS



b. MILITARY TARGET GUIDANCE

Fig. I-2 Guidance

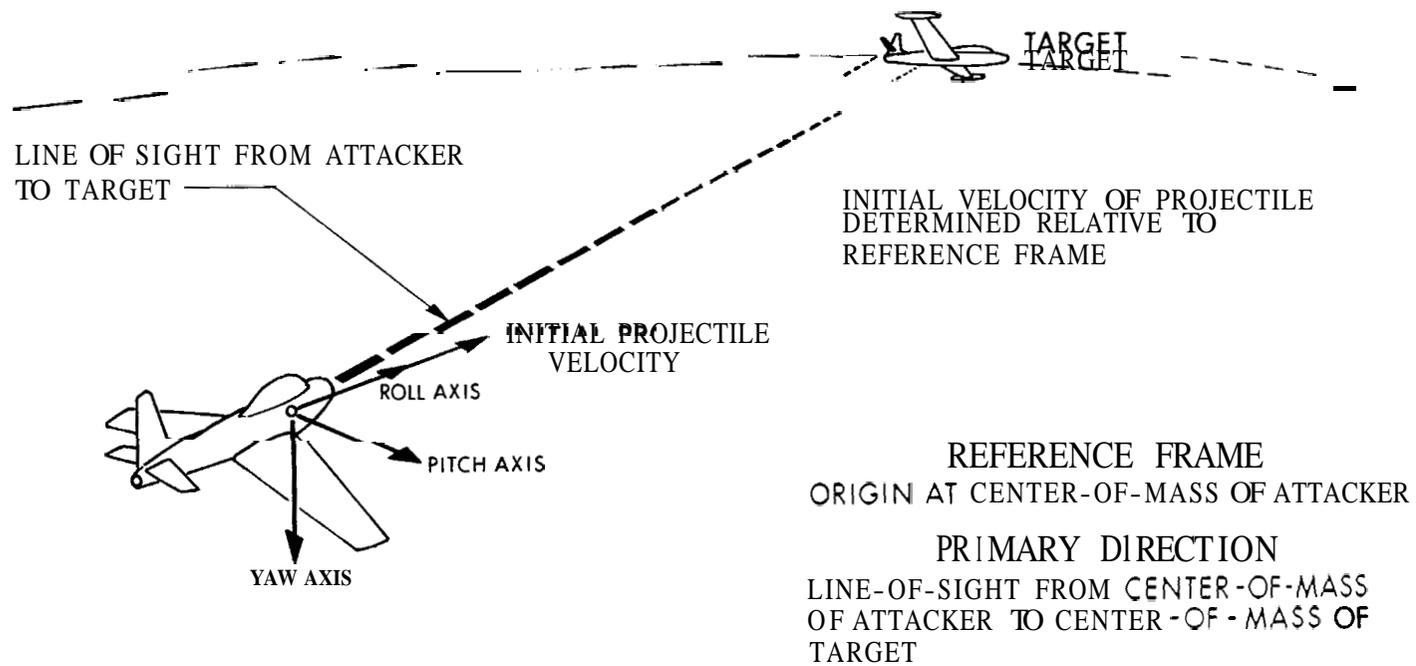
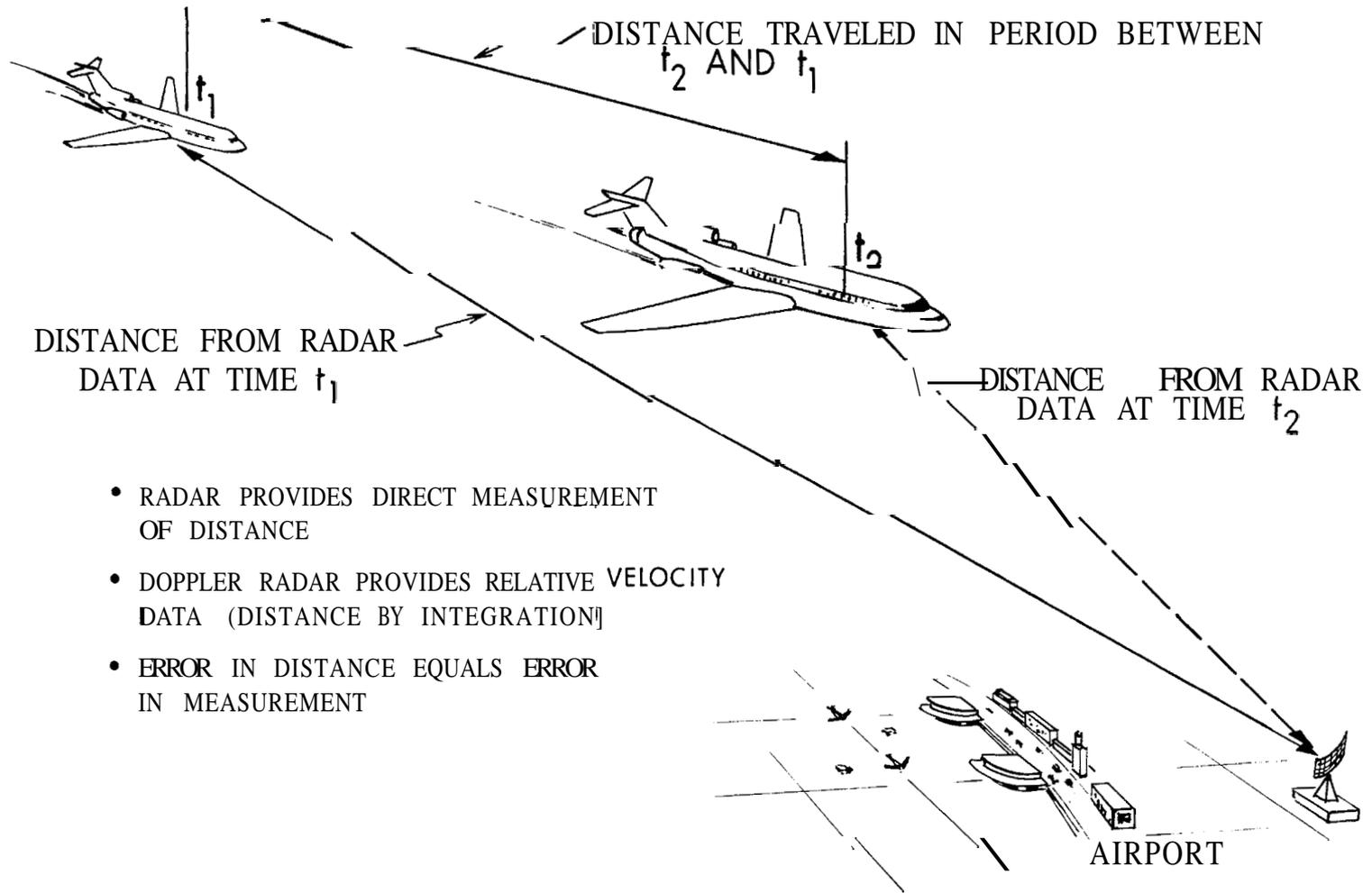


Fig. I-3 Line of Sight Guidance - Reference Frame in Vehicle



- RADAR PROVIDES DIRECT MEASUREMENT OF DISTANCE
- DOPPLER RADAR PROVIDES RELATIVE VELOCITY DATA (DISTANCE BY INTEGRATION)
- ERROR IN DISTANCE EQUALS ERROR IN MEASUREMENT

Fig. 1-4 Position Change Information from Radiation Contact Measurements

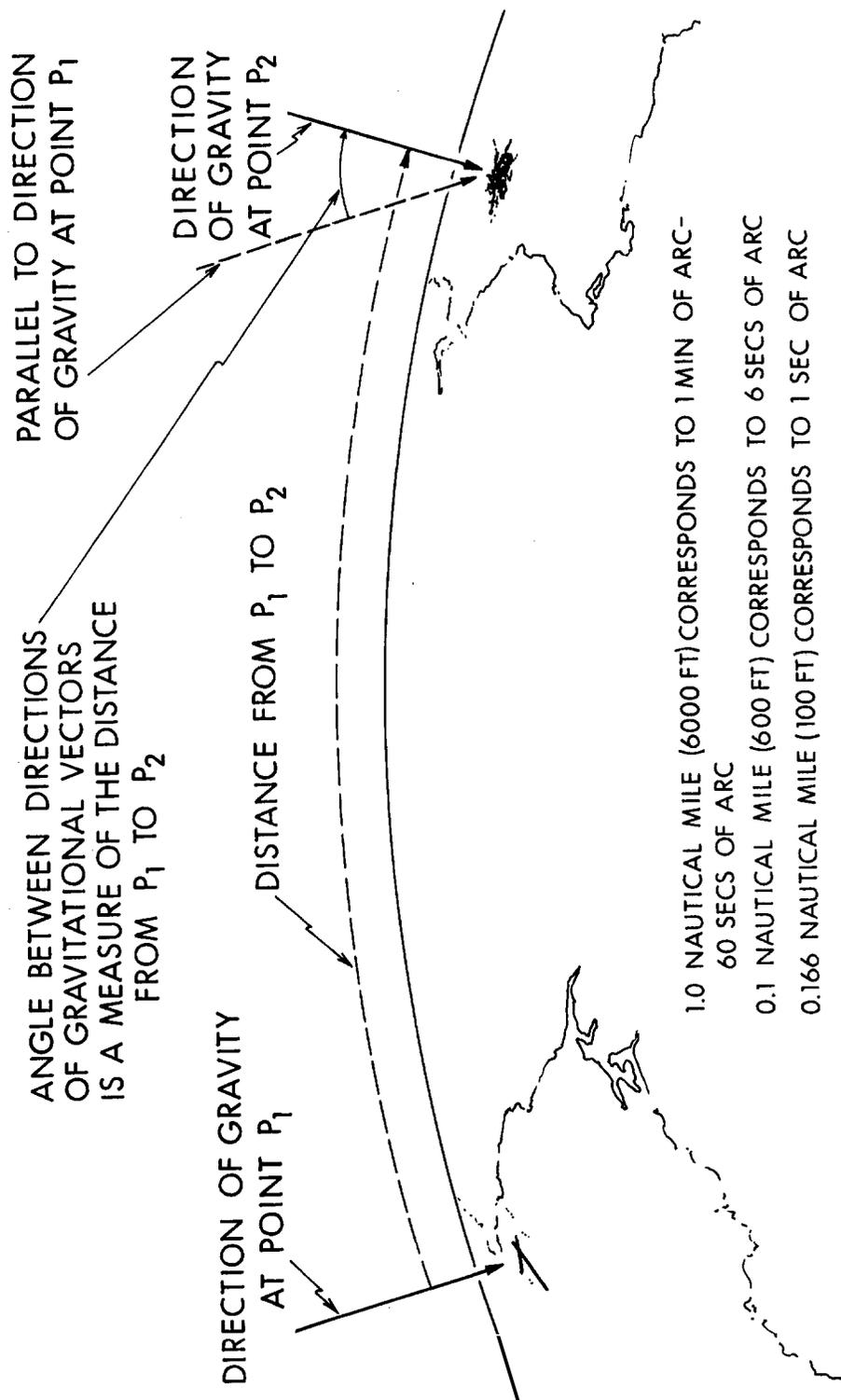


Fig. I-5 Position Information from Measurements of Angle Between Gravity Vector Directions

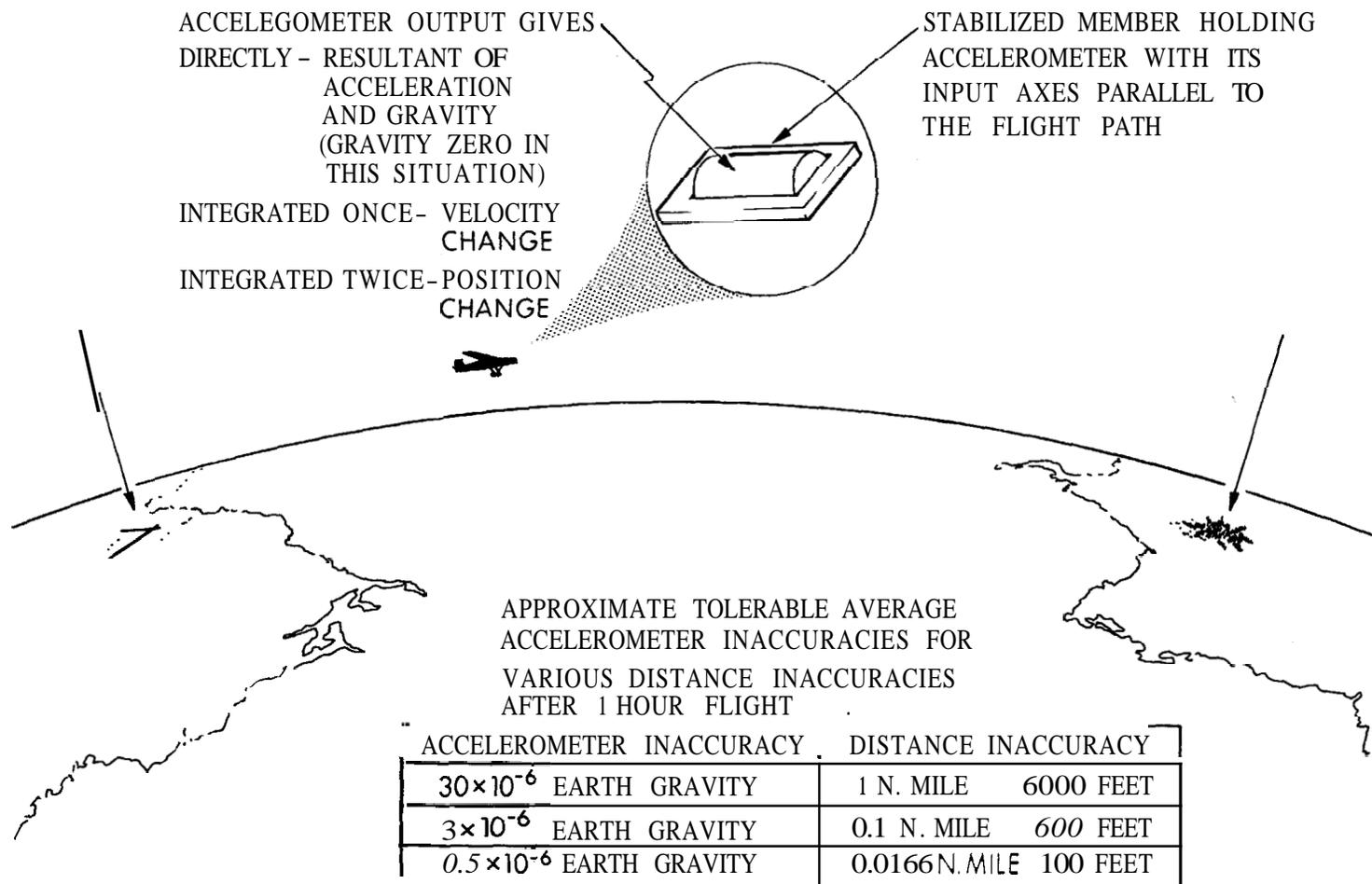


Fig. 1-6 Position and Velocity Changes from Integration of Accelerometer Measurements Taken along Flight Paths

## CHAPTER I-2

### GEOMETRICAL ASPECTS OF GUIDANCE AND CONTROL

From the standpoint of basic geometry the problem of navigation and guidance is that of commanding vehicles to move so that they reach the vicinity of destinations or effectively hit pre-selected targets. In order for this process to be at all possible a reference space in which knowledge of relative location and motion between the guided vehicle and its goal must be available. With this knowledge in hand it may be processed and compared with desired location and motion to determine indicated deviations from which correction maneuver commands can be generated. Geometrical reference space for navigation and guidance is not unique, but may be chosen in many ways to be convenient for the problem under consideration.

Figure I-7 illustrates a simple situation in which a guided vehicle and its target are linked by a direct line of sight. This line is the geometrical entity that determines the maneuvers carried out by the pilot as he flies to the vicinity of his target. His own airplane acts for him as the reference space for these maneuvers, no outside body is involved. Here his reference space is naturally provided by the earth in good weather with the horizon and landmarks to give him the vertical and north as a setting for the location of his destination. In effect, the earth supplies three coordinate directions for judging angles and a ground-fixed point at the destination for estimating distance and velocity along these axes,

Figure I-8 shows how the situation changes when clear visual contacts with the ground are lost because of night, weather or terrain. Under these circumstances it is universal practice to use gyroscopic instruments responsive to gravity for vertical indications and controlled by north-seeking devices for azimuth to establish a set of coordinates for directional reference purposes. It is significant to note that these instruments by-pass the structure of the vehicle by which they are carried and act as self-contained equipment in providing orientational reference coordinates. With visual contacts eliminated, artificial radiation links at radio and radar frequencies are used to establish distances and directions from known ground-based stations to the vehicle. Thus, directional information in earth coordinates is provided by both on-board and remote equipment while indications of distance come from the radiation links with known points on the earth.

Figure I-9 illustrates the situation that exists when both visual contacts for judging orientation and artificial radiation links for indications of position are not available. The devices providing orientational reference information must be improved to have several orders of magnitude smaller error rates than the orientational reference needed for the situation of Fig. I-8. By giving the geometrical reference member an initial alignment accurately related to earth coordinates at a known point, changes in location may be indicated by effectively carrying out the double integration of accelerometer output, or by following changes in direction of the earth's gravity vector. This means, in effect, that earth coordinates for a selected time and place must have been transferred to a self-contained system aboard the guided vehicle. With the good equipment performance necessary to provide information continuously and accurately on earth space directions and properly mounted accelerometers with output signals processed by computers, a self-contained system will indicate location and velocity with respect to its point of departure. A system which operates in this way is called an INERTIAL GUIDANCE SYSTEM. Systems of this kind are universally used in ballistic missiles and in submarines when guidance without outside contacts is important. Inertial guidance systems for service over the earth are implemented in many ways, but the necessity remains for an accurate, continuously available, self-contained geometrical reference related in a known way to the external space in which guidance is to be performed. A number of typical mechanizations are described briefly in a later section of this report.

When guidance is considered for vehicles to operate not in the near vicinity of the earth but in regions for which the earth, moon, planets and stars effectively approach mass points, coordinates aligned in earth space lose their usefulness. Rather, it is necessary to employ geometrical reference coordinates associated with stars and planets. For example, celestial sphere coordinates, or some other directions such as a line directed toward the sun, may be set into an inertially stabilized reference member. Figure I-10 shows the essential features of the Apollo Guidance System which is to be used on manned flights to the moon and return. This system must deal reliably and accurately with some fifteen or twenty different problems of guidance ranging from earth launching through earth orbit, mid-course to the moon, moon orbit, moon landing and return to orbit, and finally through space to landing at a pre-selected point on the earth.

An inertial member with provisions for either manual or automatic alignment with earth coordinates or celestial coordinates is used as the geometrical reference during acceleration phases of the trip. Visually or automatically established lines of sight to known stars and to landmarks on the earth and the moon are used as the basis for determining location and velocity during mid-course flight. Data from these

observations are processed by a digital computer which supplies both navigational information and guidance commands for system operation, which may be completely automatic as self-contained equipment, completely manual or partially automatic with monitoring by on-board human pilots, or with remote monitoring by ground-based supervisors, through radio and radar links. It is probable that the Apollo Guidance Systems which have already been conceived, designed, built, tested and delivered will be useful models for Space Guidance Systems of the future,

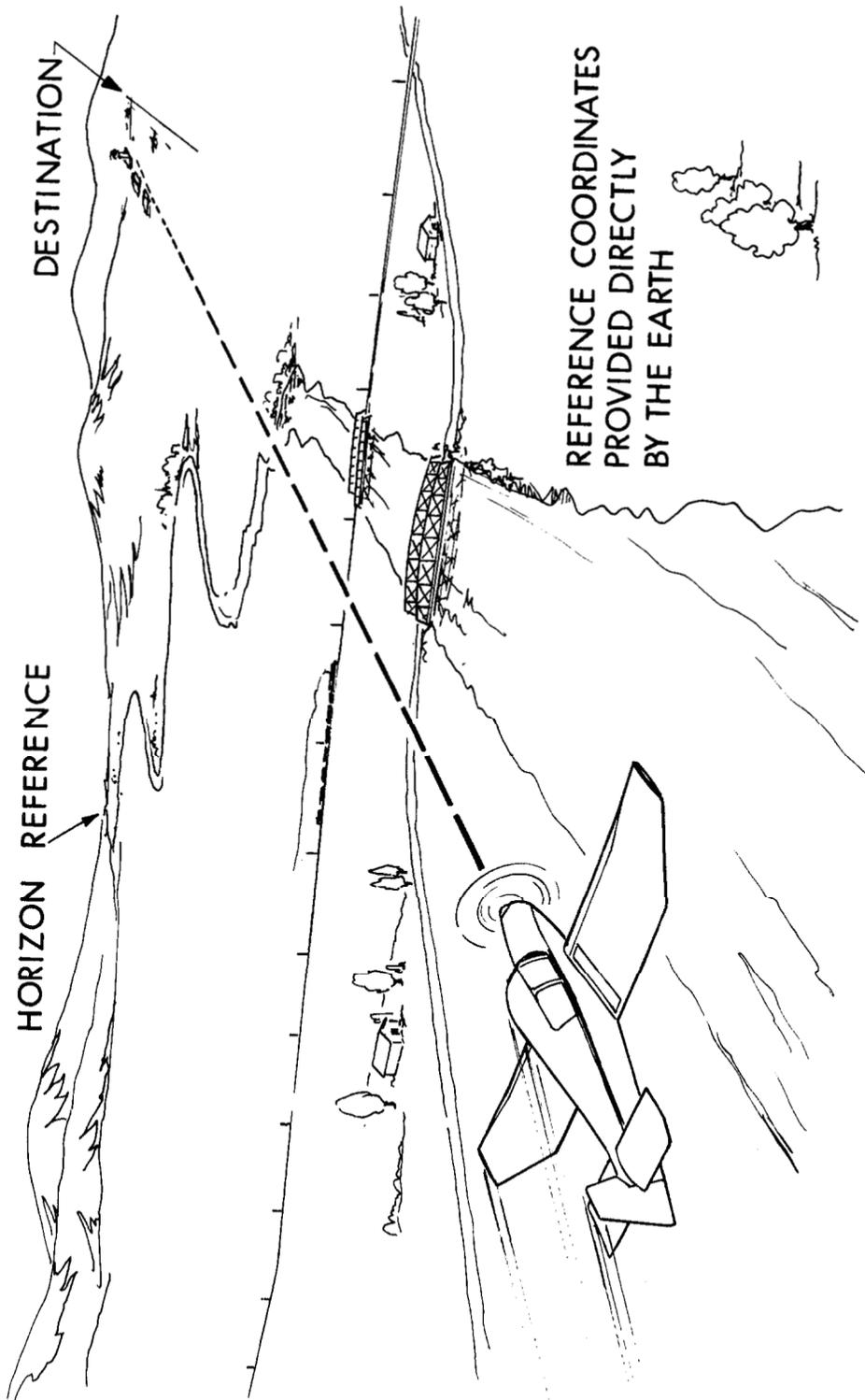


Fig. I-7 Navigation by Visual Contact with Earth

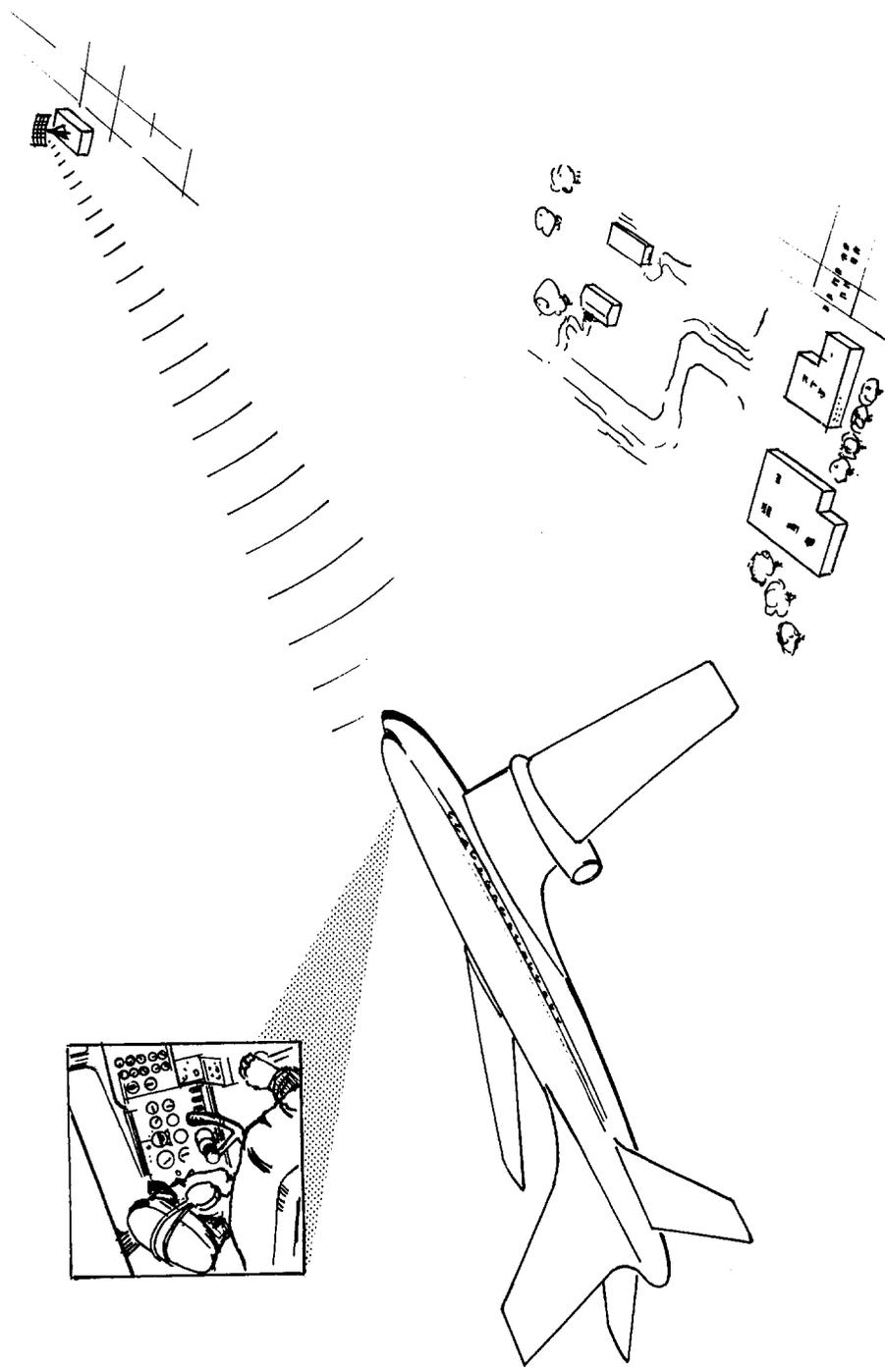


Fig. I-8 Navigation Without Visual Contact to Earth

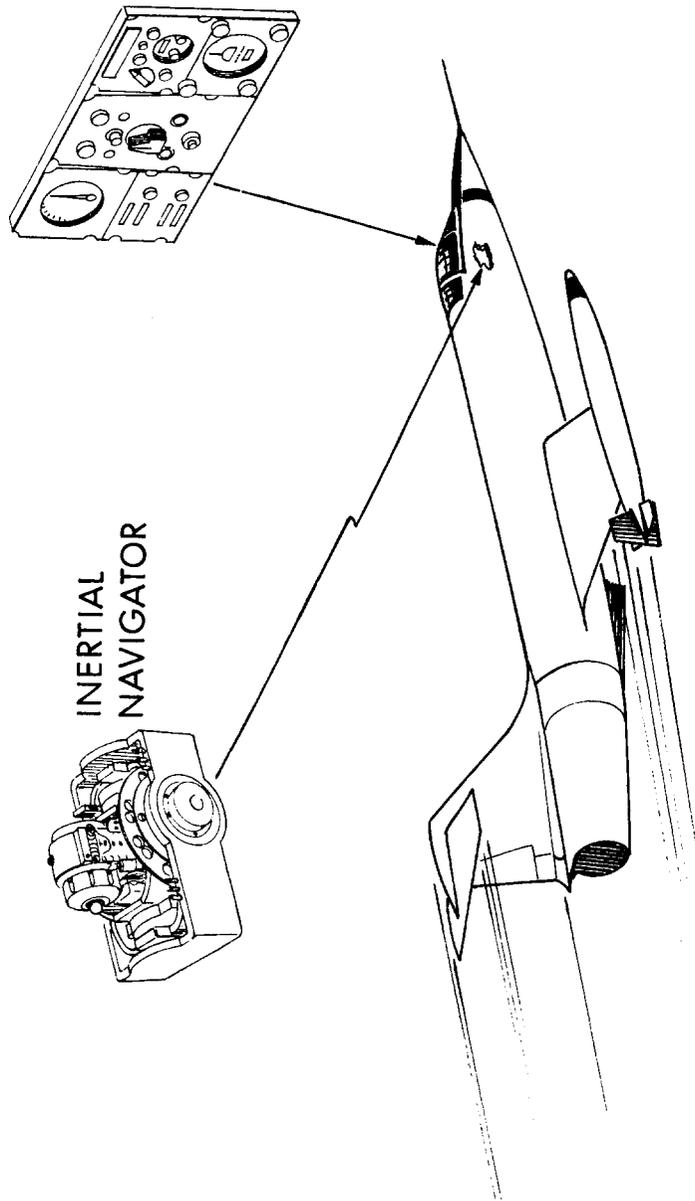


Fig. I-9 Navigation with Both Visual and Artificial Radiation Contacts with Ground Loast

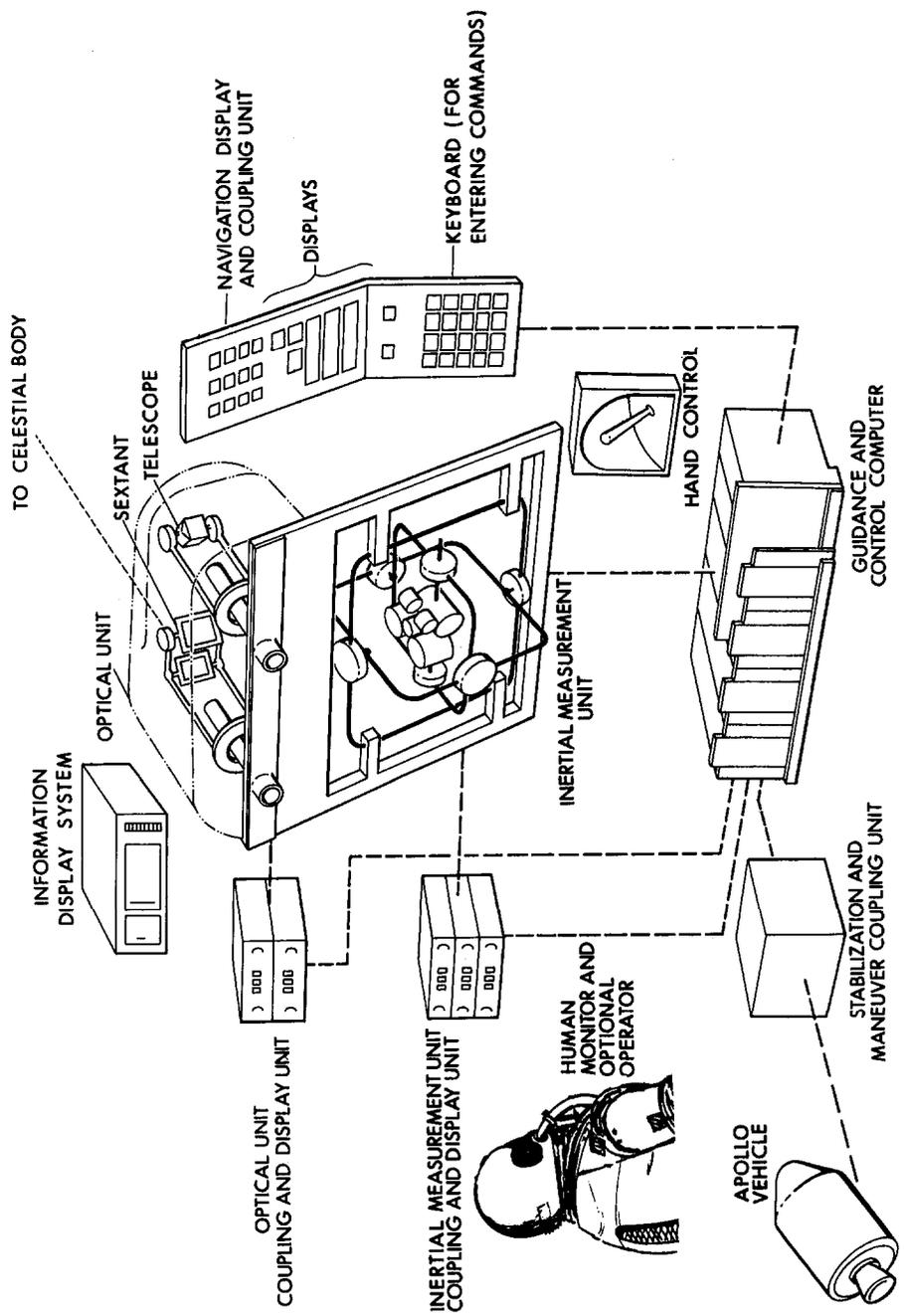


Fig. I-10 Mechanism Elements of Apollo Guidance System

## CHAPTER I-3

### FUNCTIONAL REQUIREMENTS OF SYSTEMS AND THEIR COMPONENTS FOR CONTROL AND GUIDANCE

Figure I-11 represents the essential geometrical function of stabilization in terms of right-angled three-coordinate axis systems associated with the spaces concerned. A set of stabilization reference coordinates is established by some instrumental means or by direct visual contact to a space either identical with or related to the space in which vehicle motion is to be guided. An essential duty of the control system is to cause axes fixed to the vehicle structure to remain continually close to the stabilization reference axes, and also to keep the vehicle velocity vector substantially identical with a stabilization reference velocity vector which has a direction and a magnitude established in some way with respect to the reference space in which the vehicle path is defined.

Figure I-12 represents maneuver, a second control system function in which the stabilization reference coordinates and the stabilization reference motion are changed with respect to the vehicle path reference space in the ways necessary to accomplish missions. The control system inputs that serve this purpose are maneuver commands generated from plans, programs, feedback data, environmental data, and other information by a guidance system. The nature and functions of this system are illustrated by the diagram of Fig. I-13.

Figures I-14 and I-15 suggest the control and guidance situation that existed during the early days of manned flight. Without a pilot to complete the information-handling feedback loop of control and guidance, an airplane was completely useless. Plans and programs were stored in the man's brain, stabilization references and airplane conditions were noted by human senses and processed in the pilot's mind to generate maneuver commands that were applied to the airplane control levers by his hands and feet.

Figures I-16 and I-17 illustrate the control and guidance situation that commonly exists today in jet aircraft. Human pilots continue to be used in the control and guidance systems but their senses are greatly extended by radio, radar and many instruments, their muscle forces are boosted by servo power and their ability to solve complex problems rapidly is extended by computers. All of these appurtenances

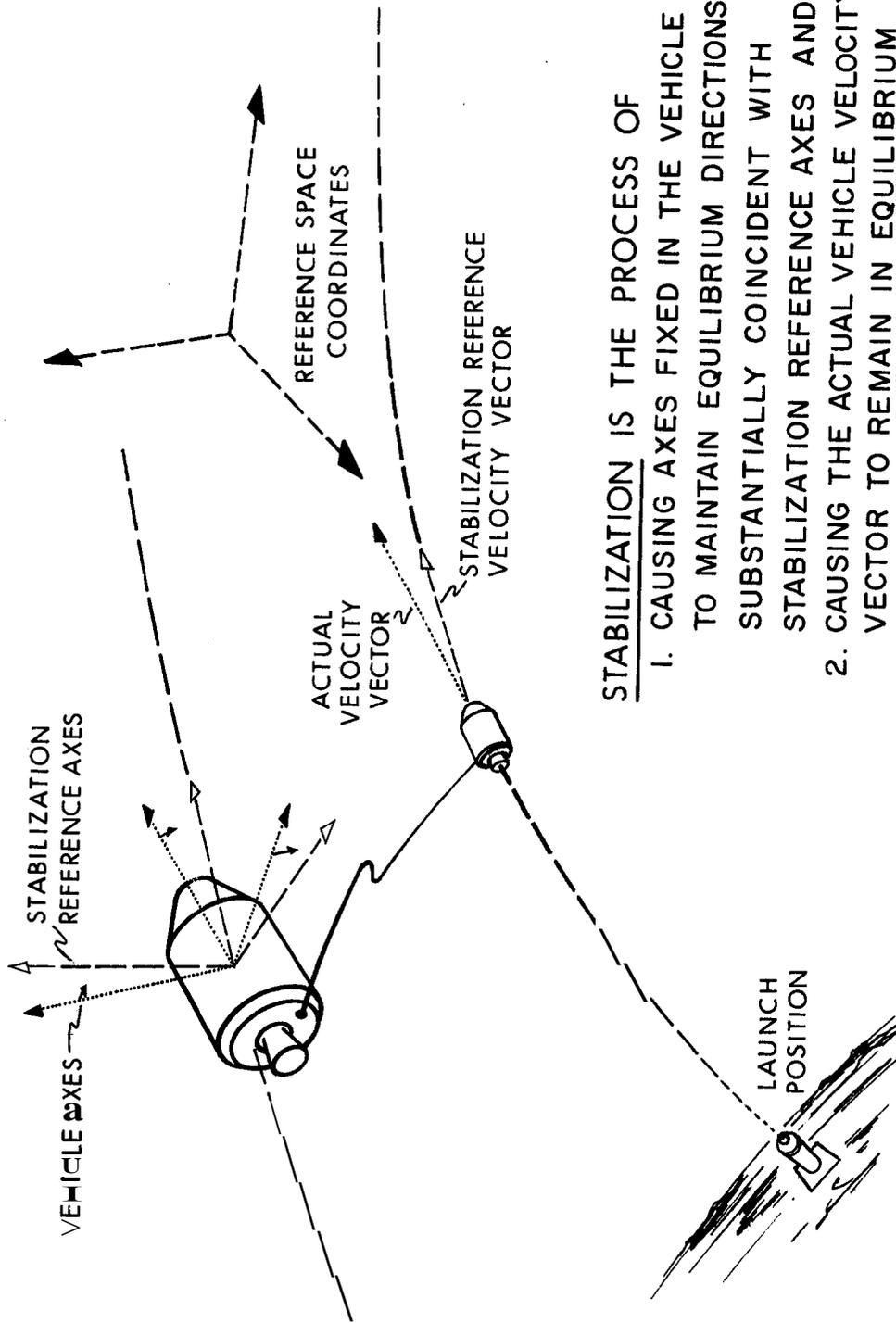
certainly improve the effectiveness of control and guidance, but the pilot's position as an "on-line" component in both the control and guidance loops means that his limitations in ability to solve complex problems rapidly and properly handle situations requiring too rapid responses, set boundaries to the possible performance of the overall system.

Figures I-18 and I-19 suggest the circumstances that exist in rocket powered vehicles that, because of limited payload capacities, one-way missions, hostile environments, and severe programs must operate with self-contained automatic control and guidance systems. The absence of restrictions imposed by human limitations makes it possible to realize ballistic missiles and other vehicles with capabilities well beyond those in which men provide control and guidance functions as "on-line" components.

Figures 1-20 and 1-21 illustrate the situation that exists in the control and guidance system of a manned vehicle to operate in the astronomical regions above the earth's atmosphere and beyond the earth's gravitational field. The control and guidance systems are automatic with orientational and translational references provided by an inertially stabilized member, and a set of three accelerometers rigidly mounted on this member with input axes set in an orthogonal configuration. A telescope and a space sextant with their line-of-sight directions adjustable and transferable through a computer to the inertial reference member give information for correcting reference member alignment. Observations by the human pilot or by automatic optional tracking also supply data for a computing system to calculate positions in space and to generate correction maneuver commands.

Flight condition data displayed by the automatic control and guidance system to the human pilot provide the information for monitoring system operation. A set of controls forming the operation mode selector and optional command system make it possible for the pilot to determine the mode in which the overall system works. He may select any sort of configuration from full automatic, in which he only observes operation, to completely manual, in which he acts to close servo-loops by continued on-line operation.

Figures 1-22 and 1-23 suggest the complete configuration used by Apollo in which a ground-monitoring system connected by radio, radar and possibly visual up and down links to the flight vehicle. The earth-based system acts as an information collecting and monitoring branch in parallel with the on-board pilot monitor. Information and suggestions may be sent to the space vehicle - whether or not they are accepted in any given case depends on operating doctrine and the circumstances of particular cases.



STABILIZATION IS THE PROCESS OF

1. CAUSING AXES FIXED IN THE VEHICLE TO MAINTAIN EQUILIBRIUM DIRECTIONS SUBSTANTIALLY COINCIDENT WITH STABILIZATION REFERENCE AXES AND
2. CAUSING THE ACTUAL VELOCITY VECTOR TO REMAIN IN EQUILIBRIUM SUBSTANTIALLY COINCIDENT WITH THE STABILIZATION REFERENCE VELOCITY.

Fig. I-11 Stabilization

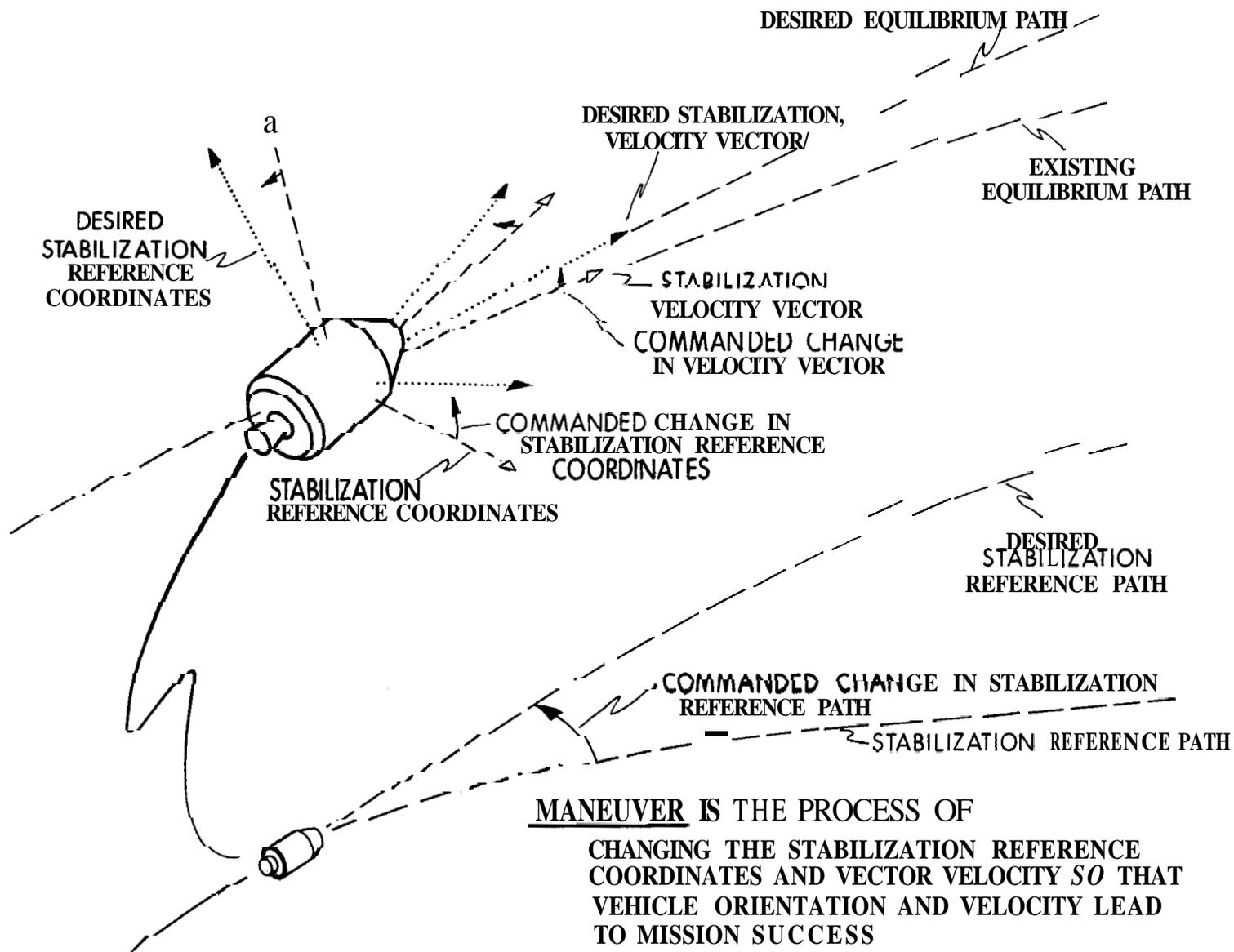
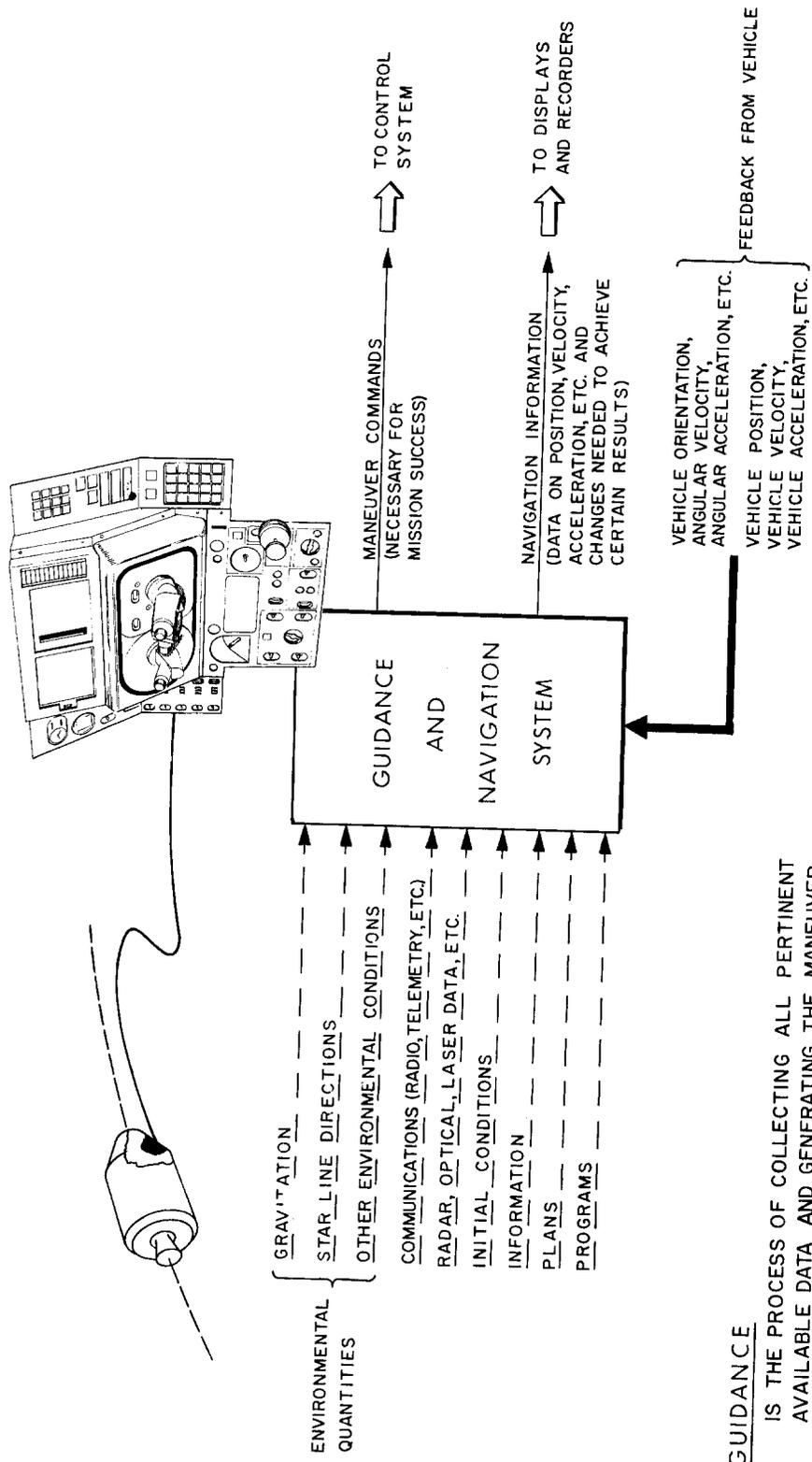


Fig. I-12 Maneuver



BECAUSE THE OPERATIONS REQUIRED ARE GENERALLY SIMILAR FOR GUIDANCE AND FOR NAVIGATION THE TWO FUNCTIONS ARE CARRIED OUT BY THE SAME SYSTEM—CALLED THE GUIDANCE AND NAVIGATION SYSTEM

GUIDANCE  
IS THE PROCESS OF COLLECTING ALL PERTINENT AVAILABLE DATA AND GENERATING THE MANEUVER COMMANDS NECESSARY FOR MISSION ACCOMPLISHMENT

NAVIGATION  
IS THE PROCESS OF COLLECTING ALL PERTINENT AVAILABLE DATA, GENERATING INFORMATION ON POSITION AND MOTION, AND INDICATING THIS INFORMATION FOR THE PURPOSES OF DISPLAY AND RECORDING

Fig. # 13 Guidance

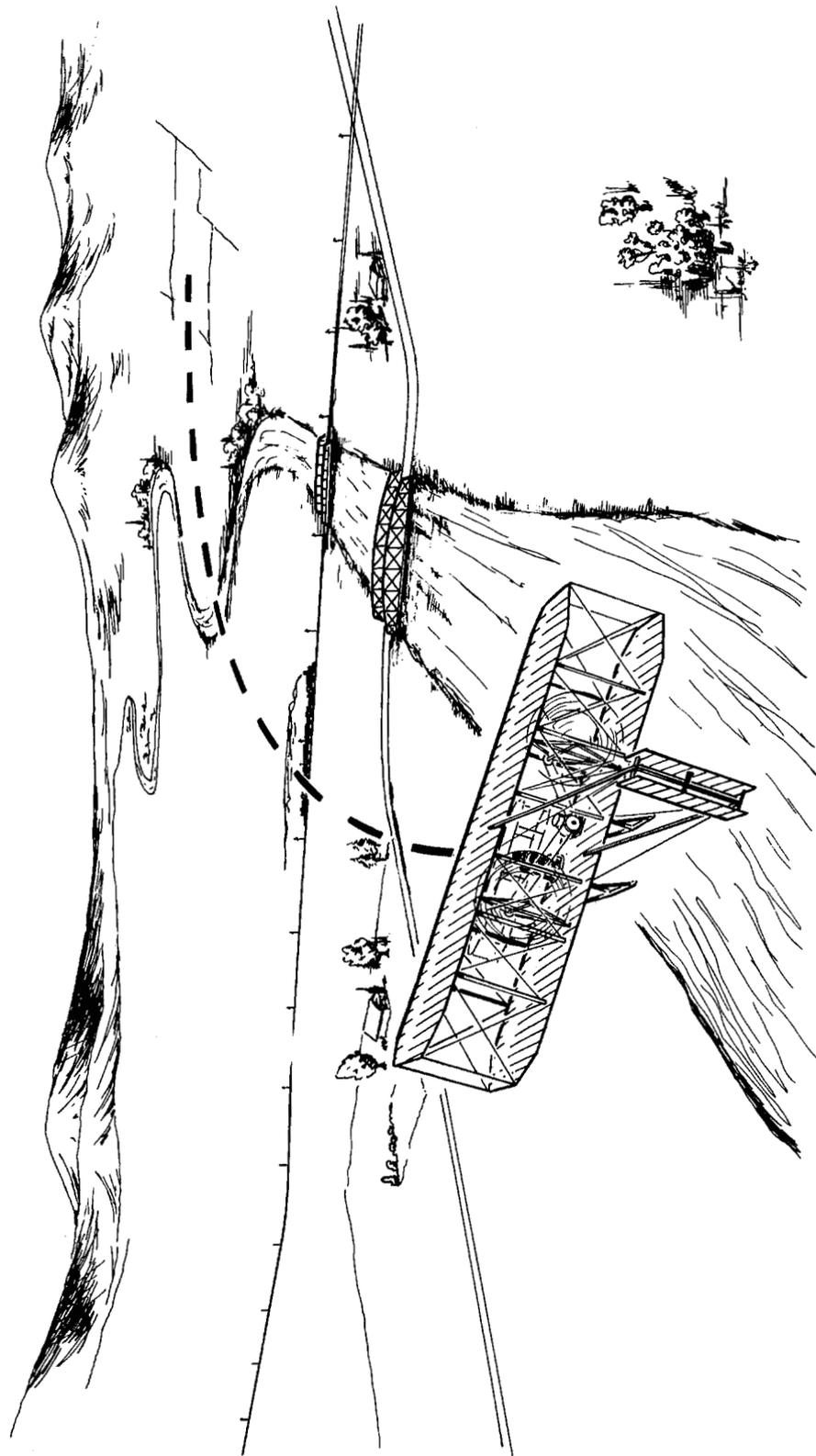


Fig. I-14 Guidance - Human Operator

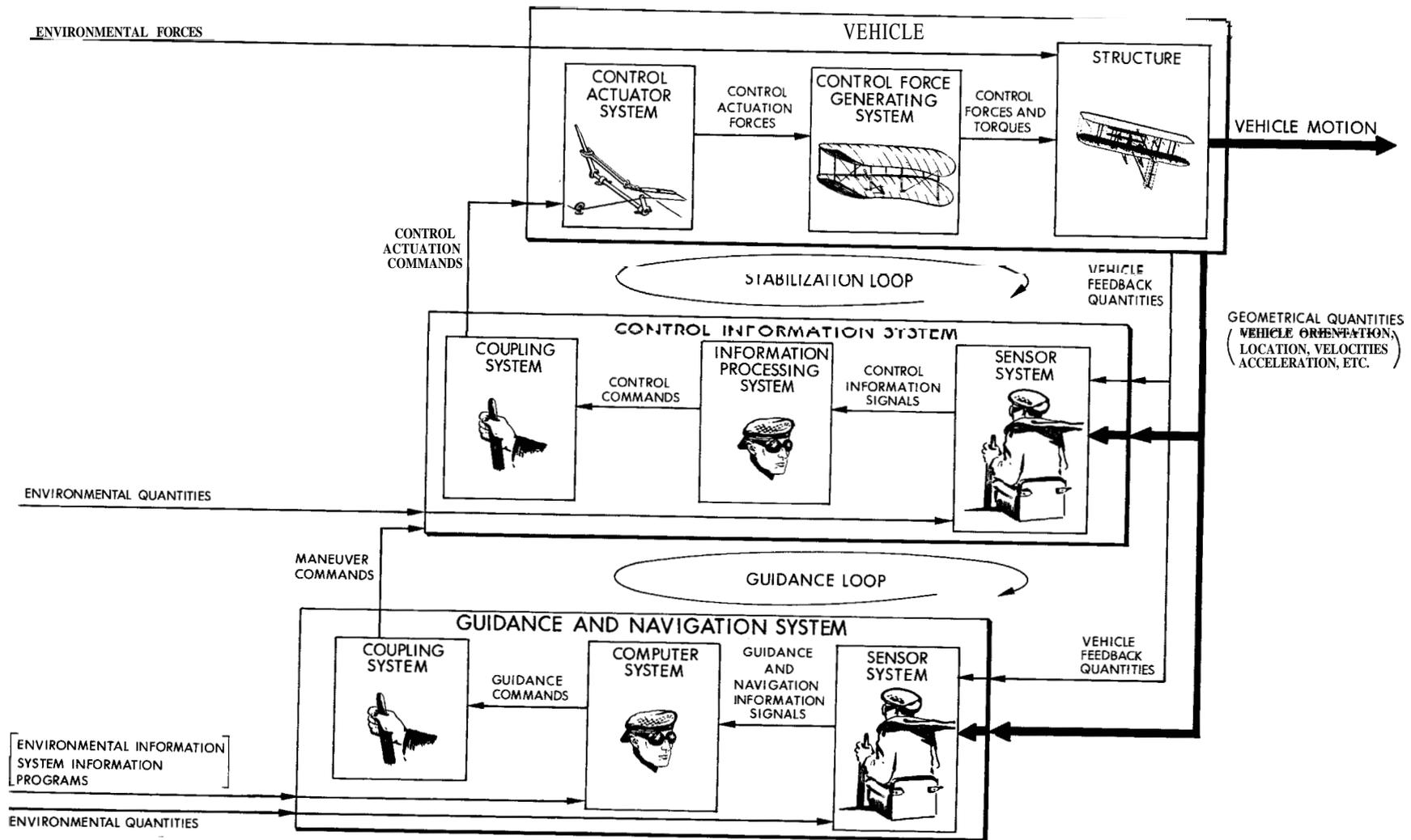


Fig. I-15 Guidance and Control System - Human Operator

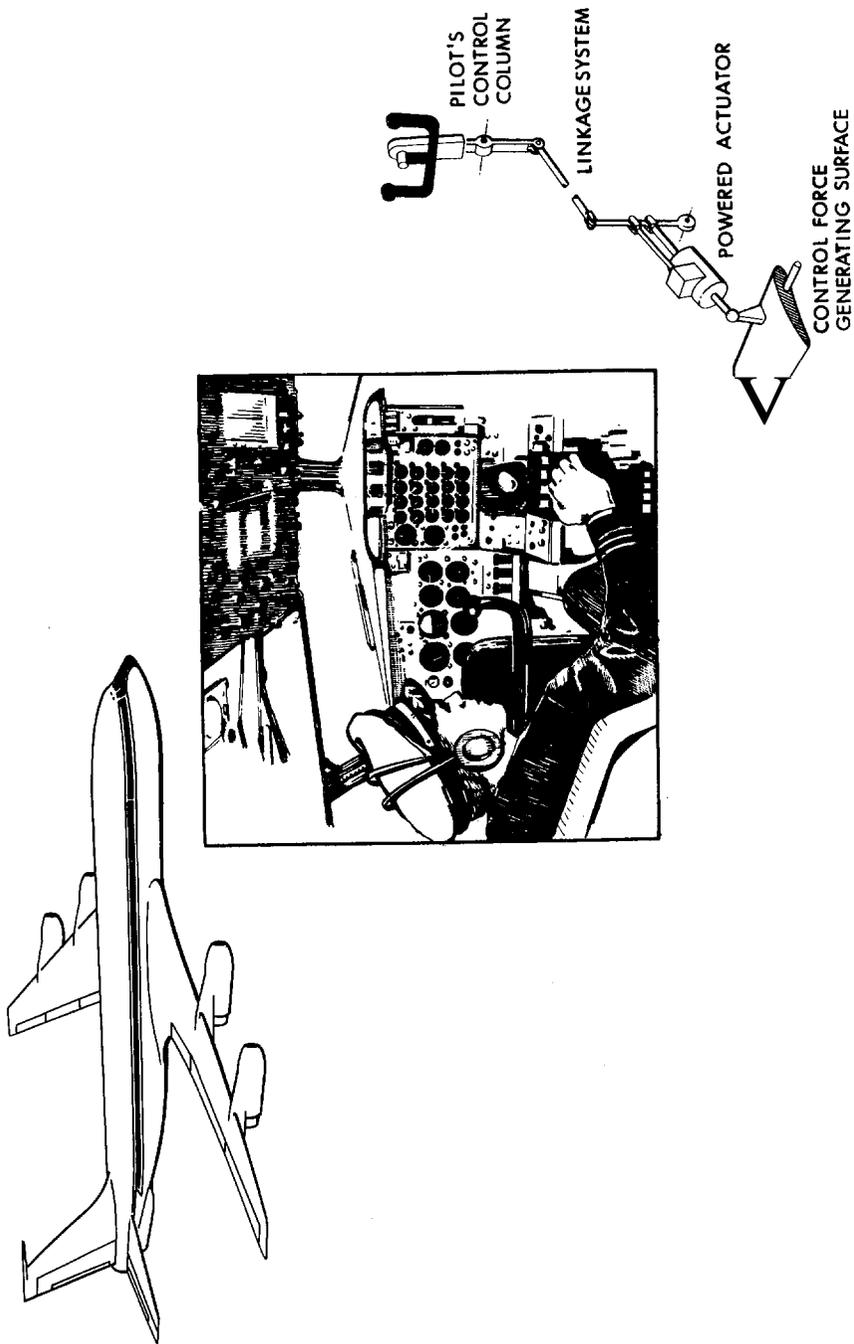


Fig. I-16 Extension of Human Operator's Senses and Power

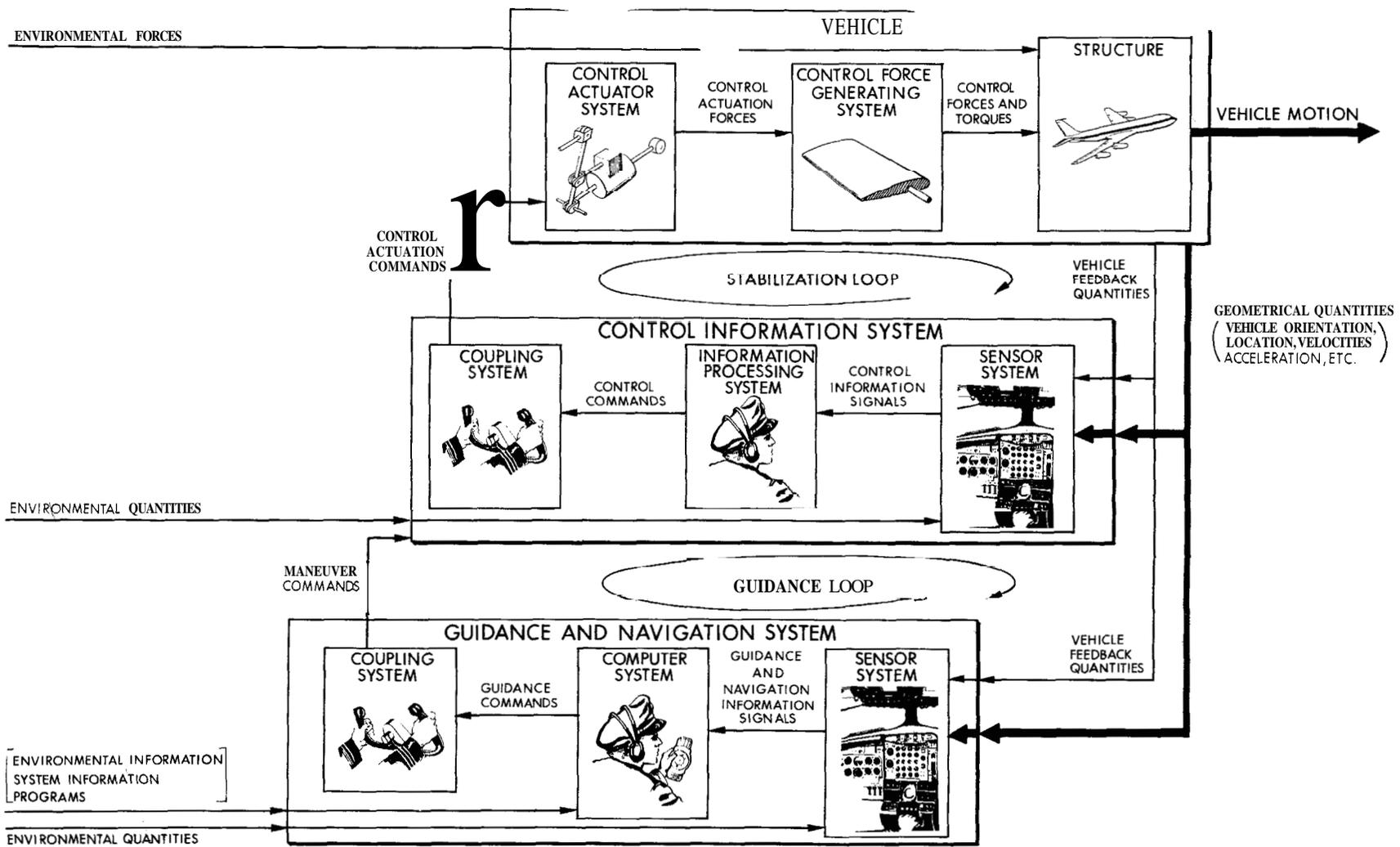


Fig. 1-17 Guidance and Control System - Human Operator's Senses and Power Extended

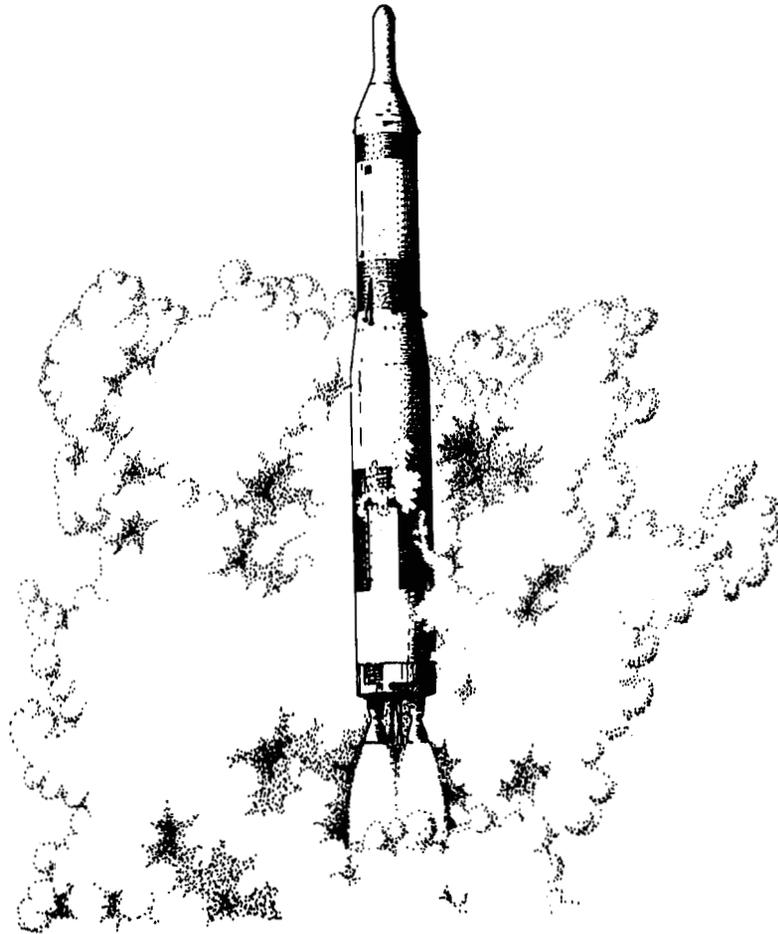


Fig. I-18 Automatically Guided Flight Vehicle

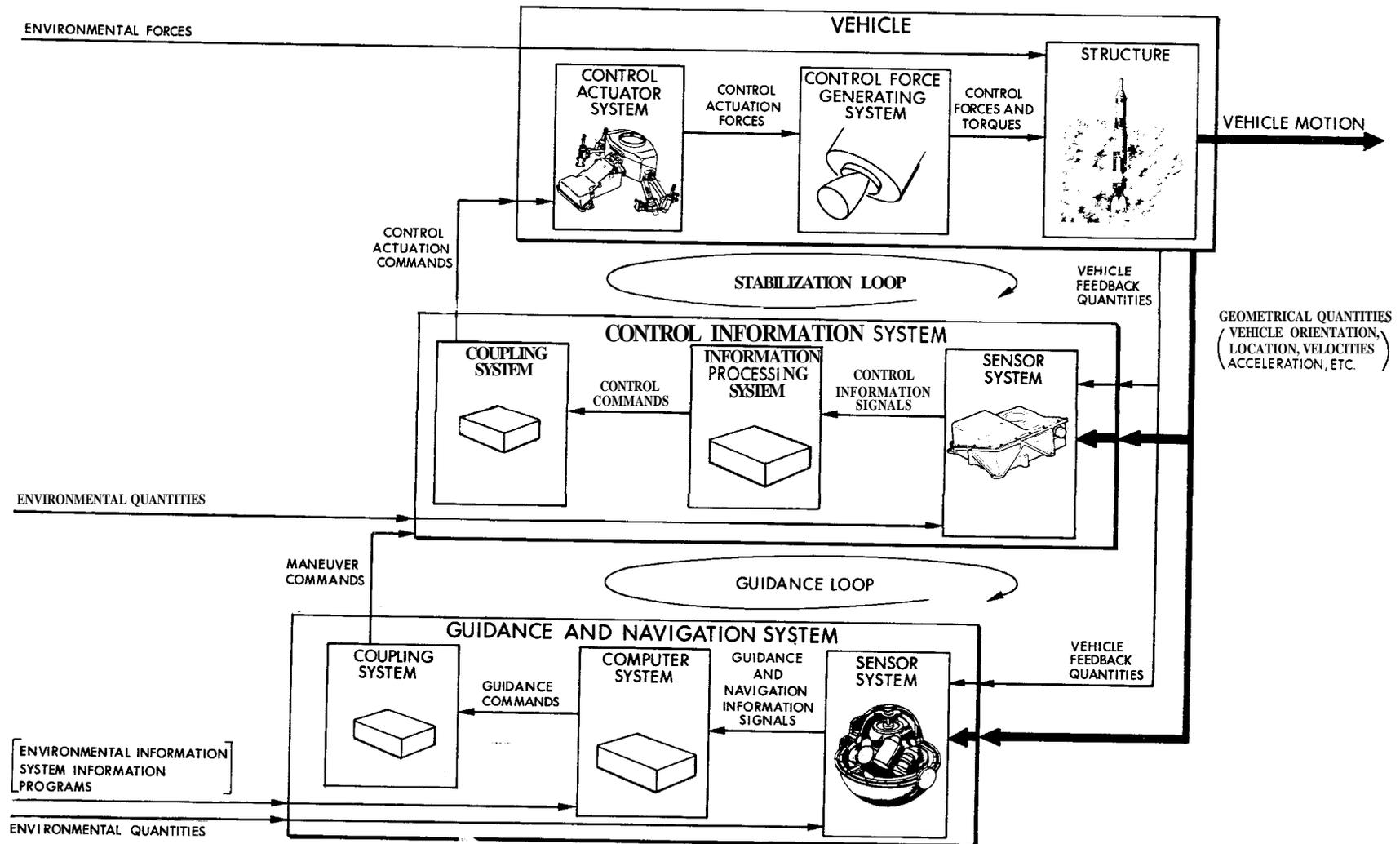


Fig. I-19 Guidance and Control System - Automatic Operator

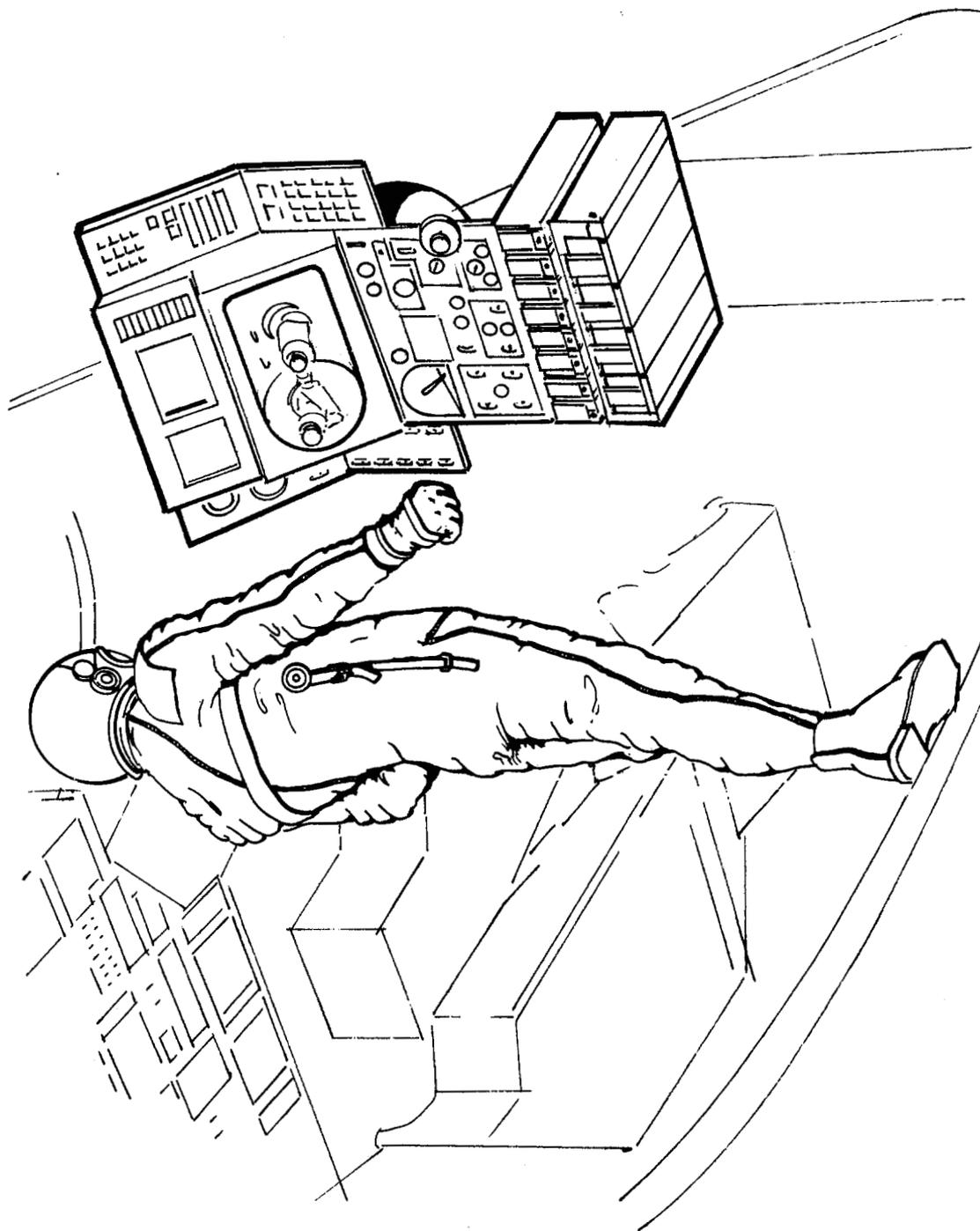


Fig. I-20 Space Vehicle Guidance System

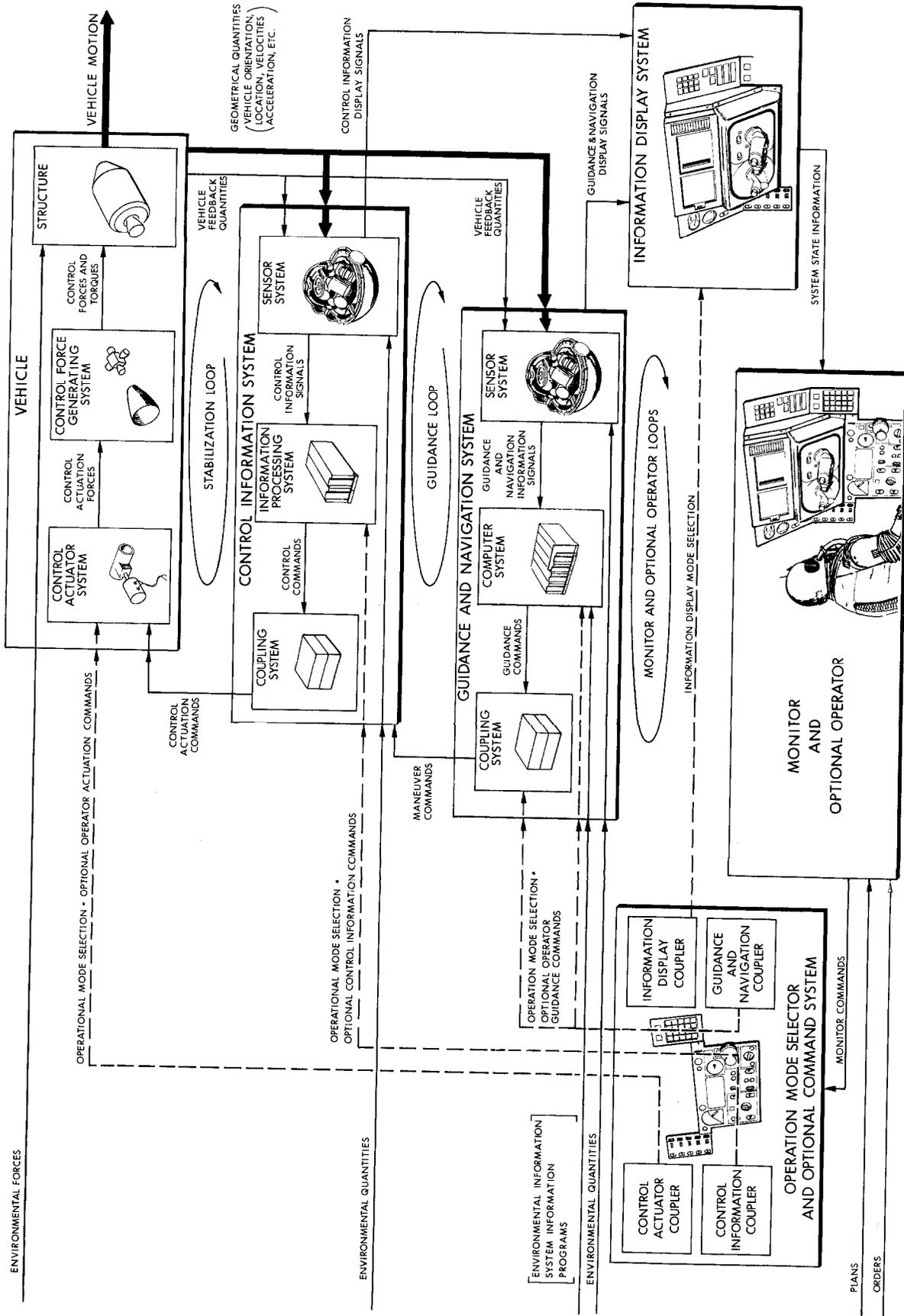


Fig. I-21 Functional Diagram of Complete Vehicle Guidance System with Optional Monitor Command Operation

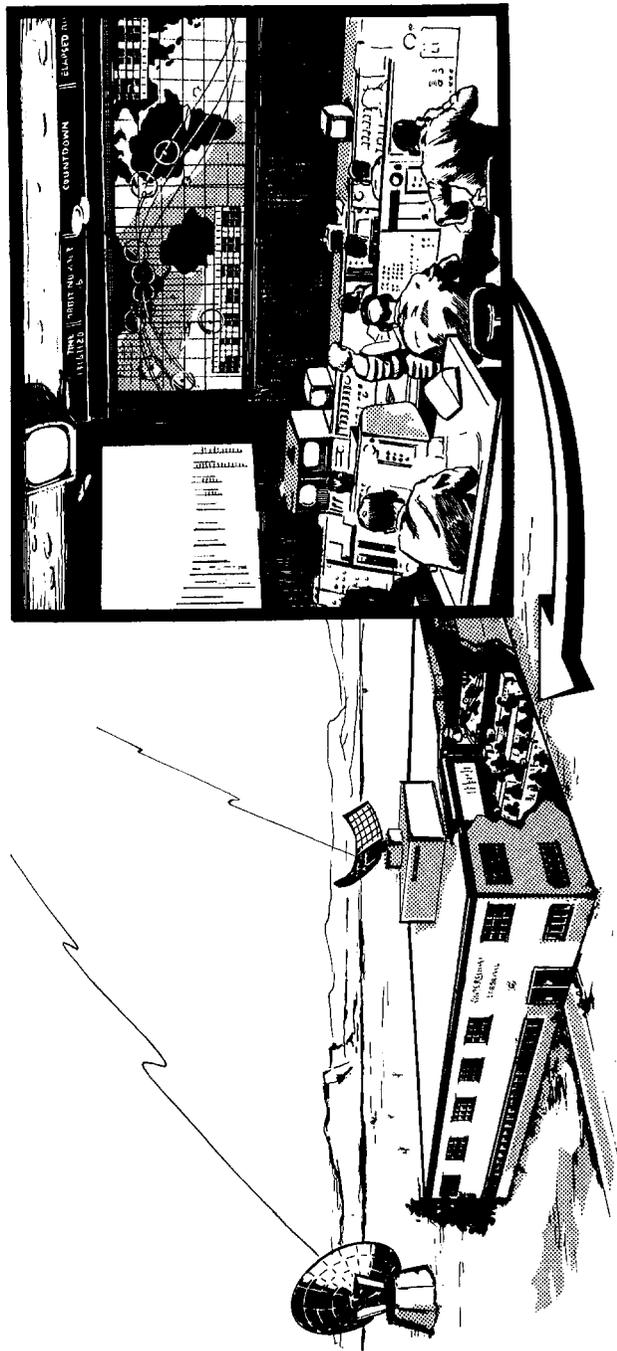
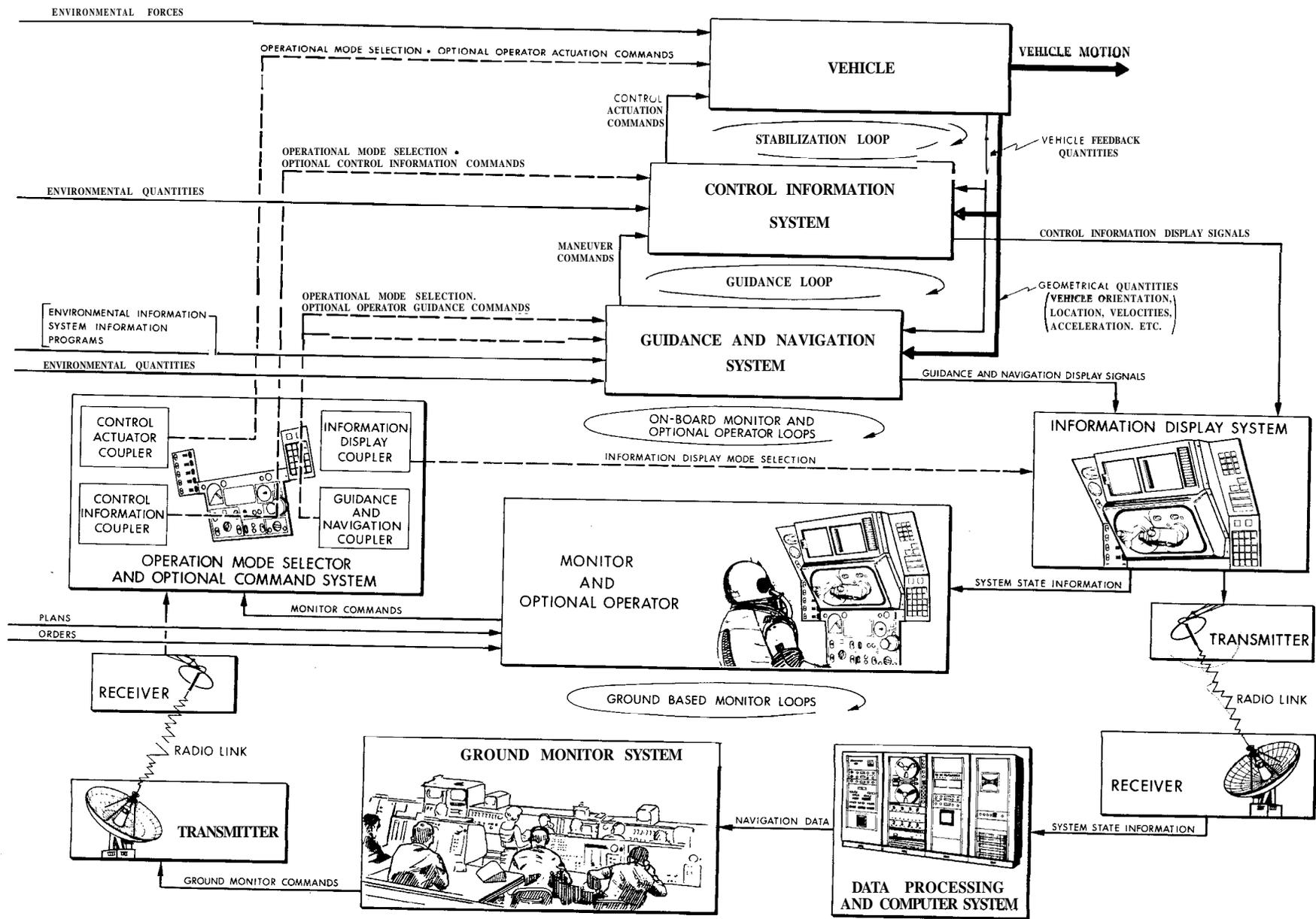


Fig. I-22 Ground Based Monitor System



1-26

Fig. 1-23 Control and Guidance System with Ground Based Monitor

## CHAPTER I-4

### STATE OF TECHNOLOGY OF COMPONENTS FOR CONTROL, NAVIGATION AND GUIDANCE SYSTEMS

1. Radiation links which are generally employed to implement friendly environments have the function of establishing contacts from transmitters to receivers, transponders and reflecting bodies. These links provide communications by voice, by telemetry and by signals of other kinds. Pulsed and continuous wave radar indicate line-of-sight directions and distances. Lasers and ordinary search-lights also offer powerful radiation links, particularly for satellites and space vehicles.

In the current state of radiation link technology which allows determination of distances within a few feet, the links themselves do not impose limiting restrictions on the performance of navigation and guidance systems as far as distance measurements are concerned. The ability of radiation links to determine line-of-sight directions with inaccuracies less than one milliradian is adequate for the needs associated with navigation and guidance.

Carefully surveyed ground station sites with commonly available indicators of the vertical provide earth reference coordinates of such high accuracy that radiation beam orientations may be taken as substantially perfect. On the other hand, radiation links established by airborne transmitters generally have adequate ability to measure distances, but have restricted ranges due to limitations on size, weight and power consumption of the geometrical stabilization member. For these reasons the accuracy of directional tracking is not so good as that obtained from ground stations. A generally more severe limitation of air-borne radiation link equipment is introduced by inaccuracies of reference coordinate equipment which will always be greater than the corresponding inaccuracies of transmitters rigidly fixed to the earth. This source of reduced performance may be serious, and, in any case, must be given careful consideration in evaluating the errors of any particular system.

2. Computing Systems receive essential information and carry out the mathematical processes necessary to generate required outputs. The problems solved range from trigonometric transformations to the determination of position and velocity from accelerometer output signals. In terms of a rough analogy,

computers perform the same functions that the brain of a pilot provides when a human being acts as the on-line data processing component in navigation and guidance equipment.

The current technology of computers, particularly those based on digital operations, is so well developed that units of ample capacity, speed and reliability with reasonable sizes and weights and power consumptions are available for use in guidance systems. Improvements in all essential computer features including resistance to environmental interference effects are now in progress. It is certain that mechanization of computing functions is not now and will not in the future be a limiting factor on navigation and guidance systems.

3. Engineering problems associated with angle sensing servomechanisms, data transmission, mechanical design, etc., have current solutions that are generally satisfactory with advances certain to appear in the near future. Except for certain special situations these factors do not limit the performance of equipment for navigation and guidance.

4. Guidance system coordinates related in a known way to reference directions of the space in which the desired path of the guided vehicle is defined are easily established when rigid *or* optical connections to the ground are available. When the guidance system is vehicle-borne its necessary reference coordinates must be established with a known relationship to external space and maintained with this relationship defined during the progress of guided flight. This situation is suggested by the diagram of Fig. 1-24 with the guidance system coordinates initially established before flight with known geometrical relationships to the flight path space reference coordinates.

For the purposes of guidance, the system coordinates must continue to provide a geometrical reference that accurately represents the flight path space as the vehicle moves to complete its mission. Because mechanical connections are impossible and radiation links between the vehicle and the flight path reference space generally absent, the only possibility for realizing satisfactory guidance system reference coordinates lies in the use of inertial principles. Properly applied, these principles make it possible to mechanize a member which either remains non-rotating with respect to inertial space, *or* moves in a quantitative way with respect to this space. Details of arrangements to accomplish such results are discussed in the next section of this paper.

Current technology is easily able to provide guidance system reference coordinates representing flight space coordinates within one minute of arc for each hour of operation. The arc-second inaccuracy required by military guidance for hard targets is more difficult to achieve, but is feasible with proper attention to

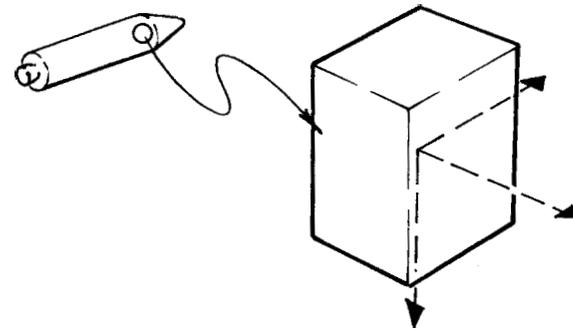
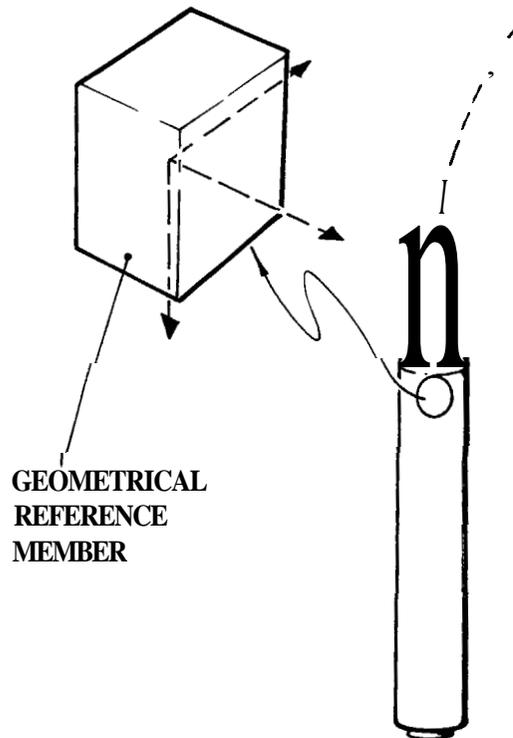
design and production of components. The principles available, typical arrangements and performance realized are discussed in the next section.

5. Specific force receivers, the devices commonly called accelerometers, are the only available means for on-board sensing translational vehicle motion when radiation links with the environment are not available. A commonly used configuration of specific force receivers is to mount three units rigidly to the geometrical reference member with their input axes aligned with the guidance reference axes. This arrangement is suggested in the diagram of Fig. I-25.

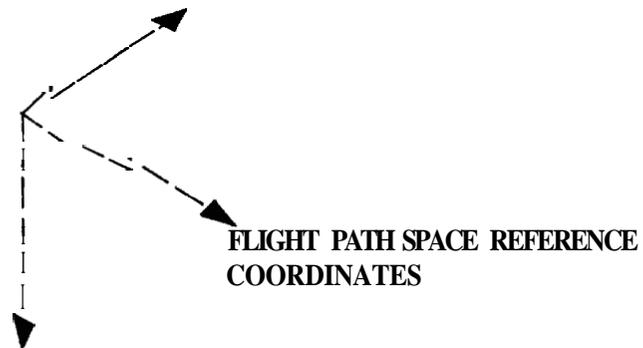
Signals from the three specific force receivers represent the resultant components of gravity force and inertia reaction force along each of the three axes. With the geometrical relationships of these axes to the flight space reference axes known, calculations based on these signals make it possible for the computer to generate output signals giving the changes in vehicle location and velocity occurring after the start of system operation.

Assuming perfect alignment of the guidance system reference coordinates with the flight space reference coordinates, and perfect computer operation, errors in indicated location and motion are due to imperfections in specific force receiver performance. Performance matching the requirements of navigation is easy to realize, while military guidance for hard point targets is within the capabilities of today's advanced technology. The mechanizations that afford these results are described in a later section.

**GUIDANCE SYSTEM REFERENCE COORDINATES  
INITIAL CONFIGURATION ESTABLISHED  
WITH RESPECT TO FLIGHT PATH REF-  
ERENCE COORDINATES**



**GUIDANCE SYSTEM REFERENCE COORDINATES MAINTAINED  
DURING FLIGHT WITH A KNOWN RELATIONSHIP TO FLIGHT  
PATH SPACE REFERENCE COORDINATES**



1-40

**GEOMETRICAL  
REFERENCE  
MEMBER**

**FLIGHT PATH SPACE REFERENCE  
COORDINATES**

Fig. 1-24 Relationship of Guidance System Reference Coordinates to Flight Path Reference Coordinates

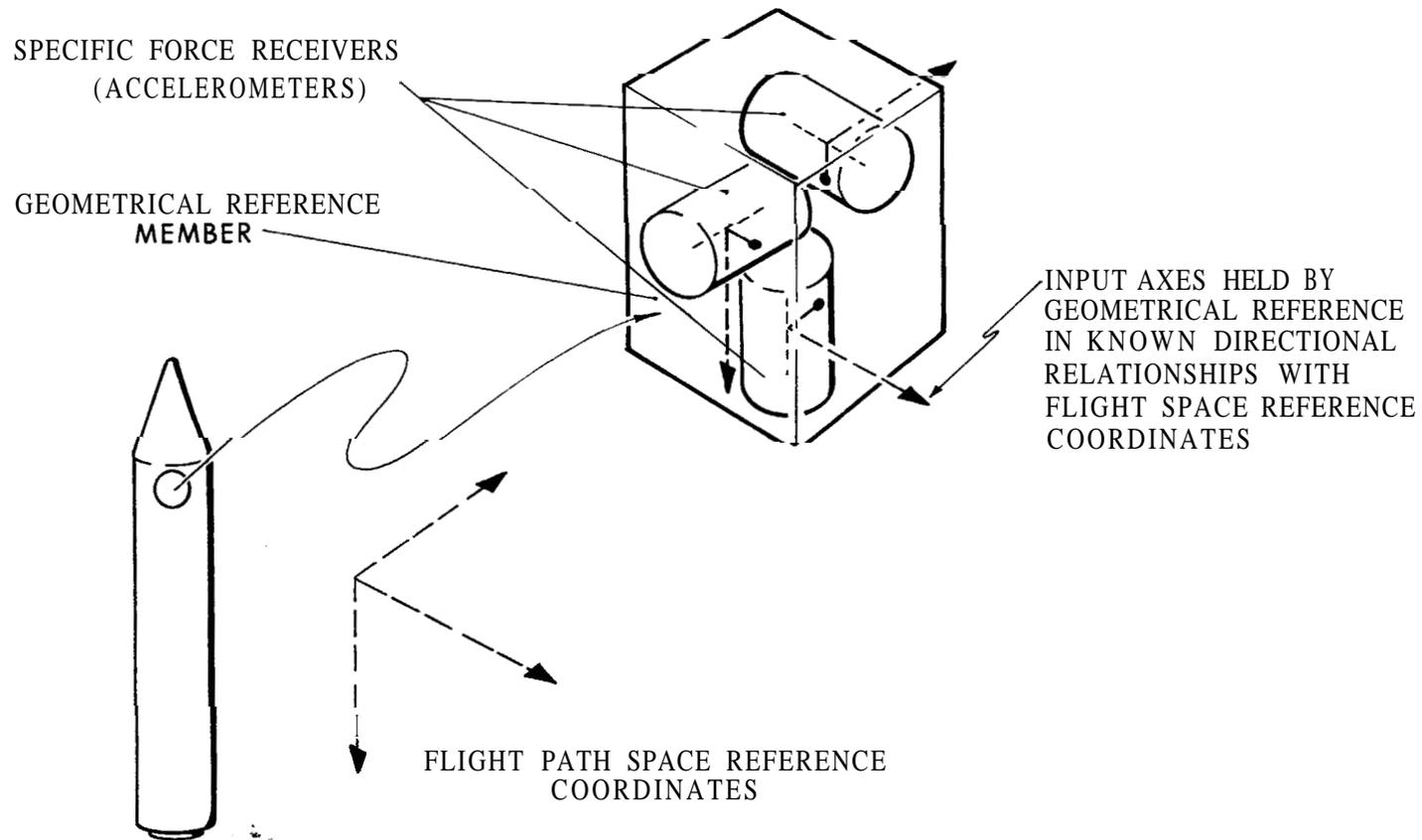


Fig. 1-25 Specific Force Receiver (Accelerometer) System

## CHAPTER I-5

### GYROSCOPIC UNITS FOR REALIZATION OF GUIDANCE SYSTEM REFERENCE COORDINATES

Reference coordinates for vehicle-borne guidance systems with satisfactory performance must fulfill the general requirements summarized in Fig, 1-26. The basic functions are: 1) to provide continuously available mechanism reference directions; 2) to provide accurate control of the reference directions with respect to a selected external reference space in response to command signals; 3) to provide angular output signals that accurately represent deviations of case fixed reference directions from the mechanism reference directions. In addition to these essential performance characteristics, practical instruments must be reliable, of reasonable size and weight, and be available at acceptable cost.

Gyroscopic principles may be applied to mechanize practical instruments for providing guidance system reference directions, The theory involved is associated with applications of the Newtonian Laws of Mechanics to rapidly spinning symmetrical rotors, It is relatively simple to discuss this theory in terms of vectors representing rotational quantities in accordance with the commonly used "right hand" conventions that are summarized in Fig. 1-27. The central idea is that a rotational quantity such as angular velocity or angular momentum may be described by a vector along the axis of rotation with its length proportional to the magnitude of the quantity, and the head of its arrow related to the direction of rotation by the "right-hand screw rule, "

Figure 1-28 suggests the basic operating principle of a gyroscopic element. When the gyroscopic element definition condition of constant spin velocity exists, Newton's Law of dynamics leads to the conclusion that a torque applied to the rotor at right angles to the spin axis causes the angular momentum vector to change its orientation with respect to inertial space with an angular velocity of precession proportional to the magnitude of the torque and having a sense that always turns the angular momentum vector toward the torque vector.

One way of using the gyroscopic element to realize mechanism reference directions for guidance system purposes is to set up an angular momentum vector, and then carefully to reduce all torque components on the rotor to zero about any axis at right angles to the spin axis, In this torque-less condition the angular

momentum vector maintains an orientation with respect to inertial space that is completely determined by the direction about which the spinning torque originally built up the angular momentum of the rotor. Once the spin angular momentum vector direction is established it becomes useful for reference purposes only through the medium of signals that represent angles between the spin axis and reference directions fixed to the case within which the rotor spins. Figure 1-29 suggests the situation that exists when a case is rigidly fixed to some base that may have any arbitrary changes in orientation with respect to inertial space. The position of the case with respect to the reference direction can be indicated in terms of the two angles between perpendicular lines fixed to the case, and the spin axis. Signals representing these two angles may be processed by a computer to give information about the position of the case with respect to the spin axis in terms suitable for any particular problem of control and guidance.

The situation that is generally of practical interest requires information on the orientation of the case with respect to some specified external reference space. It is obvious that any single direction such as a spin axis can not specify angular positions of the case about the spin direction so that a second gyroscopic element with its spin axis having some projection at right angles to the first spin axis is required for any complete indication of case orientation in three dimensional space. When two spin axis directions are available for mechanism reference purposes, the common position of two gyro unit cases rigidly connected together gives all the needed information. It is to be noted that the interpretation of this information depends on the continuous operation of a computer to carry out complex trigonometrical calculations. The relationships of spherical trigonometry are such that computations generally give results of varying accuracy as case orientations change by large angles with respect to spin axis directions. This fact, coupled with difficulties of achieving satisfactory signals from large components of case-to-gyro rotor angles in the fractional arc second region means that the "large deviation angle" configuration illustrated in Fig. 1-29 is not suitable for the mechanism reference coordinate indications of high performance guidance systems.

Problems of mechanism reference coordinates associated with sensing and transformation of large angles are usually eliminated, so far as gyro units themselves are concerned, by mounting the unit cases on a member having three degrees of angular freedom with respect to the base by which it is carried, and providing power drives of some kind to overcome inaccuracy-producing torques due to inertia and friction. Figure I-30 suggests the essential features of such an arrangement for two-degrees-of-freedom with angular deviation signals between the case and the spin axis direction used to energize gimbal torquer drives through the operation of electrical servo-systems not shown in the diagram. With proper servo designs, the

case-to-spin axis angular deviations may be maintained small; in practice to the order of one arc-second by using good angle sensors and tight servo-loops. Because only small angles are involved and accurately matched sensitivities are unimportant, difficulties associated with accurate signals are greatly reduced from those involved in collecting data on large angles. Combinations of the signals from two perpendicular case directions in a plane at right angles to the spin axis are required to command proper responses from the two torquers. Relatively low performance resolvers on the gimbals are sufficient to serve the needs of servo loop control, It is unnecessary to include a separate trigonometric computer in the system.

The diagram of Fig. 1-30 illustrates a single rotor arrangement providing two-degree-of-freedom isolation of a controlled member (the case) from base motion. Any complete system requires three-degree-of-freedom angular isolation. In practice this situation is very often met by an arrangement like that suggested in the diagram of Fig. 1-31. In this figure, three gyro units, each with one degree-of-freedom, are shown on the inertial reference package instead of the two that would be needed if gyro elements like that of Fig. 1-30 were used. For the situation illustrated, the nature of the gyro units is immaterial so long as they provide the function of sensing and representing angular deviations in terms of usable signals,

Gyro units of many types have been conceived and a few have been reduced to successful practice. Strong discussions of relative merit for various mechanizations continue and are not likely to be settled until working equipment is tested under operational conditions. However, an understanding of the patterns in which fundamental principles may be applied is surely helpful for effective evaluations of performance data from test results. The discussion that follows is intended to help with this understanding by describing the features and problems associated with typical classes of gyro units.

All gyro units are designed around a relatively strong component of angular momentum with its direction rigidly fixed to some member which has freedom to move within the instrument case. For the instruments that have proved to be successful in the present state of technology, this angular momentum is generated by a spinning rotor of some kind. Figure 1-32 illustrates the simplest arrangement in which a single moving part, a rotor having a generally spherical form, is supported on forces generated by electromagnetic or electrostatic fields so configured that the resultant damping forces acting on the rotor are very low. This characteristic makes it possible for a rotor forced into rotation by eddy-current motor acting to continue coasting with a high velocity spin for considerable periods of time such as days, weeks or months after the driving torque has been removed. When a spherical rotor runs within a spherical case in the arrangement suggested by Fig. 1-32, the frictionless support not

only provides a spin bearing but also allows for complete angular freedom of the case with respect to the spin axis. Sensors for angles between references fixed to the case and the spin axis supply signals that yield orientational reference information after processing by a computer,

The inviscid field supported rotor gyro unit is attractive because of its simplicity but start-up of spin is an awkward process requiring considerable time with the spin direction determined by the orientation of the case during the time the spin torque is acting. Nutation, i. e. , an oscillatory change in direction of the angular momentum vector is generated during the starting operation and must be damped out. The inviscid field supported rotor arrangement does not lend itself to the controlled application of torque components for adjusting the direction of the angular momentum vector, a circumstance which makes it practically impossible to align the spin axis direction directly and accurately with external reference case coordinates. This means that auxiliary means to provide special positioning of the case with respect to external coordinates must be used. The required equipment tends to be cumbersome and difficult to use. This circumstance makes it unlikely that inviscid field supported sphere gyro units will be as satisfactory for the purposes of high performance guidance systems as other units that allow self-alignment of the system in which they operate,

Gyro units with inviscid field supported spherical rotors also present certain other difficulties. These problems stem from the inaccessibility of spinning rotors for balancing and other adjustments while the complete gyro unit is in operation with all the environmental conditions adjusted to those of operational use. Another matter of basic importance that remains to be resolved is that of the effects of vibration and acceleration on a gyroscopic system with substantially zero damping. It is certainly very desirable to measure environmental effects and to determine overall system performance as soon as possible.

Because gyro rotor behavior can not be refined by mass changes made directly on spinning spheres, it is not possible to refine gyro performance by adjustments. In practice the operation of each individual sphere must be calibrated in combination with a computing system to determine performance coefficients that can be applied during operation to reduce imperfections in behavior by calculation rather than through adjustment or compensation of the mechanism.

Practical problems of design, engineering, production and operation for gyro units may be simplified by separating the various functions that must be provided within a gyro unit in ways that allow each aspect of performance to be given individual adjustments and compensation with a minimum of coupling effects that lead to

inaccuracies in operation. Figure 1-33 suggests the basic features of the typical two-degree-of-freedom gyro unit with:

a) Angular momentum provided by the rotation of a wheel-like rotor spinning, carried by an inner gimbal through shaft and journal bearings that may be ball, roller or hydro-dynamic with gaseous or fluid lubricant.

b) Spin angular velocity sustained (no coasting in operation) with constant speed by a continuously acting motor.

c) Two-degrees of angular freedom with respect to the case provided by gimbals (fluid, ball or roller supported.)

d) Generation of angular deviation signals, restricted to small magnitudes, by reception of spherical displacements of the case with respect to the spin axis.

e) Accurate changes in angular momentum orientation with respect to inertial space by direct response to command input for gimbal torquers.

f) Balance adjustments available during unit operation by means of threaded nuts on the two gimbals. These adjustments make it possible to approach the ideal condition of gyro unit insensitivity to gravity and acceleration by adjusting the center of mass so that it approaches coincidence with the point of support provided by the gimbals,

In some designs this mechanical support may be supplemented by flotation forces provided by liquid within the clearance between hermetically sealed thin shell gimbals.

When its base is mounted on a structure which rotates with respect to inertial space, the two-degree-of-freedom gyro unit gives output signals which represent spin axis angular deviations about axes perpendicular to the spin axis, from a reference position of the case. This reference position is determined by the orientation of the case in which the angle output signal has its null level. Command signals to the torquers make it possible to change the reference orientation as desired without any need for taking base orientation into account. This possibility of directly relating gyro unit angular momentum to an external space reference direction gives the torqued two-degree-of-freedom gimbal supported gyro unit a considerable advantage over inviscid field supported spherical rotor units,

The gimbal supported two-degree-of-freedom gyro unit overcomes some of the difficulties stemming from the multiple function characteristics of inviscid field spherical rotor supports. Adjustments can be made during operation by gimbal balancing adjustments, indefinite operating periods are achieved, accurate control of angular momentum directions is made possible and various other results of practical importance are attained. However, the difficulties associated with accurately maintaining coincidence between the point of support provided by two

gimbals coincident and the center of mass of an articulated structure, limit the quality of performance available from the two-degree-of-freedom gyro unit and considerably increase the difficulty of manufacture for units of even medium performance levels. The output of deviations in terms of a conical angle subject to the coupling effects that tend to accompany the precession and nutation of a two-degree-of-freedom gyro rotor is also troublesome when inaccuracies in the region of fractional arc-seconds are desired.

Some difficulties of two-degree-of-freedom units are reduced when the mechanical gimbal system is replaced by a spherical gas bearing arrangement which allows both rotor spin action and angular freedom between the spin reference direction and the case. Balancing problems still exist and the gas bearing is always subject to sharply defined upper limits of resistance to shock and vibration, but many practical gyro units using this mechanization are in operational use.

All two-degree-of-freedom gyro units are essentially untorqued for the purposes of sensing angular deviations. This means that the obtainable resolution in terms of angular velocity components about axes of sensitivity for the unit depends on the angle defined by the minimum usable output from the signal generator. This generally corresponds to angles *so* large that detection of small components of earth's rate (in the region of one arc second per hour) is generally not practical,

Single-degree-of-freedom gyro units with the basic features illustrated in Figs. I-34 and I-35 make it possible to realize practical gyro units with the characteristics required of angular deviation sensing instruments for guidance systems able to reliably provide the low CEP range needed for hard point military targets. The design philosophy involved is the direct antithesis of the single moving part philosophy of the inviscid field supported sphere gyro unit in which refining adjustments to the rotor are impossible during operation, and accurate direct alignment of angular momentum axis to external reference space is not available. In the floated integrating single-degree-of-freedom gyro unit each function is carefully separated from others, and with the exception of dynamic balancing for the rotor, may be refined toward ultimate performance with the complete gyro unit in normal operation.

As suggested by Figs, 1-34 and 1-35:

a) Angular momentum is provided by a rotor with its spin sustained indefinitely by a driving motor. The spin axis bearings which support the rotor from the single gimbal may be either ball bearings or hydrodynamic journal bearings lubricated by air. With good design and manufacture both types have demonstrated high performance and life times of many thousands of hours. Gas bearings consume somewhat more power at starting and in operation than ball bearings. Gas bearings

are much more liable to damage than balls when subjected to torques before their full supporting power has been developed during start-up, and they are also more vulnerable to catastrophic failure under either steady or vibratory high accelerations. It appears that gas bearings are suitable for environments of limited severity, while ball bearings are adaptable to wider ranges of environmental conditions. Both types are now in use and can be applied in single-degree-of-freedom gyro units at the preference of the designer.

b) Single-degree-of-freedom motion is provided by a chamber enclosing the gyro rotor which is largely supported on the flotation pressure gradients built up by gravitational and inertia reaction forces in a dense, highly viscous fluid contained in the clearance volume between the float and the hermetically sealed case. In operation, the temperatures of the solid parts and the fluid are closely controlled so that the buoyancy support remains substantially constant. The small remaining imperfection in flotation is effectively reduced to zero by single axis magnetic support units at either end of the float which is thus suspended within the case without even the slightest rubbing contact between solid parts.

c) Complementing the buoyancy and magnetic supporting forces are forces generated by hydrodynamic forces and torques generated when the heavy fluid is forced to flow between parts of the clearance space. The resultant support minimizes distorting stress on the gimbal because of the distributed nature of the loads that are acting. The overall result is a system effectively immune to the mechanical effects of acceleration, vibration and shock within selected design ranges.

d) The use of high viscosity fluids for gimbal support means that high level drag forces are developed by motion of the float within the case. The forces not only provide support during dynamic conditions, but also develop a drag torque about the gimbal freedom axis of the float. This torque is proportional to the angular velocity of the float with respect to the case about this axis which is carefully made at right angles to the spin axis of the gyro rotor. Under gyroscopic principles a rotation of the angular momentum vector about the input axis which is at right angles to the gimbal axis, and also the spin axis causes the gyro rotor to exert a torque on the gimbal about its axis of freedom which, for this reason, is called the ~~output axis~~. This torque is absorbed by the accelerational inertia reaction of the float, and by viscous drag in the fluid. The inertia reaction torque causes the gyro unit to exhibit a time constant with the order of one thousandth second, while the viscous drag torque causes the float angular velocity within the case to be proportional to the case angular velocity with respect to inertial space about the input axis. The result is that the float output angle with respect to the case is proportional to the angle turned through by the case with respect to inertial space about the input axis. This action leads the name "integrating gyro unit" for instruments with the features of Fig. 1-34.

Because of the absence of rubbing friction between solid parts and the utilization of viscous drag as a primary factor in operation, the proportionality between input angle and float output angle is effectively perfect over the operating range of a few minutes of arc down to a lower limit that is surely less than one thousandth of an arc second.

e) In operation, the single-degree-of-freedom integrating gyro unit is suitable for use only under circumstances in which gimbal deflection angles are limited to a few seconds of arc. This condition is favorable for signal generator designs which can be constructed to give very low null signals and high sensitivity outputs when connected to feasible electronic circuits. Signals defining less than 0.01 arc seconds are achievable with carefully designed generators and good electronics.

f) As shown in Fig. 1-34 balance adjustments for the float have the form of nuts on screws attached to the float. By providing means for turning these nuts from outside the case with the unit in full operation, it becomes possible to place the center of mass on the output axis so accurately that the total effects of unbalance torques under one earth gravity may be reduced to the level of one meru or less.

g) Flexible leads carefully selected for low hysteresis and substantially floated in the suspension fluid are used to carry power to the gyro rotor within the gimbal float. By careful design and the use of a refining adjustment accessible from outside the case, the effects of power lead torque may be reduced to a level of one meru or less.

With single axis operation and accurately controlled centralization of the moving element, torque generators are feasible in which the output is very closely proportional to the electrical input. This characteristic makes the single-degree-of-freedom integrating gyro unit a very useful and flexible component for applications of many kinds.

The features described above in general terms coupled with an absolutely necessary careful control of temperature, rotor power, electrical excitation, mechanical mounting, connectors, etc., make it possible to reduce gyro performance uncertainties to levels between 0.01 and 0.001 meru. By proper compensation and/or correction of various basic effects which are measurable by inspection techniques, gyro performance substantially identical with the uncertainty levels may be achieved in practical operation.

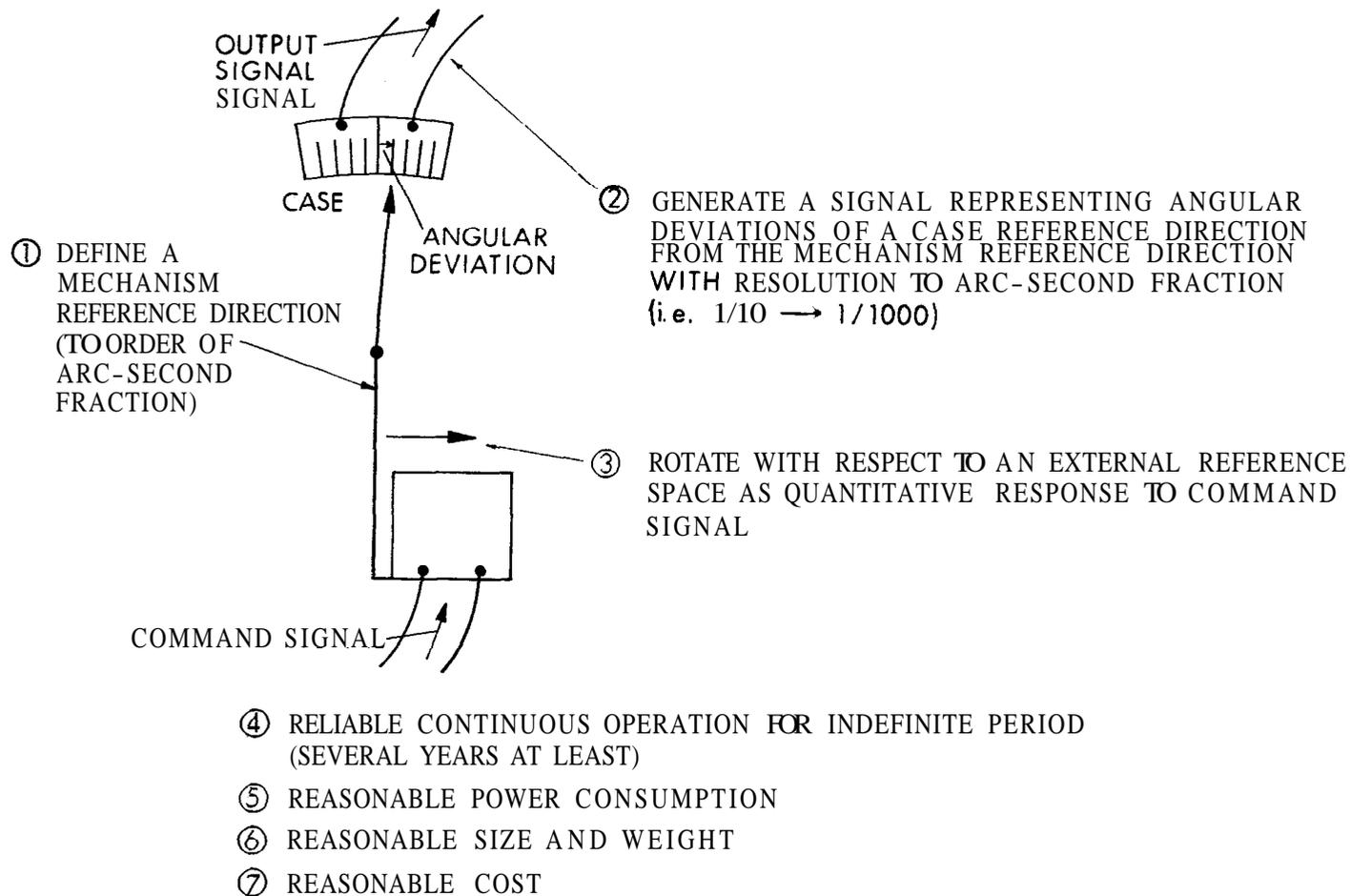


Fig. 1-26 General Requirements for Instrumental Components to Mechanize Guidance System Reference Directions

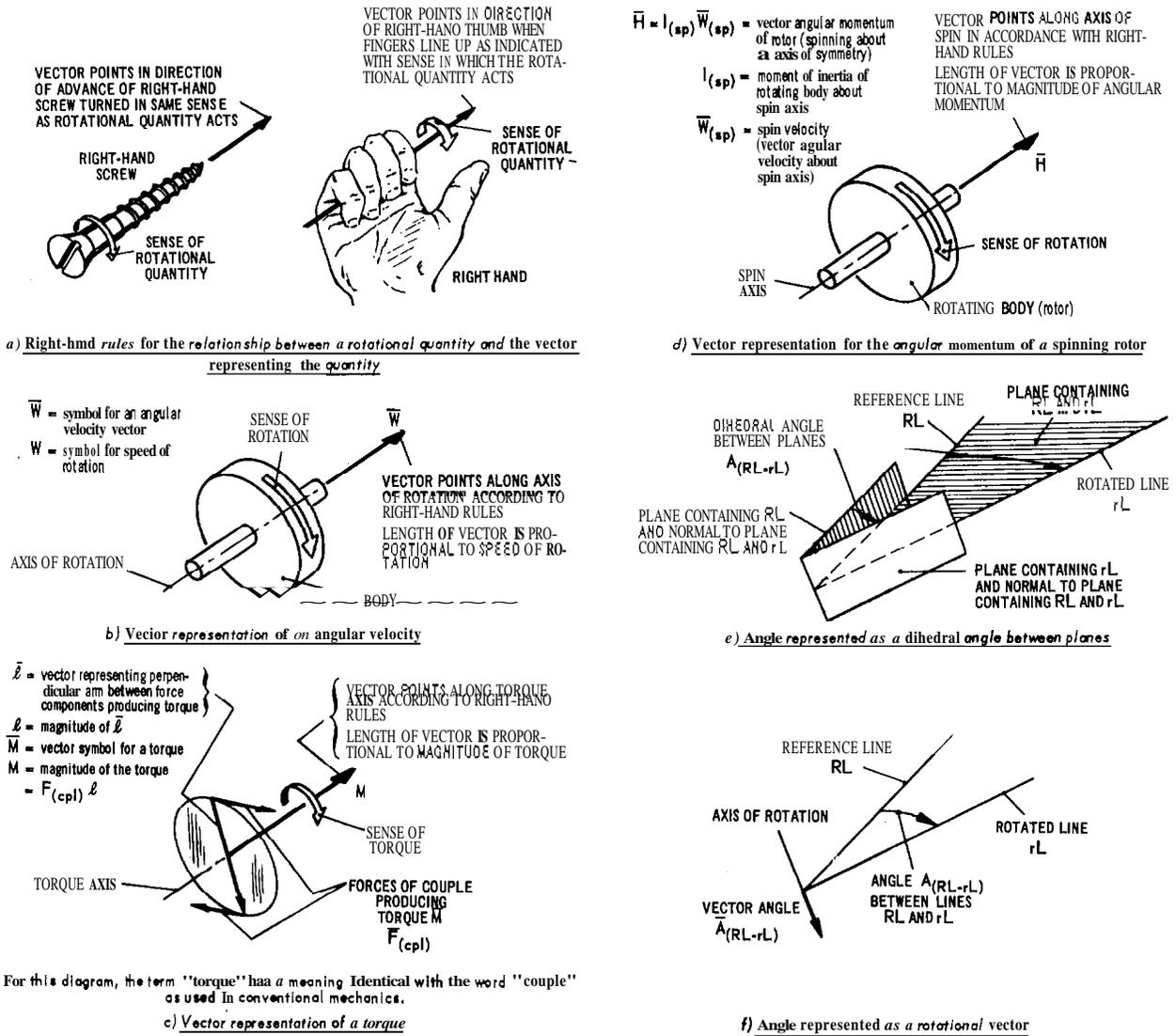
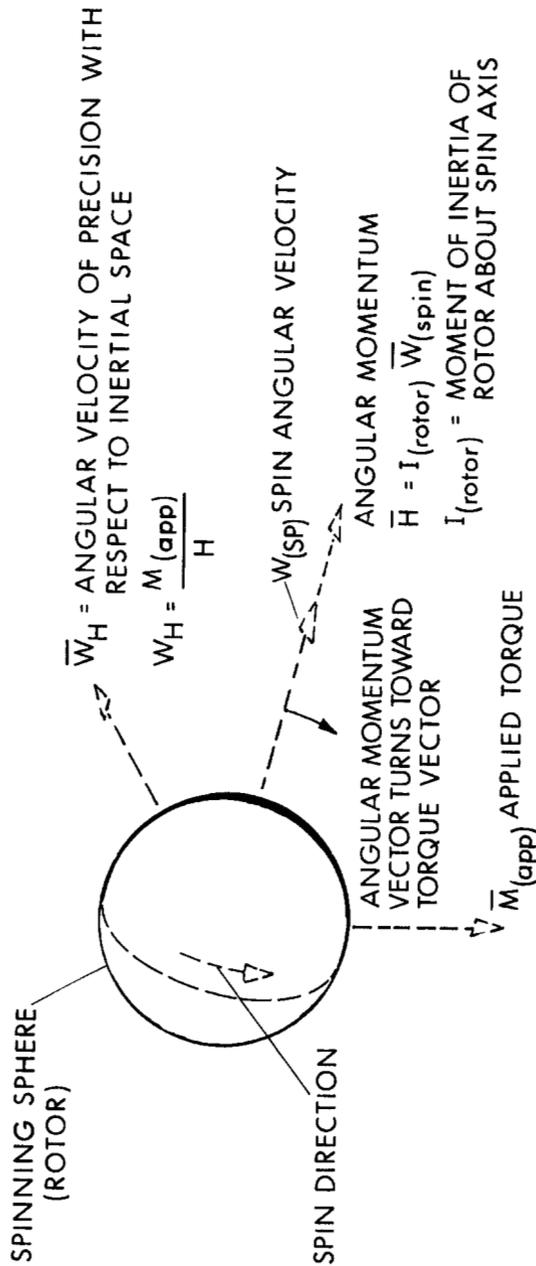


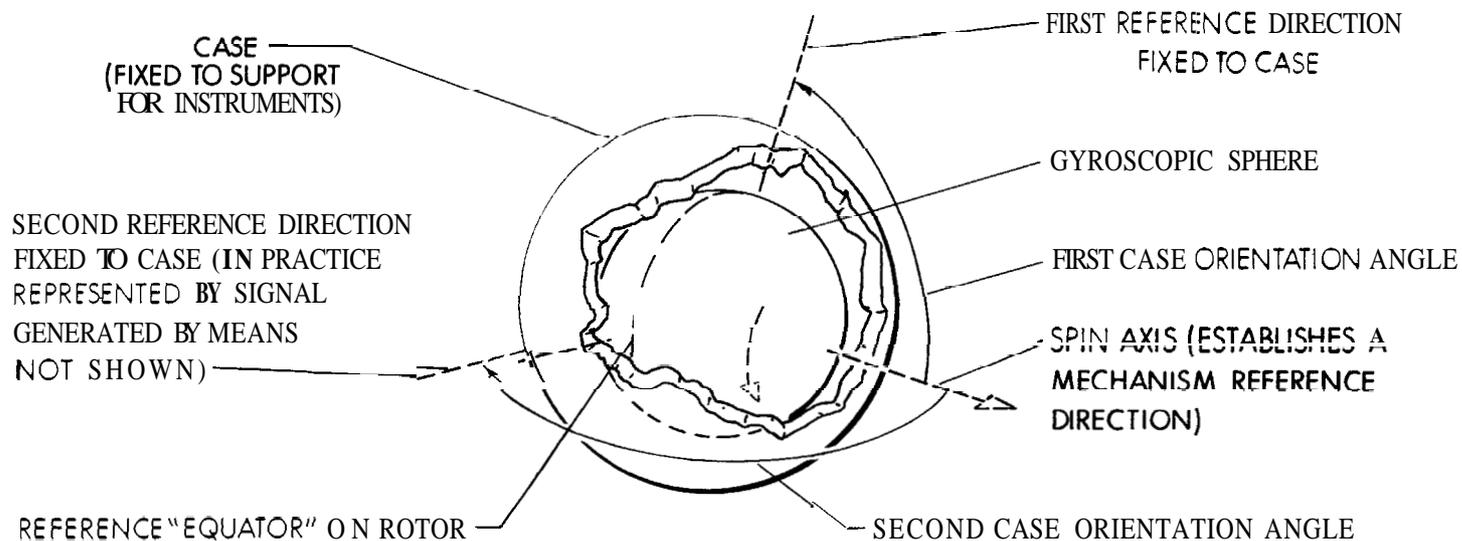
Fig. 1-27 Vector Conventions for Rotational Quantities



SPIN VELOCITY IS ASSUMED TO BE SO GREAT IN RELATION TO ALL OTHER ANGULAR VELOCITY COMPONENTS THAT  $\vec{H}_{(rotor)}$  REPRESENTS THE TOTAL ANGULAR MOMENTUM OF THE SYSTEM.

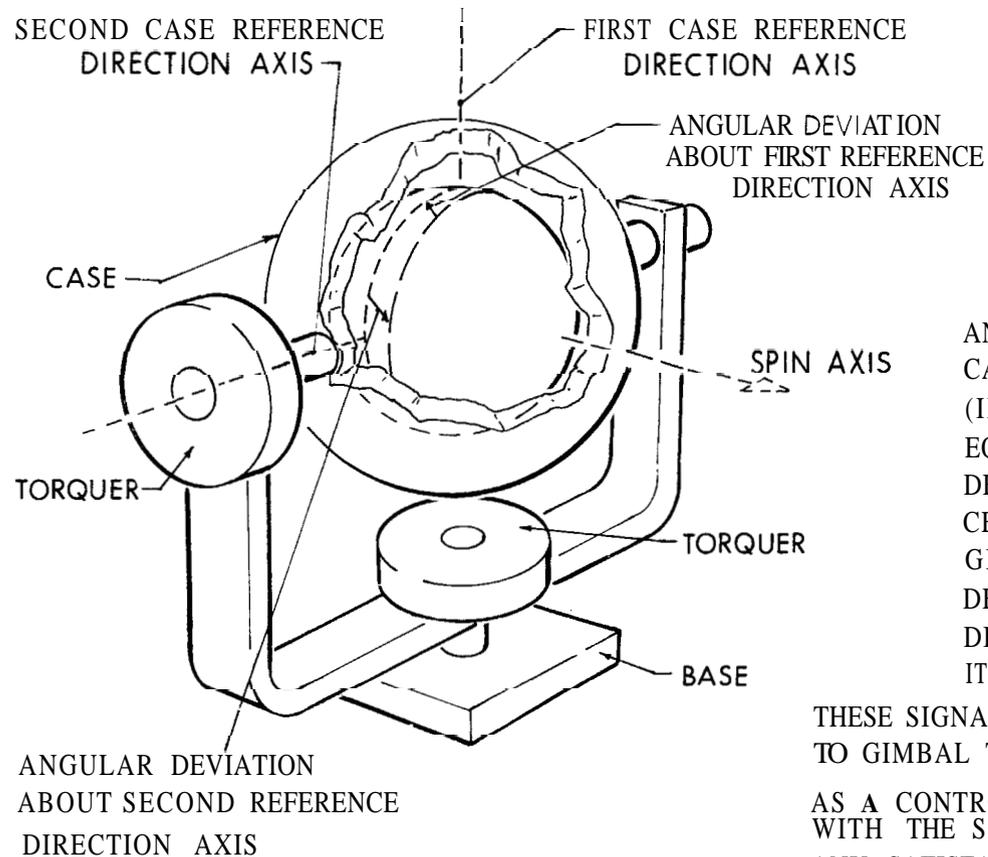
MECHANISM FOR SUPPORT AND SPINNING ROTOR NOT SHOWN.

Fig. I-28 Basic Operating Principle of the Gyroscopic Element



- INDICATIONS OF ORIENTATION OF THE CASE WITH RESPECT TO THE SPIN AXIS DIRECTION DEPEND ON COMPUTER PROCESSING OF ANGLE SIGNALS BY RELATIONSHIPS OF SPHERICAL TRIGONOMETRY.
- FOR COMPLETE DEFINITION OF INDICATED CASE ORIENTATION WITH RESPECT TO INERTIAL SPACE TWO GYROSCOPIC ELEMENTS WITH DIFFERENT SPIN AXIS DIRECTIONS ARE REQUIRED.
- ARBITRARY CASE ORIENTATION INDICATIONS WITH INACCURACIES WITH THE ORDER OF ONE ARC-SECOND ARE DIFFICULT BECAUSE OF THE HIGH SIGNAL GENERATOR ACCURACY AND COMPUTER PERFORMANCE REQUIRED.

Fig. I-29 Illustrative Reference Direction Mechanization with Untorqued Gyroscopic Elements and Computed Indications of Case Orientation from Direct Rotor-Case Angle Signals



### READOUT OF ANGLES

ANGULAR DEVIATIONS ABOUT THE CASE REFERENCE DIRECTION AXES (IN THE PLANE OF THE ROTOR SPIN EQUATOR WHEN THE ANGULAR DEVIATIONS ARE ZERO) ARE RECEIVED BY SIGNAL GENERATORS GIVING OUTPUTS PROPORTIONAL TO DEVIATIONS OF THE REFERENCE DIRECTIONS FROM PERPENDICULARITY TO THE SPIN AXIS.

THESE SIGNALS APPLIED AS SERVO-DRIVE INPUTS TO GIMBAL TORQUERS ACT TO KEEP THE CASE

AS A CONTROLLED MEMBER IN ALIGNMENT WITH THE SPIN AXIS.

ANY SATISFACTORY SERVO-SYSTEM MAINTAINS THE ANGULAR DEVIATIONS SMALL-ONE SEC. OF ARC REPRESENTS REASONABLE PERFORMANCE.

Fig. 1-30 Illustrative Two-Axis Mechanism Reference System Based on Servo-Driven Gimbals and the Untorqued Gyroscopic Element

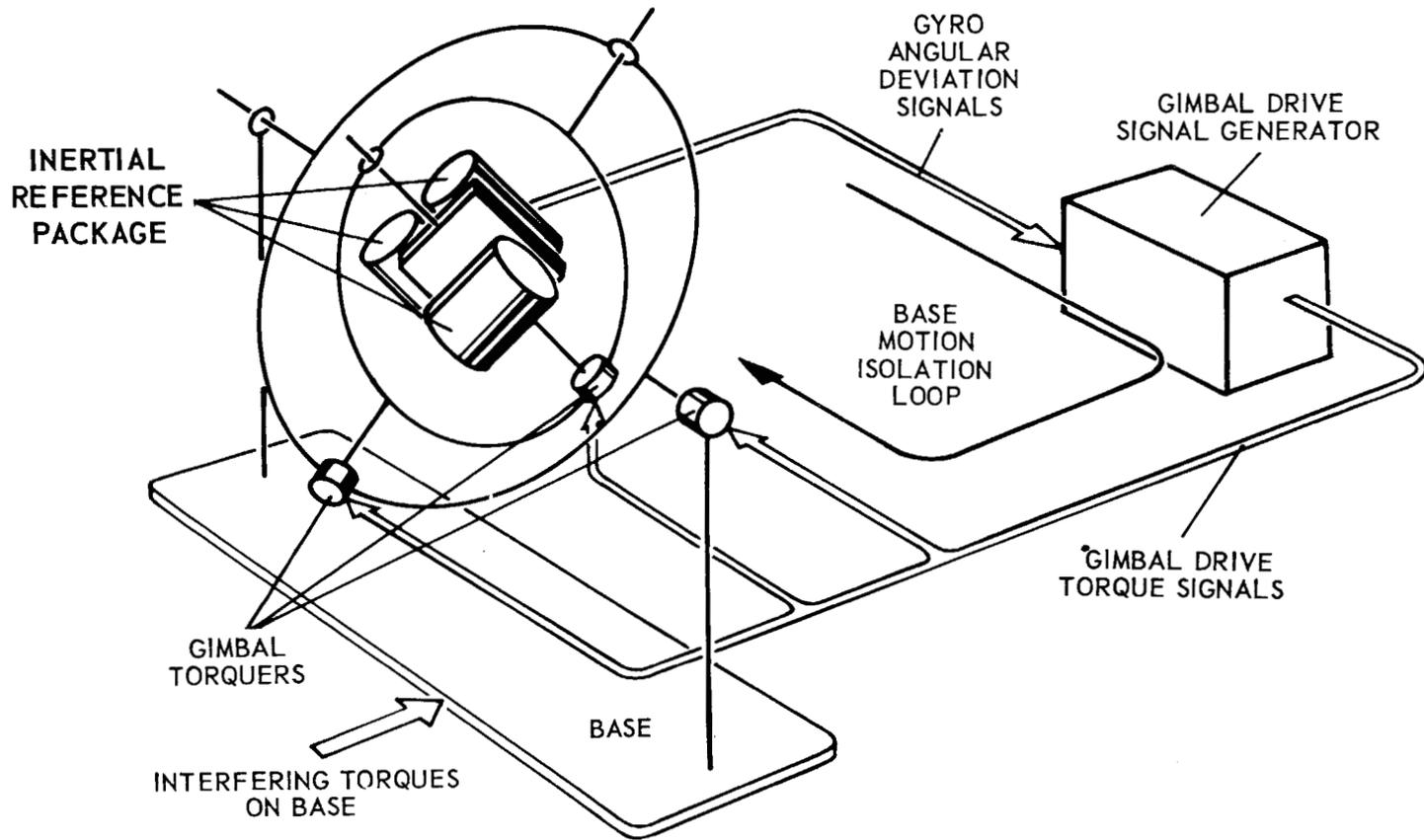


Fig. 1-31 Illustrative Arrangement of Three-Degrees-of-Freedom Servo-Driven Gimbals to Isolate Mechanism Reference Members from Base Motion

## FEATURES

- INITIAL SPIN TORQUE ONLY (ROTOR COASTS DURING OPERATION)
- ONE MOVING PART
- FIELD PROVIDES BEARING FOR SPINNING MEMBER
- FIELD SUPPORT ALSO PROVIDES TWO DEGREES OF FREEDOM FOR ANGULAR MOMENTUM VECTOR
- ANGLE SIGNAL GENERATOR RECEIVES COMPONENTS OF SINGLE ANGLE
- SENSITIVE TO ANGULAR MOTION ABOUT ALL AXES AT RIGHT ANGLE TO SPIN AXIS
- ADAPTED TO TORQUE-FREE OPERATION ONLY

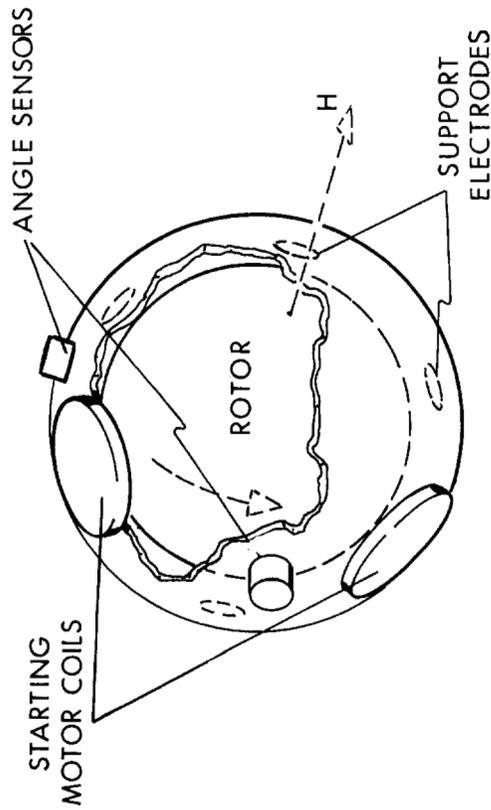
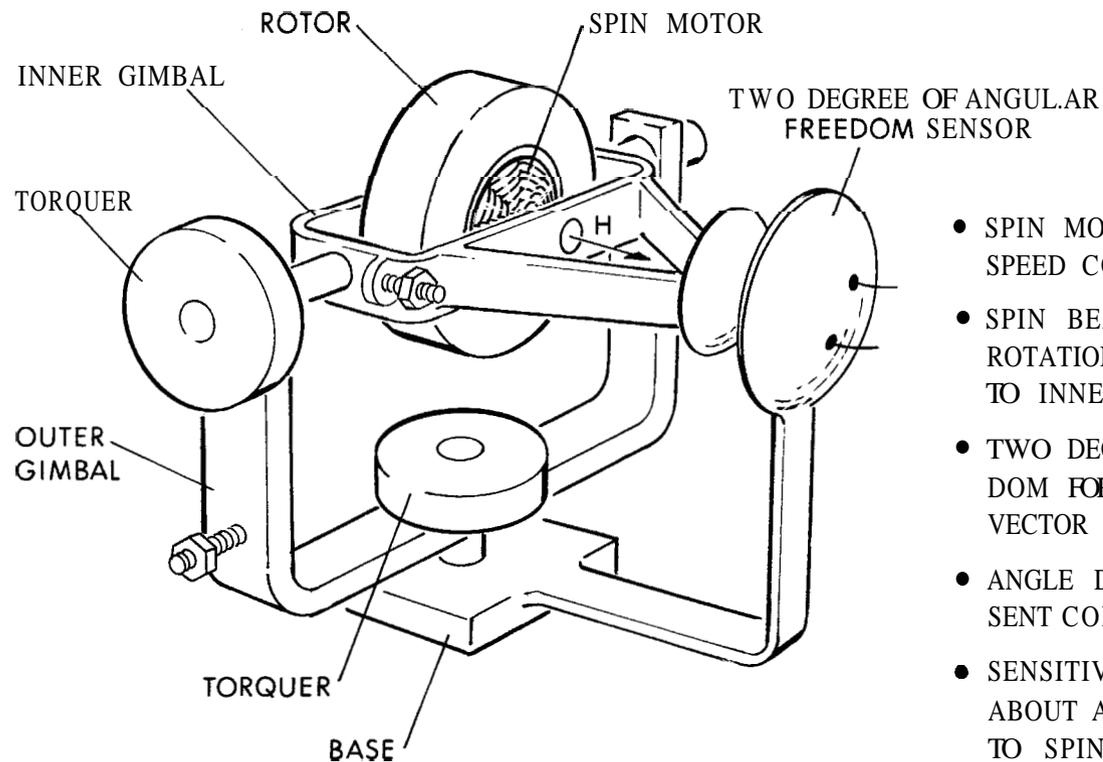


Fig. I-32 Basic Features of the Non-Viscous Field (Electromagnetic, Electrostatic) Supported Two-Degrees-of-Freedom Gyro Unit



### FEATURES

- SPIN MOTOR SUSTAINS ROTOR SPEED CONSTANT INDEFINITELY
- SPIN BEARINGS LIMIT ROTOR TO ROTATION ABOUT SPIN AXIS FIXED TO INNER GIMBAL
- TWO DEGREES OF ANGULAR FREEDOM FOR ANGULAR MOMENTUM VECTOR PROVIDED BY GIMBALS
- ANGLE DEVIATION SIGNALS REPRESENT COMPONENTS OF SINGLE ANGLE
- SENSITIVE TO ANGULAR MOTION ABOUT ALL AXES AT RIGHT ANGLES TO SPIN AXIS
- TORQUERS ON GIMBALS MAKE IT POSSIBLE TO CHANGE ORIENTATION OF SPIN AXIS AS A QUANTITATIVELY ACCURATE RESPONSE TO COMMAND SIGNALS

NOTE: ROTOR AND ITS BEARINGS MAY BE ENCLOSED IN A HERMETICALLY SEALED CAN AND SUBSTANTIALLY SUPPORTED BY FLOTATION IN A FLUID CONTAINED WITHIN AN OUTER CASE

Fig. 1-33 Basic Features of the Gimbal Supported Two-Degrees-of-Freedom Gyro Unit

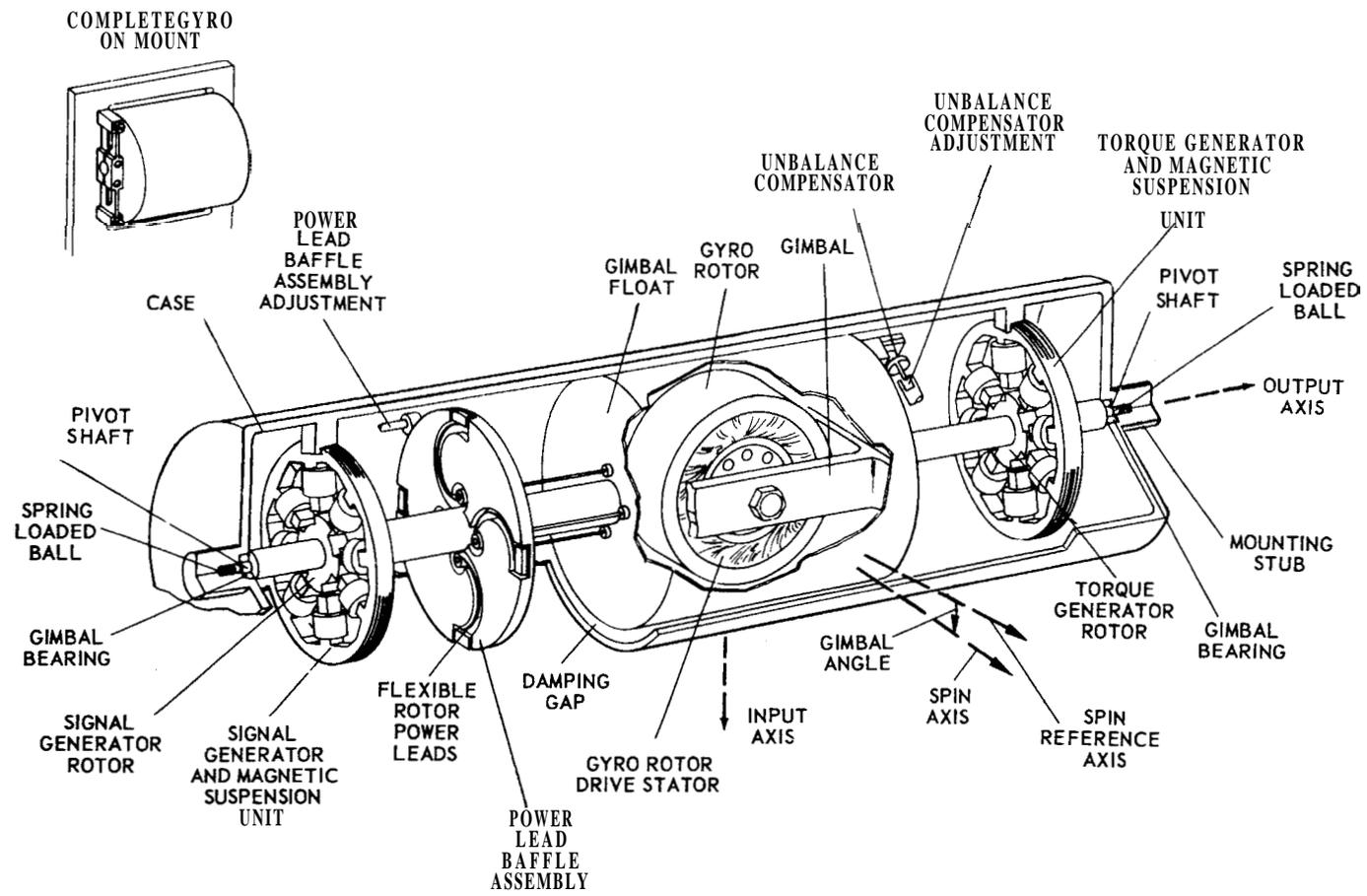
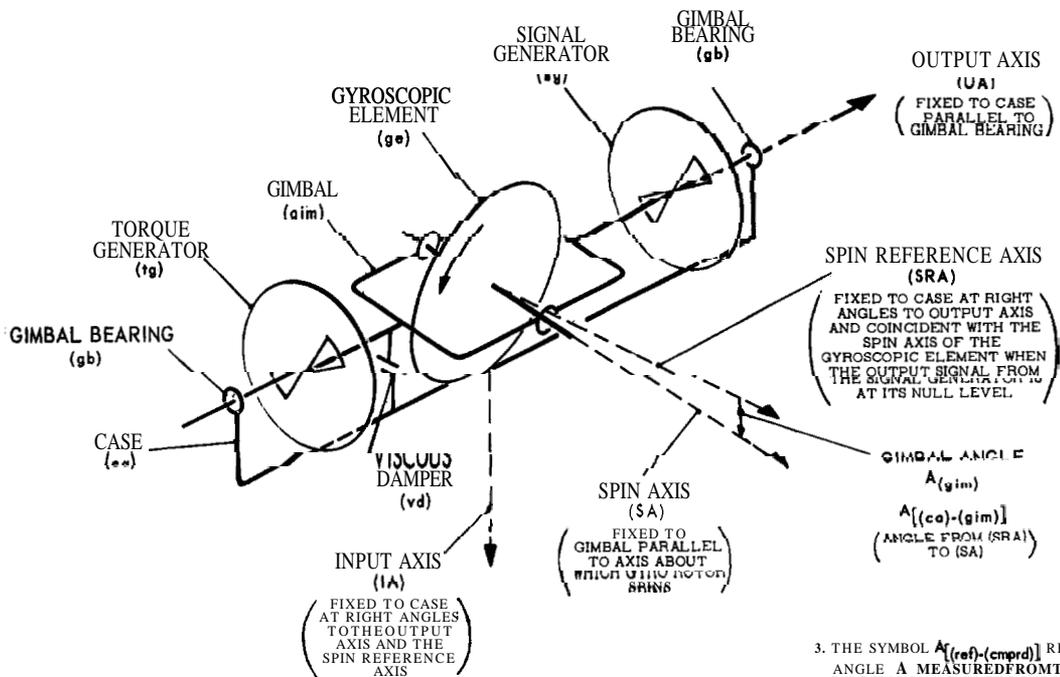


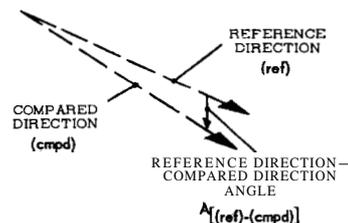
Fig. 1-34 Basic Features of Single-Degree-of-Freedom Floated Integrating Gyro Unit



NOTES: 1. POSITIVE SENSES SHOWN BY THE ARROWS ARE CHOSEN SO THAT (IA), (SRA), AND (OA) FORM A RIGHT-HANDED SYSTEM.

2. THE GYRO UNIT TEMPERATURE CONTROL POWER IS SUPPLIED TO A MOUNTING BLOCK ADAPTED TO RECEIVE THE GYRO UNIT CASE. THE FLOW OF POWER IS CONTROLLED BY THE DAMPER TEMPERATURE SETTING.

3. THE SYMBOL  $A^{(ref)-(cmpd)}$  REPRESENTS THE ANGLE  $A$  MEASURED FROM THE REFERENCE DIRECTION (ref) IN THE SUBSCRIPT TO THE COMPARED DIRECTION (cmpd) IN THE SUBSCRIPT.



- CASE - (ca)** - THE STRUCTURE THAT GIVES SUPPORT FOR THE INTERNAL WORKING PARTS OF THE GYRO UNIT, ENCLOSES THE PARTS AND CARRIES PROVISIONS FOR EXTERNAL CONNECTIONS OF ALL KINDS.
- TORQUE GENERATOR - (tg)** - COMPONENT FOR RECEIVING INPUT SIGNALS AND PRODUCING CORRESPONDING OUTPUT TORQUE APPLIED TO THE GIMBAL ABOUT THE OUTPUT AXIS.
- DAMPER - (dmp)** - SUBSYSTEM RECEIVING ANGULAR VELOCITY OF THE GIMBAL WITH RESPECT TO THE CASE AS ITS INPUT AND PRODUCING AS OUTPUT A RETARDING TORQUE ACTING ON THE GIMBAL ABOUT THE OUTPUT AXIS WITH A MAGNITUDE PROPORTIONAL TO THE MAGNITUDE OF THE ANGULAR VELOCITY OF THE GIMBAL WITH RESPECT TO THE CASE.

- GYRO UNIT - (gu)** - THIS ENTITY MADE UP OF THE COMPONENTS REPRESENTED IN THIS DIAGRAM AND ALL THE ADDITIONAL PARTS NECESSARY FOR A SINGLE PACKAGE TO CARRY OUT THE FUNCTIONS OF A GYRO UNIT.
- SIGNAL GENERATOR - (sg)** - COMPONENT FOR RECEIVING THE ANGLE OF THE SPIN AXIS WITH RESPECT TO THE CASE AS INPUT AND PRODUCING A CORRESPONDING SIGNAL THAT SERVES AS THE OUTPUT SIGNAL FROM THE GYRO UNIT.
- GIMBAL - (gim)** - STRUCTURE CARRYING THE BEARINGS FOR THE SPINNING ROTOR OF THE GYROSCOPIC ELEMENT, ROTORS FOR THE TORQUE GENERATOR AND SIGNAL GENERATOR, PART OF THE DAMPER, FLOAT SEALS AND STRUCTURE, BALANCE ADJUSTMENTS, STOPS, PIVOTS, ETC.

Fig. I-35 Line Schematic Diagram of Single-Degree-of-Freedom Floated Integrating Gyro Unit

## CHAPTER I-6

### BASIC PRINCIPLES OF GYRO UNIT APPLICATIONS

Single-degree-of-freedom gyro unit systems as components are combined with many more devices to produce a coordinated overall result in guidance systems. The particular function of any single gyro unit is to translate the resultant of rotations about its input axis with respect to inertial space and command inputs to its torque generator into a resultant signal that represents the angular deviation of the float from the position for which the output signal has its null level. In effect, the output signal represents the angular deviation of the case about the input axis from a reference position established by the null level of the signal. Command inputs to the torque generator have the effect of rotating the reference position with an angular velocity proportional to the signal.

Figure 1-36 is an illustrative pictorial schematic diagram in terms of a single axis system suggesting the basic features and operating principles of a typical gyro unit - servodriven controlled member combination. For the arrangement of this figure which provides functions similar to those of the geometrical reference member of an inertial guidance system, it is assumed that the command input is zero except, perhaps, for small compensations for calibrated imperfections of the particular gyro unit involved.

When, for any reason, the base of the servo-drive moves so that the gyro unit is rotated away from its reference orientation about the input axis, or a torque is imposed from any other source, a gyro output signal is generated. Through slip rings this signal is applied as input to the servo-drive system which applies torque to the controlled member to force it back to the orientation for which the gyro unit case has its reference position. As a result of this continued action of the servo in overcoming disturbing torques, the case effectively holds its reference position about the input axis no matter how the base may move. Operation of this is typical of geometrical stabilization.

The servo-drive-gyro unit combination of Fig. 1-36 provides several functions for inertial guidance systems. The broad natures of these functions are suggested by the diagrams of Fig. 1-37,

a) When no command signal is applied to the gyro unit, the servo-drive stabilizes the input axis orientation of a controlled member on the basis of angular deviation signals from the gyro unit.

b) Command signal integration appears when an input is supplied to the gyro unit torque motor. The resulting torque on the float causes rotation which produces an output signal. This signal acts as an input to the servo so that the controlled member turns in the proper direction to reduce the output signal. Except for dynamic response effects which may be reduced to negligible levels by proper servo design, the operation described may be made to rotate the controlled member about the input axis with an angular velocity effectively proportional in magnitude to the magnitude of the command signal. Directional senses are determined by phasing or polarity changes of the command signal. The resulting effect of the signal is that the angle turned through with respect to inertial space by the controlled member about the input axis is proportional to the time integral of the command signal.

Another mode of gyro-unit operation that is generally similar to the space integrator result described above is that in which the command signals are applied to the gyro unit to keep the output signal at null as the controlled member is forced to rotate in any arbitrary way with respect to inertial space about the input axis. With this mode of operation, integration of the command signal over any given time interval represents the total angle turned through by the controlled member during the same interval. It is to be noted that with this type of operation, the servo-drive of Fig. 1-37 has no significant function beyond providing a shaft for controlled member rotation,

In practice, command signals may be direct current, alternating current, or electric pulses. Pulses are especially suitable for command signals because they may be easily adapted as inputs-outputs for digital computers. Integration with pulses is particularly easy as the process involved is a matter of simple counting,

c) When the axis of base rotation is about an axis perpendicular to the servo-drive axis and the gyro unit is adjusted on the controlled member so that its input axis is at right angles to the servo axis, the arrangement may be used to indicate the direction of the axis about which the base is being rotated. In effect, if the input axis is inclined so that it receives a component of the base angular velocity, the gyro unit case is forced to rotate about its input axis so that it generates an output signal which causes the servo to turn the case so that its input axis is at right angles to the base rotation axis. By giving the gyro unit performance that enables it to respond consistently to say, one millionth of the magnitude of the base angular velocity, the direction of the gyro unit input axis may be made to indicate perpendicularity to the base rotation within an inaccuracy with the order of one arc second.

This mode of operation using components now available, makes it possible to align controlled members to north with sufficient accuracy for the operation of high performance guidance systems.

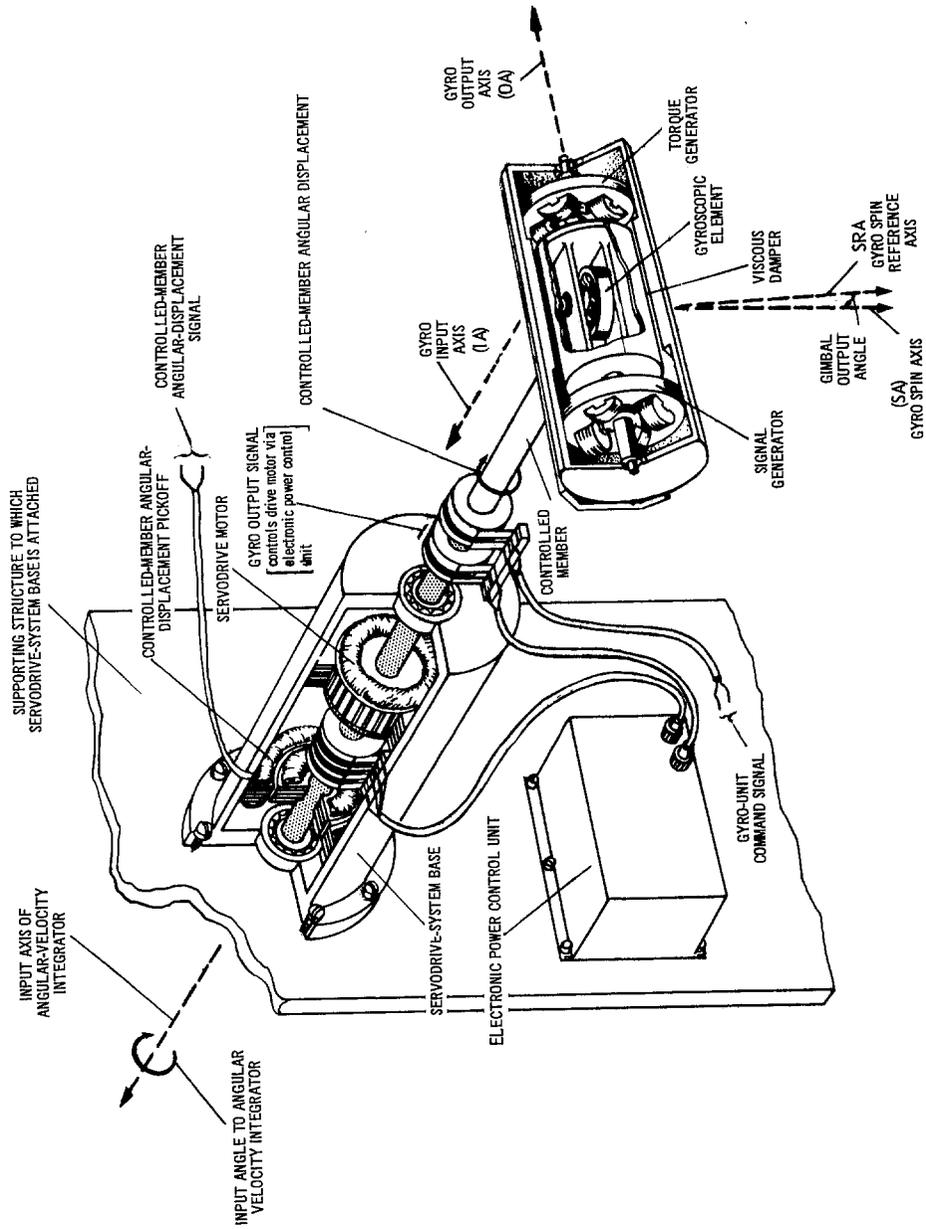


Fig 1-36 Pictorial Schematic Diagram Illustrating Basic Features of the Single Axis Servo-Driven Stabilization with Inertial Space Angular Deviations Sensed by the Single-Degree-of-Freedom Floated Integrating Gyro Unit

STABILIZATION —  
BASE MOTION ISOLATION

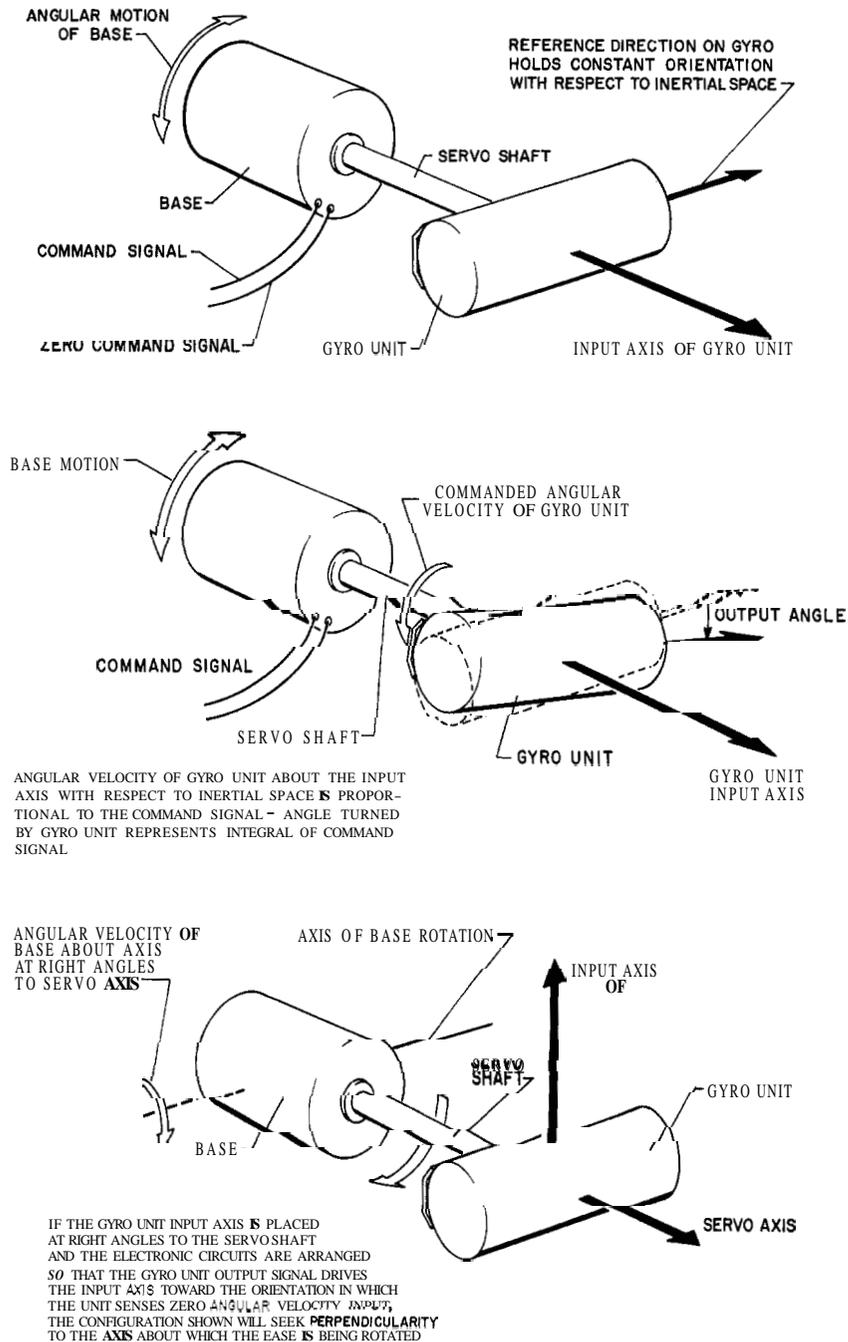


Fig. 1-37 Basic Features of Illustrative Single Axis Configurations of Servo-Drives and the Single-Degree-of-Freedom Gyro Unit to Provide: a) Stabilization, b) Space Integrators for Angular Motion, c) Angular Velocity Axis Direction Sensor

## CHAPTER I-7

### SPECIFIC FORCE RECEIVERS

All instruments designed to receive specific force depend upon a sensitive element that is essentially an unbalanced mass. This mass is arranged so that it imposes a force, or a torque, on some member that is restrained in a calibratable way. Figure I-38 illustrates the use of a floated-magnetic suspension carried pendulum restrained by a torque generator fed by pulses under the control of float angle signals. The integral of specific force (the resultant of gravity and inertia reaction force) is obtained by counting the pulses required to keep the float angle on its null position.

Figure I-39 suggests the features of a pendulous gyro-servo-drive specific force receiver in which the calibrated balancing torque for the output of the unbalanced mass is provided by the gyroscopic output torque from a constant speed spinning rotor driven by a servo to keep the output signal at its null level. Since the required angular velocity about the gyro input axis is proportional the balancing torque, the angle turned by the gyro unit case in a given time interval is a measure of the specific force integral for the same time. This calibration relationship depends upon interactions between two absolutely linear, non-saturable effects (both depending only on Newton's Laws of Dynamics) so that the range of accurate operation is basically limited only by servo design considerations. By using digitalizing signal generators for the servo-axis, the integrated output appears in terms of pulses suitable for direct use as digital computer inputs.

The requirements of position location for center-of-mass positions to be maintained in order to realize various drift rates with typical gyro units are summarized in the table of Fig. I-41. These numbers also suggest that specific force receivers must have their unbalanced mass centers positioned within very small tolerances if inaccuracies of  $10^{-6}$  earth's gravity are to be maintained over a range including some ten's of gravity.

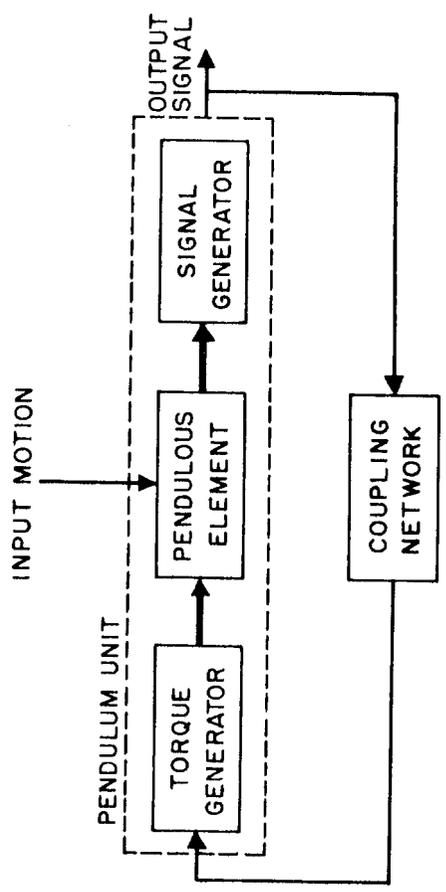
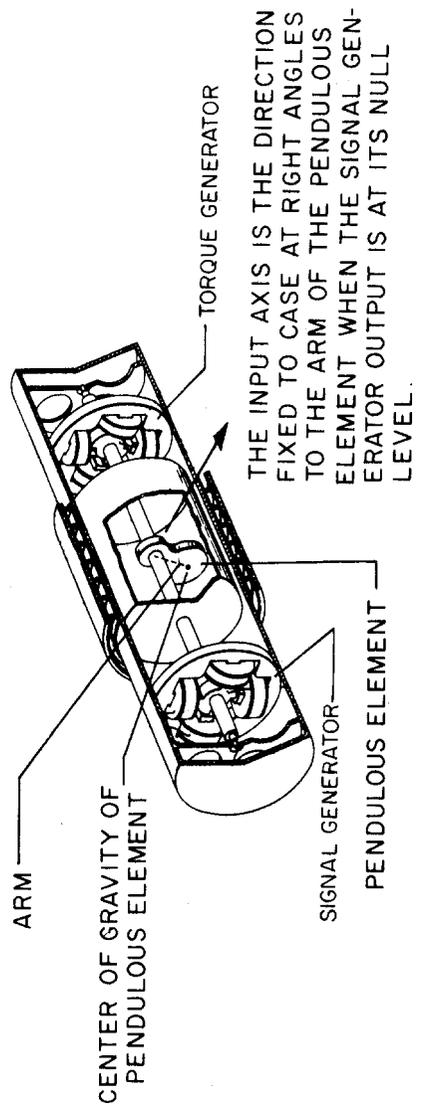


Fig. I-38 Basic Features of the Single-Degree-of-Freedom Pulsed Integrating Pendulum for Receiving Specific Force

NOTE: THE SEISMIC ELEMENT IS SYMMETRICALLY LOCATED ABOUT THE SPIN AXIS, BUT SHIFTS THE CENTER OF GRAVITY OF THE GYROSCOPIC ELEMENT AWAY FROM THE SPIN AXIS.

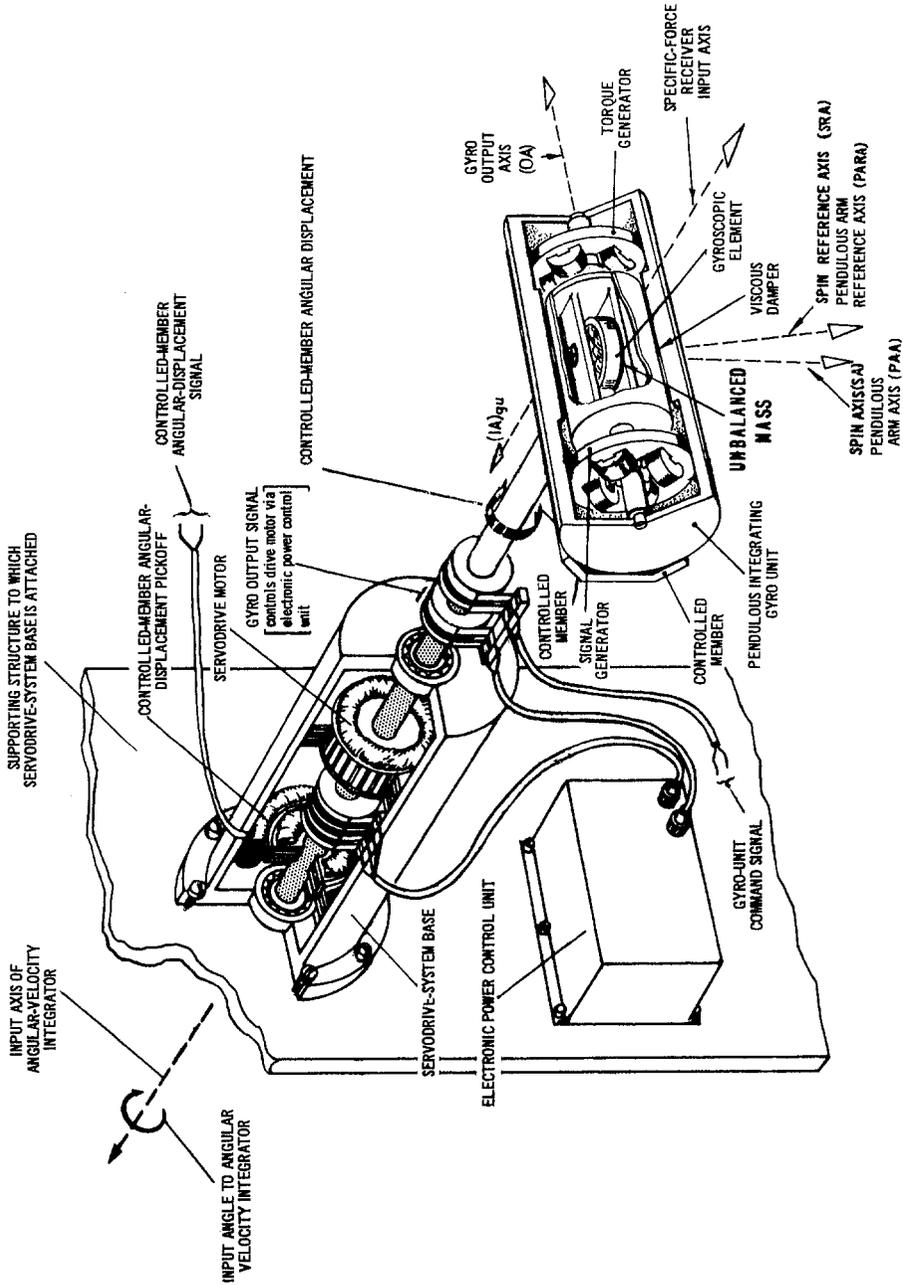
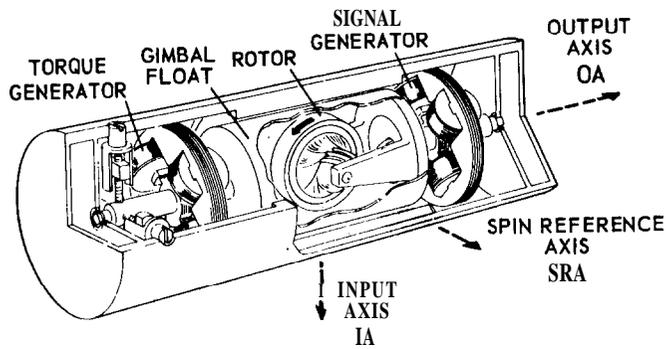
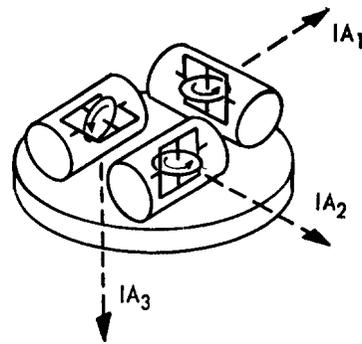


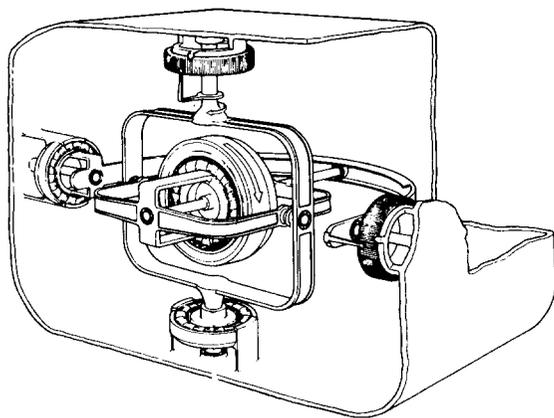
Fig. I-39 Pictorial Schematic Diagram Showing Basic Features of the Single Axis Single-Degree-of-Freedom Pendulous Single-Degree-of-Freedom Gyro Unit Integrating Specific Force Receiver



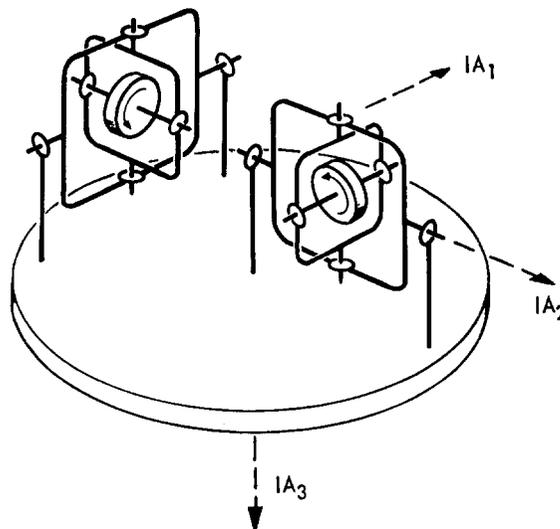
Schematic drawing of single-degree-of-freedom gyro unit



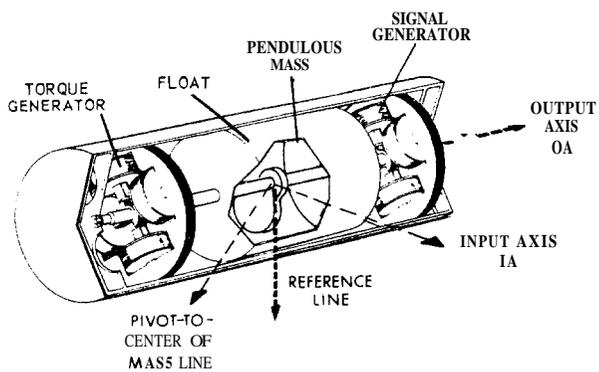
Single-degree-of-freedom gyro package



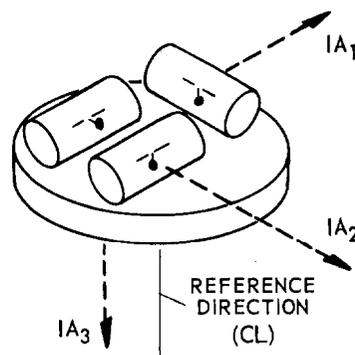
Schematic two-degree-of-freedom gyro



Two-degree-of-freedom gyro package



Pictorial schematic of a single-degree-of-freedom specific-force receiving unit



Accelerometer package

Fig. 1-40 Basic Elements of Typical Geometrical Reference Packages

Drift Rate		Center-of-Mass Position with Respect to Output Axis (arm) (cg)					Lattice Constants (approx) of Aluminum, Steel, or Beryllium*
In Earth Angular Velocity Units	In Radians Second	In Centimeters	In Microinches	In Angstroms	In		
1 eru	$0.73 \times 10^{-4}$	$1.46 \times 10^{-4}$	±7.5	$1.46 \times 10^3$	$\approx 0.5 \times 10^4$ or 5000		
1 deru	$0.73 \times 10^{-5}$	$1.46 \times 10^{-5}$	5.75	$1.46 \times 10^3$	$0.5 \times 10^3$ 500		
1 ceru	$0.73 \times 10^{-6}$	$1.46 \times 10^{-6}$	0.575	$1.46 \times 10^2$	$0.5 \times 10^2$ 50		
1 meru	$0.73 \times 10^{-7}$	$1.46 \times 10^{-7}$	0.0575	$1.46 \times 10$	$0.5 \times 10$ 5		
1 d meru	$0.73 \times 10^{-8}$	$1.46 \times 10^{-8}$	0.00575	1.46	0.5 1/2		
1 e meru	$0.73 \times 10^{-9}$	$1.46 \times 10^{-9}$	0.000575	0.146	0.05 1/20		
1 m meru	$0.73 \times 10^{-10}$	$1.46 \times 10^{-10}$	0.0000575	0.0146	0.005 1/200		

\* The lattice constants of aluminum, steel and beryllium are approximately 3 angstrom units, that is,  $3 \times 10^{-8}$  centimeter.

Fig. I-41 Approximate Center-of-Mass Positions in Terms of Distance Along the Spin Axis for Typical Gyro Units of Any Type

## CHAPTER I-8

### INERTIAL SYSTEMS

Inertial guidance systems all require the instrumentation of reference coordinates accurately aligned with the external space used for flight path reference purposes. In addition, specific force receivers rigidly mounted on the reference member are needed to produce signals that represent specific force components. A typical arrangement is suggested by the diagram of Fig. I-42. Controlled member stabilization maintains the reference coordinates and specific force receiver outputs give computer inputs representing components along known axes. These outputs, processed by the computer, give control, navigation and guidance outputs without the necessity of geometrical transformations based on gimbal orientations.

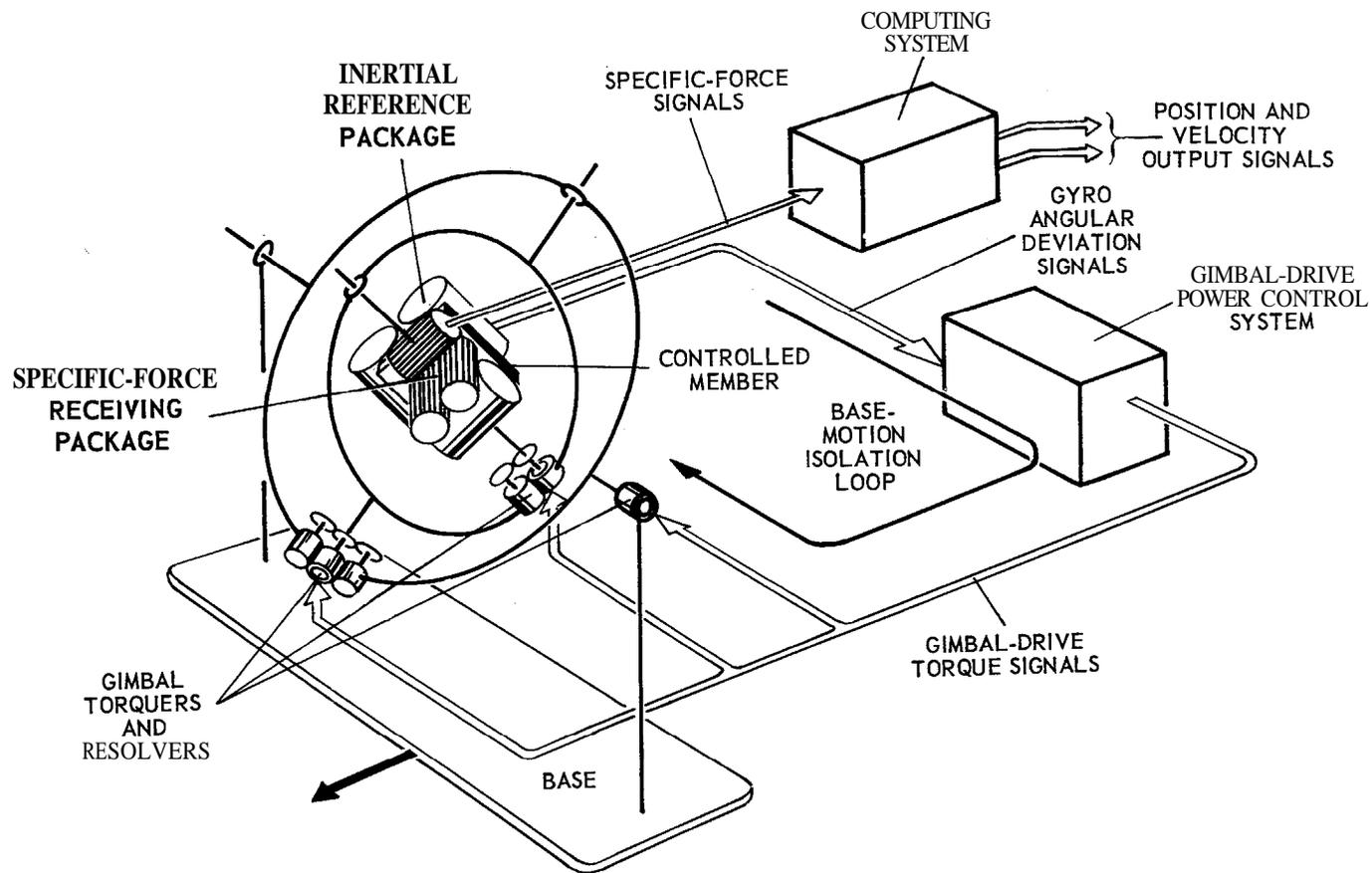


Fig. 1-42 Basic Features of Inertial Guidance System with Inertial Reference Package and Specific Force Receiving Package Rigidly Mounted on a Common Controlled Member

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**PART II**

**THE NAVIGATION, GUIDANCE AND CONTROL  
OF A MANNED LUNAR LANDING**

**by**

**David G. Hoag**

DAVID G. HOAG

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Mr. Hoag was born October 11, 1925, in Boston. He was graduated from Chauncy Hall School, Boston, in 1943, and entered M. I. T. that year as a member of the Navy V-12. He received his Bachelor of Science degree in Electrical Engineering from M. I. T. in 1946, and his Master of Science degree in Instrumentation from M. I. T. in 1950.

Mr. Hoag joined the Instrumentation Laboratory in 1946, and worked for several years on gun fire control systems and ship-launched missile fire control systems for the Navy. He became an Assistant Director of the Laboratory in 1955, and was placed in charge of all technical design aspects of the Polaris guidance system when the Laboratory later assumed responsibility for that work. He was appointed an Associate Director in 1963.

Part II  
THE NAVIGATION, GUIDANCE, AND CONTROL  
OF A MANNED LUNAR LANDING  
CHAPTER II-1  
THE BACKGROUND AND THE PROBLEM  
OF SPACECRAFT GUIDANCE, NAVIGATION, AND CONTROL

INTRODUCTION

Among the many extensions of old disciplines and development of new technologies needed in man's present rush into space flight is that of the subject of this book: the measurement and control of spacecraft position, velocity, and orientation in support of space mission objectives. In this chapter, we will introduce more specifically the nature of the problem in order to provide a background of definition and approach for the following chapters which deal with actual details of specific problems and their solutions.

One might choose the words "spacecraft rotational and translational management" as being descriptive of the subject. The parameters of concern are the time history of the three degrees of freedom describing spacecraft orientation and the time history of the three degrees of freedom describing spacecraft position.

Spacecraft missions such as those being flown today operate in phases which alternate with a short period of powered or accelerated\* flight followed by a long period of free-fall coasting. This is a consequence of the character of available propulsion typical in the chemical rocket. The nature of the rotational and translational management problems differ markedly between the free-fall and thrusting accelerated conditions. Thus it becomes convenient to separate discussions and base definitions on paired combinations of the "rotational" or "translational" and the "free-fall" or "accelerated" aspects of the subject. This results in the following definitions of four often-used terms:

- A. "Navigation"  
Translational measurement and control in free fall
- B. "Attitude Control"  
Rotational measurement and control in free fall
- C. "Guidance"  
Translational measurement and control during acceleration
- D. "Thrust Vector Control"  
Rotational measurement and control during acceleration

\*"Accelerated" here refers to that motion with respect to the free-fall arising from non-gravitational forces.

Unfortunately two minor flaws mar this symmetrical array of definitions. First, the process of "navigation" probably ought not to be constrained only to free-fall flight. Indeed the determination of position and velocity during any phase of flight might be a better definition of navigation. We can take the view then that navigation is one of the functions of the guidance process - as will be seen. Second, using "thrust vector control" for the title associated with rotational measurement and control during accelerated flight appears to exclude similar operations during phases where aerodynamic forces - not rocket thrust - are causing the acceleration. This occurs during the important phases of planetary atmospheric entry using drag and lift forces for deceleration and steering.

Recognizing these qualifications, the following sections cover the problems of each of the above four situations.

#### NAVIGATION (Translational Measurement and Control in Free Fall Coasting Flight)

Navigation as defined herein is the process of measurement and computation to determine the existing present position and probable future position of a vehicle. It is concerned only with the translational aspects of motion - i. e. position and velocity - and is considered here temporarily to be applicable only to the free-fall coasting conditions of spacecraft. It includes those processes necessary to determine needed trajectory corrections as well as to compute the initial conditions of major powered maneuvers. In this sense it has "control" aspects as well as "measurement" in that it includes activity to modify the spacecraft's path.

In non-thrusting flight out in space, the forces on the craft which determine its motion are dominated by the Newtonian gravitation attraction of the near bodies - the earth, moon, sun, and planets. Generally the vehicle is influenced primarily by one body and follows nearly the classical Keplerian conic path. The effects of forces other than that of the point mass central body can usually be treated as deviations or perturbations to the simpler motion. A non-exhaustive listing of typical perturbing effects are: (1) Mass distribution within central body, e. g. oblateness of earth, triaxiality of moon, (2) Attraction of more remote bodies, (3) Atmospheric drag, (4) Solar radiation pressure, (5) Meteoroid impact, and (6) Magnetic and electric field interactions with spacecraft.

In a given situation it is usually possible to ignore all but a few of the perturbing effects and predict the future trajectory of the vehicle with satisfactory accuracy many hours to many days into the future using knowledge of present position, and velocity. However, for a given accuracy, the prediction finally deteriorates due to ignored perturbing effects and due to the accuracy limitations of the initial conditions and the extrapolation model.

Because of the relatively predictable nature of spacecraft trajectories in free coasting flight, continuous measurement of position and velocity is unnecessary. Measurements are needed periodically to correct for the slow deviation of the actual spacecraft from the predicted path.

Practical navigation measurements in free coasting flight all utilize electromagnetic radiation at appropriate wavelengths to sense spacial relationships among the spacecraft and the near bodies of the solar system. These measurements can be categorized into two types: First, those made earth-based by remote tracking of the spacecraft from suitable stations on the earth, and second, those made from on board using sensing devices on the craft itself. Only the first of these has yet been applied; all U. S. spacecraft and as far as we know all Soviet vehicles have been navigated using earth-based tracking measurements only.

Earth-Based Navigation - Earth-based tracking for navigation usually uses radar frequencies with the cooperative use of transmitting beacons or transponders on the spacecraft being tracked. Optical wavelengths have seen use but suffer from the problem of obscuring cloud cover,

Radio tracking for navigation is founded upon: (1) the fixed and well-known speed of light in space, (2) the use of highly accurate time bases and stable frequency sources, and (3) the ingenuity and accuracy with which precise phase measurement can be made between two signals in the presence of interfering noise.

Earth-Based Range Measurement - For measurement of spacecraft range, the earth station transmits a periodic waveform on a high frequency carrier to the spacecraft which in turn is equipped to re-radiate this waveform back to the earth. The distance to and from the spacecraft is proportional to the phase lag of the waveform as received from the spacecraft with respect to the transmitted waveform to the spacecraft. Range resolution then is that fraction of a wavelength with which the phase can be measured. A 100 mc carrier, for instance, has a wavelength of 3 meters, and range resolution well inside this dimension is straightforward. Lower frequency modulation tones with longer wavelengths must be used to resolve the ambiguities and thereby determine the more significant figures of the number representing measured range. For spacecraft this technique depends upon a transponder in the spacecraft which will receive the transmission and re-radiate with controlled phase-shift an appropriate correlated signal to the ground station.

Although range tracking, as defined, has almost micrometer resolution capability, several limitations on the total overall accuracy exist. The most apparent, of course, is our knowledge of the exact speed of light. This is currently known to about 1 part in  $10^6$ . Without calibration correction as discussed below, this means a range error of 150 kilometers in a spacecraft distance of one astronomical unit. At lunar distances this reduces to the order of 400 meters.

Range rate information is measured by the rate of change of phase shift of the received signal, or more familiarly by the equivalent doppler frequency shift,

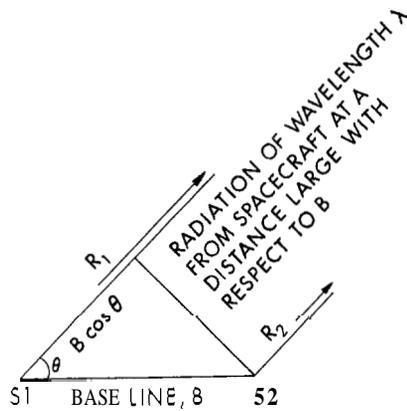
Earth-Based Direction Measurement - The most common direction measurement technique from earth stations is a sort of inverted triangulation using multiple receivers on accurately known baselines. If this baseline array is suitably short, the received signals can be simultaneously processed in the same earth-based equipment to perform an interferometric measure of the differences in range of the spacecraft from the various receivers, as shown on Fig. II-1. It is a technique which still offers many advantages, particularly the fact that the spacecraft need carry only a radio beacon transmitter which does not need to be interrogated from the ground.

For these short baselength systems the differences in phases of the various received waveforms can be measured with extreme precision. A 3-meter signal wavelength (100 mc) can be resolved by phase measurement to 3 millimeters, for example, utilizing techniques such as heterodyning to a lower frequency and precision timing. On a 150 meter baselength this corresponds to 20 microradians (4 seconds of arc) of angular resolution of spacecraft directions which lie near normal to the baseline. If a spacecraft is at one astronomical unit distance, for instance, this is position resolution of across the line of sight of 3000 kilometers. At the shorter lunar distances this reduces to about 8 kilometers,

For the usual horizontal array of receivers, it is seen that best directional accuracy is obtained for conditions with the spacecraft direction near perpendicular to the baseline. As the vehicle gets near in line with the baseline the angular resolution degenerates inversely as the sine of the angle from the baseline. Moreover, near the horizon, earth atmospheric refraction uncertainty degenerates the total indicated direction accuracy,

For greater accuracy in direction measurement, the baseline can be increased. However, several problems interfere with proportional accuracy improvement of longer baselines in comparison with the short baseline interferometric systems. First, wide separation of the receiving stations prevents accurate, simultaneous, direct phase comparison of the received signals. Also the direction is no longer a direct function of the range difference, as illustrated in Fig. II-2. So rather than using range differences obtained directly by phase comparison, the total range values from the several stations must be individually collected and then processed for determination of direction.

Realization of directional measurement accuracy improvement by increasing baselength requires that the range measurement and baseline errors accumulate less rapidly than does the baseline increase. The practicability of this can be seen by examining the accuracy needed in the baseline to maintain and improve the previous 20 microradian error derived above for the short 150 meter baselength. If the baselength is increased

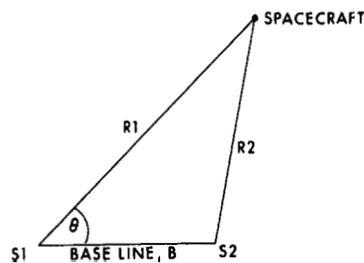


$$B \cos \theta = R_1 - R_2 \quad \Delta \phi = \text{PHASE DIFFERENCE (S1, S2)}$$

$$\cos \theta = \frac{R_1 - R_2}{B} \quad R_1 - R_2 = \frac{\lambda}{2\pi} \Delta \phi$$

$$\cos \theta = \frac{\lambda}{B} \frac{\Delta \phi}{2\pi}$$

## II-1 Short Base Line Interferometric Direction Measurement



$$R_2^2 = R_1^2 + B^2 - 2R_1 B \cos \theta$$

$$\cos \theta = \frac{R_1^2 - R_2^2 + B^2}{2R_1 B} = \frac{R_1^2 - R_2^2}{2R_1 B} + \frac{B}{2R_1}$$

$$\cos \theta = \frac{(R_1 - R_2)}{B} \left(1 - \frac{R_1 - R_2}{2R_1}\right) + \frac{B}{2R_1}$$

$$\cong \frac{R_1 - R_2}{B} \text{ when } B \ll R_1, R_2$$

## 11-2 Long Base Line Inverse Triangulation Direction Measurement

to 1500 kilometers, a 20 microradian error results from baseline errors of 30 meters. To obtain improvement to 2 microradians direction error the 1500 kilometer baseline must be known to 3 meters. However, at this precision and better, a serious question arises about achieving this necessary accuracy of earth station location. This is clearly a problem of survey and geodesy. The precise knowledge of the size and the shape of the earth is a question actively being pursued and about which agreement does not now exist.

Earth-based tracking ranging and directional measurements described above provide the basis for determining directly all components of the position and velocity of a spacecraft. The error values of our hypothetical models above by no means provide the accuracy limit from ground tracking. Considerable improvement for a given station array can be demonstrated by calibration techniques in tracking targets and applying corrections to fit the known target motions.

Spacecraft-Based Navigation - Spacecraft-borne navigation measurement tends more to optical frequency direction measurement rather than the radio frequency direct ranging that is *so* accurate for ground tracking. For relatively close work, however, direct ranging using radio frequencies with rendezvous or landing radars becomes possible, albeit necessary. But further from the planets and other targets, direct measurement of range or range rate, or the use of radio frequencies has not appeared attractive to the designers due to the weight and power penalties.

Spacecraft onboard directional measurements are those made to the near bodies . . . the sun, moon, earth, and other planets. The stars provide no position data because of their extreme distances, But because of this distance they are most excellent references against which to measure directions to the nearer bodies.

In a sense, then, onboard navigation is performed by observing the near bodies relative to the background stars. This can be done indirectly by measuring the angles sequentially from a gyro stabilized base to the stars and the near body. Alternately a direct and simultaneous measurement of the angle between a reference star and the near body with a suitable sextant-like instrument avoids an accumulation of errors with which the former sequential technique must cope.

The ancient sextant, updated and refined with a suitable telescope for image resolution and with a precision angle readout of the deflecting mirror, can provide in a reasonable size an accurate measure of the angle between a feature of a near body and a star superimposed upon that feature in the field of view.

The "feature" alluded to above is some distinct point of known coordinates on the planet to which the direction is being measured. The center of the planetary disk naturally comes to mind, but identifiable surface landmark features and horizons which can be related to planet coordinates are easier and more accurate for visual use, particularly under crescent illumination.

From sextant and sextant-like measurements, directions can be determined with accuracies, for instance, of the order of 50 microradians to targets with an additional target feature positional accuracy of the order of 1000 meters. For distances greater than 20,000 kilometers the 50 microradians dominates. Closer to the planet, however, navigation is limited by the location knowledge of the target features being used.

Each such angle measurement from the spacecraft provides a locus surface of spacecraft position at the time of measurement. Several together, if made simultaneously, define position uniquely at the common loci intersection, as shown in Fig. 11-3. In this hypothetical situation we see that range information is determined indirectly from the combination of direction data in a fashion not unlike triangulation, where the baseline is the known distance and direction between the target features of the planets. This also, in effect, is of the same nature as stadiometric ranging, made by measuring the apparent diameter of a planet disk.

Measurements separated in time can provide the basis for velocity determination. To obtain three components of position in the presence of spacecraft motion, one would desire the simultaneous measurement of at least three directional components. Practical considerations make time sequential directional measurements easier, and no direct computation of position or velocity is possible by purely geometric calculations. Schemes such as used in Apollo and described in Part III of this book depend upon the use of an onboard computer, programmed to accept the sequence of single coordinate navigation data and the precise time each measurement occurred. Each datum point is received and used to update and improve in an optimum fashion the six dimensional state vector of the spacecraft recognizing the expected error in each measurement, the current estimate of state vector error, and the motion constraints of the spacecraft in free fall.

Ground-Based and Spacecraft-Based Navigation Measurement Comparison - We can compare the similarities and differences between navigation of a spacecraft in free fall using earth-based tracking measurements and using vehicle-borne direction measurements as follows:

- A. The two categories of navigation measurement complement each other in that earth-based tracking gives strong results along the line of sight from earth, while onboard measurement can add strength across the line of sight. The latter is particularly accurate at distances far from earth and with respect to a target planet.
- B. Both categories depend upon optimum processing of data points taken over a period of time, recognizing known measurement uncertainties and spacecraft motion as constrained by orbital mechanics. Both categories use the past history of data to determine present position and velocity as limited by data uncertainty and can predict future motion further limited by the imperfect knowledge of the forces on the spacecraft due to the space environment.

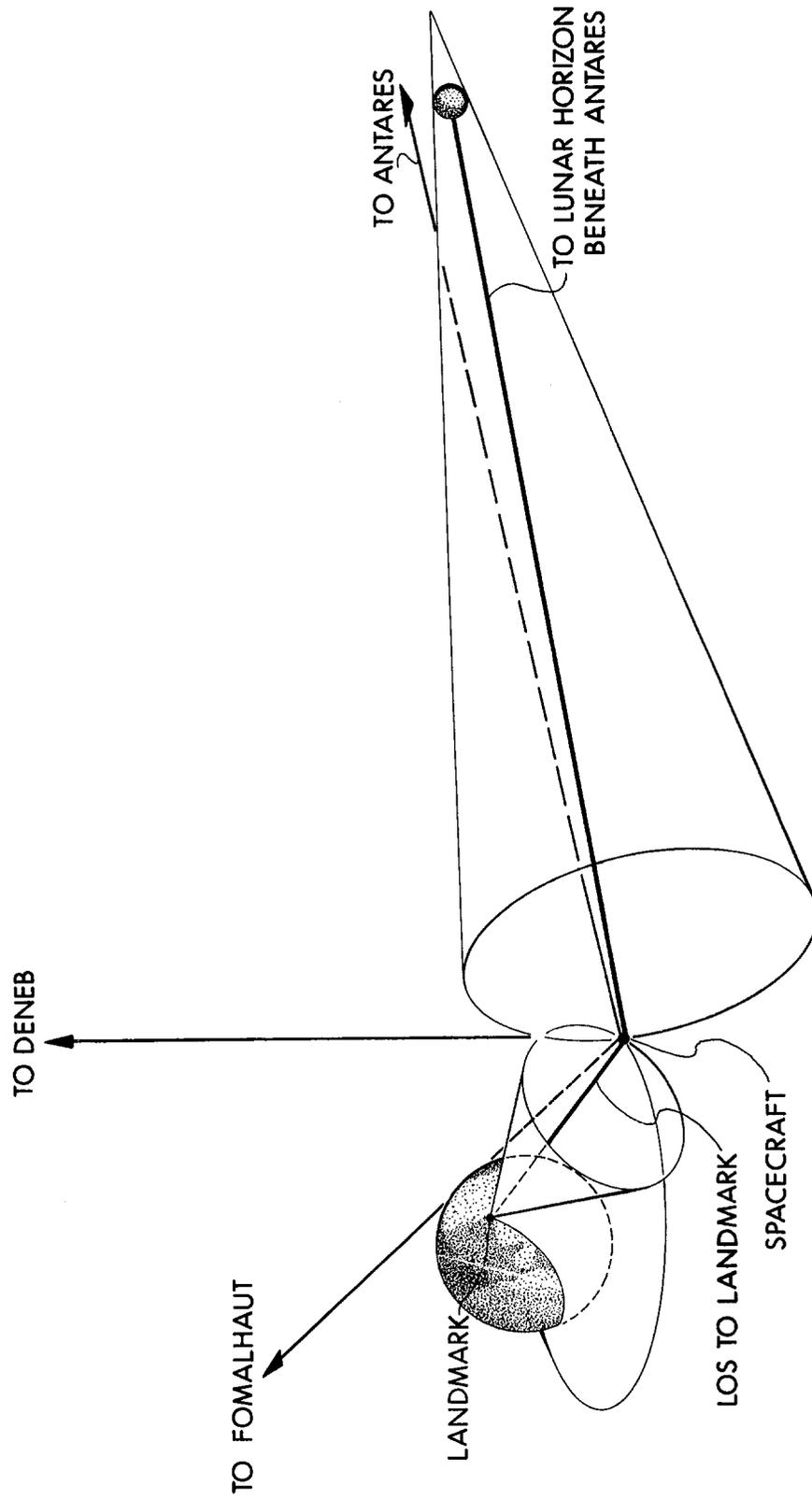
- C. Availability of earth-based navigation data from a given station is dependent upon the spacecraft being sufficiently above the horizon for that station. Spacecraft-based navigation measurements must compete for control availability with other operations of the spacecraft.
- D. Earth-based navigation stations can support simultaneously only a limited number of missions. Spacecraft-based equipment, of course, is solely available for use of that mission.
- E. Earth-based navigation tracking facilities are most limited by economic factor: in the attempt to gain more capability by the use of many large radio tracking installations with complex communication networks and data processing centers. Spacecraft-based navigation is limited more by the weight that can be carried in the sensors and data-processing computers on board.
- F. Earth-based navigation tracking facilities have the strong advantage of multiple use and re-use in sequential support of many types of missions. Spacecraft based navigation equipment is, in a sense, consumed, and only in missions where the equipment is recovered would re-use be possible.
- G. Earth-based navigation measurements fail while the spacecraft is passing in back of its target planet. This is unfortunate since efficient orbital insertion and transearth orbital escape maneuvers always occur in back of the moon and have a strong probability of being out of sight for other planets.
- H. Earth-based navigation is vulnerable to enemy action against military spacecraft. Spacecraft-based navigation measurement can be strictly passive for military use and is invulnerable to jamming or sabotage.

#### ATTITUDE CONTROL (Rotational Measurement and Control in Free-Fall Flight)

Attitude control is the process of aligning the spacecraft to a desired orientation with respect to a suitable reference framework and in response to input commands. As defined here the operation of attitude control applies to free fall coasting flight only. The diverse nature of the problem is seen in terms of: (1) the orientation requirements, (b) the attitude sensing techniques, (c) the nature of the disturbing torques, and (d) the techniques of applying the control torques. These will be discussed briefly to show the wide spectrum of problems and solutions that appear in designing attitude control systems for spacecraft.

The orientation requirements are naturally a function of the vehicle's mission and the associated operating constraints:

- A. Scientific payloads of a radiation or field sensing nature generally have pointing requirements for the sensitive axis of the instrument. Often these aiming requirements are not particularly stringent, but again others such as



II-3 Geometry of Navigational Fix in Space

astronomical telescopes can require the utmost in accuracy and stability of aiming.

- B. Spacecraft management orientation constraints generally are of a low order of accuracy. These include (1) aiming of solar cells for gathering energy to support power consuming equipment, (2) aiming of communication, telemetry, transponder, and beacon antennas toward earth, and (3) the maintenance of thermal balance by controlling attitude with respect to the sun.
- C. Navigation and guidance functions require attitude control arising from (1) the need to point the operating field of the navigation sensors towards the desired portion of the sky, and (2) the need for initial pointing of the rocket thrust axis just prior to ignition for a trajectory correction or major maneuver.

This multitude of possible requirements can lead to impossible conflicting situations which are sometimes relieved only by mounting the lightweight instruments on articulating gimbals to make them at least partially independent of spacecraft attitude.

The attitude sensing function is also performed a number of ways:

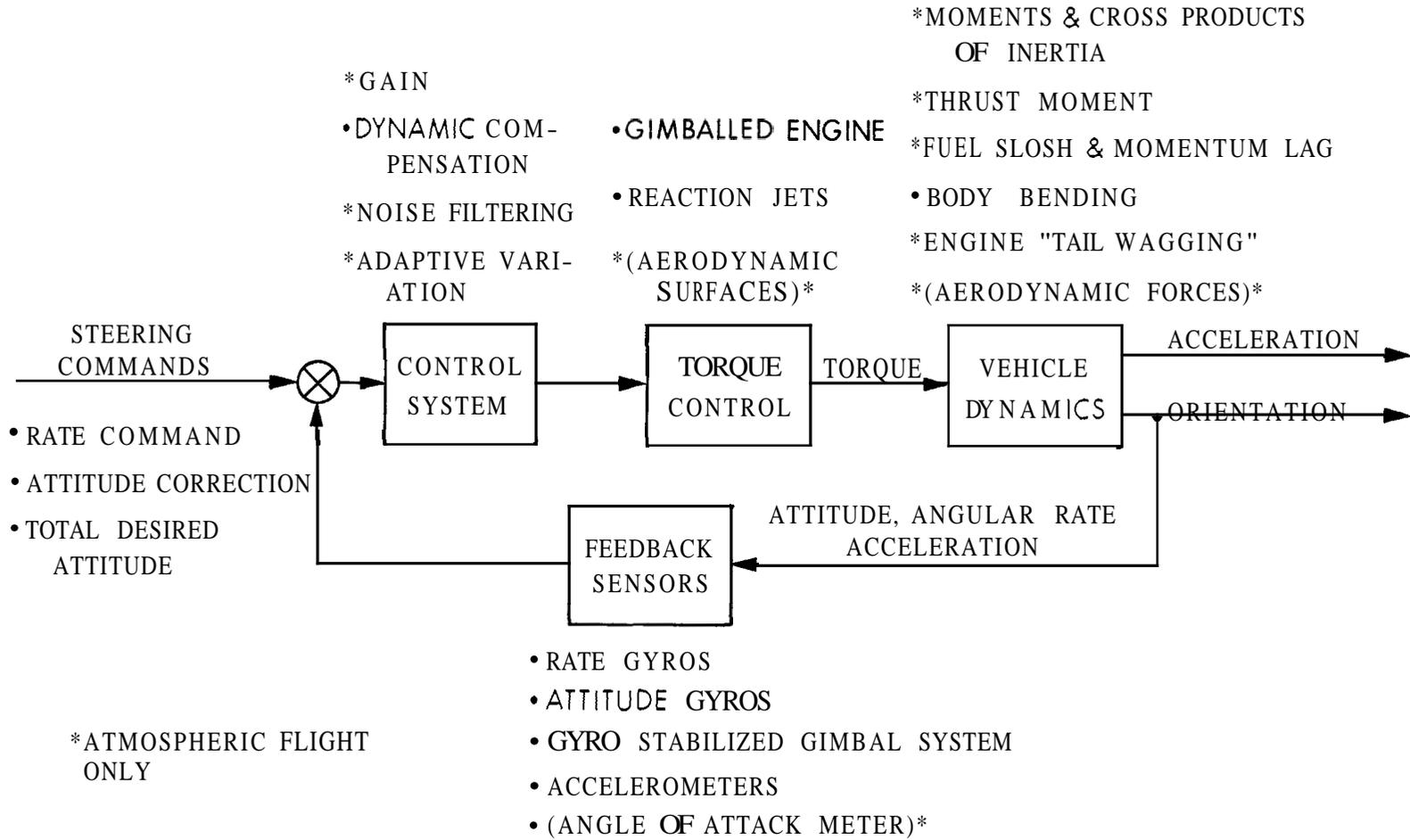
- A. In some cases radiation sensing instruments requiring pointing can be made to track the sensed flux themselves by providing error signals to the control system.
- B. For earth orbital spacecraft the most common attitude sensing uses infrared horizon detectors to indicate spacecraft orientation deviations from local vertical. These, used in conjunction with a gyroscope reference, can also provide the attitude about the local vertical with respect to the orbital plane. This process is similar to the earthbound gyrocompass in that the pendulum is replaced by the horizon detectors and the earth's rotation is replaced by the rotation in orbit.
- C. Basic attitude sensing for small cislunar and interplanetary vehicles most often depends upon a sun seeker/tracker to set up a vehicle axis with respect to the sun, combined with a star tracker offset by an adjustable angle to acquire and track a star so as to provide attitude sensing about that sun line.
- D. Once an orientation reference is established this can be maintained by the use of gyroscopes to detect deviations from the reference. Gyroscopes also provide capability to meter orientation changes accurately from the attitude established by other means.

The disturbance torques upset spacecraft orientation and cause the need for correction from the control system:

- A. Lightweight vehicles can be affected by the relatively weak forces associated with the space environment. For spacecraft with large unsymmetric surfaces with respect to the center of mass, radiation pressure from the sun is a significant torque disturbance. Lightweight vehicles also may be affected by interaction of electrical current loops or other spacecraft magnetic sources with the earth's field. Electrostatic forces, unsymmetric atmospheric drag, and the integrated effect of micrometeoroids have also been suggested as a source of disturbance torques,
- B. Vehicles having one long dimension resulting in a wide difference in the principal moments of inertia can be strongly affected by differential gravity forces when near a massive planet.
- C. Spacecraft will experience disturbance torques any time mass is thrown off. This can occur, for instance, by the boiloff venting of cryogenic fuel or oxidizer or the offloading of other wastes.
- D. Relative acceleration of masses within the vehicle cause a redistribution of angular momentum arising from associated torques. Speed changes of on-board rotating machinery, the pumping or sloshing of fluids, or the process of erection of solar panels or antennas are examples. On manned craft the movements of the crew cause significant disturbance.

Control torques to counteract these disturbances or to re-orient the vehicle can utilize any of three phenomena.

- A. The weak forces associated with the space environment can be utilized in a passive or semipassive attitude control. Self-aligning mechanisms based upon solar radiation pressure, magnetic field torques, or gravity gradient unbalances can provide weak but often adequate restoring torques to a stable orientation satisfactory for some missions. Some form of energy dissipation for damping oscillations must be provided.
- B. Small reaction rocket engines arrayed to provide suitable torque couples depend upon angular momentum transfer to the exhausted gas. These are usually chemical or cold gas low thrust engines designed for many on-off cycles as demanded by the control loop. Control is characterized by pulsed operation of the jet and limit cycle oscillation about the desired attitude.
- C. Flywheel or gyroscope momentum exchange systems achieve control torque by either accelerating a heavy flywheel or precessing a spinning gyro. Unlike the jet or rocket systems above, only power is consumed and operation is not limited by the amount of working fluid carried. However, there is a capacity limit in the sense that there is a maximum momentum that can be stored by



11-4 Generalization of Thrust Vector Stabilization and Control

practical speeds of heavy flywheels or gyrowheels. Thus, in application, these momentum exchange systems are used in conjunction with periodic use of a jet or other type of external torques to "desaturate" the system back within its control range. Finally, a simple spin of the whole spacecraft itself can often provide adequate simple means of stabilization.

The design of attitude control systems is complicated by a number of factors. The classical equations of motion under assumptions of spacecraft rigidity are straightforward. But even though it is theoretically possible to predict the rotational motions of the vehicle, a simple control system cannot make large rotational changes directly when the desired axis of rotation does not coincide with one of the principal axes of inertia. Usually the torquing axes are close to these principal axes and large rotational maneuvers are made sequentially axis by axis. This is admittedly less efficient than a hypothetical control system that would control to the shortest path achieved by applying a single initial torque impulse on the necessary axis. This impulse would create that angular momentum vector which will carry the vehicle into the desired orientation by free tumbling rotation where a second impulse could stop it.

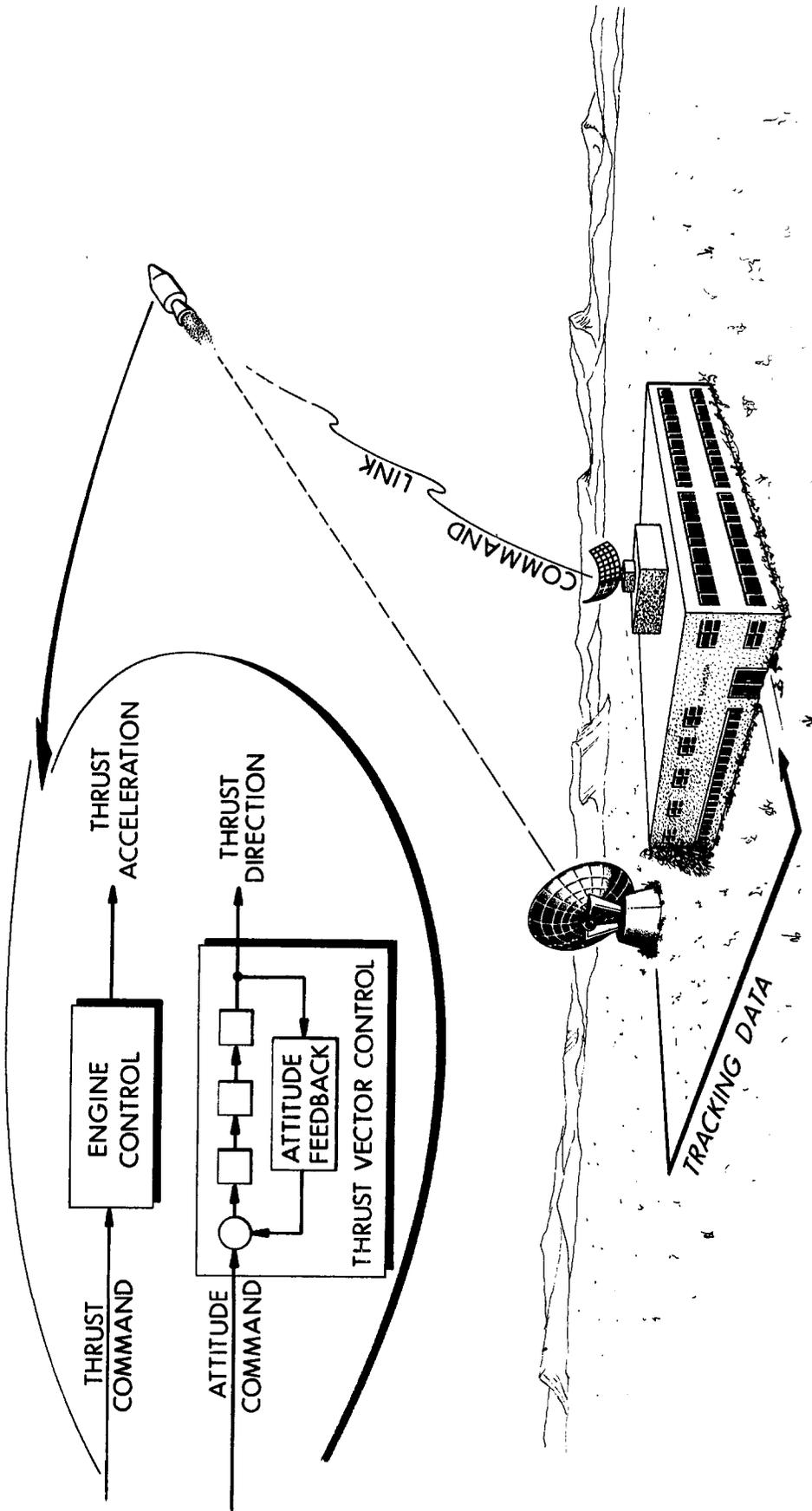
The energy used in a rotational maneuver is directly a function of the speed with which the maneuver must be accomplished. Rather than build up kinetic energy in a fast turn only to cancel it again at the destination orientation with an opposite impulse, the designers tend towards very slow rotation rates for the large turns when mission requirements permit.

Another complicating factor occurs when the spacecraft carries a significant mass of fluid fuel. A practical attitude control cannot measure the angular momentum contribution of this fluid since its loose coupling to spacecraft allows it independent motion. Again the theoretically most efficient application of control torque cannot be achieved by simple attitude control systems.

Reaction jet control engines are characterized by fixed torque levels during firing and a wearout or lifetime limit on the number or duration of individual firings. Since disturbance torques are generally less than the available control torque, the attitude control system provides on-off cycles of firing resulting in a limit cycle oscillation about the desired orientation. The total jet fuel used and the number of on-off cycles should be minimized by optimization of the control system.

#### THRUST VECTOR CONTROL (Rotational Measurement and Control During Accelerated Flight)

Thrust vector stabilization and control is the closed-loop process which (1) keeps the vehicle attitude from tumbling under the high forces of engine firing and (2) accepts turning or guidance steering commands to change the direction of engine-caused acceleration.



II-5 Radio Command Guidance

Figure 11-4 illustrates a generalization of acceleration vector stabilization and control. In order to illustrate the variations possible, the boxes in the figure may contain one or more of the aspects listed with "dot" prefix adjacent to the boxes. These systems are characterized by appropriate feedback to provide a stable control of angle or angular velocity of the thrusting vehicle. The loop also accepts input steering commands from guidance to achieve a particular desired thrust direction.

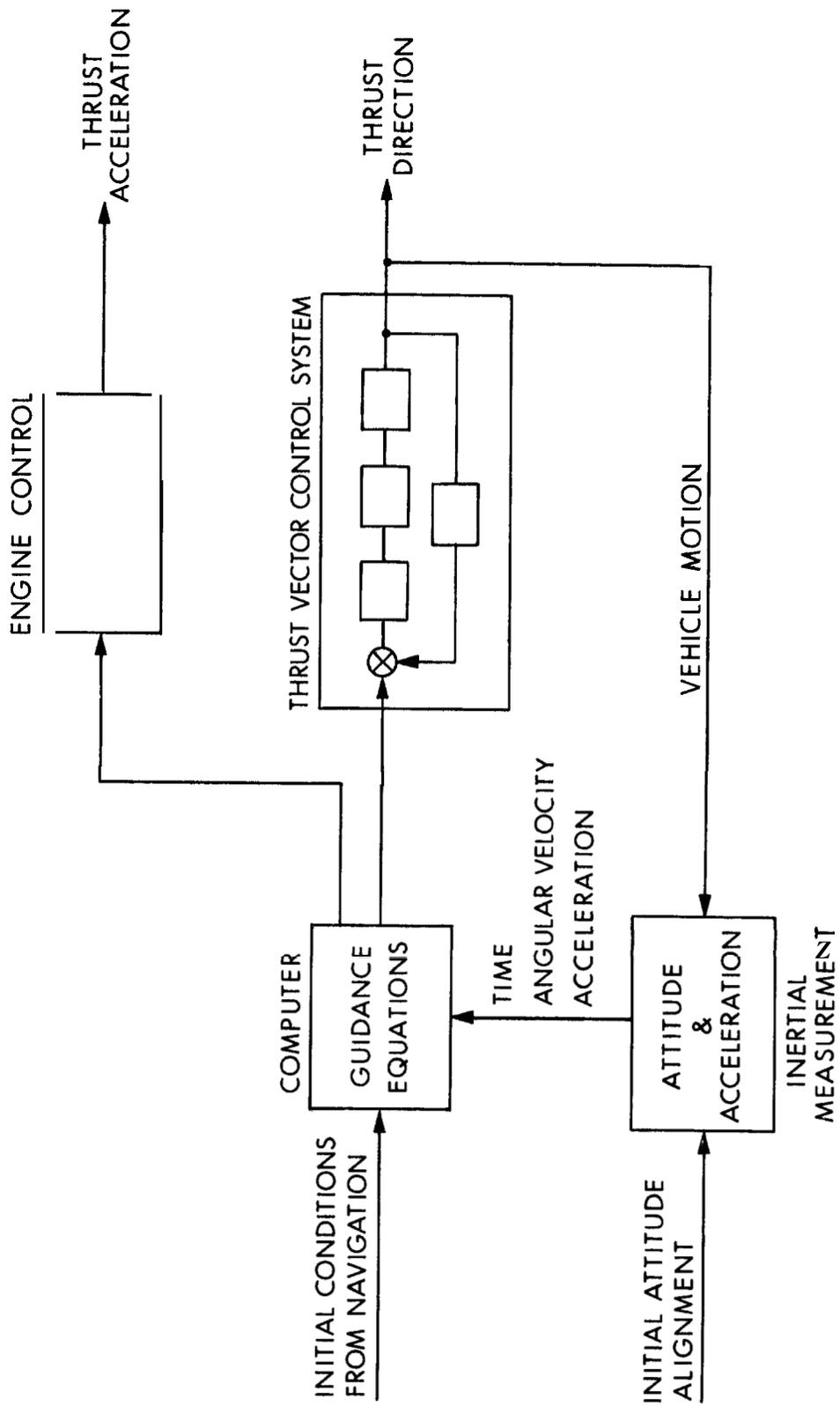
The design constraints on thrust vector stabilization and control systems represented by Fig. II-4 vary considerably. The figure lists typical variations possible in the spacecraft body dynamics and the torque producing control devices. Most of these characteristics are not only gross nonlinearities but are time variant and interacting as well. The design is further complicated by necessary constraints on dynamic response to inputs and disturbances. It is usually restricted by allowable limits on angular acceleration, angle of attack, and other variables depending on structural and controllability considerations. All this and the usual concern about reliability, weight, cost, etc. makes design particularly difficult.

#### GUIDANCE (Translation Measurement and Control During Accelerated Flight)

Guidance is the process of measurement and computation necessary to provide steering signals to the thrust vector control system and signals to modulate engine thrust level in order to achieve vehicle acceleration to a desired trajectory. Modulation of engine thrust level in the more common case of a non-throttleable engine consists only of turn-on and cutoff commands.

Earth-Based Tracking Guidance - Powered steering of some of the early USA ballistic missiles and of workhorse spacecraft launch vehicles used ground tracking data in a radio command guidance illustrated in Fig. 11-5. This type of guidance is characterized by a continuous ground tracking monitor of position and velocity changes during the powered phases and a radio command to the vehicle to change the direction of thrust appropriately - and finally to signal thrust termination. A basic requirement is an attitude reference system carried aboard the vehicle. This is illustrated in Fig. 11-5 as the attitude feedback, implemented with gyros for instance, as part of the thrust vector control system.

Far from the earth, delays occur associated with necessary longer smoothing of the noisier tracking signals and delays associated with the finite speed of electromagnetic propagation. For deep space spacecraft requiring short burn trajectory corrections of moderate accuracy these delays are not significant since the ground command need only specify the direction and length of burn required. However, for precise, long duration maneuvers the thrust vector control alone cannot assure accuracy in metering the direction or magnitude of the specified velocity change. And far from the earth the mentioned delays in the receipt of the steering commands make loop closure corrections



11-8 Inertial Guidance

of questionable effectiveness. Here inertial guidance is the only practical method of powered steering control.

On-Board Inertial Guidance - Inertial guidance, Fig. II-6, is based upon measurements of vehicle motion using self-contained instruments which do not depend upon radiation sensing. In every inertial guidance system three types of measurements are made involving distinctive instruments: (1) angular rate or direction using gyroscopic devices, (2) linear acceleration using restrained test masses in accelerometers, and (3) time using precision reference frequency sources. The integration with time of the sensed acceleration in the indicated direction with proper recognition of known gravity field forces is the essence of the navigation portion of inertial guidance. The implied processes are accomplished in a computer with the result of generating corrective steering commands to the thrust vector control system. Since inertial guidance of the type described can only integrate vehicle motion into changes in position, velocity, and orientation, accurate initial conditions are required in these parameters before the accelerated guidance phase is started. Initial conditions in position and velocity are provided by navigation prior to the accelerated phase. Initial conditions in orientation come from the attitude control systems or directly from stellar references,

One well-debated problem with inertial guidance is the presence of an increasing error of the inertially derived orientation and navigation with time. When an error in the gyro data, commonly called gyro drift, is processed in the computer the direction of the controlled acceleration is in error. When an error in the acceleration sensing exists, again the direction of acceleration as well as magnitude of acceleration and the controlled length of motor burning are affected undesirably. However, due to the motivation to perfect inertial instruments for their well adapted use in military guidance and navigation, the technology is advanced to the point that inertial guidance performance can be kept well ahead of needs for spacecraft missions in controlling accelerated flight. Furthermore, spacecraft inertial guidance can be tolerant of error, in the sense that errors in the resulting trajectory usually can be measured later by navigation and corrected with a short burn of the propulsion.

It is perhaps pertinent to examine these last statements with respect to two propulsion situations, which undoubtedly will exist in the future. The first is that of the high specific impulse, low thrust electric engines. Here the very low thrust to mass ratio requires long periods of controlled engine operation - measured in weeks. In such long periods, inertial guidance measurement alone without recourse to periodic external navigation measurements would be unacceptable, even if the inertial sensing were perfect. The inertial system cannot sense the perturbations in trajectory caused by the imperfectly known gravitation forces. Such systems then require periodic navigation by onboard or ground-tracking measurements. It is doubtful whether these navigation checks would be needed any more often than during the coasting free-fall phases with the more conventional chemical engine mission,

The second future propulsion situation is that which will exist with high thrust nuclear rockets providing more abundant total impulse. In this realm, mission times will be shortened by longer burning to higher interplanetary velocities than permitted with current chemical engines. In spite of the larger velocity changes to be measured during thrust by the inertial sensing, the dramatic shortening of the subsequent time of flight is enough to decrease required measurement precision for the same accuracy in arrival at the destination planet or orbit.

## CHAPTER II-2

### GUIDANCE, NAVIGATION, AND CONTROL TASKS IN THE APOLLO MISSION

Much insight into problems of space flight has been gained from the extensive study and hardware development during the last four years in the Apollo program for a manned lunar landing. Using Apollo as an example, specific spacecraft guidance, navigation, and control tasks are illustrated in this chapter.

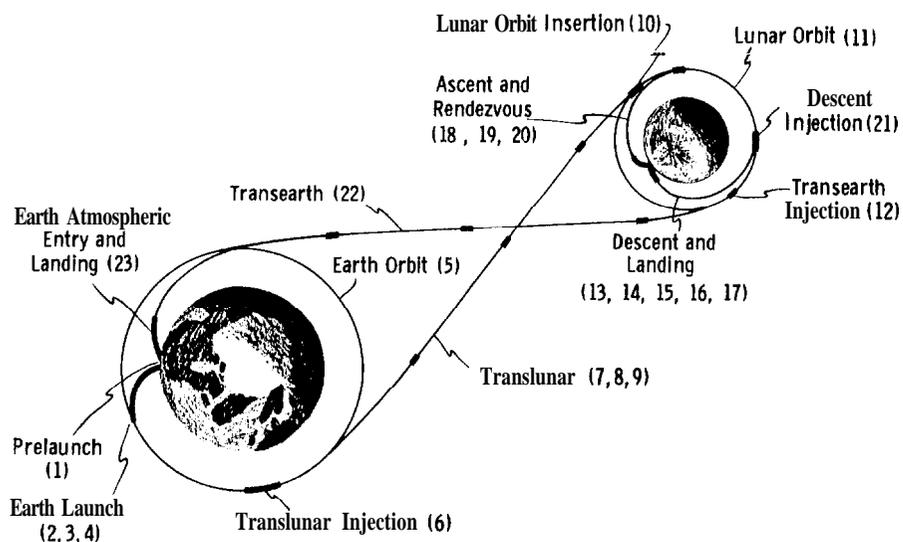


Fig. 11-7 The Overall Apollo Mission

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The overall Apollo mission trajectory is summarized above. The heavy lines correspond to the short accelerated maneuvers which are separated by the much longer free coasting phases. The trajectory on this figure is purposefully distorted, as is also some of the following figures, in order to show features of the phases more clearly. The numbers on this figure relate to the following mission phase subdivisions:

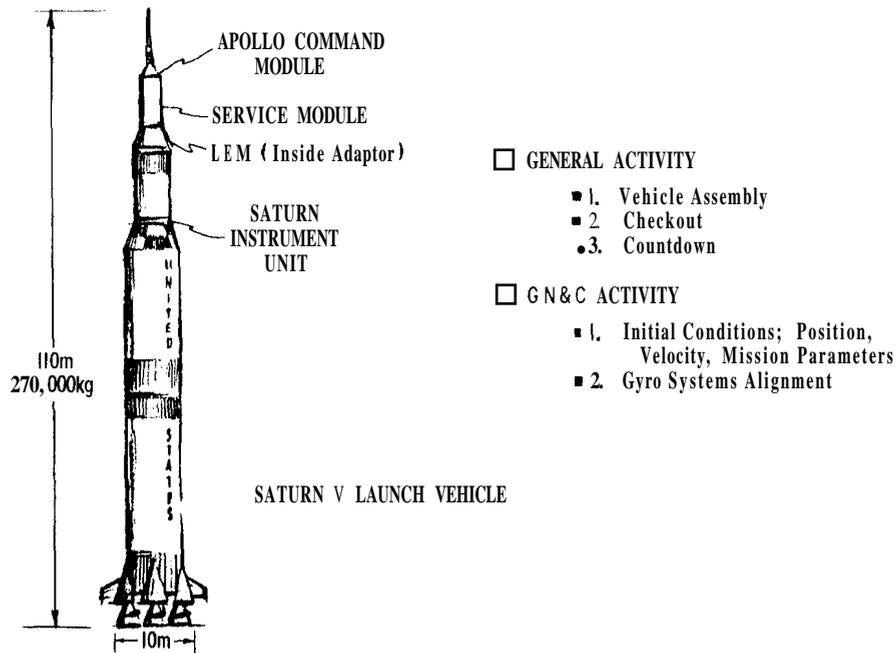


Fig. II-8 Phase 1 - Prelaunch *6/10/65 30249*

The prelaunch phase includes an intensive and intricate schedule of activity to prepare and verify the equipment for flight. Automatic programmed checkout equipment perform the exhaustive tests of the major subassemblies.

During the final countdown, testing continues. Activity of interest here concerns the preparation of the two operating sets of guidance equipment for the launch. The Saturn guidance equipment located in the Saturn Instrument Unit will control the launch vehicle. The Apollo guidance equipment located in the Command Module (CM), where the crew of three lie in their protective couches will provide a monitor of Saturn guidance during launch. A third set of guidance equipment located in the Lunar Excursion Module (LEM), which is inside the protective LEM adaptor is used later near the moon.

Ground support equipment communicates directly with the Saturn and Apollo CM guidance computers to read in initial conditions and mission and trajectory constants as they vary as a function of countdown status. Both sets of inertial guidance sensors are aligned to a common vertical and launch azimuth framework. The vertical is achieved in both cases by erection loops sensing gravity, Azimuth in Saturn is measured optically from the ground and controlled by means of an adjustable prism mounted on the stable member. Azimuth in Apollo is aligned optically on board by the astronauts and held by gyro compassing action, During countdown, both systems are tied to an earth frame reference, Just before liftoff, both systems respond to signals to release the coordinate frames simultaneously from the earth reference to the non-rotating inertial reference to be used during boost flight.

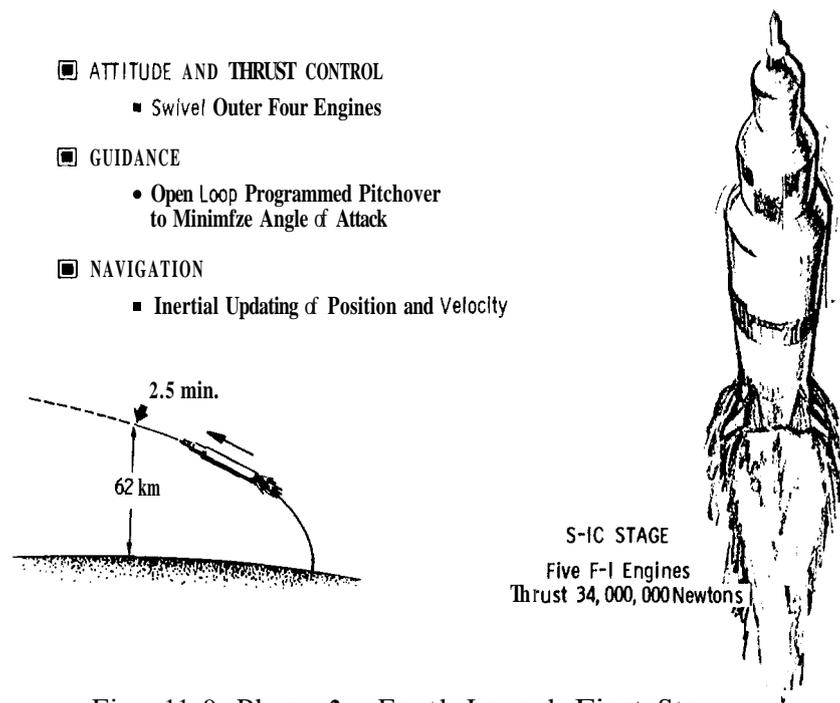


Fig. 11-9 Phase 2 - Earth Launch First Stage

During first stage flight the Saturn guidance system controls the vehicle by swiveling the outer four rocket engines. During the initial vertical flight the vehicle is rolled from its launch azimuth to the flight path azimuth. Following this the Saturn guidance controls the vehicle in an open loop pre-programmed pitch designed to pass safely through the period of high aerodynamic loading. Inertial sensed acceleration signals are not used during this phase to guide to desired path, but rather the lateral accelerometers help control the vehicle to stay within the maximum allowed angle of attack. Stable control is achieved in overcoming the effects of flexure bending, fuel slosh, and aerodynamic loading by the use of properly located sensors and control networks.

Both the Saturn and Apollo Command Module guidance systems continuously measure vehicle motion and compute position and velocity. In addition, the Apollo system compares the actual motion history with that to be expected from the Saturn control equations so as to generate an error display to the crew. This and many other sensing and display arrangements monitor the flight. If abort criteria indicate, the crew can fire the launch escape system. This is a rocket attached on a tower to the top of the command module to lift it rapidly away from the rest of the vehicle. Parachutes are later deployed for the landing.

In a normal flight the first stage is allowed to burn to near complete fuel depletion as sensed by fuel level meters before first stage engine shutdown is commanded.

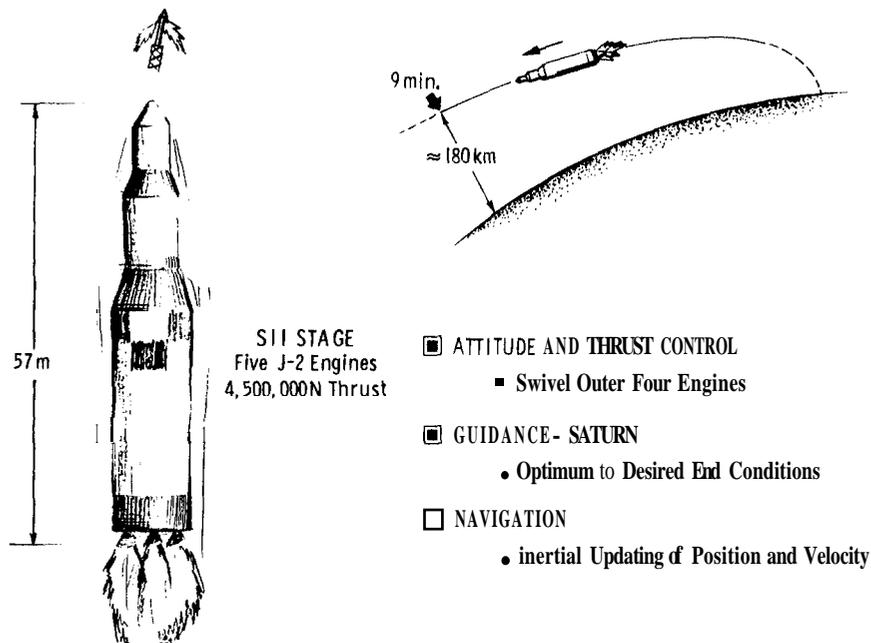


Fig. 11-10 Phase 3 - Earth Launch Second Stage

Shortly after the initial fuel settling ullage and the firing of second stage thrust, the aerodynamic pressure reduces to zero as the vehicle passes out of the atmosphere. At this time the launch escape system is jettisoned. Aborts now, if necessary, would normally be accomplished using the Apollo Service Module propulsion to accelerate the Command Module away from the rest of the vehicle.

Since the problems of aerodynamic structure loading are unimportant in second stage flight, the Saturn guidance system now steers the vehicle towards the desired orbital insertion conditions using propellant optimizing guidance equations. Thrust control is achieved by swiveling the outer four engines of the second stage.

During second stage flight the Apollo Command Module guidance system continues to compute vehicle position and velocity. Also this system computes any of several other possible parameters of the flight to be displayed to the crew for monitoring purposes. In addition, the free-fall time to atmospheric entry and the corresponding entry peak acceleration are displayed to allow the crew to judge the abort conditions existing.

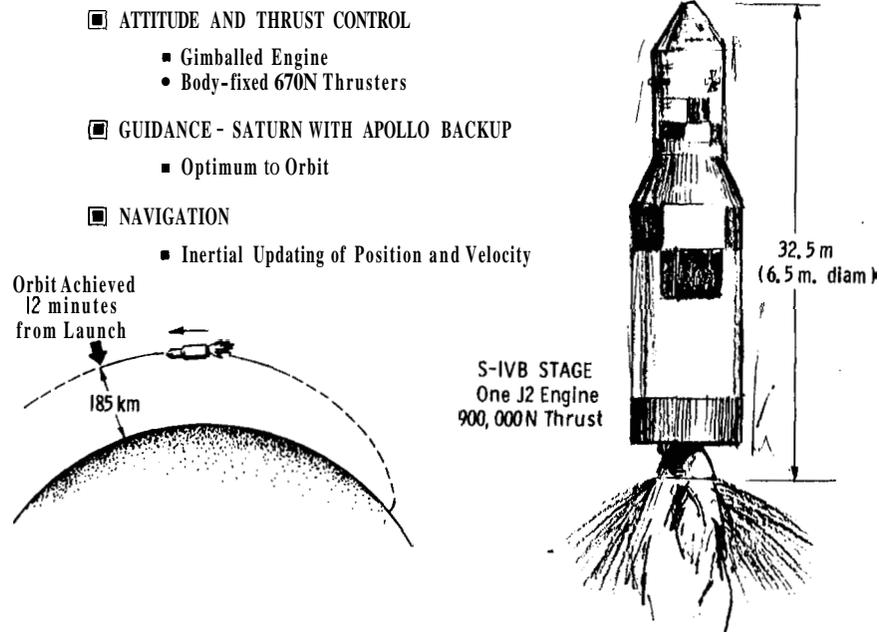


Fig. 11-11 Phase 4 - Earth Launch Third Stage

The third Saturn stage or SIVB has a single engine for main propulsion which is gimballed for thrust vector control. Roll control is achieved by use of the SIVB roll attitude control thrusters.

The Saturn guidance system continues to steer the vehicle to orbital altitude and speed. When orbit is achieved, the main SIVB propulsion is shut down,

During second and third stage boost flight, the Apollo Command Module has the capability, on astronaut option, to take over the SIVB stage guidance function if the Saturn guidance system indicates failure. If this switchover occurs, the mission presumably could be continued. More drastic failures would require an abort using the Service Module propulsion. In this case the Apollo computer is programmed to provide several abort trajectories: (1) immediate safe return to earth, (2) return to a designated landing site, or (3) abort into orbit for later return to earth.

SIVB engine shutdown occurs about 12 minutes after liftoff at 185 km altitude near circular orbit,

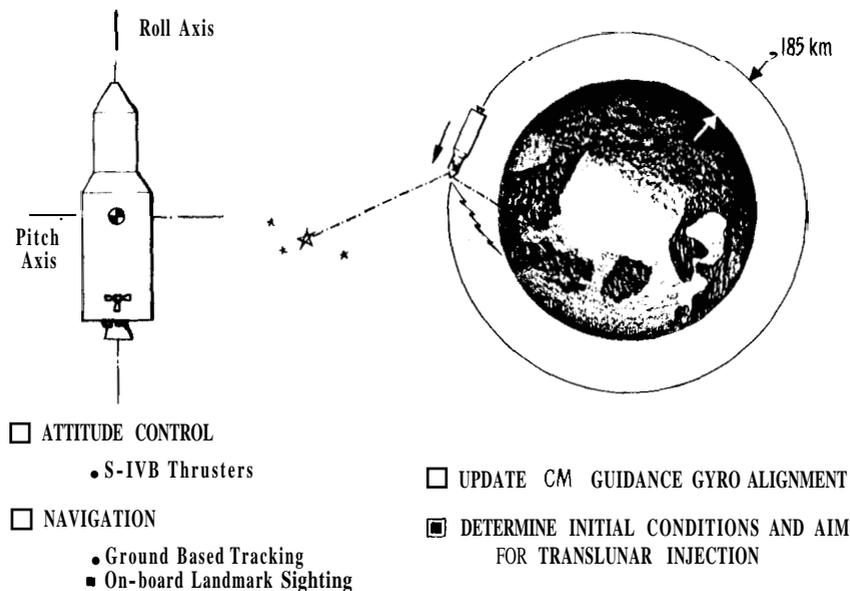


Fig. 11-12 Phase 5 - Earth Orbit

The Apollo spacecraft configuration remains attached to the Saturn SIVB stage in earth orbit. The Saturn system controls attitude by on-off commands to two of the small fixed attitude thrusters for pitch and to four more shared for yaw and roll.

Ground tracking navigation data telemetered from the Manned Space Flight Network (MSFN) stations is available to correct the position and velocity of the Saturn navigation system and provide navigation data for the Apollo navigation system. In Apollo the crew also can make onboard navigation measurements for onboard determination of the ephemeris by making landmark or horizon direction sightings using a special optical system. The Apollo inertial equipment alignment will also be updated by star sightings with the same optical system. For these measurements the crew has manual command control of attitude through the Saturn system. Normally, limited roll maneuvers are required to provide optical system visibility to both stars and earth.

The Apollo onboard navigation measurements include accelerometer measurement of the small thrust occurring during the pressure venting of the cryogenic propellant tanks of the SIVB.

Typically, the earth orbital phase lasts for several hours before the crew signals the Saturn system to initiate the translunar injection at the next opportunity.

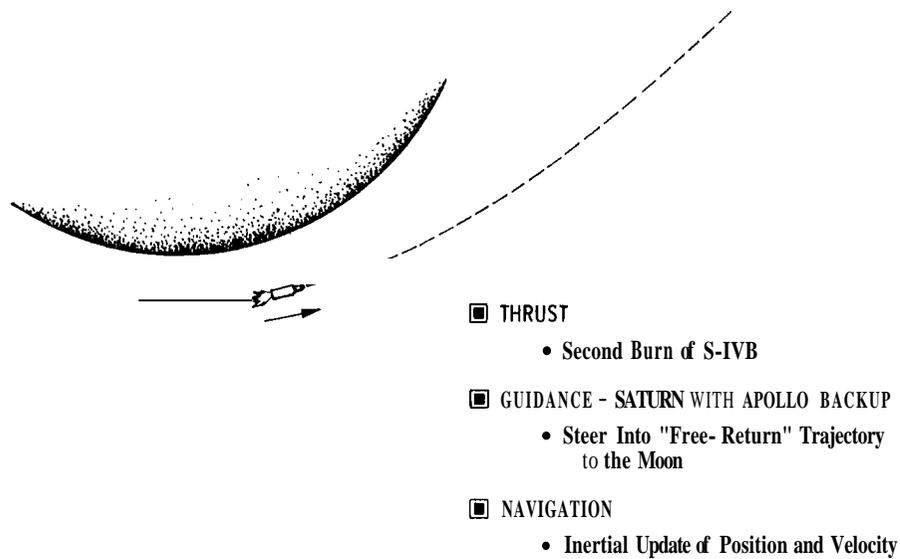


Fig. 11-13 Phase 6 - Translunar Injection

Translunar injection is performed using a second burn of the Saturn SIVB propulsion, preceded, of course, by an ullage maneuver using the small thrusters. Saturn guidance and control systems again provide the necessary steering and thrust vector control to the near parabolic velocity which for crew safety considerations puts the vehicle on a so-called "free return" trajectory to the moon. The system aims to this trajectory which ideally is constrained to pass in back of the moon and return to earth entry conditions without additional propulsion.

As before, the Apollo guidance system independently generates appropriate parameters for display to the crew for monitoring purposes. It is recognized that a display of a large error by Apollo does not necessarily indicate Saturn system malfunction because an error in Apollo system operation could instead be the fault. The identification of the failed system may be indicated by another of the available displays or by ground tracking information relayed to the crew. If the Saturn guidance system indicates failure, steering takeover by the Apollo is possible without need for aborting the mission.

The translunar injection thrusting maneuver continues for slightly over 5 minutes duration before the SIVB stage is commanded its final shutdown.

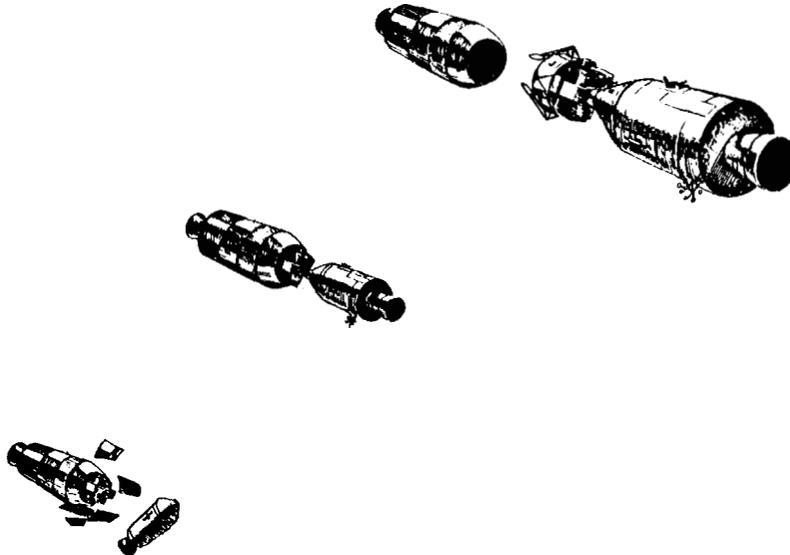


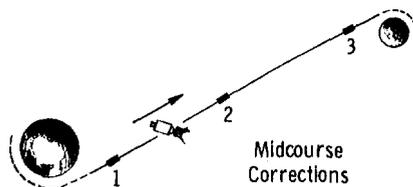
Fig. 11-14 Phase 7 - Transposition and Docking

The spacecraft configuration injected onto the translunar free-fall path must be re-assembled for the remaining operations.

The astronaut pilot separates the Command and Service Modules (CSM) from the LEM which is housed inside the adapter in front of the SIVB stage. He then turns around the CSM for docking to the LEM. To do this the pilot has a three-axis left-hand translation controller and a three-axis right-hand rotational controller. Output signals from these controllers are processed to modulate appropriately the firing of the 16 low thrust reaction control jets for the maneuver. The normal response from the translation controller is proportional vehicle acceleration in the indicated direction. The normal response from the rotational controller is proportional vehicle angular velocity about the indicated axis.

During the separation and turnaround maneuver of the CSM, the SIVB control system holds the LEM attitude stationary. This allows for a simple docking maneuver of the command module to the LEM docking hatch. The SIVB, Saturn instrument unit, and LEM adapter are staged to leave the Apollo spacecraft in the translunar configuration.

Final docking is complete less than 6.5 hours from liftoff at the launching pad.



ATTITUDE CONTROL

- 16 Attitude Jets of 445N Thrust each on Service Module

NAVIGATION

- Ground Based Tracking and On-board Star/Landmark Sextant Sightings

GUIDANCE

- Used Only for Small Midcourse Corrections

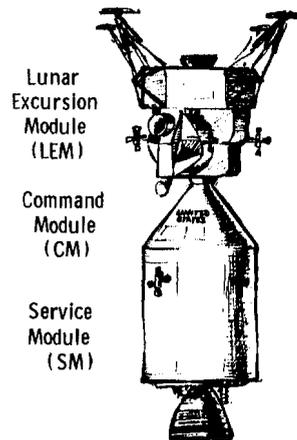


Fig. 11-15 Phase 8 - Translunar Coast

6/19/65 36256

Very soon after injection into the translunar free-fall coast phase, navigation measurements are made and processed to examine the acceptability of the trajectory. These data will probably indicate the need for an early midcourse maneuver to correct error in the flight path before it propagates with time into larger values which would needlessly waste correction maneuver fuel.

Once this first correction is made - perhaps a couple of hours from injection - the navigation activity on board can proceed at a more leisurely pace. Ground tracking data can be telemetered to the craft anytime it is available. Using this ground data and/or onboard sextant type of landmark to star angle measurements the onboard computer can correct the knowledge of the spacecraft state vector - position and velocity.

The astronaut navigator can examine with the help of the computer each datum input available - whether from ground tracking telemetered to the craft or taken on board - to see how it could change the indicated position and velocity before he accepts it into the computer state vector correction program. In this way the effects of mistakes in data gathering or transmission can be minimized.

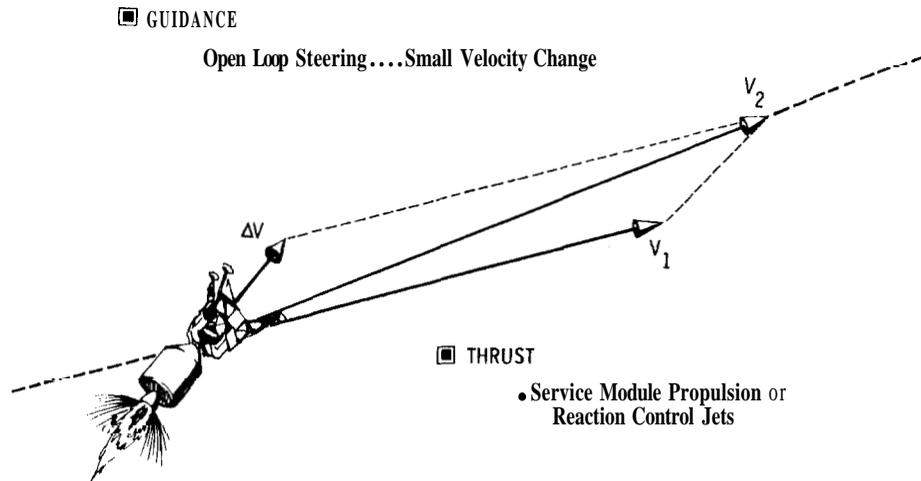


Fig. 11-16 Phase 9 - Midcourse Corrections

The navigator will examine periodically the computer's estimate of indicated uncertainty in position and velocity and the estimate of indicated velocity correction required to improve the present trajectory. If the indicated position and velocity uncertainty is suitably small and the indicated correction is large enough to be worth the effort in making, then the crew will prepare for the indicated midcourse correction. Each midcourse velocity correction will first require initial spacecraft orientation to put the estimated direction of the thrust axis along the desired acceleration direction. Once thrust direction is aimed, the rocket is fired under measurement and control of the guidance system. Use of the guidance system requires the inertial measurement system be aligned. This latter is done by optical star direction sightings.

Typical midcourse corrections are expected to be of the order of 10 meters/sec. If the required correction happens to be very small, it would be made by using the small reaction control thrusters. Larger corrections would be made with a short burn of the main service propulsion rocket. It is expected that about three of these midcourse velocity corrections will be made on the way to the moon. The direction and magnitude of each will adjust the trajectory so that the moon is finally approached near a desired plane and pericyinthian altitude which provides for satisfactory conditions for the lunar orbit insertion.

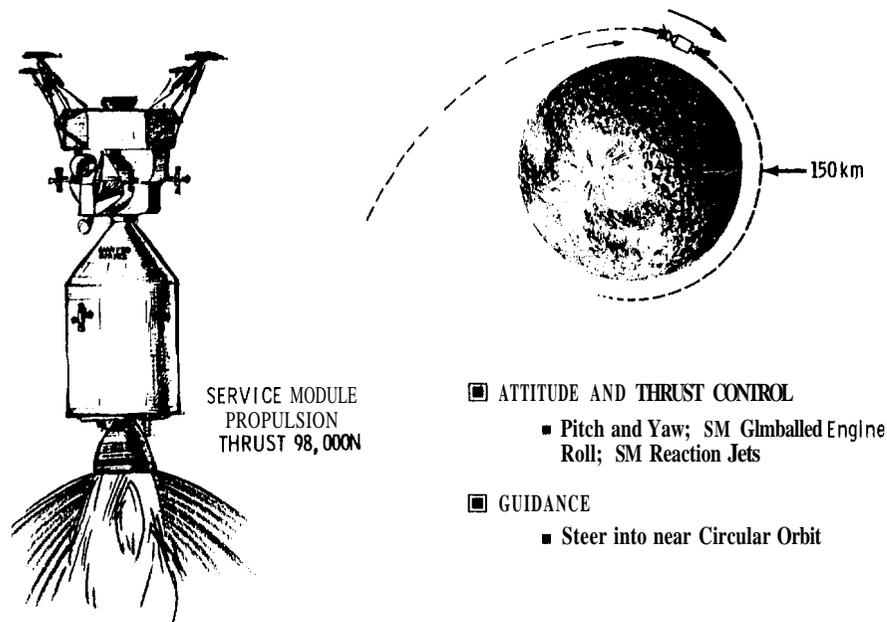


Fig. 11-17 Phase 10 - Lunar Orbit Insertion *6/11/65 36298*

For lunar orbit insertion, as with all normal thrusting maneuvers using the service propulsion of the spacecraft, the inertial guidance system is first aligned using star sightings. Then the system generates initial conditions and steering parameters based upon the navigation measure of position and velocity and the requirements of the maneuver. The guidance initiates engine turn-on, controls the direction of the acceleration appropriately, and signals engine shut-down when the maneuver is complete. *40756?*

The lunar orbit insertion maneuver is intended to put the spacecraft in a near circular orbit of approximately 150 km altitude. The plane of the orbit is selected to pass over the landing region on the front of the moon.



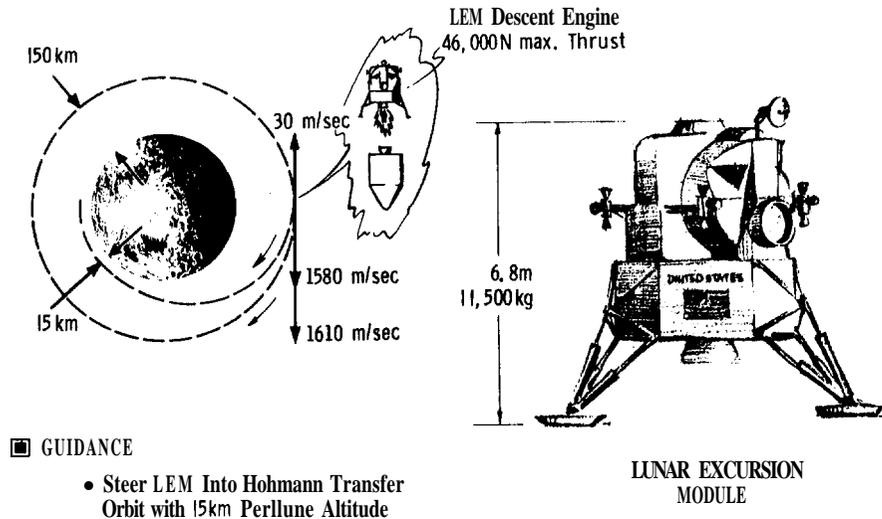


Fig. II-19 Phase 12 - LEM Descent Orbit Injection 4/19/65 36260

The LEM guidance system will have been turned on and received a checkout earlier in lunar orbit before separation and received initial conditions from the CSM. Starting about twenty minutes before initiation of the LEM descent injection maneuver the vehicles are separated, the LEM guidance system receives final alignment from star sightings, and the attitude for the maneuver is assumed. The maneuver is made using the LEM descent stage propulsion under control of the LEM guidance system, During the short burn, the throttling capability of the descent engine is exercised as a check of its operation. The maneuver is a 30 meter per second velocity change to reduce the velocity from the 1600 meter/sec orbital velocity for a near Hohmann transfer to a 15 km altitude pericynthian which is timed to occur at a range of about 370 kilometers short of the final landing area.

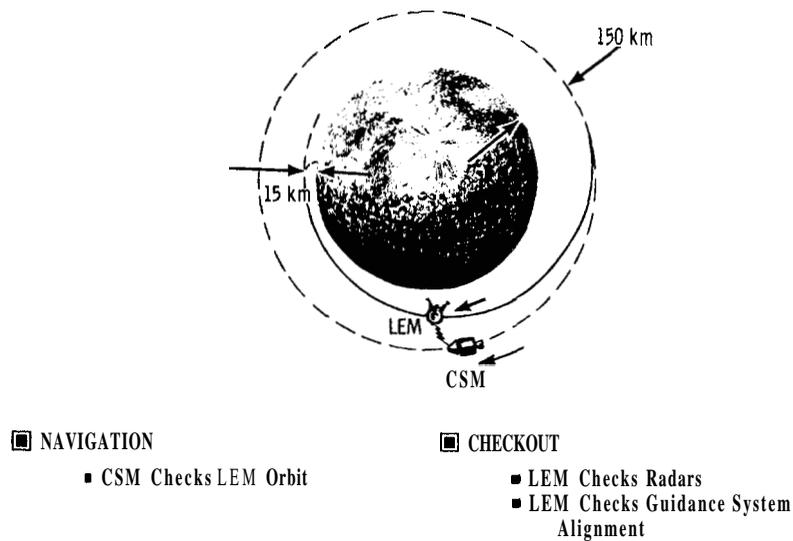


Fig. 11-20 Phase 13 - LEM Descent Orbit Coast

During the free fall phases of the LEM descent, the CSM can make tracking measurements of the LEM direction for confirmation of LEM orbit with respect to the CM. For that part of the trajectory in the front of the moon the earth tracking can also provide an independent check. The LEM, during appropriate parts of this coasting orbit, will check the operation of its radar equipment. The directional tracking and ranging operation of the Rendezvous Radar is checked against the radar transponder on the CSM. This also provides data to the LEM computer for an added descent orbit check. At lower altitudes the LEM landing radar on the descent stage is operated for checks using the moon surface return. Alignment updating of the LEM guidance system can be performed if desired.

The CM from orbit can monitor this phase of the LEM descent using the tracking systems and onboard computer.

As pericynthian is approached, the proper LEM attitude for the powered descent phase is achieved by signals from the guidance system.

▣ THRUST VECTOR CONTROL

- 3 - Axis Control Using Reaction Jets
- Engine Gimballed for Trim Correction

▣ ENGINE

- Near Full Throttle Under Guidance System Control

▣ GUIDANCE

- Inertial Sensing with Some Radar data at Lower Altitudes

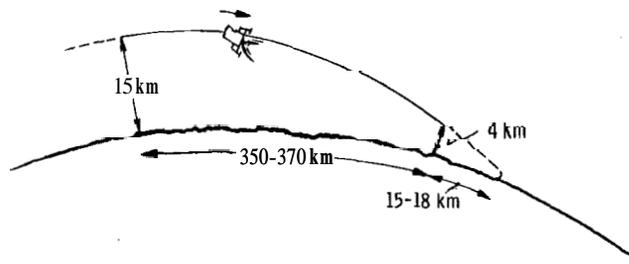


Fig. 11-21 Phase 14 - LEM Powered Descent Braking Phase

This phase starts at the 15 kilometer altitude pericynthian of the descent coast phase. The descent engine is re-ignited, and this velocity and altitude reducing maneuver is controlled by the LEM inertial guidance system,

The descent stage engine is capable of thrust level throttling over the range necessary to provide initial braking and to provide controlled hover above the lunar surface. Engine throttle setting is commanded by the guidance system to achieve proper path control although the pilot can override this signal if desired,

Thrust vector direction control of the descent stage is achieved by a combination of body-fixed reaction jets and limited gimbaling of the engine. The engine gimbal angles follow guidance commands in a slow loop so as to cause the thrust direction to pass through the vehicle center of gravity. This minimizes the need for continuous fuel wasting torques from the reaction jets,

During all phases of the descent the operations of the various systems are monitored. The mission could be aborted for a number of reasons. If the primary guidance system performing the descent control is still operating satisfactorily, it would control the abort back to rendezvous with the CSM. If the primary guidance system has failed, a simple independent abort guidance system can steer the vehicle back to conditions for rendezvous.

For a normal mission, the braking phase continues until the altitude drops to about 4 kilometers or so. Then guidance control and trajectory enter the final approach operation.

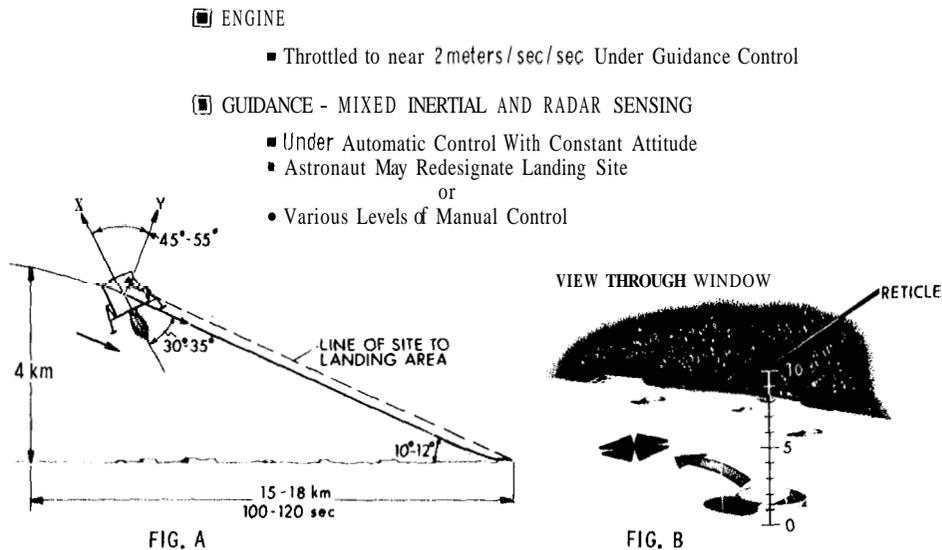


Fig. 11-22 Phase 15 - LEM Powered Descent Final Approach 4/19/65 36263

One significant feature of this phase is that the controlled trajectory is selected to provide visibility of the landing area to the LEM crew. The vehicle attitude, descent rate, and direction of flight are all essentially constant so that the landing point being controlled by the guidance appears fixed with relation to the window. A simple reticle pattern in the window, as shown, indicates this landing point in line with the number indicated by computer display. The pilot may observe that the landing point being indicated is in an area of unsatisfactory surface features with relation to other areas nearby. He can then elect to select a new landing point for the computer control by turning the vehicle about the thrust axis until the reticle intersects the better area. He then hits a "mark" button to signal the computer, reads the reticle number which is in line with this area into the computer, and then allows the guidance to redirect the path appropriately. This capability allows early change of landing area and fuel efficient control to the new area which otherwise might have to be performed wastefully later during hover.

Automatic guidance control during the terminal phase uses weighted combinations of inertial sensing and landing radar data, the weighting depending upon expected uncertainties in the measurements. The landing radar include altitude measurement and a three-beam doppler measurement of three components of LEM velocity with respect to the lunar surface.

▣ GUIDANCE

- Various Mixtures of Manual and Automatic
- Radar and Visual Correction Until Surface Obscured by Dust, Then.. .
- All Inertial For Touchdown at Low Velocity

▣ TRAJECTORY

- Mission Groundrules and Pilot Option

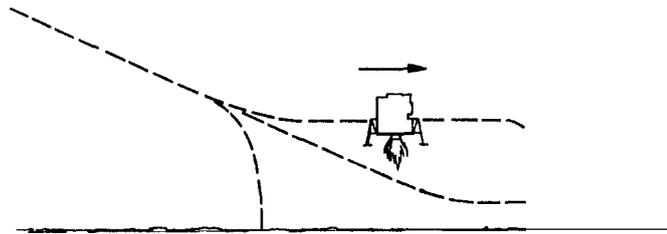


Fig. 11-23 Phase 16 - Landing and Touchdown

At any point in the landing the pilot can elect to take over partial or complete control of the vehicle. For instance, one logical mixed mode would have altitude descent rate controlled automatically by modulation of the thrust magnitude and pilot manual control of attitude for maneuvering horizontally.

The final approach phase will end near the lunar surface, and the spacecraft will enter a hover phase. This phase can have various possibilities of initial altitude and forward velocity depending upon mission groundrules, pilot option, and computer program yet to be decided. Descent stage fuel allowance provides for approximately two minutes of hover before touchdown must be accomplished or abort on the ascent stage initiated. The crew will make final selection of the landing point and maneuver to it either by tilting the vehicle or by operating the reaction jets for translation acceleration. The inertial system altitude and velocity computation is updated by the landing radar so that as touchdown is approached good data are available from the inertial sensors as the flying dust and debris caused by the rocket exhaust degrade radar and visual information. Touchdown must be made with the craft near vertical and at sufficiently low velocity.

■ ACTIVITY

- Checkout Systems
- Exploration, Experimentation, etc

■ GUIDANCE AND NAVIGATION

- Track CSM Overhead
- Align Guidance for Ascent

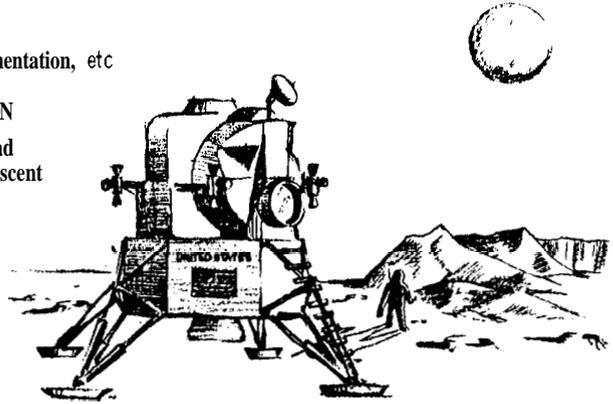
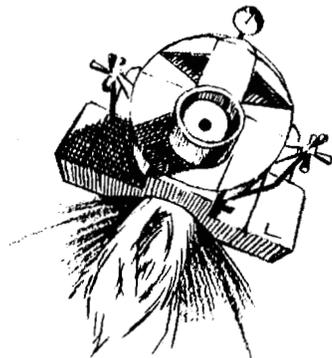


Fig. 11-24 Phase 17 - Lunar Surface Operations

The period on the moon will naturally include considerable activity in exploration, experimentation, and sample gatherings. Also during this stay time, LEM spacecraft systems will be checked and prepared for the return. The ephemeris of the CSM in orbit is continually updated and the information relayed to the LEM crew and computer. The LEM rendezvous radar also can track the CSM as it passes overhead to provide further data upon which to base the ascent guidance parameters. The inertial guidance gets final alignment from optical star direction sightings prior to the start of ascent. The vertical components of this alignment could also be achieved by accelerometer sensing of lunar gravity in a vertical erection loop, Liftoff must be timed to achieve the desired trajectory for rendezvous with the CSM.



LEM Ascent Engine  
Thrust 15,000N



THRUST VECTOR CONTROL

- 3-Axis Control Using Reaction Jets

■ GUIDANCE

- Inertial Sensing
- Steer to Rendezvous Transfer Orbit

Fig. 11-25 Phase 18 - LEM Ascent 6/14/63 36266

Normal direct ascent launches are timed and controlled to cutoff conditions resulting in a coasting intercept with the CSM. Emergency launches from the lunar surface can be initiated at any time by entering a holding orbit at low altitude until the phasing is proper for transfer to the CSM. A desirable constraint on all ascent powered maneuvers as well as abort maneuvers during the landing is that the following coasting trajectory be near enough circular so as to be clear of intersection with the lunar surface. This is a safety consideration to allow for the possibility of failure of the engine to restart. If the LEM engine thus fails, the LEM can then safely coast until a pickup maneuver by the CSM is accomplished.

The initial part of the ascent trajectory is a vertical rise followed by pitchover as commanded by the guidance equations. The ascent engine maneuvers are under the control of the LEM inertial guidance system. The engine has a fixed mounted nozzle. Thrust vector control is achieved by operation of the sixteen reaction jets which are mounted on the ascent stage. The engine thrust cannot be throttled but the necessary signals from guidance will terminate burning when a suitable rendezvous coast trajectory is achieved.

NAVIGATION

- Rendezvous Radar Data Used to Determine Velocity Corrections

VELOCITY CORRECTIONS

- Using Inertial Guidance; As Many as Needed to Achieve Intercept with CSM

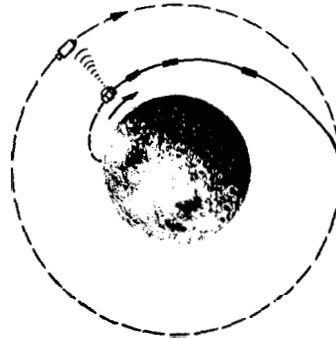


Fig. 11-26 Phase 19 - Midcourse Rendezvous

If the launch point lies in the plane of the CSM orbit, efficient ascending coasting trajectories would cover 180 degrees central angle to the rendezvous point. Several effects will cause the launch point to be removed from the CSM plane resulting in trajectories either somewhat more or somewhat less than 180 degrees.

Immediately after injection into the ascending coasting rendezvous trajectory, the rendezvous radar on the LEM will start making direction and range measurements to the CSM upon which the LEM computer will base its navigation using a process almost identical to that used in navigation of the midcourse phase between earth and moon. From this navigation the LEM computer will determine small velocity corrections to be made by LEM reaction control jets to establish the collision or intercept trajectory with the CSM more accurately. These corrections will be made as often as the radar based navigation measurements justify. The coasting continues until the range to the CSM is reduced to approximately 10 kilometers when the terminal rendezvous phase begins.

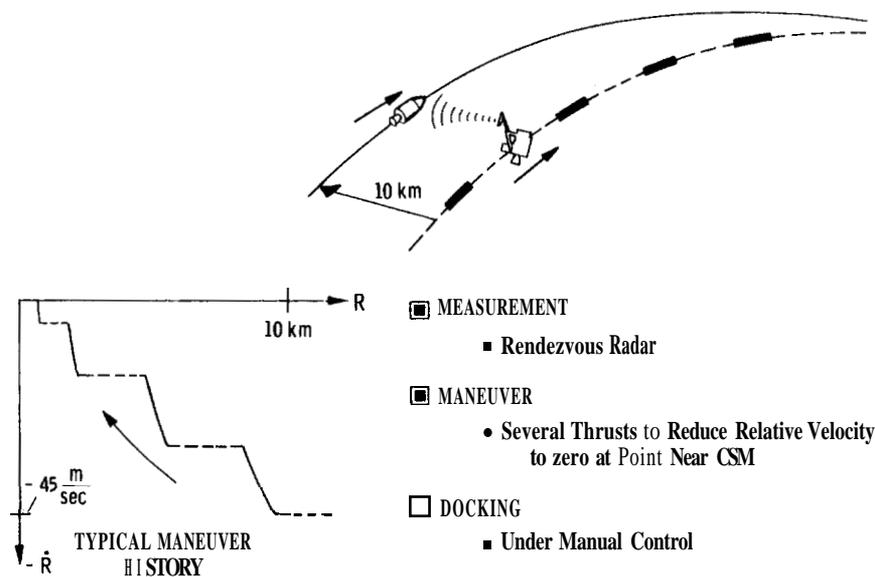


Fig. 11-27 Phase 20 - Terminal Rendezvous and Docking

The terminal rendezvous phase consists of a series of braking thrust maneuvers under control of the LEM guidance system which uses data from its inertial sensors and the rendezvous radar. The objective of these operations is to reduce the velocity of the LEM relative to the CSM to zero at a point near the CSM. This leaves the pilot in the LEM in a position to initiate a manual docking with the CSM using the translation and rotation control of the LEM reaction jets.

Although these maneuvers would normally be done with the LEM, propulsion or control problems in the LEM could require the CSM to take the active role.

After final docking the LEM crew transfer to the CSM and the LEM is then jettisoned and abandoned in lunar orbit.

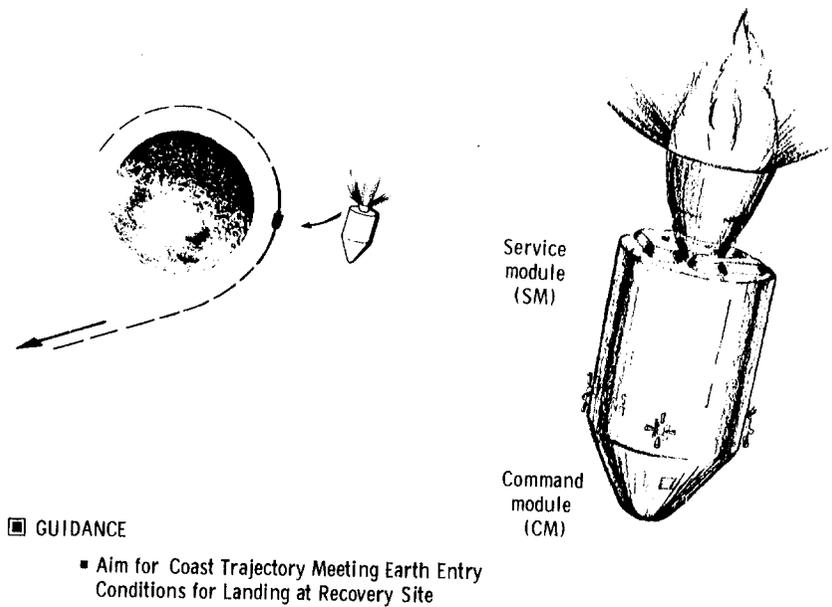


Fig. II-28 Phase 21 - Transearth Injection

Navigation measurements made while in lunar orbit determine the proper initial conditions for transearth injection. These are performed as before using onboard and ground-based tracking data as available.

The guided transearth injection maneuver is made normally under the control of the primary inertial guidance system. Several backup means are available to cover possible failures in the primary system. The injection maneuver is controlled to put the spacecraft on a free-fall coast to satisfactory entry conditions near earth. The time of midcourse transearth coast must be adjusted by the injection to account for earth's rotation motion of the recovery area and as limited by the entry maneuver capability.

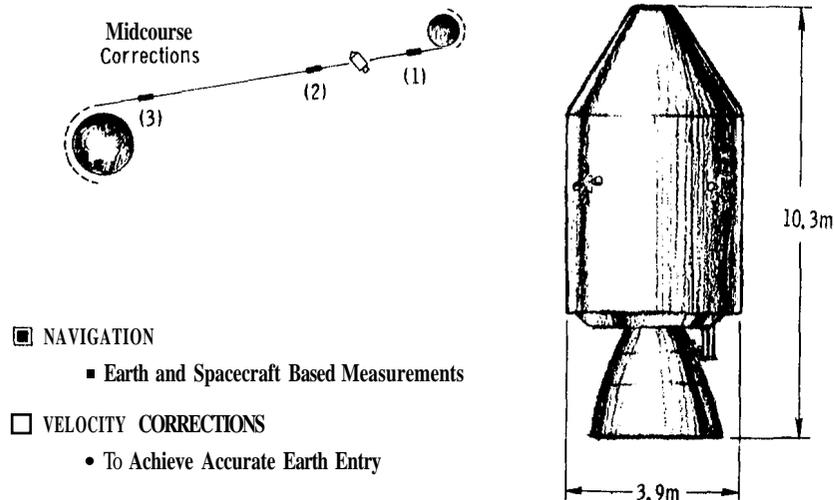


Fig. 11-29 Phase 22 - Transearth Coast

The transearth coast is very similar to the translunar coast phase. During the long coasting phases going to and from the moon the systems and crew must control the spacecraft orientation as required. Typical midcourse orientation constraints are those to assure the high gain communication antenna is within its gimbal limits to point to earth or that the spacecraft attitude is not held fixed to the local heating effect of the sun for too long a period.

During the long periods of free-fall flight going to and from the moon when the inertial measurement system is not being used for controlling velocity corrections, the inertial system is turned off to conserve power supply energy.

Onboard and ground-based measurements provide for navigation upon which is based a series - normally three - of midcourse correction maneuvers during transearth flight. The aim point of these corrections is the center of the safe earth entry corridor suitable for the desired landing area. This safe corridor is expressed as a variation of approximately  $\pm 32$  kilometers in the vacuum perigee. A too-high entry could lead to an uncontrolled skipout of the atmosphere; a too-low entry might lead to atmospheric drag accelerations exceeding the crew tolerance,

After the final safe entry conditions are confirmed by the navigation before entry phase starts, the inertial guidance is aligned, the Service Module is jettisoned, and the initial entry attitude of the Command Module is achieved.

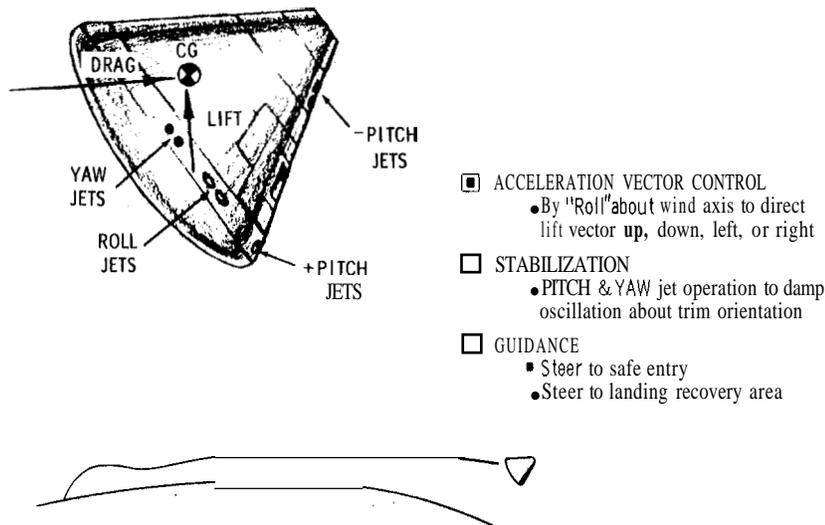


Fig. 11-30 Phase 23 - Earth Atmospheric Entry *01/16/36.771*

Initial control of entry attitude is achieved by guidance system commands to the 12 reaction jets on the command module surface. As the atmosphere is entered, aerodynamic forces create torques determined by the shape and center of mass location. If initial orientation was correct, these torques are in a direction towards a stable trim orientation with heat shield forward and flight path nearly parallel to one edge of the conical surface. The control system now operates the reaction jets to damp out oscillation about this trim orientation. The resulting angle of attack of the entry shape causes an aerodynamic lift force which can be used for entry path control by rolling the vehicle about the wind axis under control of the guidance system. Range control is achieved by rolling so that an appropriate component of that lift is either up or down as required. Track or across range control is achieved by alternately choosing as required the side the horizontal lift component appears.

The early part of the entry guidance is concerned with the safe reduction of the high velocity through the energy dissipation effect of the drag forces. Later at lower velocity the objective of controlling to the earth recovery landing area is included in the guidance. This continues until velocity is reduced and position achieved for deployment of a drag parachute. Final letdown is normally by three parachutes to a water landing.

## CHAPTER 11-3

### GUIDANCE, NAVIGATION, AND CONTROL INSTRUMENTATION IN APOLLO

The choice of sensors and data processors for guidance, navigation, and control used in Apollo is governed by the nature of the spacecraft and its mission as described in the previous chapter. Two Apollo design guidelines will be mentioned at the outset.

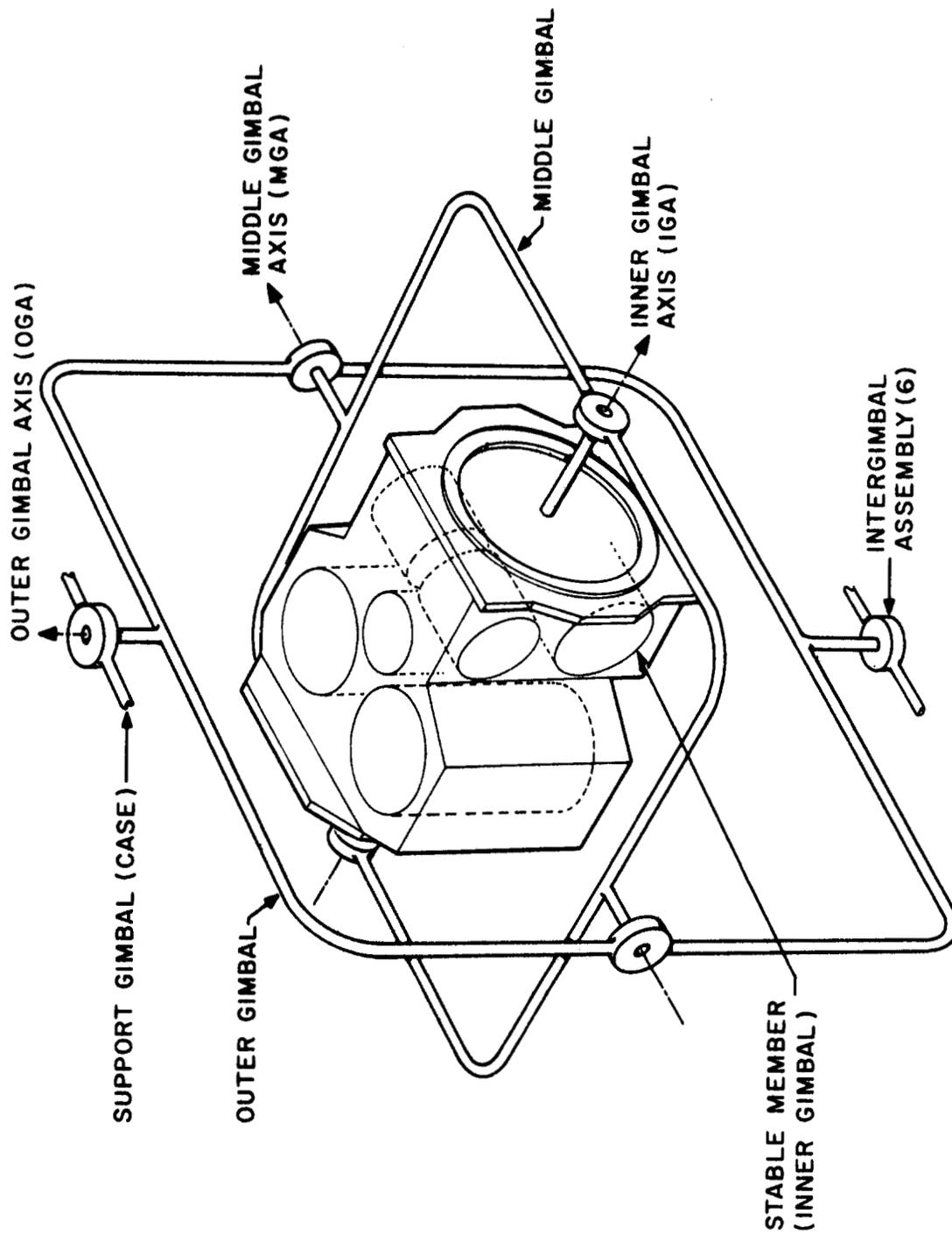
First, although full use will be made of all earth-based help, the spacecraft systems are designed to have the capability of completing the mission and returning without the use of earth-based tracking data or computation support. This provides protection against critical lack of earth coverage or failure in communication. However, earth-based data will be available most of the time which will be supported by measurements from the onboard equipment.

The second guideline recognizes the diverse nature of the mission and the variations in spacecraft configuration. The guidance, navigation, and control equipment is designed to provide a great deal of flexibility in its utilization. This is manifest in the fact that identical subsystems are used in the two independent systems controlling the command module and the lunar excursion module. This flexibility extends also to the development of the necessary operation equations expressed in the flight computer programs. The unified approach of these to handle the various thrusting and coasting computation chores with a universal compact set of programs is described in Part III. In this chapter we will describe the selection and design of hardware.

#### INERTIAL MEASUREMENT SYSTEM

The choice of inertial guidance over radio command guidance can be easily justified . . . perhaps most dramatically by recognition of the velocity change maneuvers which necessarily must occur in back of the moon. Here the guidance measurements must be made by onboard sensors during the lunar orbit insertion and escape maneuvers out of sight of the earth where ground data are not available. Even were it not for the fact that the earth is blind with respect to these maneuvers, it is extremely doubtful radio command could function for large velocity change maneuvers at lunar distances.

The choice of inertial guidance mechanization might not be so obvious. The two major configurations for inertial measurements are: (1) gyro stabilized gimbal-mounted platform and (2) vehicle frame mounted sensors. Each has advantages.



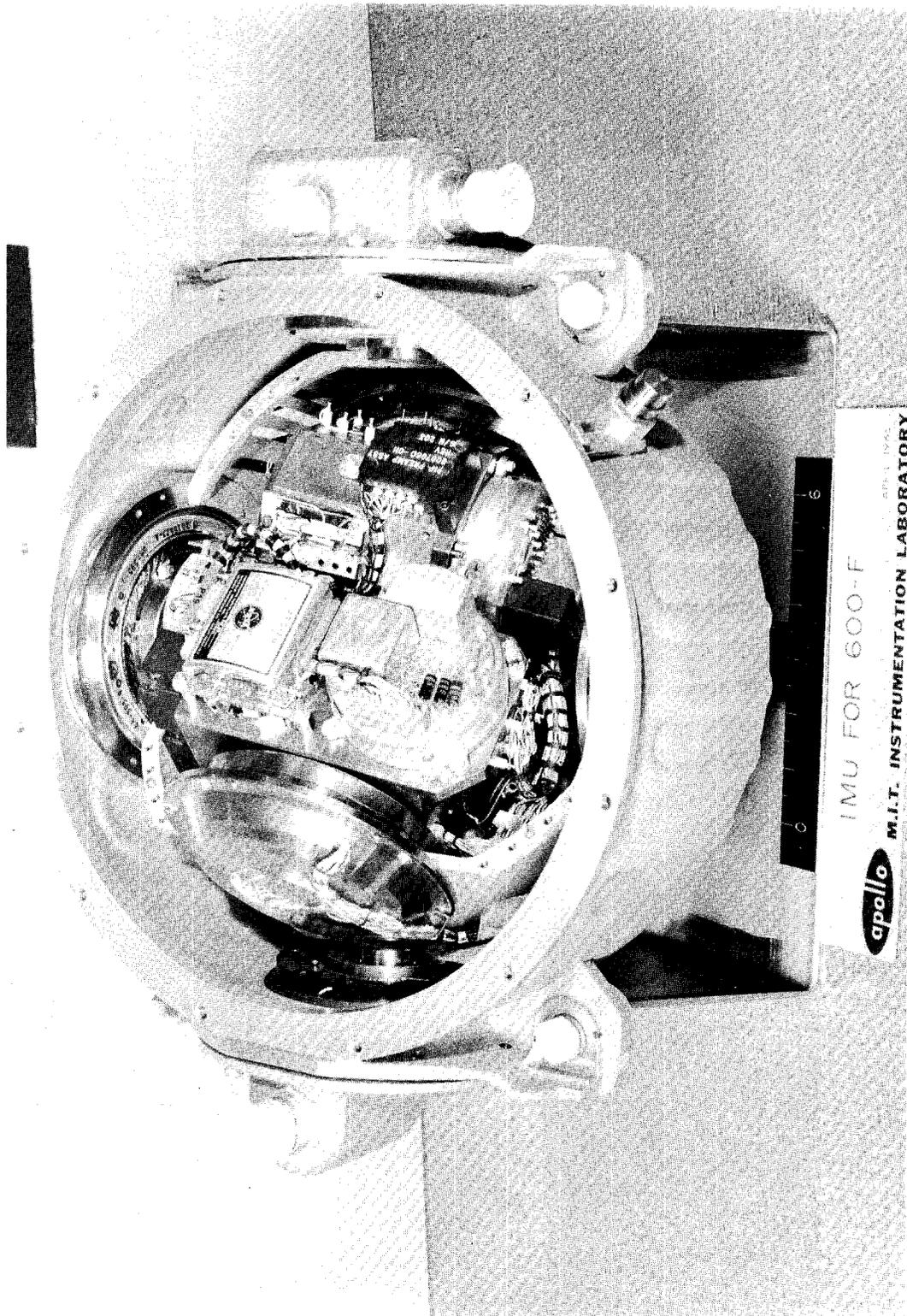
II-31 Schematic of the Inertial Measurement Unit

The gyro stabilized gimballed platform has had many years of success and experienced gained primarily by its use in guidance of military ballistic missiles. It clearly has most superior performance due, in large part, because the gyros and accelerometers are kept non-rotating by the isolation provided by the gimbals and their servos. Finally, the outputs are in a convenient form. Vehicle attitude Euler angles appear directly as the angles of the gimbals. Acceleration measurement appears directly as components in the non-rotating coordinate frame of the stable member "platform".

Alternately, the vehicle frame or body-mounted inertial sensors offer promise of dramatic savings in size, weight, and convenience in mounting. Unlike the gyros on the gimballed system which merely must indicate the small deviations from initial attitude for closed loop gimbal control, the body-mounted gyros must measure precisely the whole angular velocity experienced by the vehicle. Moreover, problems are introduced in achieving good gyro and accelerometer performance because of this large angular velocity the units must tolerate about all axes. Finally the outputs are not always in a direct useful form, Angular orientation of the vehicle is indicated only by properly transforming and integrating the body-fixed coordinates of angular motion indicated by the gyros into either an Euler angle set or a matrix of direction cosines, With either of these, the body-mounted accelerometer signals can be resolved from the rotating spacecraft coordinates into an inertial frame. All these calculations require a computer of considerable speed and accuracy to prevent accumulation of excessive error.

The choice made in Apollo for both the Command Module and LEM spacecrafts was the use of the gimbal stabilized member mounting of the sensors for the primary systems. The superior demonstrated performance provides a conservative margin of safety in economical use of rocket fuel for the major mission completion maneuvers. The secondary backup or abort guidance systems in each spacecraft, however, capitalize upon the size and convenient installation advantages of body-mounted sensors, Here the more modest performance is quite ample for the crew safety abort maneuvers in case of primary guidance system failure.

The Apollo primary guidance gimbal system - or IMU for Inertial Measurement Unit - is shown schematically in Fig. 11-31. This IMU is seen to carry three single-degree-of-freedom gyros which provide necessary error signals to stabilize in space the orientation of the inner member by servo drives on each axis. There are three of these rotational axes of the gimbal system as shown in the figure. A three degree of gimbal system such as this can present problems due to a phenomena called "gimballock". Gimbal lock would occur when the outer axis is carried by spacecraft motion to be parallel to the inner axis. In this position, all three axes of gimbal freedom lie in a plane and no axis is in a direction to absorb instantaneously rotation about an axis perpendicular to this plane. Thus, at gimbal lock the inner stable member can be pulled



IT-22 Inertial Measurement Unit for System 600F  
(LEM Functional)

off of its space alignment. Even though a three-degree-of-freedom gimbal system allows geometrically any relative orientation, the required outer gimbal angular acceleration needed at gimbal lock to maintain stabilization will exceed servo capability.

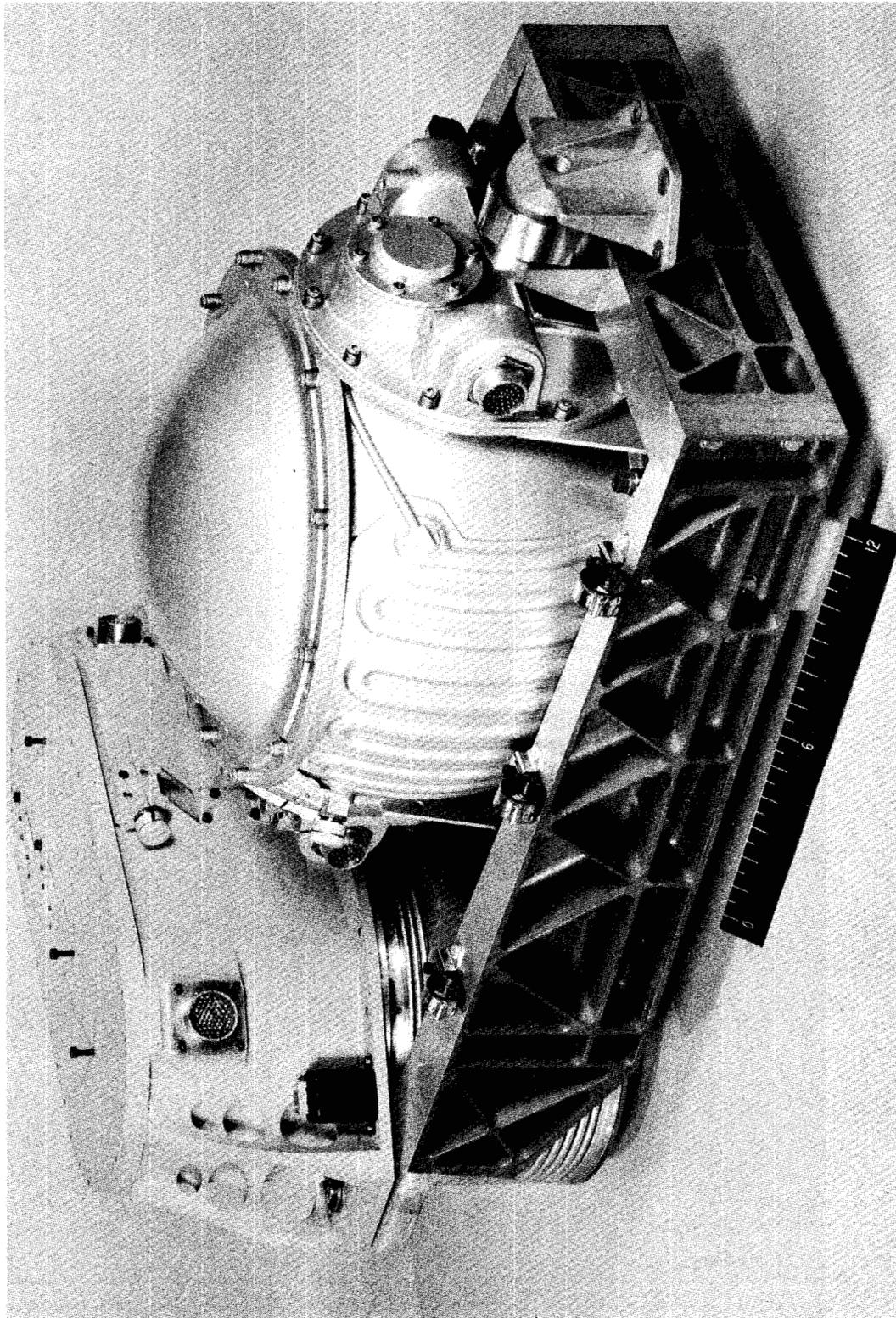
One direct solution to gimbal lock problems is to add a fourth gimbal and axis of freedom which can be driven *so* as to keep the other three axes from getting near a common plane. However, the cost in complexity and weight for a fourth gimbal is considerable. Fortunately, in Apollo the operations with the IMU are such that gimbal lock can be easily avoided, and a simple three-degree-of-freedom gimbal system is entirely satisfactory. This will be made clear in the following paragraph.

The Apollo IMU will normally be turned off during all long coasting periods not requiring its use. This is done primarily to save power and corresponding fuel cell battery reactant. (Reactant savings of the order of 20 kilograms have been estimated.) For this reason, the guidance system provides for inflight inertial system alignment against star references before the start of each accelerated phase of the mission. This allows the inner stable member alignment to be chosen for each use in the most logical orientation. Simplifications can result in the computer generation of steering commands if the "X" accelerometer axis on the stable member is aligned in some direction near parallel to the expected thrust (or entry atmospheric drag). This happens also to be optimum with respect to inertial sensor measurement error effects in velocity measurement. Since the X accelerometer is perpendicular to the inner gimbal axis, the direction of this inner axis can be chosen as required. For each mission phase involving rocket burning or atmospheric drag, the trajectory and the thrust or drag lie fairly close to some fixed plane. The inner gimbal axis is then aligned somewhere nearly perpendicular to this plane. All required large maneuvers result mostly in inner gimbal motion, thus avoiding the difficulty of approaching gimbal lock associated with large middle gimbal angles. Finally, because large roll maneuvers are desirable (for instance during entry for the Command Module) the outer gimbal axis is mounted to the spacecraft along or near the roll axis *so* that no restriction on roll maneuver ever exists.

Because the details of the design and operation of the critical inertial sensors - gyros and accelerometers - to be mounted on the stable member of the gimbal system are of particular importance and interest, this subject is covered separately in Part IV. However, an overall view of the inertial measurement unit is shown in Fig. 11-32. In this photograph the spherical gimbal halves and case cover are removed to show the appearance of the components mounted on the stable member and on the axes of the gimbals.

#### INERTIAL SYSTEM ALIGNMENT

As mentioned above, the inertial measurement system is turned off during the longer free-fall coasting periods to conserve power supply energy. Even were it not for this, unavoidable drift of the inertially derived attitude reference would require

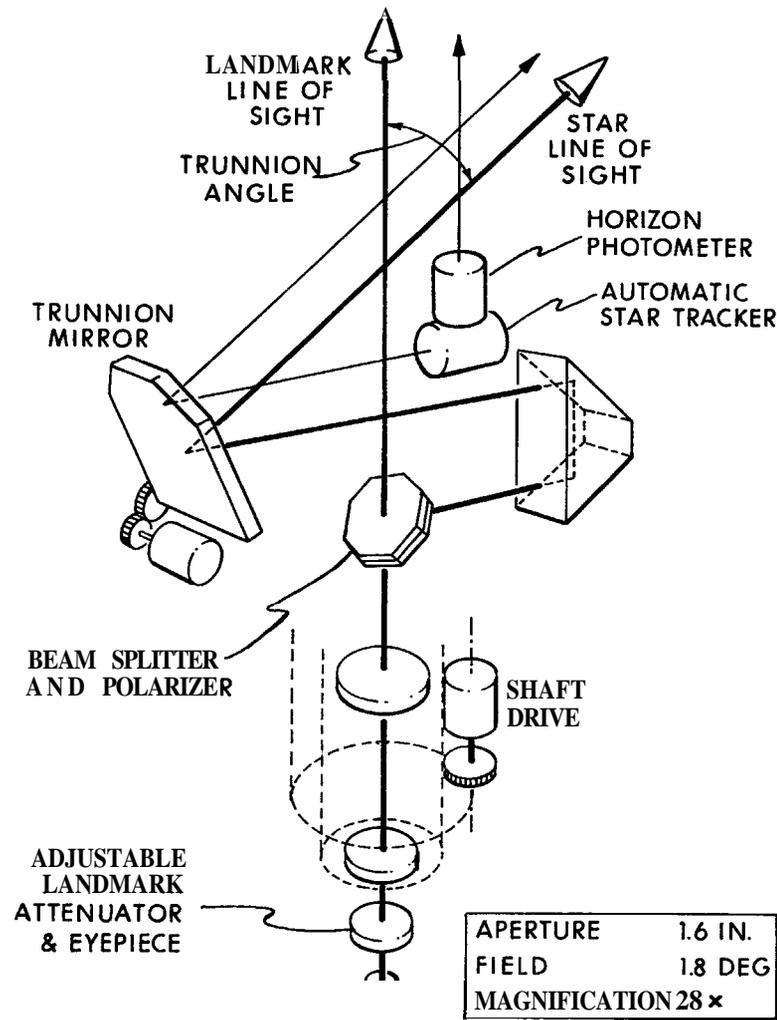


II-33 Optics (left) and IMU (right) on Navigation Base

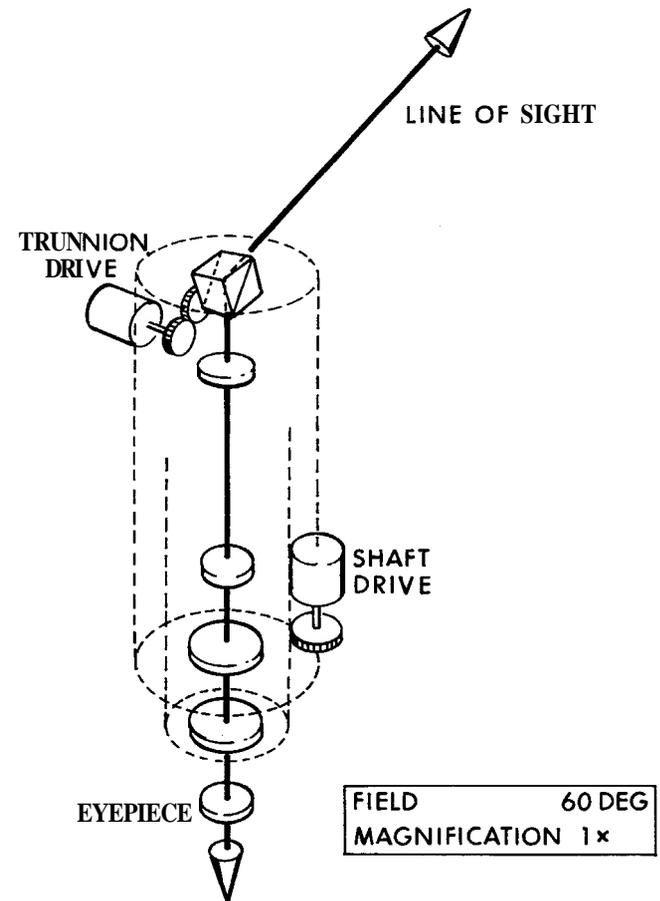
periodic in-flight alignment to the precise orientation required for measuring the large guided maneuvers. The use of identified star directions for the inertial system alignment introduces the question of physically relating the sensed star direction to inertial system stable member orientation. The problem from one point of view could be minimized by mounting the star sensor or sensors directly on the stable member itself. This would impose a most severe limitation of field of view of sky available and puts unpermissible constraints on spacecraft attitudes during the alignment. Even a measured two degree of rotational freedom of the star sensor axis on the stable member limits flexibility and compromises design more than can be tolerated.

The alternative of mounting the star sensor telescope separately near the spacecraft skin where its line of sight can be articulated to cover a large portion of the sky means far more freedom in spacecraft attitude during inertial system alignment. In Apollo a rigid structure called the navigation base which is strain-free mounted to the spacecraft provides a common mounting structure for the star alignment telescope and the base of inertial measurement gimbal system. Figure 11-33 shows this arrangement for the Command Module system. (In this photograph the eyepieces of the optics are not attached.) By means of precision angle transducers on each of the axes of the telescope and on each of the axes of the inertial system gimbals, the indicated angles can be processed in the onboard computer to generate the star direction components in inertial system stable member coordinates. This provides the computer with part of the needed stable member orientation data, except no information is provided for rotation about the star line. The use of a second star, at an angle far enough removed from in line with the first, completes the full three-axis stable member orientation measurement. With this information the stable member orientation can then be changed under computer command, if desired, to the orientation optimum for use of the guidance maneuvers.

It is recognized that the above procedure has many sources of error in achieving inertial system alignment. For example, each axis of rotation of the star telescope and the inertial system gimbals must be accurately orthogonal (or at a known angle) with respect to the adjacent axis on the same structure. This is a problem of precision machining, accurate bearings, and stable structures. Each angle transducer on each axis of the star telescope and the inertial system gimbals must have minimum error in indicated angle. This includes initial zeroing, transducer angle function errors, and digital quantization errors for the computer inputs. By careful attention to minimizing each of these and other error sources, probable Apollo inertial system alignment error of the order of 0.1 milliradian is achieved, an accuracy which exceeds requirements by a comfortable margin.



11-34 Sextant Schematic



11-35 Scanning Telescope Schematic

## OPTICAL MEASUREMENT SYSTEM

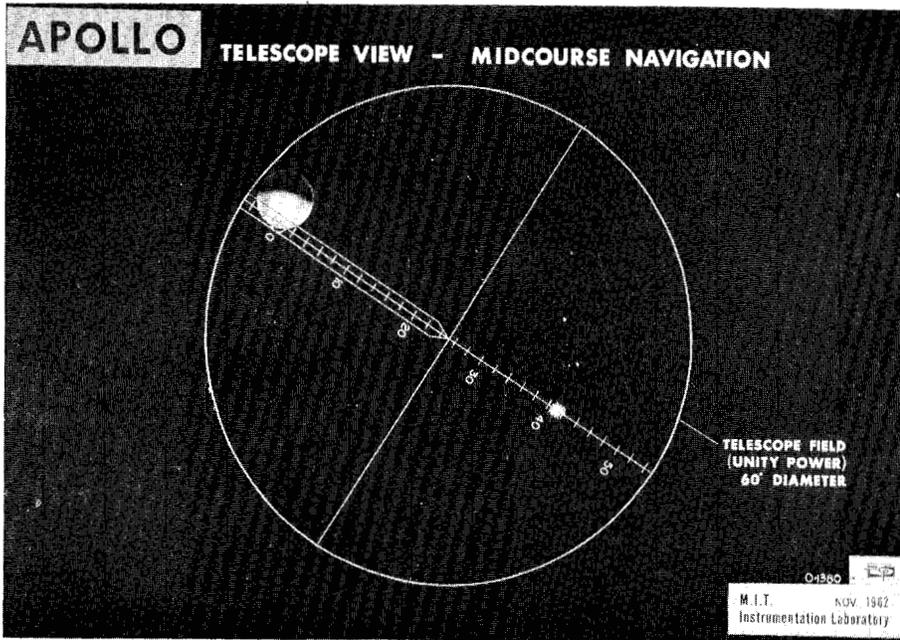
Besides providing for inertial system alignment as described above, the optical system also provides the onboard measurement capability for orbital and midcourse navigation of the command module. The single-line-of-sight direction measurement referenced to the stable member used for inertial system alignment can be well utilized also in low earth or lunar orbit for navigation. However, for onboard navigation during the translunar and transearth phases, accuracy requirements are met only by a two-line-of-sight sextant type of instrument.

Two separate Command Module optical instruments are mounted on the navigation base which also supports the inertial measurement unit. These are the two-line-of-sight sextant and the single-line-of-sight scanning telescope.

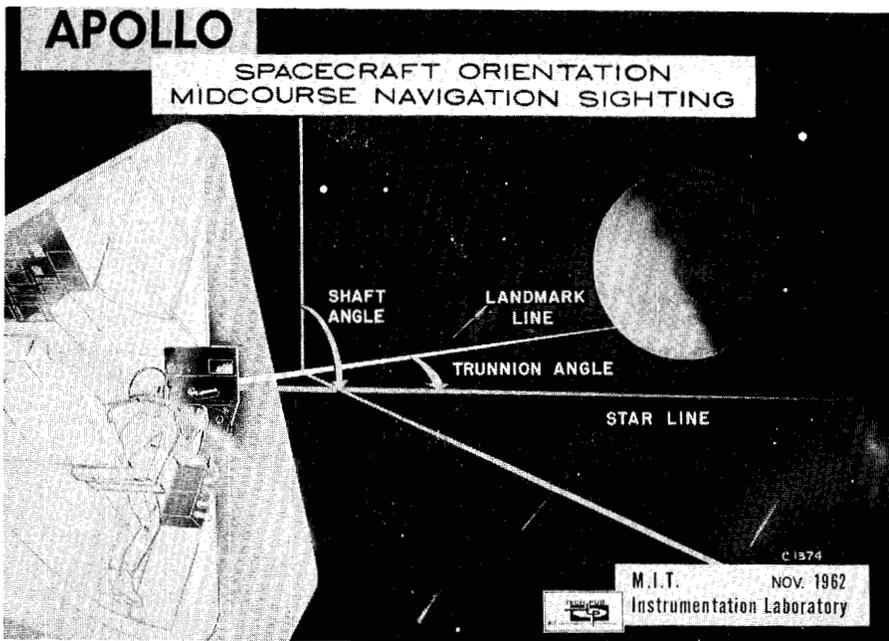
The sextant and its features are illustrated diagrammatically in Fig. II-34. It is essentially a two-line-of-sight instrument providing magnification for manual visual use as well as special sensors for automatic use. It is seen in the figure that one of the lines of sight of the sextant, identified with the landmark side of the navigation angle, is undeflected by the instrument and is thereby fixed to the spacecraft. To aim this line, then, the spacecraft must be turned in space appropriately by means of orientation commands to the attitude control system. The second line identified with the star side of the navigation angle can be pointed in space through the use of two servo motor drives illustrated schematically. One axis of this motion - called the shaft axis - is parallel to the landmark line and changes the plane in which the navigation angle is measured by rotating the head of the instrument as a whole. The second axis - trunnion axis - sets the navigation angle by tilting of the trunnion axis mirror. A precision angle data transducer on this mirror provides a direct measure of the navigation angle for the navigation routine of the computer. An angle transducer on the shaft axis completes the data needed by the computer of the indicated star direction when the instrument is used for inertial system alignment.

The light arriving along the landmark is polarized before being combined in the beam splitter mirror with the light along the star line so that the navigator can adjust the landmark background brightness relative to the star intensity by means of an eye-piece polarizer. The sextant also uses the trunnion mirror in conjunction with a star tracker sensor to provide automatic star tracking error signals to the shaft and trunnion drives.

Mounted with its sensitive axis along the landmark line is a second automatic detector called a horizon photometer. This device senses the brightness of a small portion of the sun illuminated horizon for use as one side of the navigation angle. This is described in more detail in Part V.



11-36 Telescope View - Midcourse Navigation



II-37 Spacecraft Orientation - Midcourse Navigation Sighting

Because the 28 power magnification of the visual section of the sextant results in less than a 2 degree diameter field of view, the second instrument, the scanning telescope, provides a wide field acquisition capability for the sextant to find and acquire objects in the sky. The use of an entirely separate optical instrument rather than a combined variable power instrument using one set of line-of-sight articulation drives is justified by the simpler mechanical and optical configuration and the sighting redundancy two units provide.

The scanning telescope illustrated in Fig. 11-35 has shaft and trunnion pointing of its single line of sight. The shaft angle always is made to follow the sextant shaft angle by servo action when the optics system power is on. The trunnion can be selected by the astronaut to (1) follow the sextant trunnion and hence look along the star line, (2) be driven to zero and hence look along the landmark line, or (3) be driven to a fixed angle of 25 degrees. This latter provides for ease in simultaneous acquisition of landmark and star since the scanning telescope will indicate the image along the landmark line by a reticle point 25 degrees from the center of the field and will indicate possible stars available by trunnion motion in the sextant field of view on a diametrical reticle line. Figure 11-36 shows the view through the scanning telescope during acquisition. Generally, the navigator will preset the trunnion to the expected navigation angle indicated by the computer as the preliminary step in the acquisition process.

Under manual visual control, the shaft and trunnion drives of the telescope are commanded by a left-hand two-axis controller. By this controller, the navigator can point the scanning telescope and the sextant star line. At his right hand, the navigator has spacecraft attitude controllers with which he can rotate the spacecraft to position the landmark line. His midcourse cislunar sighting strategy is to set up the measurement situation illustrated in Fig. II-37. With his right hand controls, he gets the identified landmark within the field of view of the sextant at a slow spacecraft rotation drift. He need then only provide occasional minimum impulses from the appropriate attitude jets to keep the landmark within the field while with his left hand he positions the star image to superimposition on the landmark. When this is achieved, Fig, 11-38, he pushes a "mark" button which signals the computer to record the navigation trunnion angle and time. From these data the computer updates the navigation state vector.

The unity power wide field of view of the scanning telescope is also suitable for navigation direction measurements to landmarks in low earth or moon orbit. The wide field of view makes landmark recognition easy. Landmark direction measurement accuracy of the order of 1 milliradian as referenced to the pre-aligned attitude of inertial system and as limited by the unity magnification is sufficient for landmark ranges under a few hundred kilometers.

In low orbit, the inertial measurement gimbal system must be on and pre-aligned with two star sightings. Then the navigator acquires and tracks landmarks as they

pass beneath him, pushing the mark button when he judges he is best on target, Fig. 11-39. The computer then records optics angles, inertial measurement unit gimbal angles, and time to provide the navigation data.

In all uses of stars and landmarks for navigation the computer must be told by the navigator the identifying code or coordinates of the star and/or landmark. These appear on the navigator's maps and charts to help his memory.

## ONBOARD COMPUTER

The relatively large amount of onboard data processing required for Apollo guidance, navigation, and control can be met only by the capabilities of specially designed digital computer. The special requirements define a computer which would provide for:

1. Logic, memory, word length and speed capability to fit the needs of the problems handled.
2. Real time data processing of several problems simultaneously on a priority basis.
3. Efficient and yet easily understood communication with the astronauts for display of operations and data as well as manual input provisions for instructions and data.
4. Capability of ground control through radio links as well as telemetering of onboard operations and data to the ground.
5. Multiple signal interfaces of both a discrete and continuously variable nature.

The design features of this computer are covered in detail in Part VI. But perhaps the many input and output signals should be discussed briefly here because of the large part these interfaces make in determining the system configuration and in understanding system tasks and operation. Rather than a listing of interfaces the important ones will be discussed by groups in the following paragraphs.

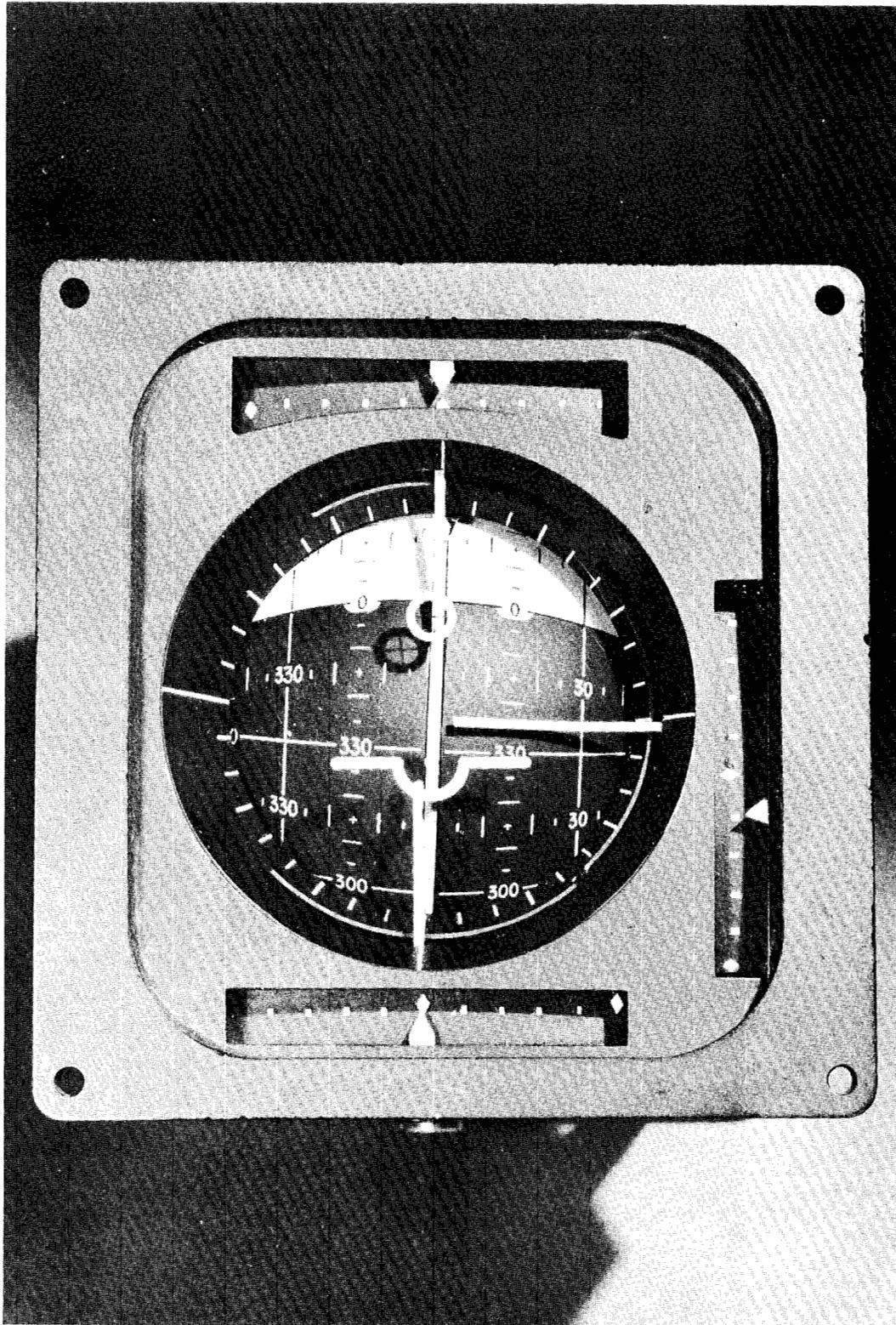
Inputs to the computer of a discrete or two-state nature are handled as contact closures or voltage signals. These offer no difficulty except for the computer activity needed to keep apprised of them. Important urgent signals of this nature - such as an abort command or the time critical "mark" signals go to special circuits which interrupt computer activity to be processed before other activity is resumed or modified. Less critical signals indicating states of the various equipment or requiring less urgent action are examined by the program periodically as necessary.



11-38 Sextant View - Midcourse Navigation



11-39 Telescope View - Orbital Navigation



II-40 Ball Attitude Indicator

Discrete signal outputs are of two types. Time critical ones such as that which signals engine thrust cutoff consist of high frequency pulse trains which are gated on at the time of the programmed event and detected remotely where the action is requested. Slower discrete outputs are either gated d-c voltages or relay contact closures set by a state matrix which drives the appropriate relay coils. The majority of these relays are used to set the states of the electroluminescent number display readout of the computer display and keyboard. Others change operating modes of the associated spacecraft systems or are used to light status or warning lights.

Direct earth communications to and from the computer requires circuits associated with the interface with the radio receiver and transmitter to convert between the serial code of the telemetry and the parallel format of the computer.

Other variables into the computer are handled by input counters which sum pulses transmitted as the indicated variable changes through fixed increments. Velocity increments, for instance, measured by the inertial system accelerometers are handled in this way.

Some variable outputs, such as the command torquing of the inertial system gyros to change alignment, appear as output increment pulses on appropriate lines.

Perhaps the most difficult class of computer interfaces is handled by the use of auxiliary pieces of equipment called Coupling Data Units - or CDUs for short. The CDUs provide the means for coupling with the digital computer the sine and cosine analog signals from the resolver type of angle transducers used on the optics and inertial gimbal system axes. There are five of these CDUs, one each associated with optics shaft, optics trunnion, and the three axes of the inertial unit gimbal system. The details of the operation and construction of the CDUs are described in Part IV.

## DISPLAYS AND CONTROLS

The preceding sections have introduced the needs and characteristics of three Apollo guidance, navigation, and control subsystems: (1) the inertial measuring equipment, (2) the optical measurement equipment, and (3) the digital computer data processing equipment. Because Apollo is by its very purpose a manned mission, the provision for system operation by the crew is essential. This identifies the need for the fourth subsystem: the displays and controls.

The provisions to involve the astronaut might appear as an unnecessary complication. Indeed many tasks are best left to the machine: those that are too tedious or require too much energy, speed of response, or accuracy outside man's capabilities. But the utilization of man in many of the tasks of guidance, navigation, and control more than pays for the display and control hardware needed. His involvement without any doubt enhances mission success significantly. Consider man's judgement and

adaptability, his decision-making capability in the face of the unanticipated, and his unique ability to recognize and evaluate patterns. Of this latter, consider his unsurpassed faculty to pick out a particular navigation star from the heavens or to evaluate a suitable touchdown spot on the moon.

Displays and controls were designed in Apollo to provide the crew with visibility into and command over the guidance, navigation, and control tasks. In most of these tasks, then, the astronaut can select either to be intimately involved in the procedures or allow full automatic operation which he will be able to monitor at his discretion.

In the command module, the navigator has displays and controls illustrated in Fig. 11-41. The eyepieces of the sextant and scanning telescope appear prominently beside each other. Just below these eyepieces is a control panel used primarily for operating the optics. The left-hand optics hand controller and the right-hand spacecraft attitude minimum impulse controller appear at the top of this panel. At the bottom of this panel are operating mode selector switches. The inertial system mode controls and displays appear to the left and above the eyepieces. Directly to the left are the five coupling data units with a display of the associated variable of each. To the right is the numerical readout and keyboard associated with the computer. Features of this are described with the computer in Part VI.

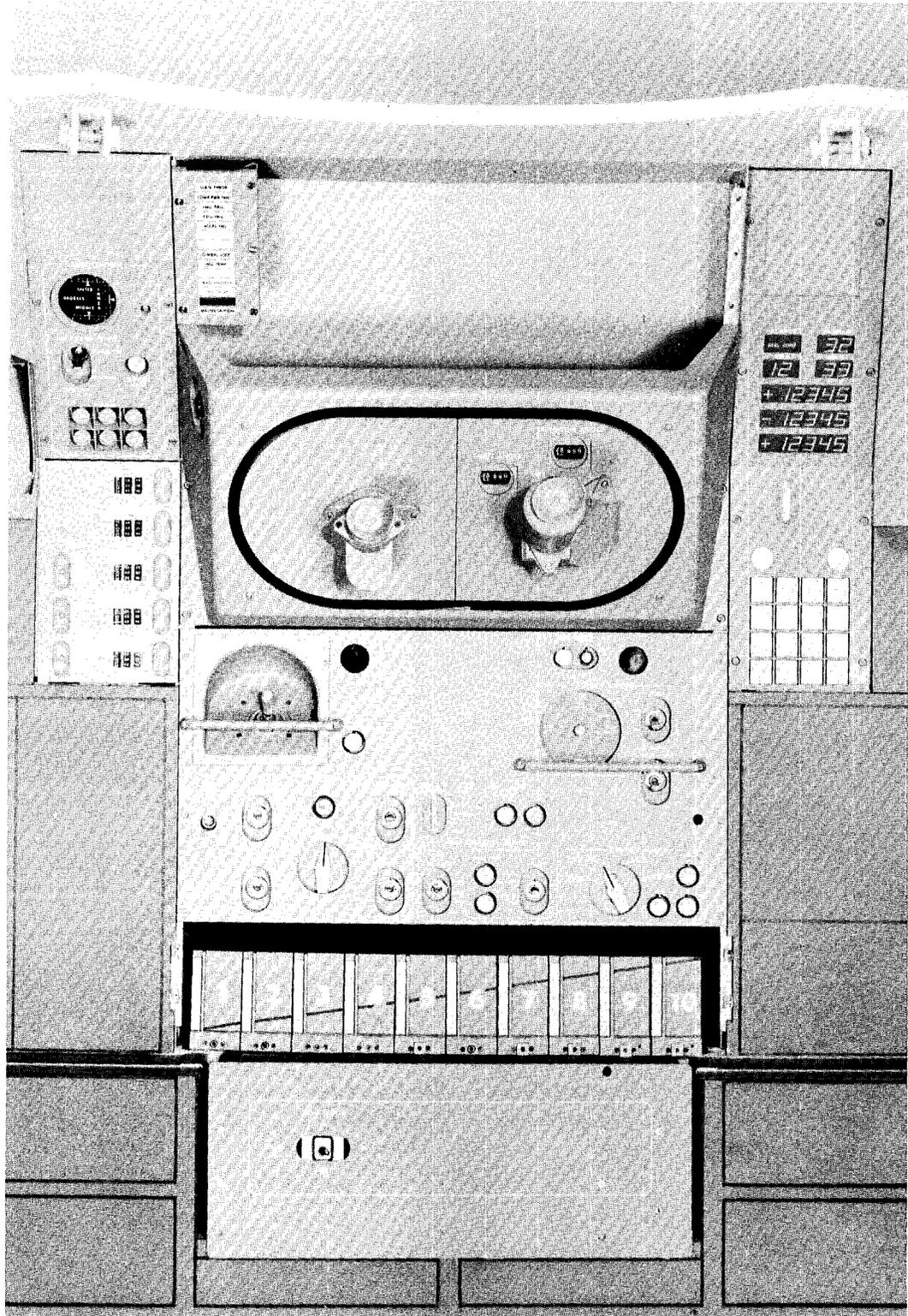
Tabulations of data, lists of procedures, and maps and charts of landmarks and stars will be provided in a bound book or could alternately be projected on a microfilm projector. Space for this projector appears near the top of Fig. 11-41. Controls to operate the film drives and projection lamp are seen on the center panel.

The numbered modules below the optics control panel contain the miscellaneous analog electronics that operate the equipment. And below this is the digital computer.

Separated from the displays and controls described above and on the main panel in front of the pilot's couch certain important guidance data are displayed. A second display and keyboard of the digital computer is mounted here. This unit is functionally in parallel with the one used by the navigator so that the majority of guidance and navigation functions can be operated and observed from either station.

Also visible to the pilot is a ball attitude indicator and associated needles, Fig. 11-40. The spacecraft orientation is indicated to the pilot by the attitude of the ball which is driven in three axes by the three axes of the inertial measurement unit gimbals. Also, the three components of attitude error generated by the guidance system are displayed by the position of three pointers which cross the face of the instrument. Vehicle attitude rates measured by three vehicle mounted rate gyros are displayed by three more pointers around the sides of the instrument.

Other displays showing guidance, navigation, and control system status also are available for the pilot on the main panel along with a complex array of equipment associated with other systems for controlmode selection, and display.



11-41 Displays & Controls - Command Module Lower Equipment Bay

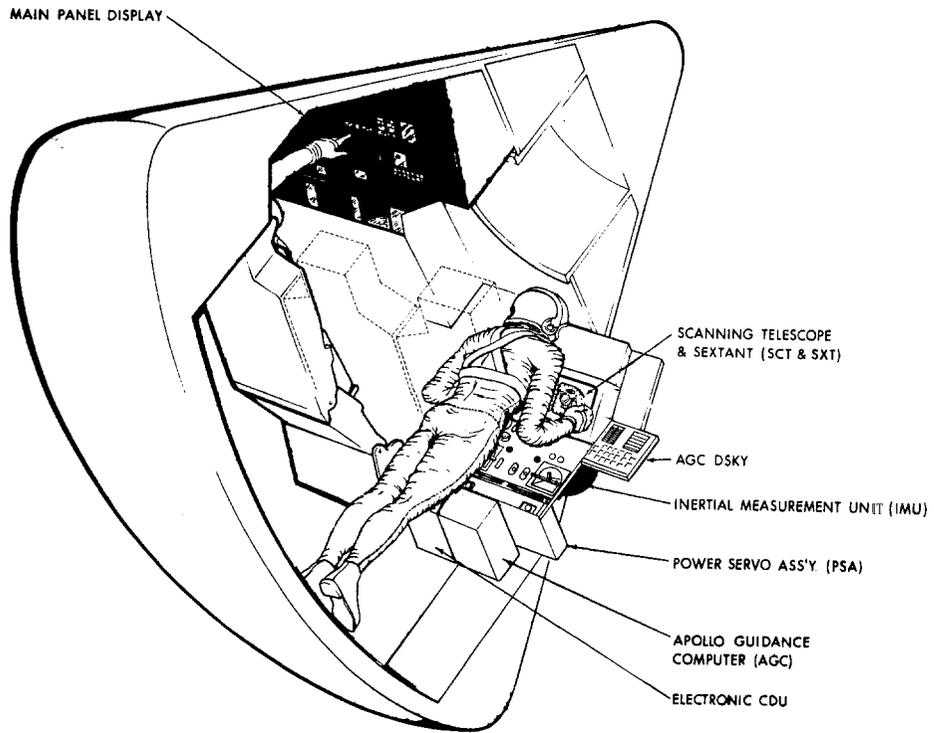
## EQUIPMENT INSTALLATION IN SPACECRAFT

The installation of the guidance and navigation equipment in the command module is illustrated in the cutaway view, Fig. II-42. This shows the navigator operating the displays and controls at the lower equipment bay where the majority of the guidance and navigation equipment is located. Other control equipment is distributed around the spacecraft. During launch boost into earth orbit and during return entry when the acceleration forces are high, the navigator must leave his station as shown and lie in the protective couch in the center between his companions. Sufficient controls and displays as described above are on the main panel in front of the couches to perform all the guidance and navigation functions except for those requiring visual use of the optics. Thus, for the limited period near the earth when the navigator cannot be stationed in front of the eyepieces at the lower equipment bay, use of optics is through the automatic features only.

The installation in the LEM is shown in Fig. 11-43. The inertial measurement unit (IMU), the LEM guidance computer (LGC), the coupling data units (CDUs), and support electronics of the power servo assembly (PSA), are all identical to those used in the command module. Since the LEM activity, when separated from the command module, does not require optical navigation sightings, a simpler optical alignment telescope is installed on a navigation base with the inertial measurement unit and is used only for aligning the stable member of the latter. Also unique to the LEM are the two radars. The rendezvous radar is mounted near the inertial unit so that direction data can be related between the two. The landing radar is on the descent stage, not shown, and is, therefore, discarded on the lunar surface after it has served its function during landing.

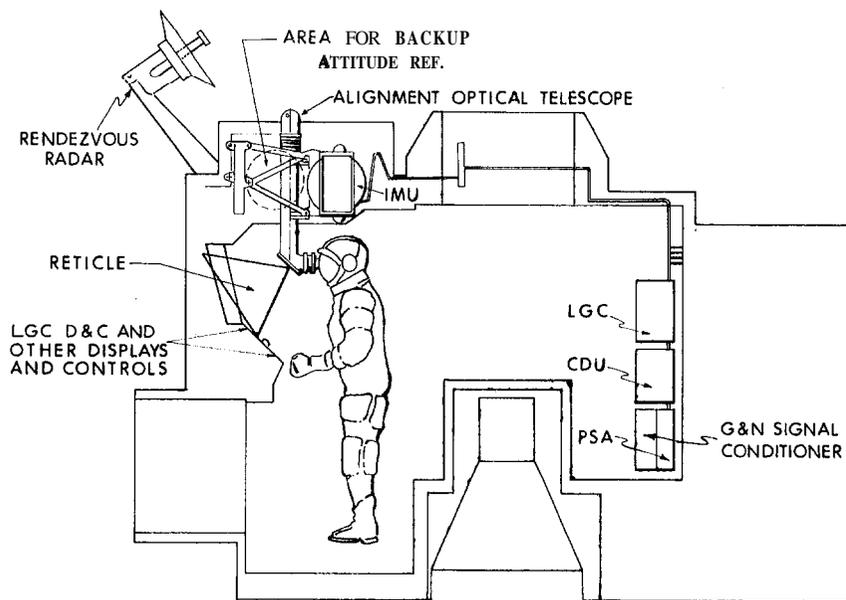
## OVERALL BLOCK DIAGRAMS

The signal interconnections among the various equipments which constitute or have some part in the guidance, navigation, and control are illustrated in Fig. II-44 and Fig. 11-45 for the command module and LEM systems, respectively. The equipment and signals shown on these figures can, for the most part, be related to material already discussed. A detailed explanation is not given here since the intent is only to show the general nature of the equipment interfaces, the similarity and differences between the command module and LEM systems, and the central role of the guidance computer in each case.



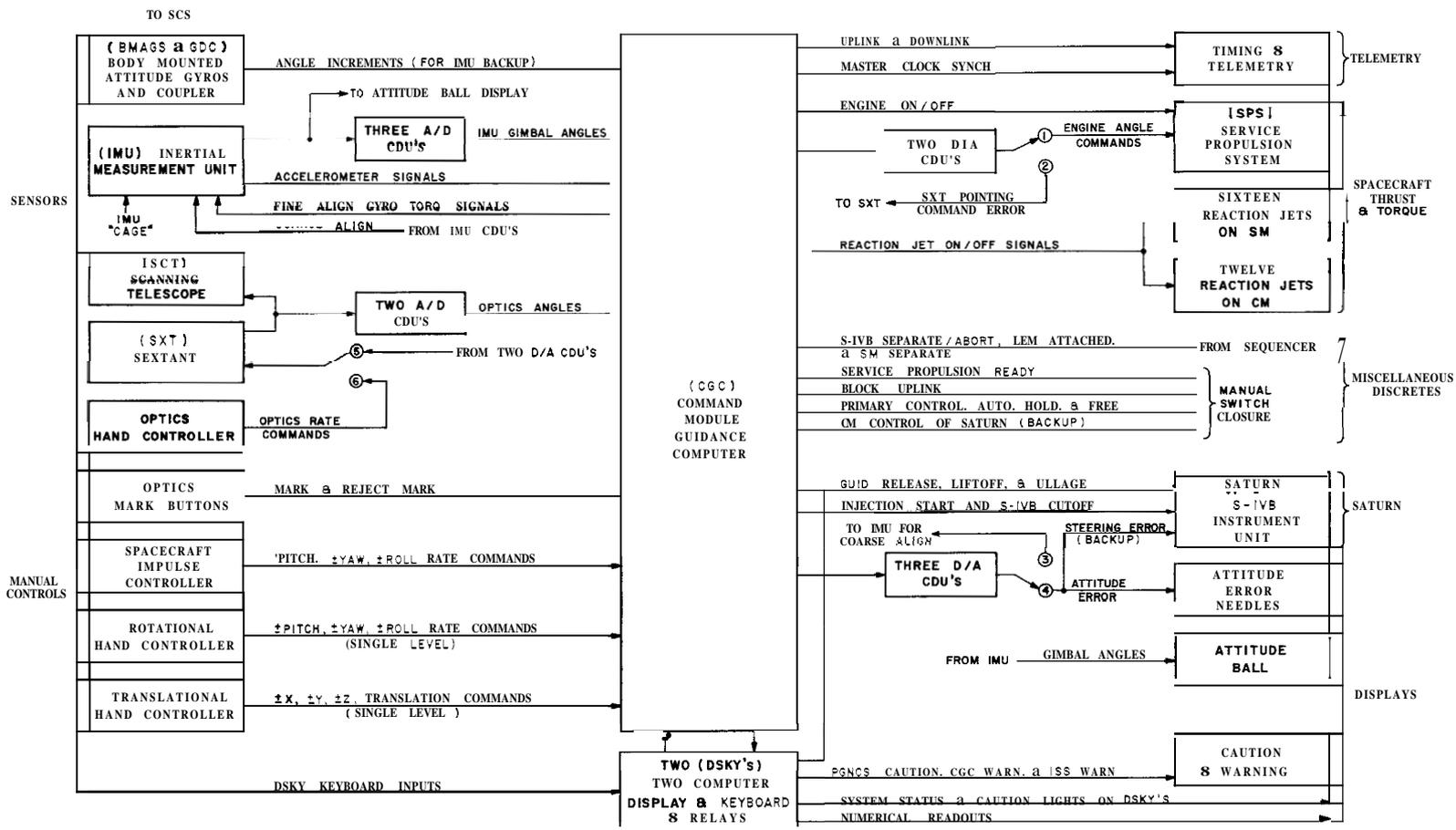
11-42 Location of the Guidance and Navigation System in the Command Module

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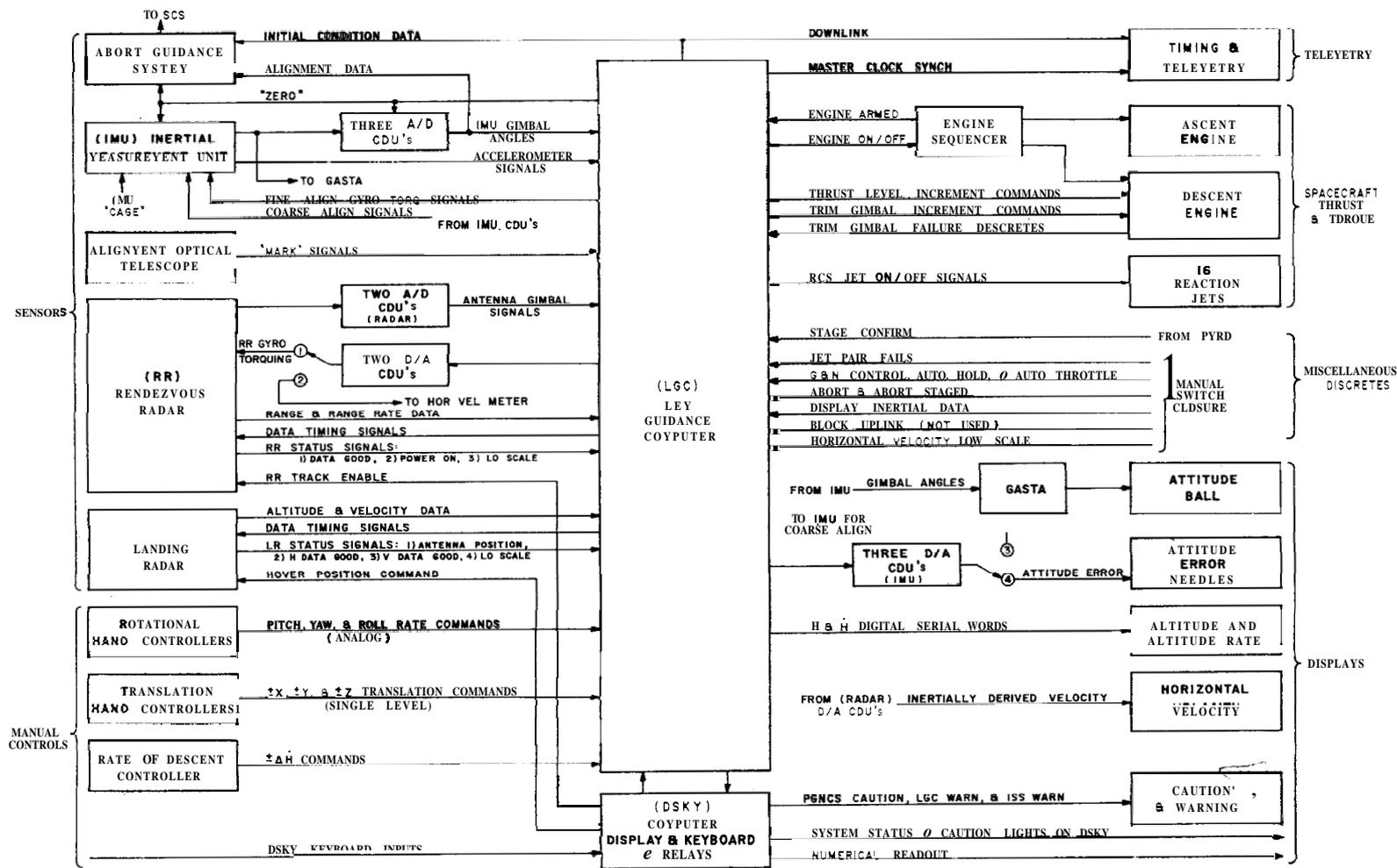


11-43 Location of the Guidance and Navigation System in the Lunar Excursion Module

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11-44 Guidance, Navigation, & Control Interconnections in Command Module



11-45 Guidance, Navigation, & Control Interconnections in LEM

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## CHAPTER II-4

### OPERATION MODES OF GUIDANCE, NAVIGATION, AND CONTROL APOLLO COMMAND MODULE BLOCK I

The system that has been described so far can be seen to have a high degree of flexibility in performing the many tasks of concern. In this chapter a series of diagrams are used to show briefly the equipment involved and the information flow in operations with these tasks.

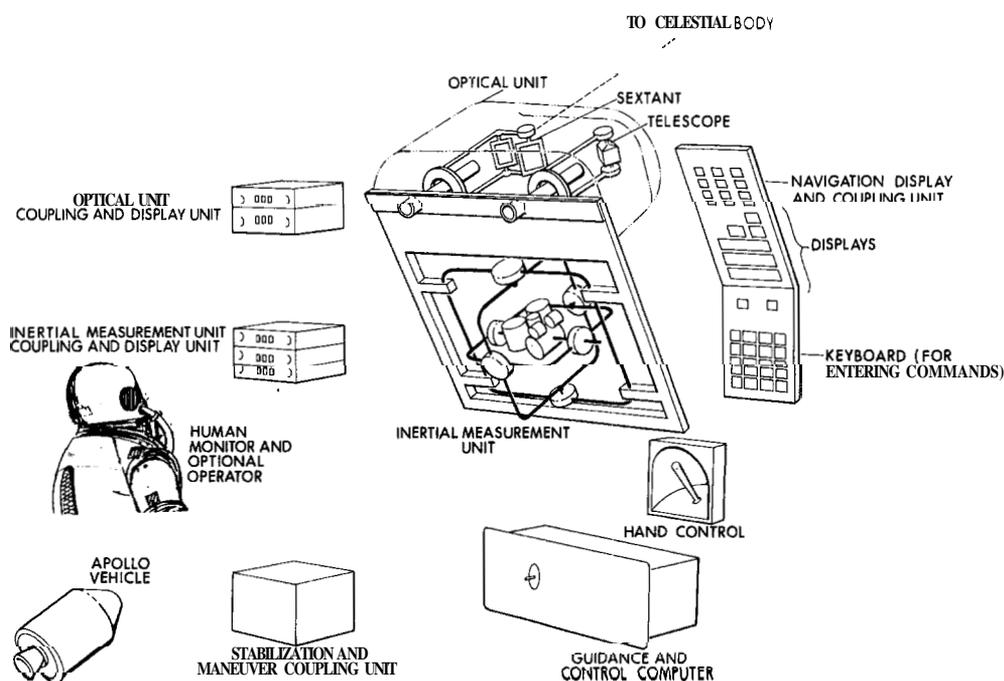


Fig. 11-47 Equipment Arrangement in the Command Module Lower Equipment Bay

The figure above shows the installed arrangement of the equipment in the Block I command module. In the following figures this equipment is shown separated in order to trace signal paths more easily. The Block I equipment configuration lends itself better to the tasks of this chapter than the later more integrated Block II equipment which will finally perform the lunar landing.

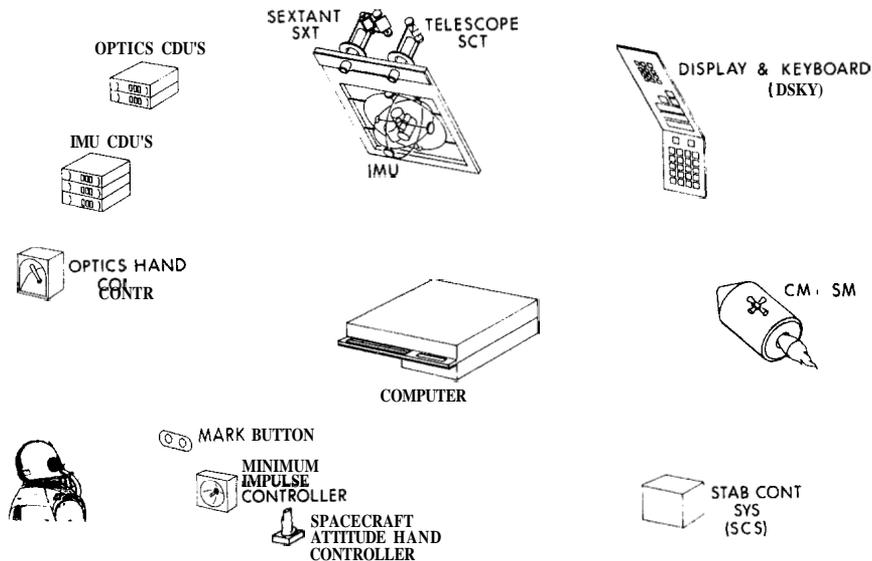


Fig. 11-48 Subsystem Identification

This is the key figure of the series. Here the principal subsystems of the Block I command module guidance and navigation are arrayed and identified for use in the subsequent figures. The sensors of the system are shown in the top center: the two optical instruments, sextant and scanning telescope, and the inertial measurement unit (IMU) all mounted on the common rigid navigation base. At top left are the two sets of coupling data units (CDUs) to provide the communication of the optics and IMU angles with the computer shown at the center. The computer display and keyboard (DSKY) is at upper right. The whole vehicle - command and service modules - is represented by the figure center right. The separate stabilization and control system of the Block I system is bottom right. The astronaut navigator is shown bottom left surrounded by several of the important controls.

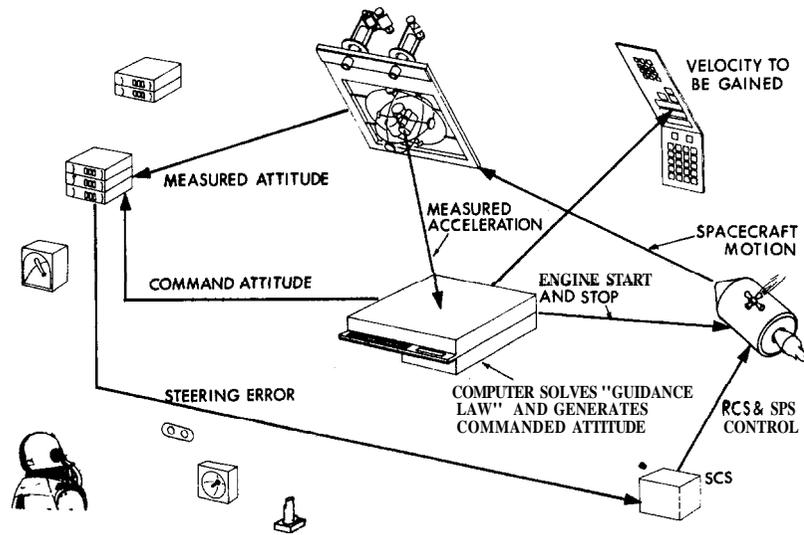


Fig. 11-49 Guidance Steering Control

This first mode is that of powered flight guidance. The signals from the accelerometers on the IMU are processed within the computer where the steering equations develop a desired thrusting attitude of the vehicle to achieve the desired direction of acceleration. This is treated as a commanded attitude which is compared in the CDUs with actual attitude measured by the IMU. The difference is a steering error which is sent to the SCS to control the vehicle. Resulting vehicle motion is sensed by the IMU to complete the feedback. When the required velocity change is achieved, the computer sends a rocket engine shutdown signal. The crew can monitor the whole operation by the display of appropriate variables on the DSKY such as the components of velocity yet to be gained.

Before the IMU can be used for an operation such as this, it must be aligned to the desired spacial orientation. This process is described next.

IMU alignment is normally performed in two stages: "coarse" and "fine". Coarse alignment is described here in two steps using the figures on the opposite page.

The first step of coarse alignment is to give the computer a reasonably accurate knowledge of spacecraft attitude with respect to the celestial framework being used. Illustrated here, the navigator sights sequentially two stars using the scanning telescope (SCT). The star image is sensed by the navigator who uses his left hand optics controller to command the SCT prism such as to center the star on the reticle. He pushes the mark button when he achieves satisfactory tracking which signals the computer to read the SCT angles being transmitted it by the optics CDUs. A second star direction at a reasonably large angle from the first is similarly measured. The navigator identifies which stars are being used to the computer through the keyboard of the DSKY. With these data the computer determines in three dimensions the spacecraft attitude which is held reasonably fixed by a gyro control attitude hold of the SCS during all of these coarse alignment operations.

In step two the computer determines desired IMU gimbal angles based upon its knowledge of spacecraft attitude and the guidance maneuver which will be next performed. These desired angle signals sent to the IMU through the CDU are quickly matched by the IMU gimbal servos in response to error signals developed on the angle transducers on each gimbal axis. At this point the IMU gimbal servos are then switched over to the gyro stabilization error signals to hold the achieved orientation.

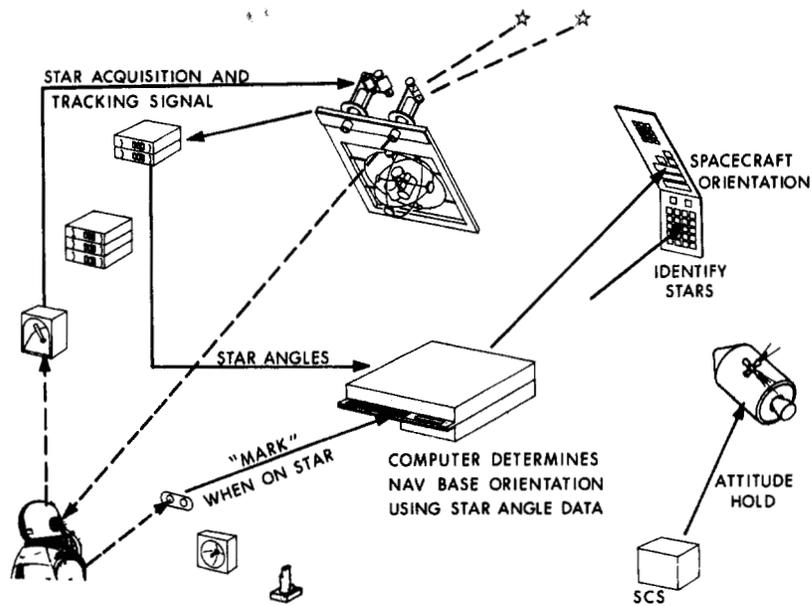


Fig. 11-50 IMU Coarse Alignment Step 1

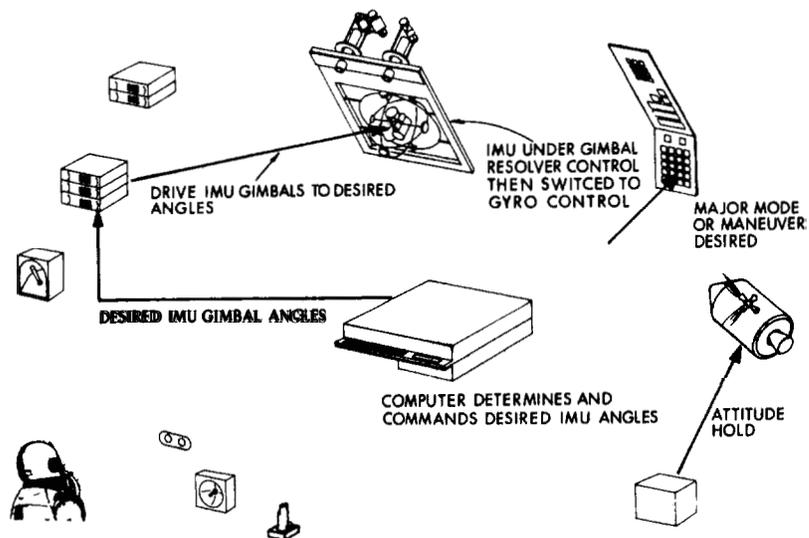


Fig. 11-51 IMU Coarse Alignment Step 2

Fine IMU alignment will also be described in two steps using the figures on the opposite page.

In the first step two star directions are again measured by the navigator. This time he uses the high magnification of the sextant (SXT) with the precision readout on the star line in order to achieve necessary accuracy. The IMU is presumed to be under gyro stabilization control and to be reasonably close to the desired orientation. On each of two stars, which the navigator identifies to the computer, the navigator signals "mark" when he achieves precise alignment on the SXT crosshair. On these signals the computer simultaneously reads the SXT and IMU angles being transmitted through the CDUs. With these data the computer determines star directions in IMU stable member coordinates from which the spacial orientation of the IMU being held by gyro control can be computed. The spacecraft attitude need not be held fixed during these fine alignment operations as long as the angular velocity is small enough to permit accurate star tracking by the navigator.

In step two the computer determines the existing IMU attitude error based upon the desired attitude as determined from the next use of the IMU such as for a particular guided maneuver. The computer then meters out the necessary number of gyro torquing pulses necessary to precess the gyros and the IMU to correct the IMU alignment error.

The above two steps can be repeated if desired to obtain more precision in the fine alignment when the torquing precession angle is large.

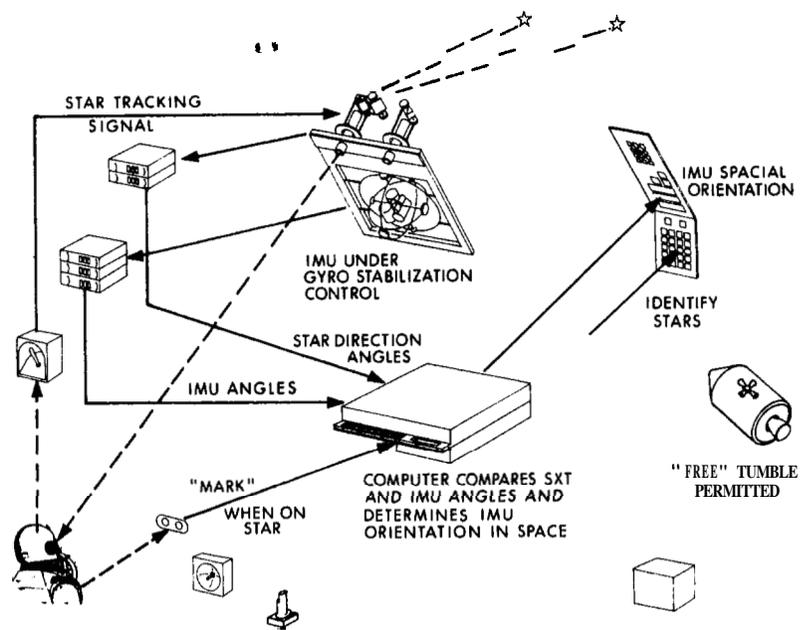


Fig. 11-52 Manual IMU Fine Alignment Step 1

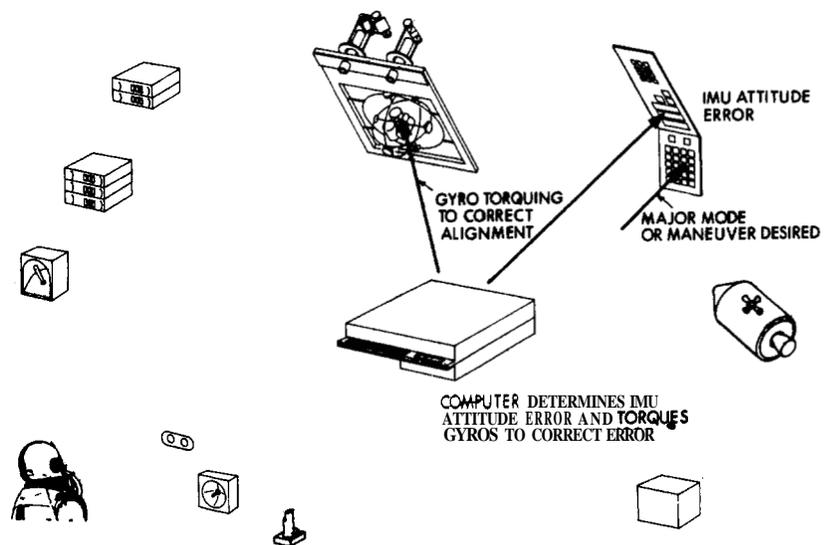


Fig. 11-53 Manual IMU Fine Alignment Step 2

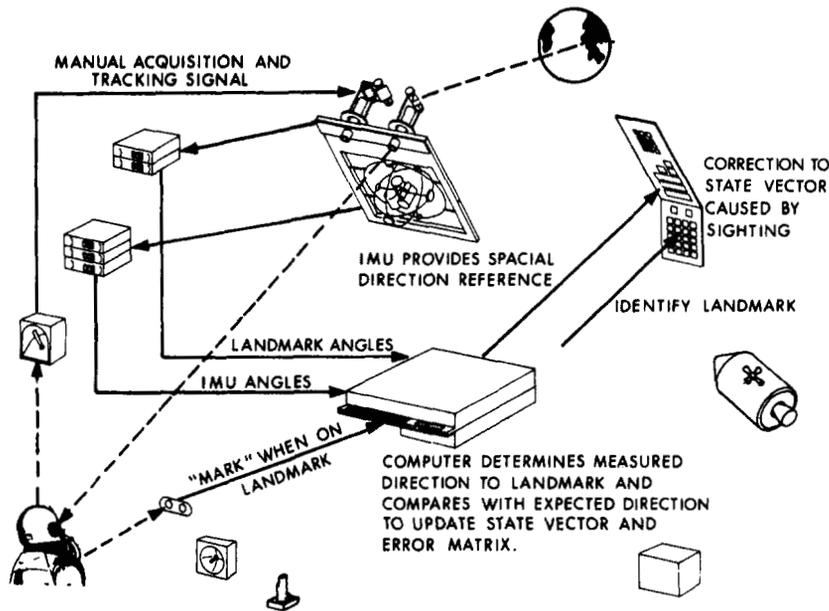


Fig. 11-54 Low Orbit Navigation - Landmark Tracking

Onboard navigation measurements in low orbit can be performed either using landmark references as shown on the above figure or using other references as described later. In the above figure, the navigator first aligns the IMU as previously described and then tracks identified landmarks as they pass beneath him using the SCT. When he is on target he signals "mark" and the computer records IMU and SCT angles and time so as to compute landmark direction in the coordinate frame of the aligned IMU. These direction measurements are then used to update the computer's estimate of position and velocity and the computer's estimate of error in these parameters. These data can be displayed to the astronauts if desired,

Although the above assumes identified landmarks of known coordinates, unidentified landmark features can be used as described in Part V.

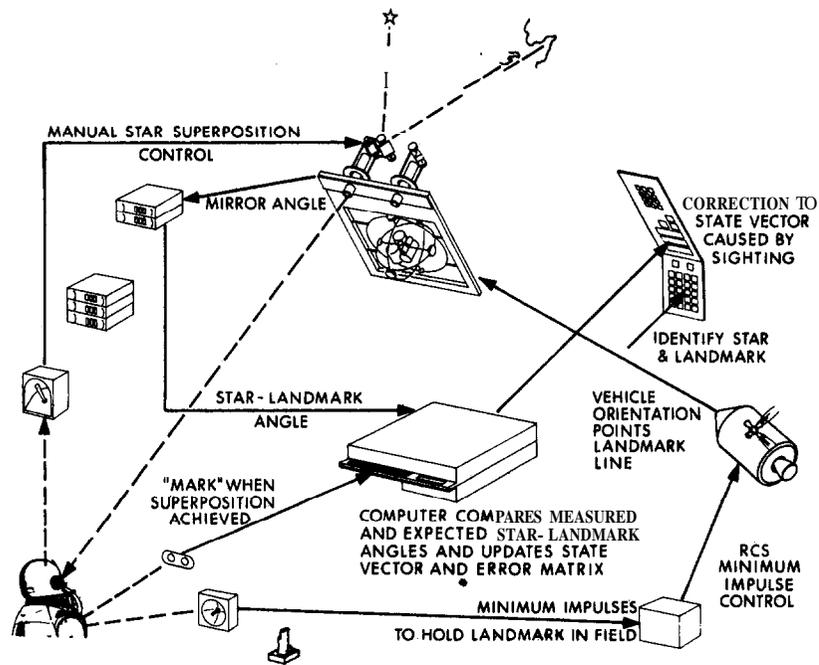


Fig. 11-55 Midcourse Navigation - Manual Star-Landmark Measurement

The use of the sextant to measure the angle between identified stars and landmarks for midcourse navigation is described with the above figure. The acquisition process using the scanning telescope is assumed already to have been performed so that the desired star and landmark images appear in the SXT field of view. With his right hand the navigator periodically commands jet impulses to hold the landmark in the field of view by controlling spacecraft attitude and the body-fixed landmark line. With his left hand he controls the sextant mirror to superimpose the star image onto the landmark. When this superposition is satisfactory he signals "mark" and the computer records the measured navigation angle and time. These numbers are then further processed in the computer navigation routines. The computer displays the correction to the state vector which would be caused by this sighting so that the navigator is given a basis to reject a faulty measurement before it is incorporated into the navigation.

The use for navigation of the automatic star tracker (AST) and photometer (PHO) is shown in two steps with the figures on the opposite page.

In the first step the navigator uses the scanning telescope (SCT) to acquire the navigation star with the automatic star tracker on the sextant. Acquisition is confirmed by a "star present" light signalled from the star tracker.

In step two the navigator maneuvers spacecraft attitude manually to point the body-fixed horizon photometer line to the illuminated horizon by observing the geometry through the SCT. The SCT has a reticle pattern which permits the navigator to judge when the photometer is looking in the plane containing the star and the center of the planet. This puts the photometer sensitive area directly beneath the star. His task is then to sweep the photometer line in this plane through the horizon. When the sensed brightness drops to half the peak value, the photometer automatically sends a "mark" to the computer so that the resulting navigation angle and time can be recorded.

This operation can be performed using the sun illuminated limb of either the moon or earth. Operation with the earth depends upon the systematic brightness of the atmospheric scattered light with altitude described in Part V.

The navigation measurement process described above uses astronaut control in positioning the photometer line. If the IMU is on and aligned, this process could be completely automatic through computer control program.

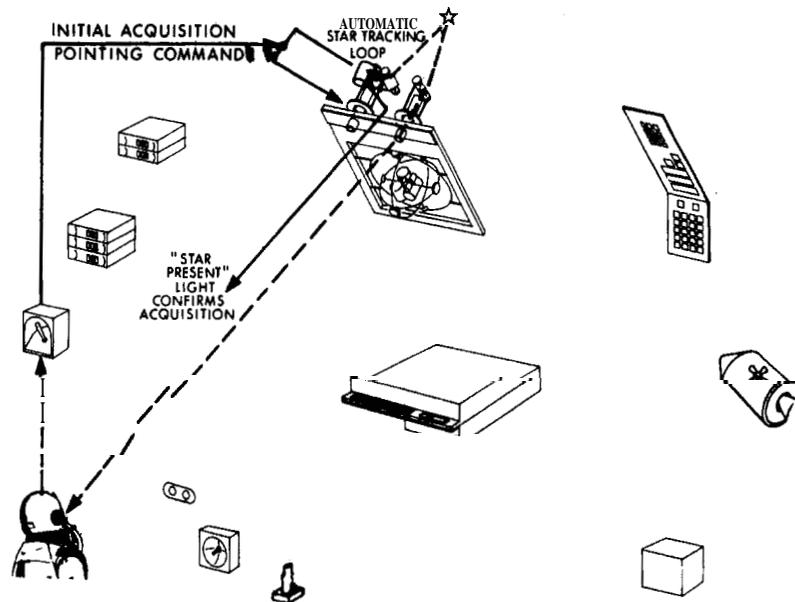


Fig. 11-56 Illuminated Horizon Manual Navigation Measurement - Step 1

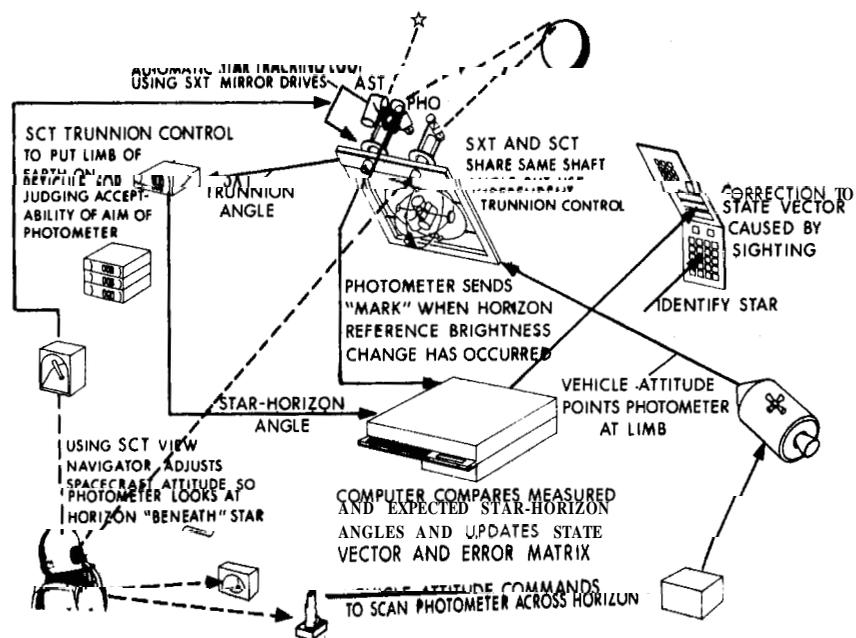


Fig. 11-57 Illuminated Horizon Manual Navigation Measurement - Step 2

The automatic star tracker on the sextant provides the capability of automatic IMU alignment as illustrated in two steps in the figures on the opposite page. Without astronaut help, however, the star tracker cannot acquire a known alignment star unless the IMU is already roughly aligned to provide a coarse direction reference. The automatic IMU alignment capability described here, then, is most useful to re-correct the IMU drift after a long period of IMU operation,

In the first step the computer points the sextant to the expected star direction through the optics CDUs based upon the vehicle attitude measured by the IMU. Presumably the star tracker now senses the desired star within its acquisition field of view and signals the computer that the star is detected.

The computer now changes equipment mode, step two, to send the star tracker error signals to the sextant drives so as to track the star automatically. The computer then reads simultaneously the sextant and IMU angles in order to determine two components of the actual IMU misalignment. This latter is corrected by computer torquing signals to the IMU gyros as described previously. Acquisition and tracking of a second star complete the automatic fine alignment in three degrees of freedom.

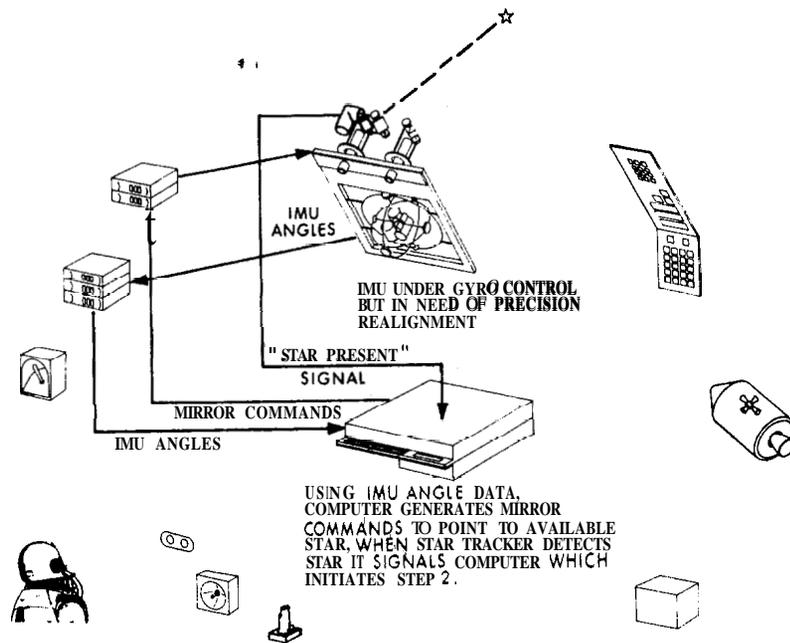


Fig. 11-58 Automatic IMU Fine Alignment - Step 1

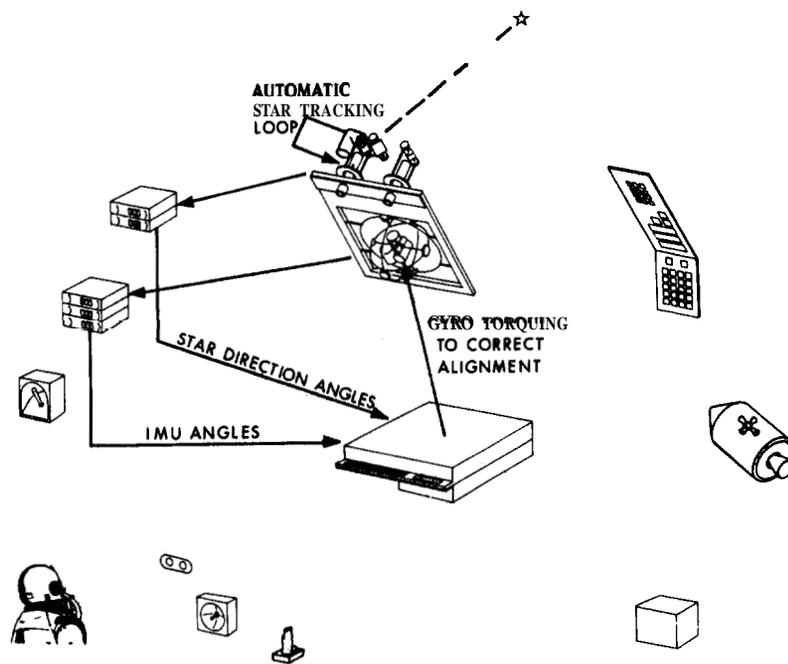


Fig. 11-59 Automatic IMU Fine Alignment - Step 2

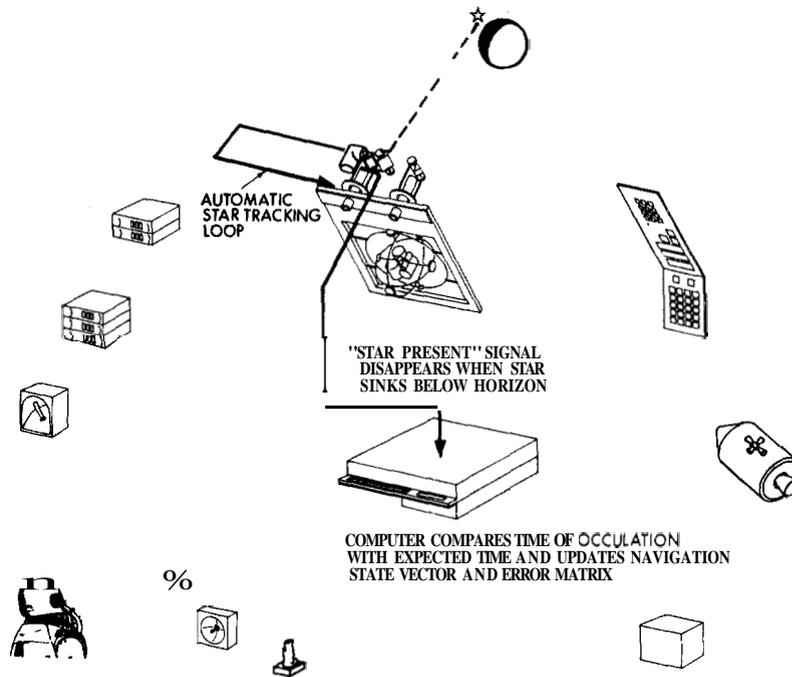


Fig. 11-60 Star Occultation by Moon - Automatic Navigation Measurement

The automatic star tracker provides the means for making automatic star occultation navigation measurements with the moon, as shown above. An acquisition by the star tracker as shown in Fig. 11-56 or Fig. 11-58 is required, of course, as an initial step. While the star is being tracked, the instrument generates a "star present" signal for the computer which is based upon the detected star light energy. As the star sinks below the lunar horizon due to the orbital motion of the spacecraft, the star present signal disappears at the moment of occultation. The time of this event is measured by the computer as a point of the navigation data.

A similar process is possible using the earth's limb, but this requires a more elaborate star present detection. The star intensity diminishes gradually due to dispersion and scattering as the beam sinks into the earth's atmosphere.

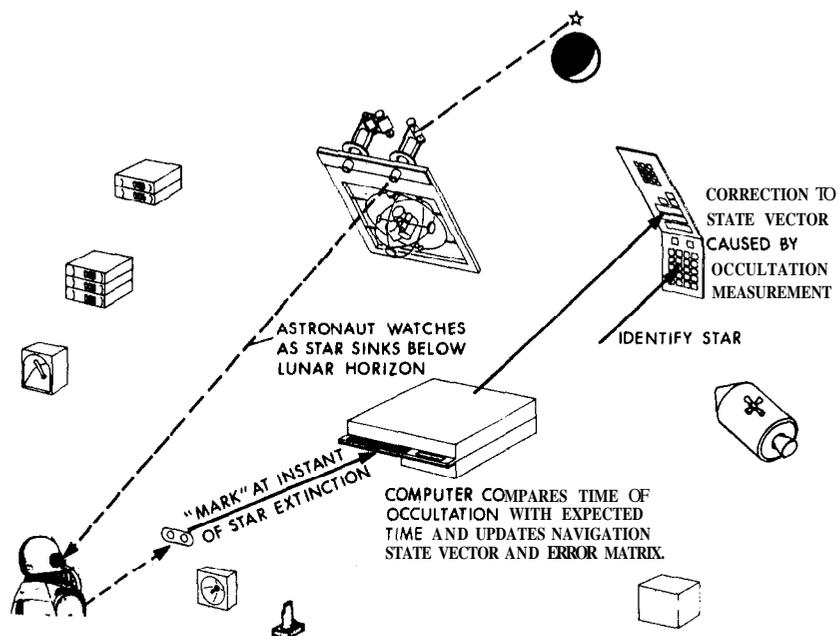
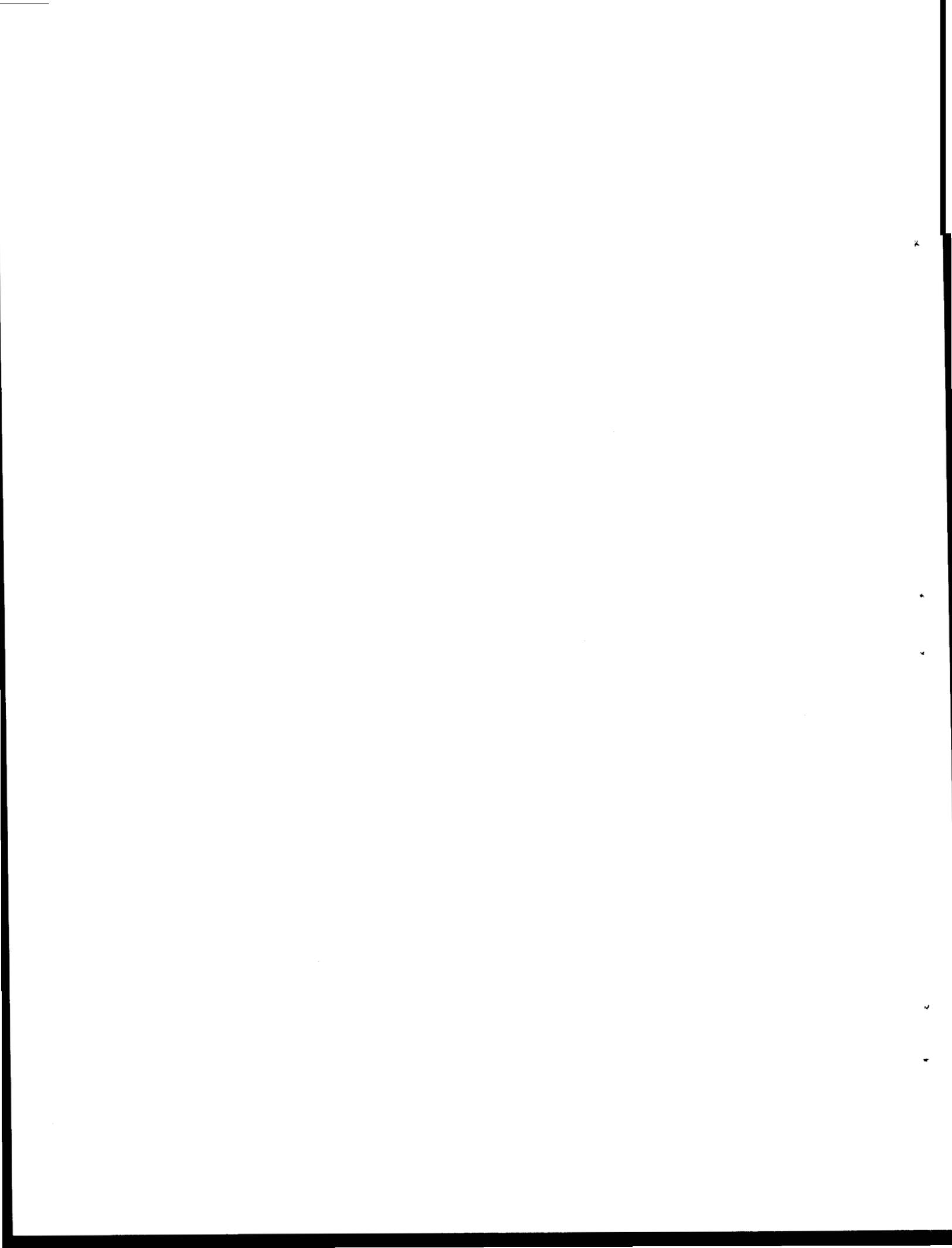


Fig. 11-61 Star Occultation by Moon - Manual Navigation Measurement

Besides the automatic occultation measurement just described, a manual detection is possible, of course, as shown above. This is of advantage since it does not require that the optics system electronics be turned on. In fact, the event can be observed by the astronaut through the window and timed with a separate stopwatch for transmission to the earth for use in aiding ground-based navigation measurement.



## CHAPTER 11-5

### SPACECRAFT SAFETY CONSIDERATIONS OF GUIDANCE, NAVIGATION, AND CONTROL

Although the risk is actually small, the Apollo crew, when they embark in their spacecraft admittedly put their life in jeopardy. However, unlike the more traditional pioneers and adventurers, the men flying the Apollo missions will leave in a spacecraft only after their safety is assured. Crew survival will be a most strong concern in the preparation for the voyage. The National Aeronautics and Space Administration has set high safety standards: The crew, in a checked-out vehicle leaving the earth launching pad for the lunar surface, should have a 90% probability of accomplishing the lunar landing mission and have a 99.9% probability of returning to earth safely whether having been able to complete the mission or not. These goals are sought by consideration of all parts of the Apollo program: mission planning, spacecraft design, crew training, testing methods, etc. In this chapter we are concerned only with the Apollo safety aspects under consideration in the guidance, navigation, and control systems.

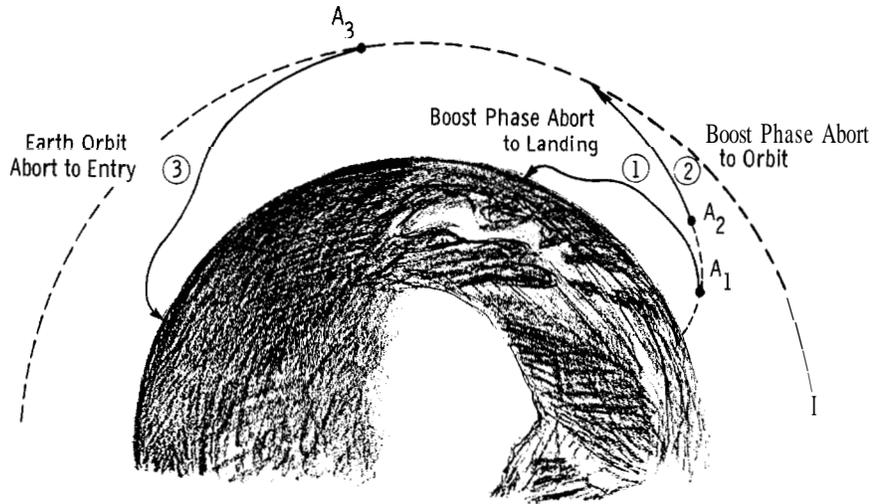
Much can be said about the means of producing complex equipment which has an extremely low failure probability. Questions of discipline in basic design, parts selection, manufacturing techniques, qualification methods, testing procedures, and other reliability enhancing techniques are much debated. We will bypass this well-treated subject and look at aspects of design and planning which accept the occurrence of failure without the occurrence of disaster,

The tolerance to failure in Apollo systems is based primarily on the deliberate design guideline that any single failure should, if at all possible, leave enough working equipment remaining to bring the crew safely home. Although for practical reasons, this guideline cannot be met everywhere, the number of safety critical flight items that have no backup is quite small.

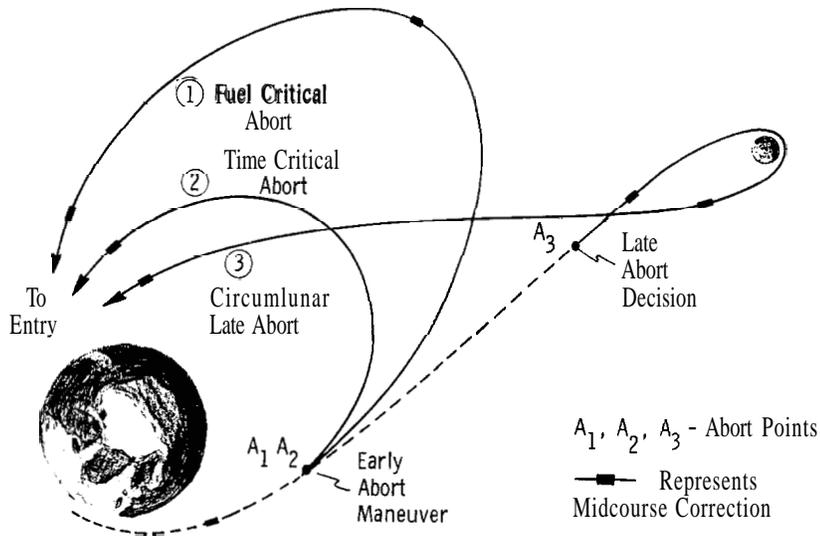
#### ABORT TRAJECTORIES

The guidance and navigation equipment is designed with enough flexibility in hardware and in computer program to support the measurements and maneuvers necessary for all reasonable mission abort trajectories required due to failures in other parts of the spacecraft. Depending upon the nature of this failure and the phase of the mission failure occurs, the crew can initiate an abort by so informing the flight computer and

$A_1, A_2, A_3$  - Abort Points



11-62 Near Earth Abort Trajectories



11-63 Translunar Coast Abort Trajectories

setting the proper condition of the appropriate propulsion systems. In some situations the pilot can inform the computer which of three types of abort he wishes: (1) time critical aborts which require fastest return using all available propulsion, (2) propulsion critical aborts which require optimum use of available fuel in energy efficient orbital transfers, and (3) normal aborts which use trajectories which are constrained to achieve a landing on one of the prepared earth recovery areas. The computer can inform the crew about the times of flight and propulsion usage for each of the above aborts so that the abort mode decision can be made.

The abort trajectory to be determined and controlled by the guidance and navigation equipment depends upon the mission phase in which the abort decision is made, Figure 11-62 illustrates the three abort trajectory types pertinent to operations near the earth. Trajectory 1 on the diagram is direct abort to earth during launch boost ascent, It is flown when the failure is of a nature requiring immediate return to earth or where sufficient propulsion isn't available to fly trajectory 2. Abort trajectory 2 continues the flight into earth orbit using an upper stage of the vehicle. It has the advantage of better choice of landing recovery area by selecting the phasing of the return maneuver. It further permits a possible continuation of the flight but obviously of more limited mission scope. The descent from orbit, trajectory 3, is similar to the earth orbit returns already flown by the Soviet and American manned orbital flights.

Aborts that can be initiated after Apollo has been committed through translunar injection are illustrated in Fig. 11-63. Trajectories 1 and 2 on this figure are typical of the paths flown for aborts initiated during the first part of the translunar coast. Trajectory 1 illustrates a fuel optimum direct return to earth. Trajectory 2 illustrates the full fuel usage quick return to earth. At some point in the translunar coast, the time to earth return is quicker if the spacecraft coasts around back of the moon and then continues home, trajectory 3. All of these cislunar aborts will require careful navigation, Navigation is required before the abort is initiated upon which to base the abort injection guided maneuver. After this maneuver, navigation is needed upon which to base small midcourse corrections to assure accurate arrival at the safe earth entry conditions.

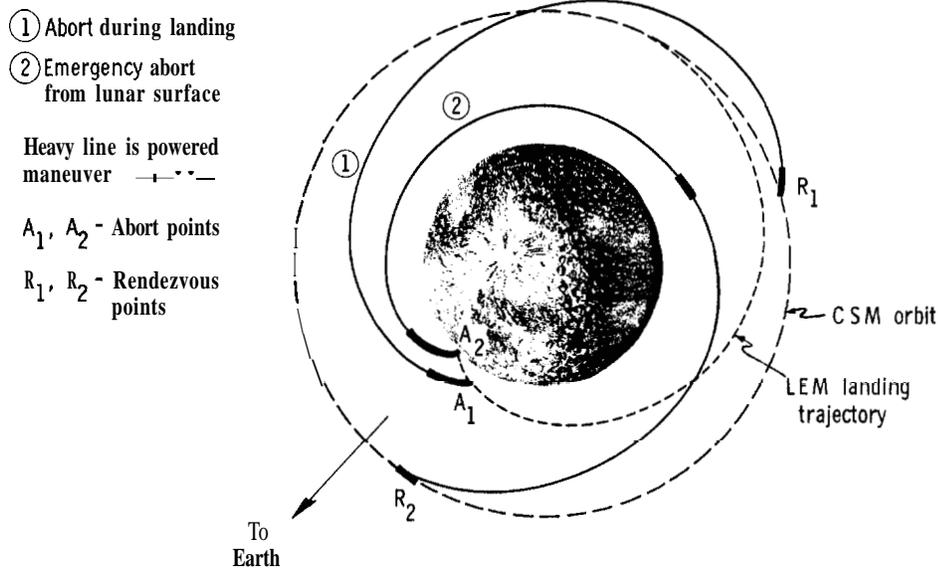
After arrival into lunar orbit, aborts either may be an immediate transearth injection or may necessarily be preceded by recovery of the two men in the LEM. Figure II-64 illustrates the trajectories and operations involved with the LEM aborts. Trajectory 1 illustrates a typical abort initiated during the LEM descent. The abort trajectory, injection, begun at point  $A_1$ , is guided and controlled to put the LEM in a fairly high elliptical trajectory so that the phasing is proper for rendezvous to meet the orbiting command module at point  $R_1$ . Midcourse corrections, not shown, are necessary based upon navigation from the rendezvous radar or earth tracking data. Unfortunately much of the trajectory occurs in back of the moon out of sight of the earth tracking facility. This might suggest use of a low altitude holding orbit such as described below to provide better phasing of the rendezvous for earth coverage.

Trajectory 2 of Fig. II-64 illustrates a typical LEM emergency abort from the lunar surface. Here it is supposed that a failure has occurred - such as fuel tank leakage or life support system failure - that requires immediate ascent without waiting until the CSM is in the proper position for a normal ascent and rendezvous. The powered phase is guided to put the LEM in a low altitude clear perilune orbit where it will hold until it catches up appropriately with the orbiting command module. At the proper point the ascent engine is fired again for transfer and rendezvous using midcourse corrections as required. Alternately, once the LEM succeeds in getting into a holding orbit, it can assume, if necessary, a passive role and allow the CSM to maneuver for rendezvous and crew pickup.

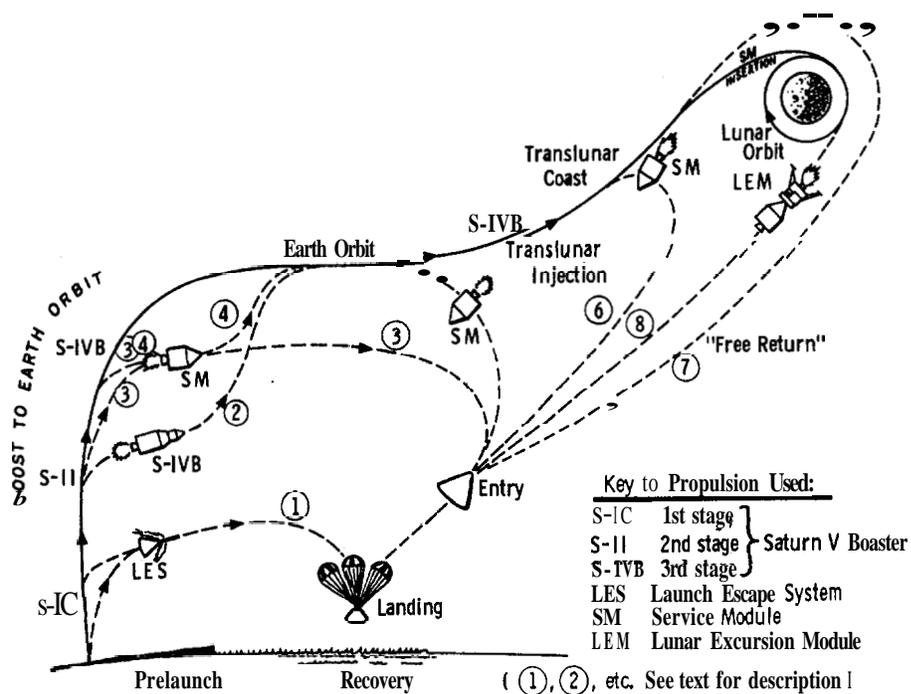
#### CONTROL OF PROPULSION FAILURE BACKUPS

The Apollo guidance and control equipment will be designed to operate with abnormal propulsion and loading configurations for given mission phases to provide abort capabilities covering failure in any of the primary rocket engines. Figure 11-65 is a rather fanciful diagram showing examples of aborts of this nature. The heavy ascending line traces out the normal mission phases from prelaunch to lunar orbit. The dashed lines trace out abort paths through alternate propulsion sources to cover failures of the normal rocket used in each phase. These paths are numbered on the diagram and are explained briefly below:

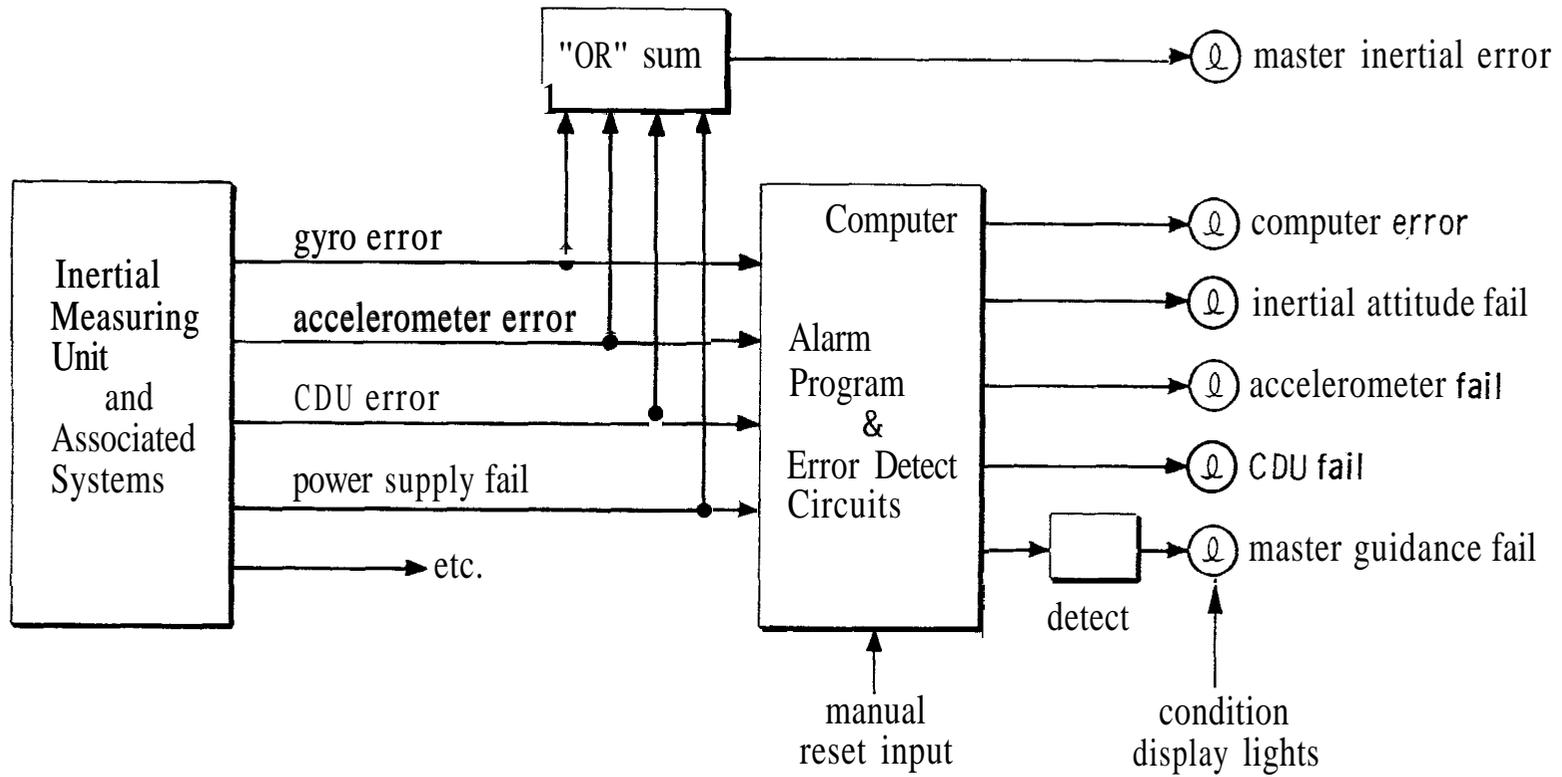
1. This is the use of the launch escape system providing aborts during the period from on the pad before liftoff until atmospheric exit during the early part of the second stage burn. No measurement is necessary by the guidance system for launch escape aborts; the system is designed to pull the command module safely past and far enough away from an exploding booster for a low velocity entry and normal CM parachute landing.
2. A failure of the second stage during ascent might be of a nature to allow thrusting in the third SIVB Saturn stage into earth orbit. This would naturally deplete SIVB fuel sufficiently to prevent continuation of a lunar mission.
3. Again during second stage boost and during third stage as well, the abort may be made to an immediate entry trajectory and landing using the command module propulsion and the spacecraft guidance and control systems.
4. Aborts using service module propulsion during third stage boost may also be made into earth orbit. A second burn of the service module would then initiate descent to a selected landing site.
5. If the abort is initiated while in earth orbit the service module propulsion would be used for descent assuming it still functions. If not, the small reaction jets could be used in a limited retrograde translational burn or series of burns so as to capture the atmosphere.



11-64 LEM Operations Abort Trajectories



11-65 Propulsion Failure Abort Paths



11-66 Failure Detection - Command Module Primary Guidance System

6. On the way to the moon the service module propulsion could be used to inject into the return orbits described previously.
7. If the service module rocket has failed the flight can continue around the moon on the "free return" path using the reaction jets in translation maneuvers to perform the necessary midcourse maneuvers determined by navigation.
8. If service module propulsion fails while in lunar orbit before the LFM separation and descent, the LEM propulsion and LFM guidance and control systems can be used to inject the command module onto the necessary transearth trajectory.

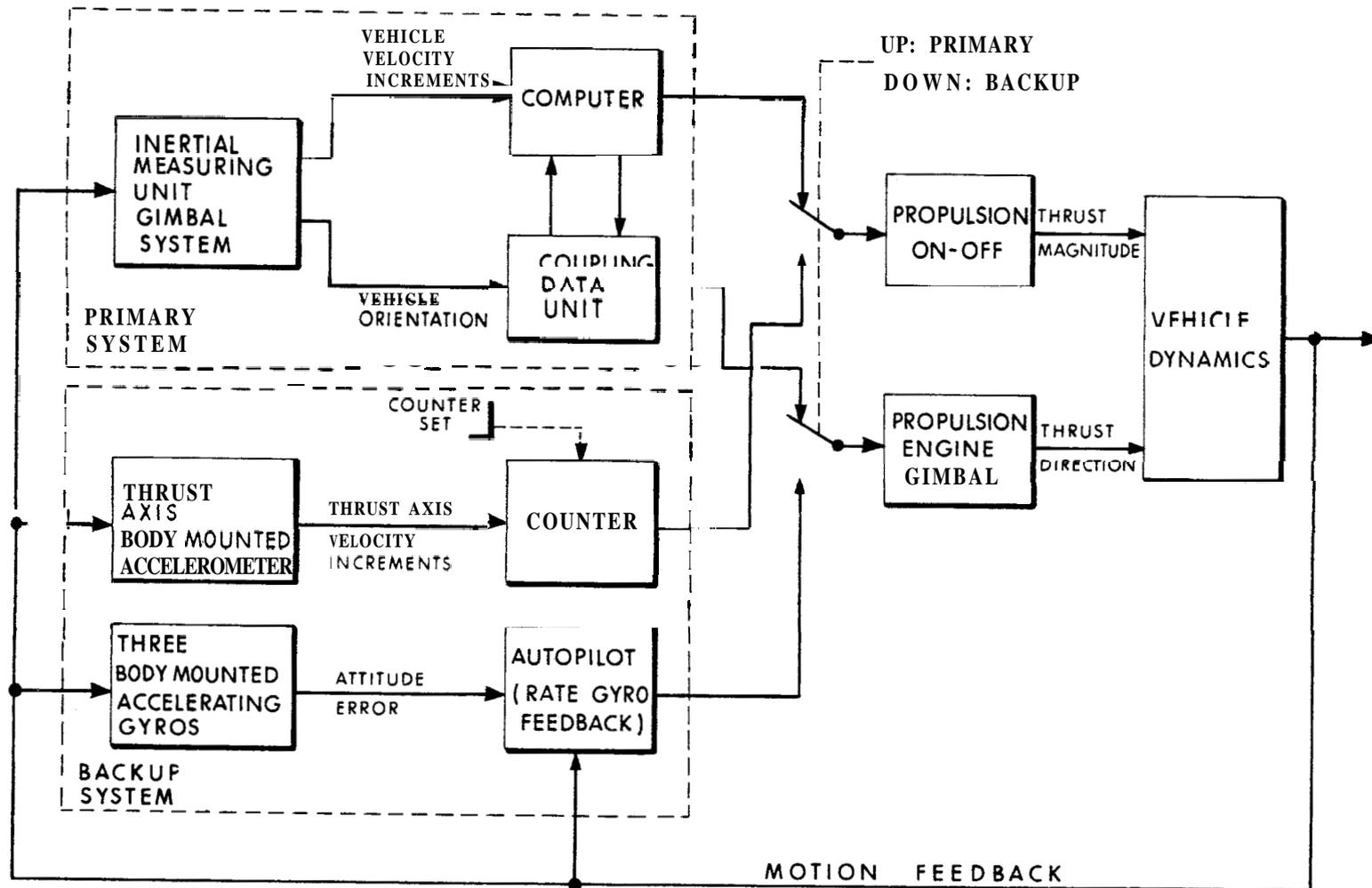
These examples of propulsion failure abort paths illustrate dramatically the necessary flexibility and universality needed of the Apollo guidance, navigation, and control systems.

#### FAILURE DETECTION AND ALARM

A central aspect of mission safety is the early detection of system failure. Part of this detection is in the systematic onboard testing during the stress-free coasting phases to assure the needed systems are functioning. Of more interest, perhaps, is the automatic failure detection features which immediately signal appropriate alarms during the stressed accelerated phases of rocket thrust or entry. It is with the help of these alarms that the crew can initiate appropriate abort action immediately as necessary. As an example, we will limit this discussion to the automatic failure detection of the guidance system in the command module as used during guided flight.

Figure 11-66 is a simplified diagram of this failure detection system. The box at the left represents the inertial system. The signals coming out are error detections. Shown is "gyro error" which is a signal which exists when any of the gyro gimbal stabilization loop servo errors exceed a preselected detection level. The "accelerometer error" and "CDU error" have similar properties with detection of any of the accelerometer servo loop errors and coupling display unit servo loop errors. The "power supply fail" signals deviations of the inertial system power supply voltages from preselected levels. Each of these detections is sent to the computer as well as being separately summed to light a master inertial system error display light. During system turn-on or mode switching this light is expected to operate briefly during the transient, but will extinguish itself in a normal system,

The computer contains its own error detection programs and circuitry which may light a master computer error light. If this occurs for transient reasons, the astronaut will succeed in extinguishing it by pushing a reset button. The computer program will examine its error and the inertial system detections and will light appropriate fail lights. The "inertial attitude fail" signals that the inertial system alignment is lost and the crew should use a backup system if re-alignment cannot be accomplished. The "accelerometer



II-67 Primary &amp; Backup Guidance Concept

fail" indicates that the acceleration data in the guidance is faulty and the primary guidance steering cannot be used. In this latter case, however, the inertial attitude data may still be correct for use in a backup mode. A similar situation occurs with the "CDU fail" light.

The last light is a "master guidance fail" which has special features which makes it fail-safe. The computer program examines periodically at a fixed frequency all of the previous failure detections and if it finds none, the program sends out a pulse of a particular duration. If this pulse keeps occurring at the expected frequency, then the detector inhibits lighting the "master guidance fail". Otherwise this signal lights up.

If any of these lights operate the crew is trained to take appropriate emergency backup action.

#### NAVIGATION REDUNDANCY

The originally stated premise that a single failure should leave the system with enough capability to return safely should now be examined. With respect to navigation this is quite straightforward with the use of both onboard and ground-based provisions.

If ground tracking navigation data are unavailable because of a loss of communications, then the onboard system can perform all the necessary navigation,

If the onboard navigation capability fails, the ground can provide the necessary data. This applies to the failure of either the optics system or the onboard computer. For a failed computer, onboard navigation data can be telemetered for ground processing to aid in the ground determination of the necessary abort maneuver. If the optics system drives have failed, star occultation events can be observed, either by spacecraft attitude aiming of the optics or directly through the window. These events are useful navigation data for the computer.

#### GUIDANCE AND CONTROL BACKUP

Failure in the primary guidance equipment requires the use of an alternate backup system to provide safe return of the crew. The redundancy concept used is illustrated in Fig. 11-67. The figure has been simplified to illustrate more clearly the principles involved.

The primary system involving the inertial measurement unit gimbal system, the flight digital computer, and the associated coupling data unit, constitutes a complete, flexible, accurate, and fuel efficient guidance and control having the capability to perform all the maneuvers required to complete the mission. The backup system is simpler, smaller, and of more modest endowment, being able to make the more simple maneuvers to return the crew after the failure of the primary system has aborted the mission. Figure 11-67 shows a backup using three body-mounted single-degree-of-freedom

integrating gyros which provide attitude error signals over a limited angular range. These errors are treated as steering commands to a simple autopilot to hold vehicle direction fixed during an abort thrusting maneuver. Engine cutoff is signalled by the integrated output of a single body axis accelerometer mounted with its sensitive axis along the nominal thrust axis. The integrator consists of a simple preset counter giving as an output an engine shutdown signal when the sum of the accelerometer output velocity increments reaches a level equal to the total velocity change desired of the maneuver. The crew, using data telemetered from the ground, will pre-align the thrust axis in the required abort maneuver direction by aiming the vehicle with respect to the stars. The ground instructions to do this will recognize the expected offset of the thrust axis from the vehicle roll axis. The maneuvers are restricted to accelerations in a fixed direction but of any magnitude set into the acceleration counter. If large magnitude velocity changes are required, then the more limited accuracy of this backup system will result in significant errors. These can be corrected by a much smaller maneuver after a short coast based upon the ground tracking of the abort trajectory.

This backup guidance system just explained is a somewhat simplified description of that used in the command module. The LEM has been given a more complex abort guidance system to perform more accurately the critical and complex abort maneuvers near the moon's surface. This LEM abort system consists of three body-mounted rate-measuring gyros, three body-mounted accelerometers, and a small computer to perform necessary transformations of gyro and accelerometer data. This computer also generates steering commands appropriate for abort at any phase of LEM operation to rendezvous conditions with the orbiting command module.

Satisfactory vehicle control also requires three-axis spacecraft torquing during the free-fall and accelerating phases. Optimum redundancy in the hardware to drive the engine gimbals is provided. The sixteen reaction jets on the service module and the sixteen reaction jets on the LEM allow a limited number of jet failures without unacceptable loss of rotational control or translational control. Likewise the necessary rotational control of the command module is provided by a redundant assembly of 12 reaction jets for use during earth atmospheric entry. Various levels of automatic, semiautomatic, and manual control can be selected by the crew to utilize the subsystems available and working.

In the limit, the pilot or a surviving companion can use direct hand control commands to the reaction jets and the engine gimbals and a view of the stars as reference directions to provide him with the last level of emergency backup.

PART III  
EXPLICIT AND UNIFIED METHODS OF  
SPACECRAFT GUIDANCE

by

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He is in charge of the Space Guidance Analysis Group which is charged with both theoretical analysis and programming of the on-board guidance computer for the Laboratory's development of the guidance system that will be used aboard the Project Apollo spacecraft.

Dr. Battin was born in Atlantic City, N. J., March 3, 1925, and was graduated from Forest Park High School, Baltimore, Md., in 1942. He received the S. B. degree in electrical engineering from M. I. T. in 1945 and served in the Navy as a Supply Corps officer for one year. He returned to M. I. T. in 1946 as an instructor in mathematics and a research assistant in meteorology, studying atmospheric circulations. He was awarded the Ph. D. degree in applied mathematics from M. I. T. in 1951.

Dr. Battin joined the Instrumentation Laboratory in 1951 as a research mathematician working on fire control and inertial navigation systems and later became head of the Laboratory's Computing Devices Group.

From 1956 to 1958, Dr. Battin was a senior staff member in the Operations Research Group of Arthur D. Little, Inc., working on digital data processing for business and industrial research. Since returning to the Instrumentation Laboratory in 1958, Dr. Battin has been engaged primarily in interplanetary navigation theory and development.

Dr. Battin is the author of the book, Astronautical Guidance (New York; McGraw-Hill, Inc., 1964) and co-author of the book, Random Processes in Automatic Control (New York; McGraw-Hill, 1956). He has published numerous articles in professional journals dealing with meteorology, analog and digital computing techniques, stochastic processes, interplanetary trajectories and navigation. He is a member of Sigma Xi and an Associate Fellow of the American Institute of Aeronautics and Astronautics.

### Part III

#### EXPLICIT AND UNIFIED METHODS OF SPACECRAFT GUIDANCE

Of fundamental importance in the design of space guidance systems is the creation of both flexible techniques and versatile instrumentation which have a wide range of applicability but neither compromise mission accuracy nor place an undue burden on propulsion requirements. Minimal constraints on the system and methods of its operation should be imposed by detailed mission objectives which are subject to frequent and last minute revision. In partial fulfillment of these goals, the development of explicit guidance techniques is warranted to reduce any dependence on precomputed reference orbits or specific rocket engine characteristics.

During the evolution of a space flight program such as Apollo, the ultimate mission objective is attained progressively by a series of intermediate flights. Each successive flight is planned as a direct extension of the previous one so that the need for special equipment and untried techniques can be minimized. The success of this approach is enhanced through the development of unified guidance methods. Then the guidance requirements for each new mission phase can be met as a specific application of a general guidance principle.

The two fundamental tasks of a guidance system are to maintain accurate knowledge of spacecraft position and velocity and to provide steering commands for required changes in course. It is the purpose here to review some of the current techniques for solving the guidance problem emphasizing those methods which are consistent with the explicit and unified philosophy of design.

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## Chapter III-1

### ACCELERATED FLIGHT NAVIGATION

The task of periodic determination of position and velocity, customarily referred to as navigation, divides naturally into two parts - accelerated flight and coasting flight. For navigation during an accelerated maneuver, the system frequently includes inertial instruments capable of measuring thrust acceleration along three mutually orthogonal axes which are nonrotating. A guidance computer is then required to perform accurate integrations and gravity calculations on a real-time basis.

A functional diagram of the basic computations required of the navigation system is shown in Fig. III-1. Incremental outputs from inertially stabilized integrating accelerometers, together with components of gravitational acceleration computed as functions of inertial position in a feedback loop, are summed to give the components of inertial velocity. The ultimate precision attainable is, of course, limited by the accuracy of the inertial instruments, the speed of the guidance computer and the knowledge of the initial conditions.

#### GRAVITY COMPUTER

The gravity calculations may be performed in a straight-forward manner. The equations of motion for a vehicle moving in a gravitational field are

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \qquad \frac{d\mathbf{v}}{dt} = \mathbf{g} + \mathbf{a}_T \qquad \text{(III-1)}$$

where  $\mathbf{r}$  and  $\mathbf{v}$  are the position and velocity vectors with respect to an inertial frame of reference. The measured acceleration vector  $\mathbf{a}_T$  of the vehicle is defined to be the vehicle acceleration resulting from the sum of rocket thrust and aerodynamic forces, if any, and would be zero if the vehicle moved under the action of gravity alone. The vector sum of  $\mathbf{a}_T$  and  $\mathbf{g}$ , the gravitational vector, represents the total vehicle acceleration.'

A simple computational algorithm, by means of which position and velocity are obtained as a first order difference equation calculation, follows

$$\Delta \underline{v}_a(t_n) = \underline{v}_a(t_n) - \underline{v}_a(t_{n-1})$$

$$\underline{r}(t_n) = \underline{r}(t_{n-1}) + \underline{v}(t_{n-1}) \Delta t + \frac{1}{2} \underline{g}_{n-1} (\Delta t)^2 + \frac{1}{2} \Delta \underline{v}_a(t_n) \Delta t \quad (\text{III-2})$$

$$\underline{v}(t_n) = \underline{v}(t_{n-1}) + \Delta \underline{v}_a(t_n) + \frac{1}{2} (\underline{g}_n + \underline{g}_{n-1}) \Delta t$$

The vector  $\underline{v}_a$  is the time integral of the non-gravitational acceleration forces, the components of which are the outputs of the three mutually orthogonal integrating accelerometers as shown in Fig. 111-2. The gravitational vector  $\underline{g}_n$  is a function of position at time  $t_n$ . In the figure, only a simple spherical gravitation field is considered.

Since velocity is updated by means of the average effective gravity over the interval of one time step, this method has been termed the "average  $g$ " method. A careful error analysis of a vehicle in earth orbit has shown this algorithm to yield errors of the order of 100 feet and 0.2 feet per second after a period of 35 minutes using a 2 second time step and rounding all additions to 8 decimal digits. The errors will increase for a smaller time step due to the effects of accumulated round off errors and will also increase for larger time steps as truncation errors become significant. When compared to typical accelerometer scale factor errors, the error in the computational algorithm seems to be several orders of magnitude smaller,

#### BODY-MOUNTED SENSORS

During recent years increasing attention has been devoted to the so-called "gimballess inertial measurement unit" in which the inertial sensors are mounted directly to the spacecraft (see Bumstead and Vander Velde<sup>(1)</sup> and Wiener<sup>(2)</sup>). Although many advantages might accrue in terms of system weight, volume, power, cost, packaging flexibility, reliability and maintainability, the realization of a satisfactory design is not without significant problems. Unlike the environment provided by a gimballed system, the body mounted inertial instruments are subjected to substantial angular velocity which tends to exaggerate performance errors. Also, the role of the guidance computer is expanded since the angular orientation of the vehicle must also be determined by integration of measured angular velocities. It is most convenient if the outputs of the body mounted accelerometers are immediately transformed into an inertially stabilized coordinate frame so that the navigation or guidance problem can be solved just as if a physically stabilized platform had been employed.

As indicated in Fig. III-3, the body fixed coordinates and the inertial coordinates of the thrust acceleration vector are related by a transformation matrix of direction cosines. The additional computations required of the guidance computer involve the updating of the matrix and using it to transform vectors from one frame of reference to the other. The transformation matrix  $R$  is readily shown to satisfy a first order differential

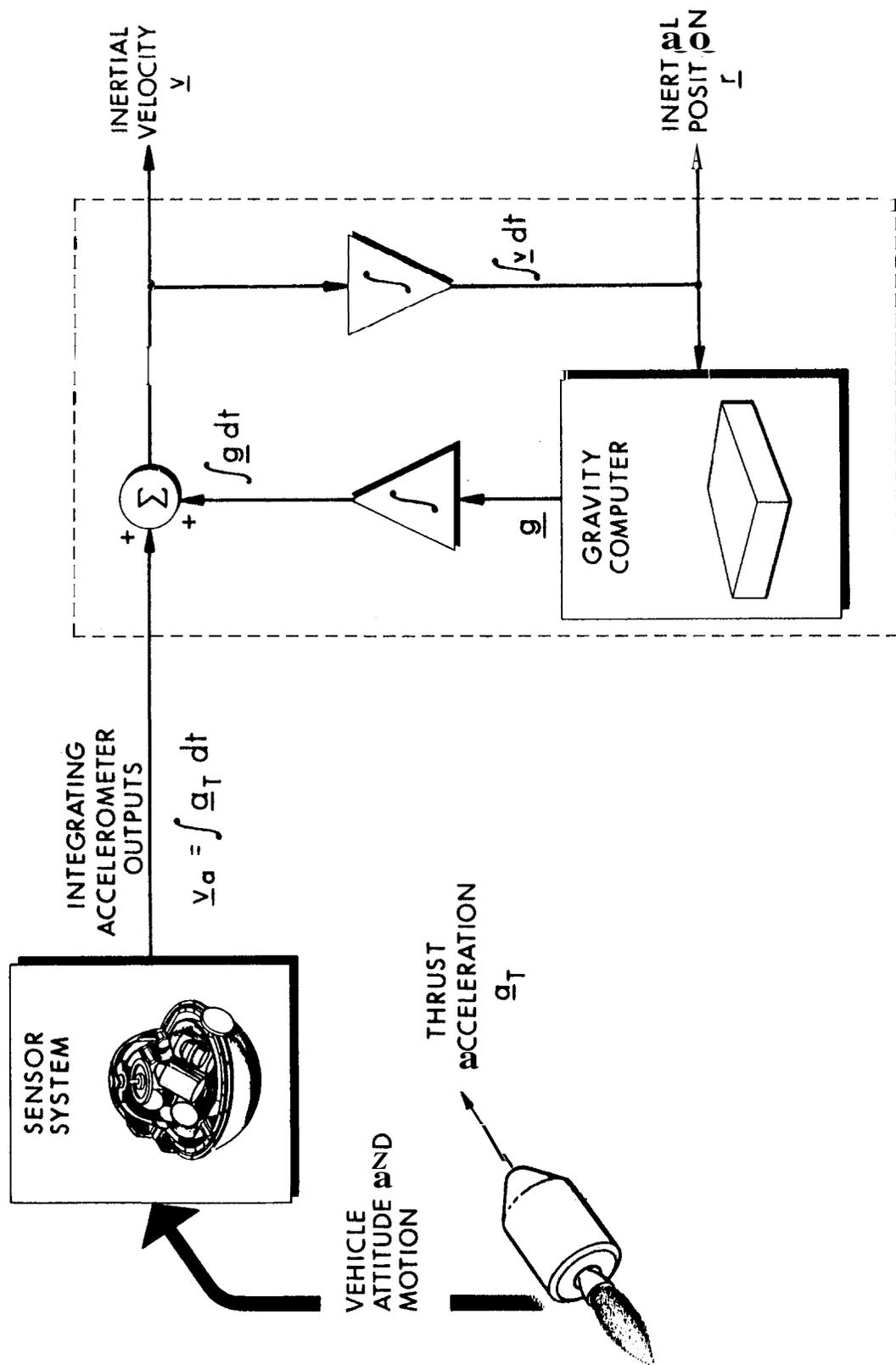
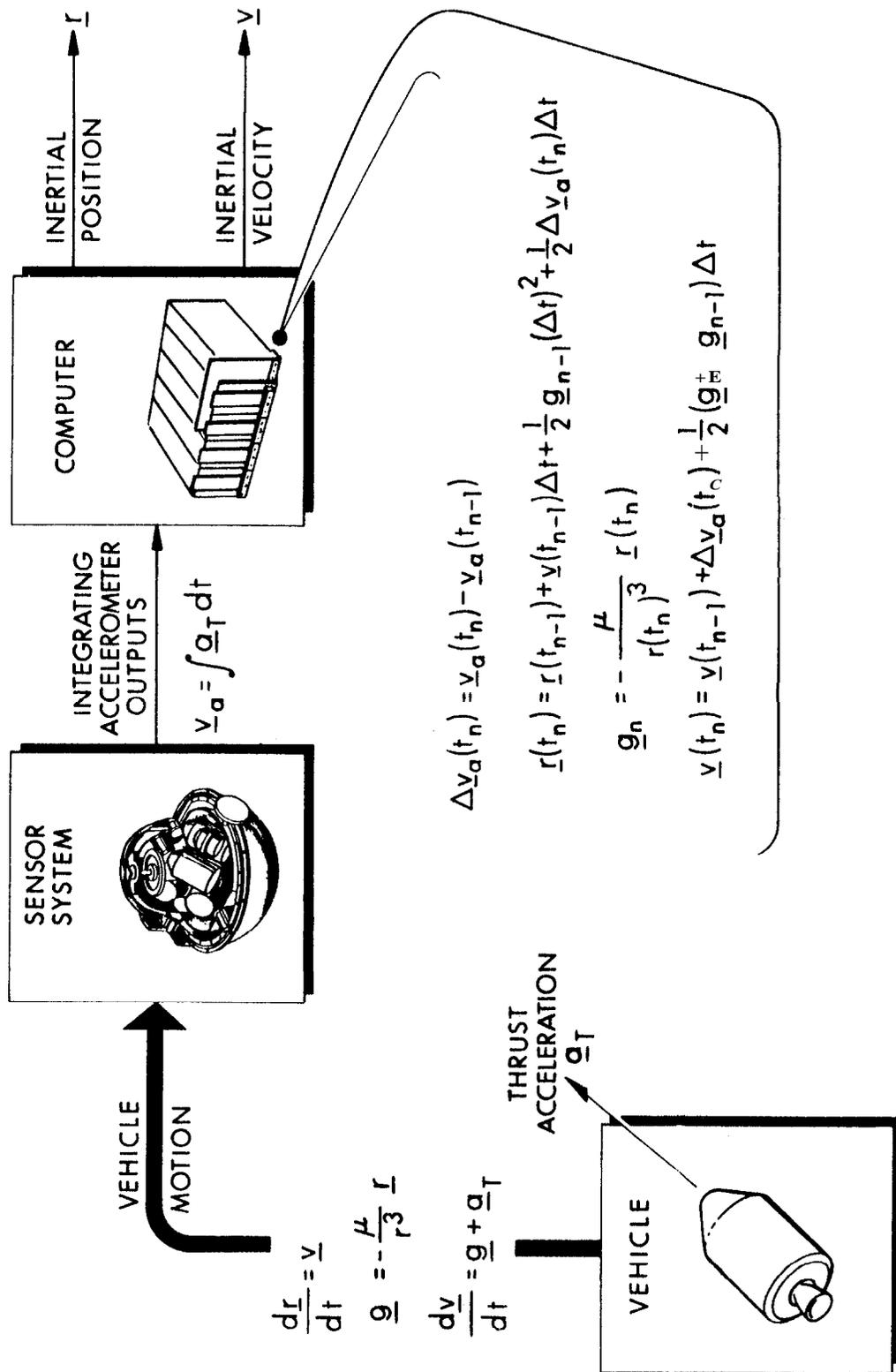


Fig. III-1 Accelerated Flight Navigation System



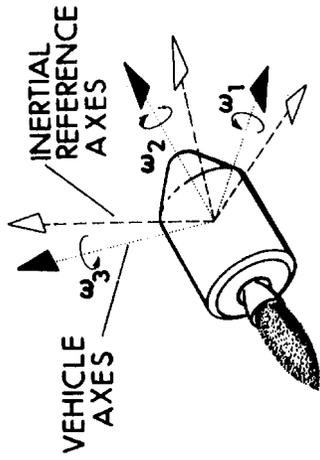
$$\Delta \bar{v}_a(t_n) = \bar{v}_a(t_n) - \bar{v}_a(t_{n-1})$$

$$\bar{r}(t_n) = \bar{r}(t_{n-1}) + \bar{v}(t_{n-1})\Delta t + \frac{1}{2} \bar{g}_{n-1}(\Delta t)^2 + \frac{1}{2} \Delta \bar{v}_a(t_n)\Delta t$$

$$\bar{g}_n = -\frac{\mu}{r(t_n)^3} \bar{r}(t_n)$$

$$\bar{v}(t_n) = \bar{v}(t_{n-1}) + \Delta \bar{v}_a(t_n) + \frac{1}{2} (\bar{g}_n + \bar{g}_{n-1})\Delta t$$

Fig III-2 Accelerated Flight Navigation



$\underline{a}_T$  (INERTIAL) =  $R \underline{a}_T$  (BODY-FIXED)  
 R: TRANSFORMATION MATRIX

$$\frac{dR}{dt} = -R\Omega$$

$$\Omega = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix}$$

$\omega_j$  = ANGULAR VELOCITY SENSED BY  $j^{\text{th}}$  GYRO

$\omega_j \Delta t$  = INCREMENTAL CHANGE IN OUTPUT OF  $j^{\text{th}}$  GYRO DURING SAMPLE TIME

$$\Delta \Theta(t_n) = \Omega(t_n) \Delta t$$

$$R(t_{n+1}) = R(t_n) - R(t_n) \Delta \Theta(t_n)$$

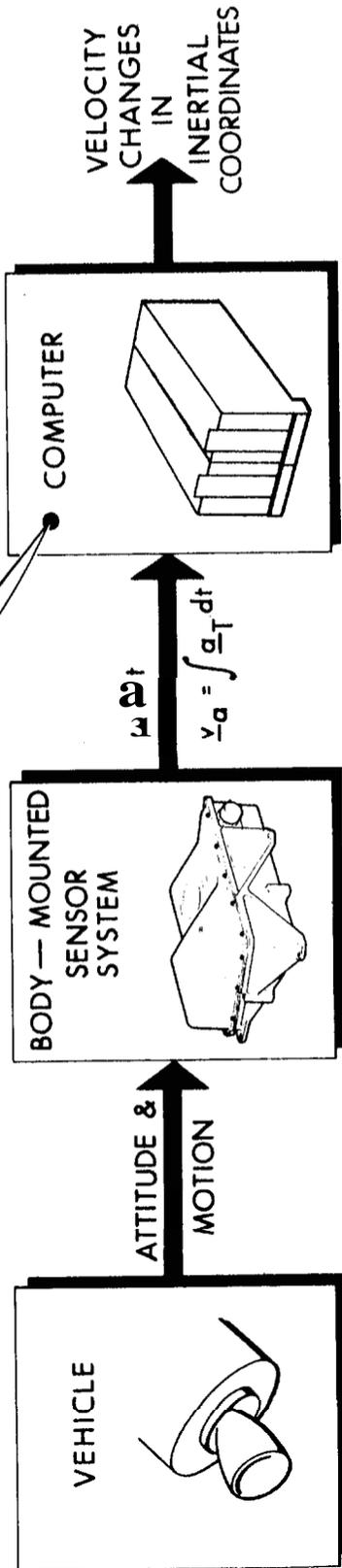


Fig. III-3 Navigation Using Body-Mounted Sensors

equation with a coefficient matrix  $\Omega$  whose elements are the components of the angular velocity of the body fixed coordinate frame measured in body coordinates.

Currently, pulse-torqued integrating gyros are the most promising candidates for angular velocity sensors. However, since their basic output consists of angular increments rather than angular velocity, the accuracy with which the transformation matrix differential equation may be integrated is adversely affected. The use of a higher order integration rule provides no advantage over simple rectangular integration since the basic data from the gyro has already an uncertainty of the order of the square of the gyro quantization error.

The accuracy attainable by a gimballess inertial system is limited primarily by the maximum angular velocities to which the vehicle is subjected. The required sampling time of the integrating gyros is inversely proportional to this maximum angular velocity and the time step used for integrating the direction cosine differential equations must be of the same order of magnitude as the gyro sampling time. If the sample time is very short, a digital differential analyzer may prove to be the best solution to the problem of selecting a guidance computer. On the other hand, if the sampling time is long enough to permit the use of a general purpose computer, there may be sufficient time remaining in which to process the navigation and/or steering equations. This is, of course, more satisfactory, for then one has the possibility of satisfying all or most of the complete system computation requirements with a single computer.

## Chapter III-2

### COASTING FLIGHT NAVIGATION

Spacecraft navigation during prolonged coasting flight is performed by appropriate utilization of periodic measurements of convenient physical quantities such as (1) distance, velocity, elevation and azimuth from well-established reference points, (2) angles between lines of sight to known celestial objects, (3) star occultations, and (4) apparent planet diameters. Since navigation measurements are more accurately made when the sensors are in proximity to the data source, vehicle-borne and ground-based instrumentation can serve in complementary roles.

#### COMPARISON OF METHODS

The data processing aspects of the navigation problem have been the subject of much research during recent years. The classical method of the astronomer, called the "method of differential corrections", is cumbersome for large amounts of observational data and is not well suited to implementation in a vehicle-borne computer. Blackman, in a recent paper<sup>(3)</sup>, gives an excellent review of several of the new methods contrasting them with the classical approach and with each other.

Currently, the statistical methods of optimum linear estimation theory seem to hold the most promise. The statistical method of maximum likelihood, which is based on the concept of maximizing a particular conditional probability, has received much attention. Optimum filter theory, whose goal is to find a linear estimator that minimizes some function of the variances and covariances of the uncertainties in the estimated state vector, provides an alternate method of attack. Although the subject may be approached from a variety of points of view, Potter and Stern<sup>(4)</sup> have shown that all such methods lead to equivalent results if the measurement uncertainties have Gaussian distributions.

#### RECURSIVE NAVIGATION

The scope of this chapter does not permit a thorough development of the mathematics underlying the so-called "recursive method" of spacecraft position and velocity estimation; however this is adequately treated in reference (5). The method, which is particularly well-suited to both vehicle-borne and ground-based computation, is under active consideration for use in the Apollo guidance computer as well as in the Mission Control Center at the Manned Spacecraft Center, Houston, Texas.

The estimate of position and velocity is maintained in the computer in non-rotating rectangular coordinates and is referenced to either the earth or the moon. An earth centered equatorial coordinate system is used when the vehicle is outside of the lunar sphere of influence. Inside of this sphere the center of coordinates coincides with the center of the moon. The extrapolation of position and velocity is made by a direct numerical integration of the equations of motion.

The basic equation may be written in vector form as

$$\frac{d^2}{dt^2} \underline{r}_{PV} + \frac{\mu_P}{r_{PV}^3} \underline{r}_{PV} = \underline{a}_d \quad (\text{III-3})$$

where  $\underline{r}_{PV}$  is the vector position of the vehicle with respect to the primary body P which is either the earth or moon and  $\mu_P$  is the gravitational constant of P. The vector  $\underline{a}_d$  is the vector acceleration which prevents the motion of the vehicle from being precisely a conic with P at the focus.

If  $\underline{a}_d$  is small compared with the central force field, direct use of Eq. (III-3) is inefficient. As an alternative, the integration may be accomplished employing the technique of differential accelerations suggested by Encke.

Encke's Method - At time  $t_0$  the position and velocity vectors  $\underline{r}_{PV}(t_0)$  and  $\underline{v}_{PV}(t_0)$  define an osculating orbit. The vector difference  $\underline{\delta}(t)$  satisfies the following differential equation

$$\frac{d^2}{dt^2} \underline{\delta} = \frac{\mu_P}{PV(C)} \left[ \left( 1 - \frac{PV(C)^3}{r_{PV}^3} \right) \underline{r}_{PV} - \underline{\delta} \right] + \underline{a}_d \quad (\text{III-4})$$

subject to the initial conditions

$$\underline{\delta}(t_0) = 0 \quad \frac{d}{dt} \underline{\delta}(t_0) = \underline{v}(t_0) = 0$$

where  $\underline{r}_{PV(C)}$  is the osculating conic position vector. The numerical difficulties which would arise from the evaluation of the coefficient of  $\underline{r}_{PV}$  in Eq. (III-4) may be avoided. Since

$$\underline{r}_{PV}(t) = \underline{r}_{PV(C)}(t) + \underline{\delta}(t) \quad (\text{III-5})$$

it follows that

$$1 - \frac{r_{PV(C)}^3}{r_{PV}^3} = -f(q_C) = 1 - (1 + q_C)^{3/2}$$

where

$$q_C = \frac{(\delta - 2r_{PV}) \cdot \delta}{r_{PV}^2} \quad (\text{III-6})$$

The function  $f(q)$  may be conveniently evaluated from

$$f(q) = q \frac{3 + 3q + q^2}{1 + (1+q)^{3/2}} \quad (\text{III-7})$$

Encke's method may now be summarized as follows:

A. Position in the osculating orbit is calculated from

$$\begin{aligned} \underline{r}_{PV(C)}(t) = & \left[ 1 - \frac{x^2}{r_{PV}(t_0)} C(\alpha_0 x^2) \right] \underline{r}_{PV}(t_0) \\ & + \left[ (t - t_0) - \frac{x^3}{\sqrt{\mu_P}} S(\alpha_0 x^2) \right] \underline{v}_{PV}(t_0) \end{aligned} \quad (\text{III-8})$$

where

$$C_0 = \frac{2}{r_{PV}(t_0)} - \frac{v_{PV}(t_0)^2}{\mu_P} \quad (\text{III-9})$$

and  $x$  is determined as the root of Kepler's equation in the form

$$\begin{aligned} \sqrt{\mu_P}(t - t_0) = & \frac{\underline{r}_{PV}(t_0) \cdot \underline{v}_{PV}(t_0)}{\sqrt{\mu_P}} x^2 C(\alpha_0 x^2) \\ & + \left[ 1 - r_{PV}(t_0) \alpha_0 \right] x^3 S(\alpha_0 x^2) + r_{PV}(t_0) x \end{aligned} \quad (\text{III-10})$$

The special transcendental functions  $S$  and  $C$  are defined by

$$\begin{aligned} S(x) = & \frac{1}{3!} - \frac{x}{5!} + \frac{x^2}{7!} - \dots \\ C(x) = & \frac{1}{2!} - \frac{x}{4!} + \frac{x^2}{6!} - \dots \end{aligned} \quad (\text{III-11})$$

- B. Deviations from the osculating orbit are obtained by a numerical integration of

$$\frac{d^2}{dt^2} \underline{\delta}(t) = - \frac{\mu_P}{r_{PV(C)}^3(t)} \left[ f(q) \underline{r}_{PV}(t) + \underline{\delta}(t) \right] + \underline{a}_d(t) \quad (\text{III-12})$$

The first term on the right hand side of the last equation must remain small, i. e. of the same order as  $\underline{a}_d(t)$ , if the method is to be efficient. As the deviation vector  $\underline{\delta}$  grows in magnitude, this term will eventually increase in size. Therefore, in order to maintain the efficiency, a new osculating orbit should be defined by the true position and velocity. The process of selecting a new conic orbit from which to calculate deviations is called rectification. When rectification occurs, the initial conditions of the differential equation for  $\underline{\delta}$  are again zero and the right hand side is simply the perturbation acceleration  $\underline{a}_d$  at the time of rectification.

- C. The position vector  $\underline{r}_{PV}(t)$  is computed from Eq. (III-5) using Eq. (III-8). The velocity vector  $\underline{v}_{PV}(t)$  is then computed as

$$\underline{v}_{PV}(t) = \underline{v}_{PV(C)}(t) + \underline{v}(t) \quad (\text{III-13})$$

where

$$\begin{aligned} \underline{v}_{PV(C)}(t) = & \frac{\sqrt{\mu_P}}{r_{PV}(t_0) r_{PV(C)}(t)} \left[ \alpha_0 x^3 S(\alpha_0 x^2) - x \right] \underline{r}_{PV}(t_0) \\ & + \left[ 1 - \frac{x^2}{r_{PV(C)}(t)} C(\alpha_0 x^2) \right] \underline{v}_{PV}(t_0) \end{aligned} \quad (\text{III-14})$$

Disturbing Acceleration- The form of the disturbing acceleration  $\underline{a}_d$  to be used depends on the phase of the mission. In earth orbit only the gravitational anomalies arising from the non-spherical shape of the earth need be considered. During trans-lunar and transearth flight, the gravitational attraction of the sun and the secondary body Q (either earth or moon) are relevant forces. In lunar orbit, it may be necessary to consider forces arising from the non-spherical shape of the moon. A summary of the various cases appears below.

- A. Earth Orbit

$$\underline{a}_d = \frac{\mu_E}{r_{EV}^2} \sum_{k=2}^4 J_k \left( \frac{r_{eq}}{r_{EV}} \right)^k \left[ P'_{k+1}(\cos \phi) \underline{i}_{EV} - P'_k(\cos \phi) \underline{i}_z \right] \quad (\text{III-15})$$

where

$$P_2'(\cos \phi) = 3 \cos \phi$$

$$P_3'(\cos \phi) = \frac{1}{2} (15 \cos^2 \phi - 3)$$

$$P_4'(\cos \phi) = \frac{1}{3} (7 \cos^3 \phi - 4 \cos \phi)$$

$$P_5'(\cos \phi) = \frac{1}{4} (9 \cos^4 \phi - 5 \cos^2 \phi)$$

are the derivatives of the Legendre polynomials;

$$\cos \phi = \underline{i}_{EV} \cdot \underline{i}_z$$

is the cosine of the angle  $\phi$  between the unit vector  $\underline{i}_{EV}$  in the direction of  $\underline{r}_{EV}$  and the unit vector  $\underline{i}_z$  in the direction of the north pole;  $r_{eq}$  is the equatorial radius of the earth; and  $J_2, J_3, J_4$  are the coefficients of the second, third and fourth harmonics of the earth's potential function. The subscript E denotes the center of the earth as the origin of coordinates.

#### B. Translunar and Transearth Flight

$$\underline{a}_d = - \frac{\mu_Q}{r_{QV}^3} \left[ f(q_Q) \underline{r}_{PQ} + \underline{r}_{PV} \right] - \frac{\mu_S}{r_{SV}^3} \left[ f(q_S) \underline{r}_{PS} + \underline{r}_{PV} \right] \quad (\text{III-16})$$

where the subscripts Q and S denote the secondary body and the sun, respectively. Thus, for example,  $\underline{r}_{PS}$  is the position vector of the sun with respect to the primary body. The arguments  $q_O$  are calculated from

$$q(\ ) = \frac{(\underline{r}_{PV} - 2 \underline{r}_{P(\ )}) \cdot \underline{r}_{PV}}{r_{P(\ )}^2} \quad (\text{III-17})$$

and the function f from Eq. (III-7).

Ephemeris data for the positions of the moon relative to the earth  $\underline{r}_{EM}$  and the sun relative to the earth-moon barycenter  $\underline{r}_{BS}$  are required as functions of time. The position of the sun relative to the primary planet  $\underline{r}_{PS}$  is then computed from

$$\underline{r}_{PS}(t) = \underline{r}_{BS}(t) + \frac{\mu_Q}{\mu_P + \mu_Q} \underline{r}_{PQ} \quad (\text{III-18})$$

In the vicinity of the lunar sphere of influence a change in origin of coordinates is made. Thus

$$\underline{r}_{PV}(t) - \underline{r}_{PQ}(t) = \underline{r}_{QV}(t) \rightarrow \underline{r}_{PV}(t) \quad (\text{III-19})$$

$$\underline{v}_{PV}(t) - \underline{v}_{PQ}(t) = \underline{v}_{QV}(t) \rightarrow \underline{v}_{PV}(t)$$

### C. Lunar Orbit

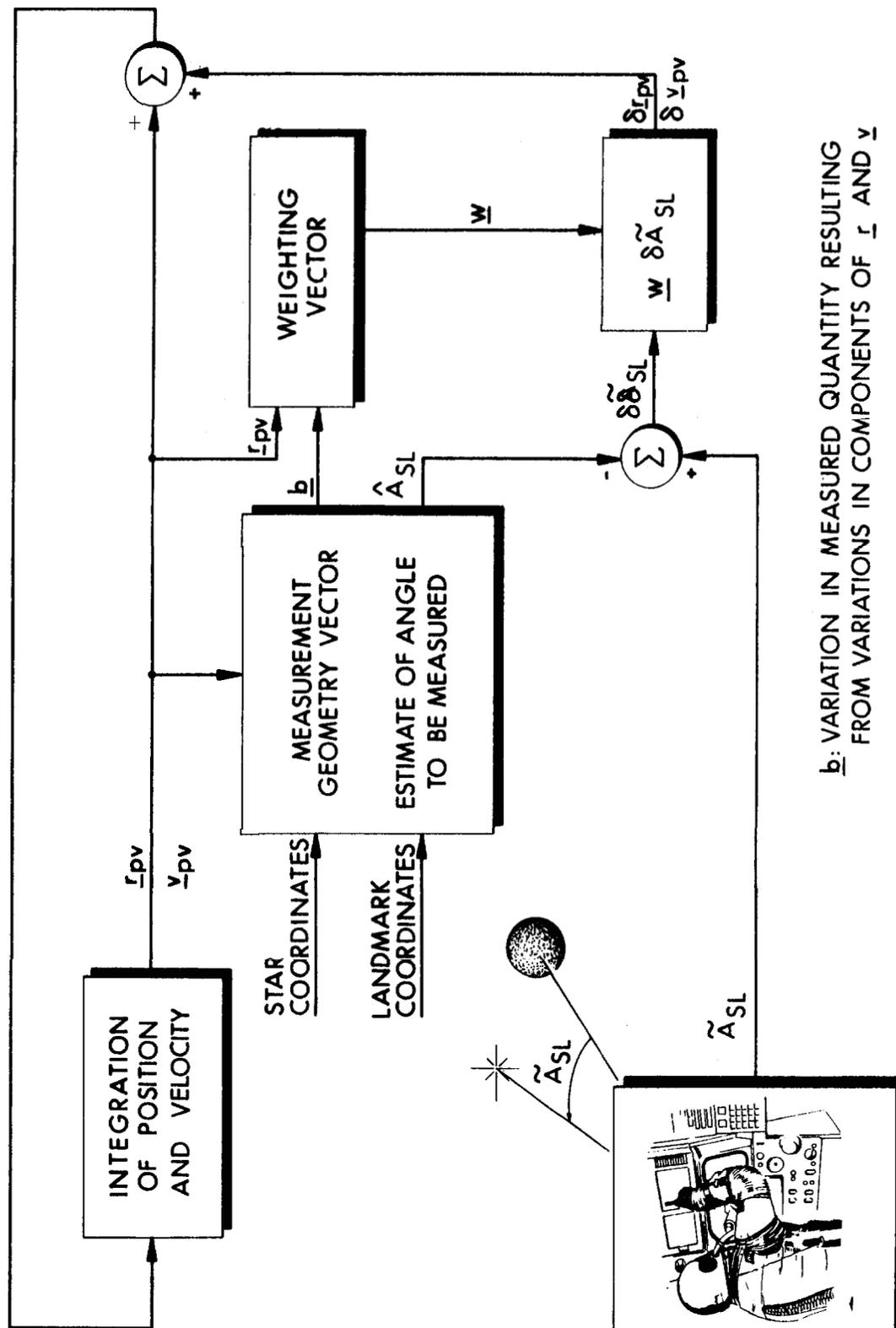
$$\begin{aligned} \underline{a}_d = & -\frac{3\mu_M r_m^2 C'}{2r_{MV}^4} \left\{ \left[ \frac{B-A}{C} \left[ 1 - 5 (\underline{i}_{MV} \cdot \underline{i}_\eta)^2 \right] + \frac{C-A}{C} \left[ 1 - 5 (\underline{i}_{MV} \cdot \underline{i}_\zeta)^2 \right] \right] (\underline{i}_{MV} \cdot \underline{i}_\xi) \underline{i}_\xi \right. \\ & + \left. \left[ \frac{B-A}{C} \left[ 3 - 5 (\underline{i}_{MV} \cdot \underline{i}_\eta)^2 \right] + \frac{C-A}{C} \left[ 1 - 5 (\underline{i}_M \cdot \underline{i}_\zeta)^2 \right] \right] (\underline{i}_{MV} \cdot \underline{i}_\eta) \underline{i}_\eta \right. \\ & \left. + \left[ \frac{B-A}{C} \left[ 1 - 5 (\underline{i}_{MV} \cdot \underline{i}_\eta)^2 \right] + \frac{C-A}{C} \left[ 3 - 5 (\underline{i}_{MV} \cdot \underline{i}_\zeta)^2 \right] \right] (\underline{i}_{MV} \cdot \underline{i}_\zeta) \underline{i}_\zeta \right\} \quad (\text{III-20}) \end{aligned}$$

where A, B, C are the principal moments of inertia of the moon,  $r_m$  is the radius of the moon,  $C'$  is C divided by the product of the mass of the moon and the square of its radius,  $\underline{i}_\xi, \underline{i}_\eta, \underline{i}_\zeta$  are the selenographic coordinate unit vectors, and  $\underline{i}_{MV}$  is the unit vector in the direction of  $\underline{r}_{MV}$ .

Navigation Measurements - Periodically, the position and velocity of the spacecraft must be brought into accord with optical or radar observations made with either onboard or ground based sensors. At the time a measurement is made, the best estimate of spacecraft position and velocity is the extrapolated estimate maintained in the computer and denoted by  $\underline{r}_{PV}$  and  $\underline{v}_{PV}$  as shown in Fig. 111-4. From this estimate, it is possible to determine an estimate of the quantity to be measured such as an angle, range from a tracking station or range rate. When the predicted value of this measurement is compared with the actual measured quantity, the difference is used to improve the estimated position and velocity vector.

#### A. The Measurement Geometry Vector

An important feature of the recursive navigation method is that measurement data from a wide variety of sources may be incorporated within the same framework of computation. Associated with each measurement is a six-dimensional vector  $\underline{h}$  representing, to a first order of approximation, the variation in the measured quantity  $\underline{q}$  which would result from variations in the components of position and velocity. Thus, each measurement establishes a component of the spacecraft state vector along the direction of the  $\underline{h}$  vector in state space,



$\underline{b}$ : VARIATION IN MEASURED QUANTITY RESULTING FROM VARIATIONS IN COMPONENTS OF  $\underline{r}$  AND  $\underline{v}$

Fig III-4 Coasting Flight Navigation Computation

Specifically, if  $\underline{b}_1$  and  $\underline{b}_2$  are the upper and lower three-dimensional partitions of the six-dimensional  $\underline{b}$  vector and if  $\delta \tilde{q}$  is the difference between the value of the quantity as actually measured and the expected value as computed from the current values of  $\underline{r}_{PV}$  and  $\underline{v}_{PV}$ , then

$$\delta \tilde{q} = \underline{b}_1 \cdot \delta \underline{r}_{PV} + \underline{b}_2 \cdot \delta \underline{v}_{PV} \quad (\text{III-21})$$

where  $\delta \underline{r}_{PV}$  and  $\delta \underline{v}_{PV}$  are the changes in the computed values of position and velocity necessary to make the estimated state of the vehicle compatible with the observation.

As examples of both ground-based and onboard measurements we may list the following.

1. Radar Range Measurement

$$\underline{b}_1 = \underline{i}_{RV}$$

$$\underline{b}_2 = 0$$

$$q = r_{RV}$$

where  $r_{RV}$  is the range of the vehicle from the radar site and  $\underline{i}_{RV}$  is a unit vector in the direction of the vehicle from the site.

2. Radar Range Rate Measurement

$$\underline{b}_1 = \frac{1}{r_{RV}} \underline{i}_{RV} \times (\underline{v}_{RV} \times \underline{i}_{RV})$$

$$\underline{b}_2 = \underline{i}_{RV}$$

$$q = \underline{v}_{RV} \cdot \underline{i}_{RV}$$

where  $\underline{v}_{RV}$  is the velocity of the vehicle with respect to the radar site.

3. Star-Landmark Measurement

$$\underline{b}_1 = \frac{1}{r_{LV}} \text{UNIT} [\underline{i}_s - (\underline{i}_s \cdot \underline{i}_{LV}) \underline{i}_{LV}]$$

$$\underline{b}_2 = 0$$

$$q = \cos^{-1} (-\underline{i}_{LV} \cdot \underline{i}_s)$$

where  $r_{LV}$  is the distance of the vehicle from the landmark and  $\underline{i}_{LV}$  is a unit vector in the direction of the vehicle from the landmark. The unit

vector  $\underline{i}_s$  gives the direction of the star. The notation UNIT( ) indicates a unit vector in the direction of the quantity ( ).

#### 4. Star-Horizon Measurement

$$\underline{b}_1 = \frac{1}{\sqrt{r_{PV}^2 - r_{PH}^2}} \text{UNIT} \left\{ \sqrt{r_{PV}^2 - r_{PH}^2} \times \right. \\ \left. \text{UNIT} [\underline{i}_s - (\underline{i}_s \cdot \underline{i}_{PV}) \underline{i}_{PV}] - r_{PH} \underline{i}_{PV} \right\}$$

$$\underline{b}_2 = 0$$

$$q = \cos^{-1} (-\underline{i}_{PV} \cdot \underline{i}_s) - \sin^{-1} \frac{r_{PH}}{r_{PV}}$$

where  $\underline{r}_{PV}$  is a vector from the selected planet to the vehicle and  $r_{PH}$  is the altitude of the horizon from the center of the planet. If the far horizon is chosen for the measurement, then  $r_{PH}$  must be negative.

#### B. The Error Transition Matrix

Six measurements made simultaneously would provide a set of six equations of the form of Eq. (III-21). If the directions of the associated  $\underline{b}$  vectors span the state space, then the vector changes  $\delta \underline{r}_{PV}$  and  $\delta \underline{v}_{PV}$  could be obtained by inversion of the six-dimensional coefficient matrix each of whose rows were elements of the  $\underline{b}$  vectors.

Both the problems of simultaneous measurements and matrix inversion can be avoided in such a manner that measurement data may be incorporated sequentially as it is obtained. For this purpose, it is necessary to maintain statistical data in the guidance computer in the form of a six-dimensional correlation matrix  $E(t)$  of estimation errors. If  $\underline{\epsilon}(t)$  and  $\underline{u}(t)$  are the errors in the estimates of the position and velocity vector, respectively, then the six-dimensional correlation matrix  $E(t)$  is defined by\*

$$E(t) = \begin{pmatrix} \overline{\underline{\epsilon}(t) \underline{\epsilon}(t)^T} & \overline{\underline{\epsilon}(t) \underline{u}(t)^T} \\ \overline{\underline{u}(t) \underline{\epsilon}(t)^T} & \overline{\underline{u}(t) \underline{u}(t)^T} \end{pmatrix} \quad (\text{III-22})$$

\*

The transpose of a vector or a matrix is denoted by a superscript T.

Because of accumulated numerical inaccuracies, it is possible that this correlation matrix may fail to remain positive definite after a large number of computations as it theoretically must. A recent innovation to avoid this problem, which has also the advantage of significantly reducing the total computational requirements, is to replace the correlation matrix by a matrix  $W(t)$ , called the error transition matrix. The  $W(t)$  matrix has the property

$$E(t) = W(t) W(t)^T \quad (\text{III-23})$$

and thus, in a sense, is the square root of the correlation matrix. If needed, the correlation matrix may be determined as the product of the matrix  $W(t)$  and its transpose, thereby guaranteeing it to be at least positive semi-definite.

Extrapolation of the matrix  $W(t)$  is made by direct numerical integration of the differential equation

$$\frac{dW}{dt} = \begin{pmatrix} O & I \\ G(t) & O \end{pmatrix} W \quad (\text{III-24})$$

where  $G(t)$  is the three-dimensional gravity gradient matrix. The matrices  $I$  and  $O$  are the three-dimensional identity and zero matrices, respectively. If the  $W$  matrix is partitioned as

$$W = \begin{pmatrix} o & \dots & \omega_6 \\ \frac{d\omega_1}{dt} & \frac{d\omega_2}{dt} & \dots & \frac{d\omega_6}{dt} \end{pmatrix} \quad (\text{III-25})$$

then the extrapolation may be accomplished by successively integrating the vector differential equations

$$\frac{d^2}{dt^2} \omega_i = \dots \omega_i \quad i = 1, 2, \dots, 6 \quad (\text{III-26})$$

The  $G(t)$  matrix for translunar and transearth flight is readily shown to be

$$G(t) = \frac{\mu_E}{r_{EV}^5(t)} \left[ 3r_{EV}(t) r_{EV}(t)^T - r_{EV}^2(t) I \right] + \frac{\mu_M}{r_{MV}^5(t)} \left[ 3r_{MV}(t) r_{MV}(t)^T - r_{MV}^2(t) I \right]$$

so that the differential equations for the  $\underline{\omega}_i(t)$  vectors are simply

$$\begin{aligned} \frac{d^2}{dt^2} \underline{\omega}_i &= \frac{\mu_E}{r_{EV}^3(t)} \left\{ 3 \left[ \underline{i}_{EV}(t) \cdot \underline{\omega}_i(t) \right] \underline{i}_{EV}(t) - \underline{\omega}_i(t) \right\} \\ &+ \frac{\mu_M}{r_{MV}^3(t)} \left\{ 3 \left[ \underline{i}_{MV}(t) \cdot \underline{\omega}_i(t) \right] \underline{i}_{MV}(t) - \underline{\omega}_i(t) \right\} \end{aligned} \quad (\text{III-27})$$

$$i = 1, 2, \dots, 6$$

### C. The Weighting Vector

By algebraically combining the W matrix, the  $\underline{h}$  vector and a mean-squared a priori estimation error  $\alpha^2$  in the measurement, there are produced a weighting vector  $\underline{w}$  and the step change to be made in the error transition matrix to reflect the changes in the uncertainties in the estimated quantities as a result of the measurement. The weighting vector  $\underline{w}$  has six components and is determined so that the observational data is utilized in a statistically optimum manner. The required calculation is given as a flow' diagram in Fig. 111-5,

The computation may be conveniently organized in terms of the vector partitions of the W matrix as given in Eq. (III-25). If  $\underline{w}_1$  and  $\underline{w}_2$  are the three-dimensional upper and lower partitions of the weighting vector, then we have

$$\underline{z}_i = \underline{\omega}_i \cdot \underline{b}_1 + \frac{d\underline{\omega}_i}{dt} \cdot \underline{b}_2 \quad i = 1, 2, \dots, 6$$

$$\beta = \frac{1}{\alpha^2 + \sum_{i=1}^6 \underline{z}_i^2} \quad (\text{III-28})$$

$$\underline{w}_1 = \sum_{i=1}^6 \underline{z}_i \underline{\omega}_i$$

$$\underline{w}_2 = \sum_{i=1}^6 \underline{z}_i \frac{d\underline{\omega}_i}{dt}$$

Finally, the navigation parameters are updated according to

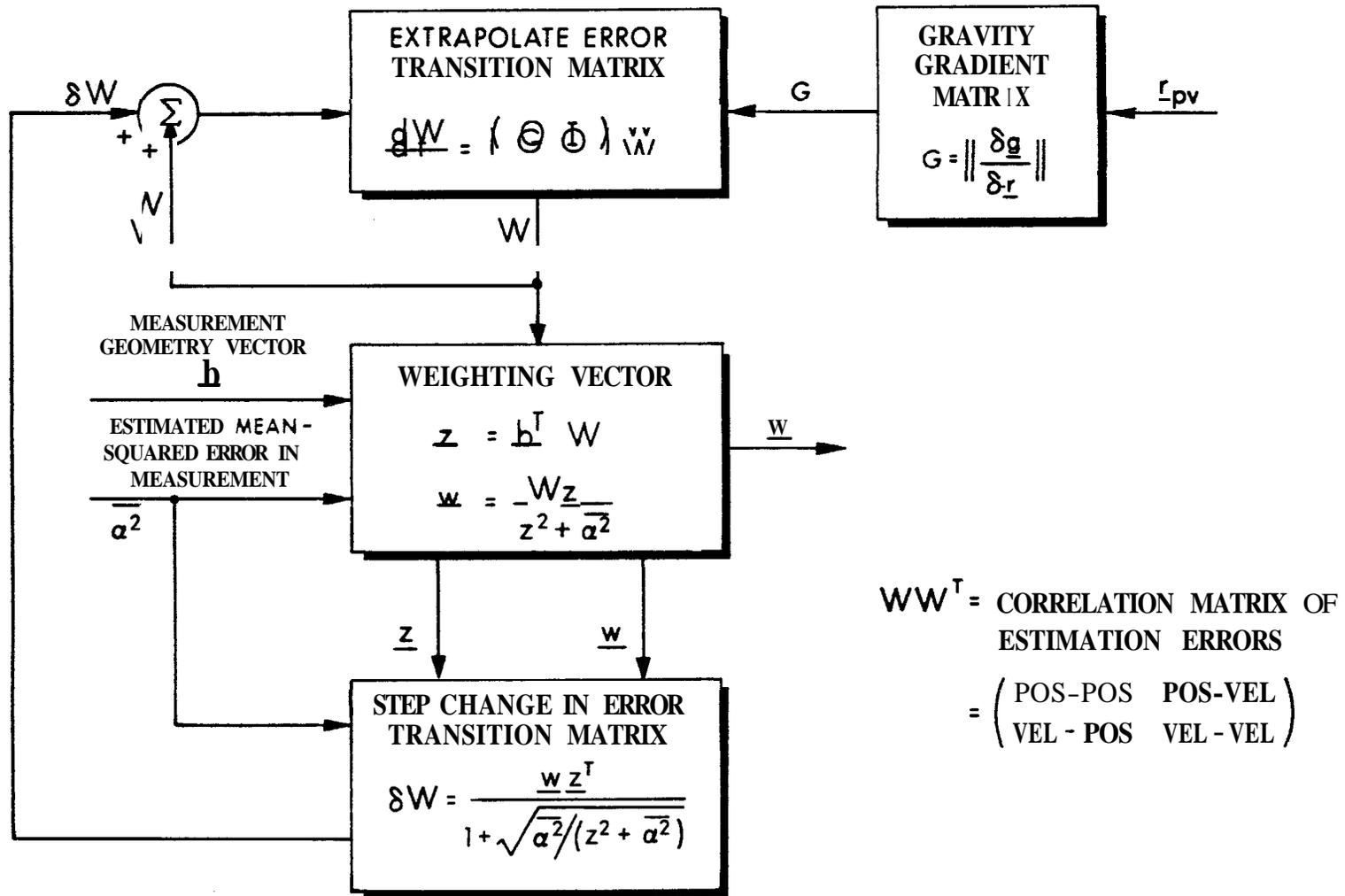


Fig. 111-5 Weighting Vector Calculation

$$\begin{aligned}
\underline{r}_{PV} + \underline{w}_1 \delta \tilde{q} &\rightarrow \underline{r}_{PV} \\
\underline{v}_{PV} + \underline{w}_2 \delta \tilde{q} &\rightarrow \underline{v}_{PV} \\
\underline{\omega}_i - \gamma z_i \underline{w}_1 &\rightarrow \underline{\omega}_i \\
\frac{d\underline{\omega}_i}{dt} - \gamma z_i \underline{w}_2 &\rightarrow \frac{d\underline{\omega}_i}{dt} \quad i = 1, 2, \dots, 6
\end{aligned}
\tag{III-29}$$

where

$$\gamma = \frac{1}{1 + \sqrt{\beta \alpha^2}}$$

#### D. Numerical Integration

The extrapolation of navigation parameters requires the solution of seven second order vector differential equations, specifically Eqs. (III-12) and (III-27). These are all special cases of the form

$$\frac{d^2}{dt^2} \underline{y} = \underline{f}(\underline{y})
\tag{III-30}$$

in which the right hand side is a function of the independent variable and time only. Nyström's method<sup>(6)</sup> is particularly well-suited to this form and gives an integration method of fourth order accuracy. The second order system is written as

$$\frac{d}{dt} \underline{y} = \underline{z} \quad \frac{d}{dt} \underline{z} = \underline{f}(\underline{y})
\tag{III-31}$$

and the formulae are summarized below

$$\begin{aligned}
\underline{y}_{n+1} &= \underline{y}_n + \underline{\phi}(\underline{y}_n) \Delta t \\
\underline{z}_{n+1} &= \underline{z}_n + \underline{\psi}(\underline{y}_n) \Delta t \\
\underline{\phi}(\underline{y}_n) &= \underline{z}_n + \frac{1}{6} (\underline{k}_1 + 2\underline{k}_2) \Delta t \\
\underline{\psi}(\underline{y}_n) &= \frac{1}{6} (\underline{k}_1 + 4\underline{k}_2 + \underline{k}_3) \\
\underline{k}_1 &= \underline{f}(\underline{y}_n) \\
\underline{k}_2 &= \underline{f}\left(\underline{y}_n + \frac{1}{2} \underline{z}_n \Delta t + \frac{1}{8} \underline{k}_1 (\Delta t)^2\right) \\
\underline{k}_3 &= \underline{f}\left(\underline{y}_n + \underline{z}_n \Delta t + \frac{1}{2} \underline{k}_2 (\Delta t)^2\right)
\end{aligned}
\tag{III-32}$$

For efficient use of computer storage as well as computing time the computations should be performed in the following order.

1. Equation (III-12) is solved using the Nyström formulae (III-32). The position of the sun and moon are required at times  $t_n$ ,  $t_n + 1/2 At$ ,  $t_n + At$  to be used in the evaluation of the vectors  $\underline{k}_1$ ,  $\underline{k}_2$ ,  $\underline{k}_3$  respectively. It is necessary to preserve the values of the vectors  $\underline{r}_{-EV}$  and  $\underline{r}_{-EM}$  at these times for use in the solution of Eq. (III-27).
2. Equations (III-27) are solved one set at a time using formulae (111-32) together with the values of  $\underline{r}_{-EV}$  and  $\underline{r}_{-EM}$  which resulted from the first step.

Many of the advantages of the recursive navigation method are now readily apparent. Although linear techniques are still employed, it has been possible to remove any dependence on a reference or pre-computed orbit. Within the framework of a single computational algorithm, measurement data from any source may be incorporated sequentially as obtained. Sensitive numerical computations, such as the inversion of matrices, are avoided.

#### PARAMETER ESTIMATION

The coasting flight navigation procedure just outlined is capable of generalization to include the estimation of quantities in addition to position and velocity by the simple expedient of increasing the dimension of the state vector beyond *six*. For example, one might wish to estimate biases or cross-correlations in the optical or radar instruments, the frequency of a satellite-borne doppler source, or even astronomical quantities such as distances and gravitational constants.

#### MEASUREMENT SCHEDULE

For effective application of this navigation method, an efficient observation schedule should be prepared. An elementary procedure, which has been found to be quite effective for this purpose, is described in reference (5). At each of a number of discrete times, appropriately spaced along the flight path, that measurement is selected, from a variety of possible observations, which would result in the greatest reduction in mean-squared position uncertainty at the destination. In order to control the number of measurements and prevent an unnecessarily lengthy schedule, a measurement is required to produce a significant reduction in the potential miss distance or it will not be made.

The simple strategy described above, in which only currently available information is exploited, does not, of course, insure an optimum schedule since the uncertainties in position and velocity at the target clearly depend on the entire measurement schedule. A method of improving a measurement schedule iteratively, employing an adjoint of the correlation matrix, has been developed. The technique has been shown

to converge always to essentially the same schedule starting from a variety of nominal measurement schedules, A numerical example, reported by Denham and Speyer<sup>(7)</sup>, gives a minimum rms uncertainty in position at the terminal point which is 10 per cent less than the value obtained using the more elementary method.

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## Chapter 111-3

### POWERED-FLIGHT GUIDANCE

The task of providing steering commands, frequently called guidance, separates naturally into two categories — major and minor maneuvers. Launch into parking orbit, transfer to lunar or interplanetary orbit, insertion into orbit, and landing are all examples of major thrusting maneuvers and differ markedly from the minor orbit changes typified by mid-course velocity corrections. In either case, the guidance problem is always a boundary value problem subject to a variety of constraints of which fuel conservation, vehicle maneuverability, and time are examples.

Explicit solutions to the problem of guidance during periods of major thrusting require relatively complex calculations to be performed in flight on a time-critical basis. Considering the modest size and capabilities of vehicle-borne computers, contrasted with the more familiar commercial machines, the design of feasible explicit methods presents a considerable challenge. Several of the more promising guidance techniques currently under development are compared in this chapter.

#### ADAPTIVE GUIDANCE MODE

The guidance method developed at Marshall Space Flight Center for the Saturn rocket and termed the Adaptive Guidance Mode<sup>(8)</sup> is conceptually simple and easily described. The form of the guidance and cut-off equations is invariant with changing missions and vehicles and, therefore, is in accord with the requirements of a unified method. However, significantly large quantities of ground computations are required to determine certain coefficients needed in the mechanization.

The vector values of position and velocity, the scalar magnitude of thrust acceleration and time are updated continuously during powered flight. At each instant the present values of these quantities may be considered as initial conditions for the remainder of the flight. Ideally, one would determine the optimum trajectory from present conditions to desired terminal conditions and command a thrust direction from this optimum solution. This is, of course, impractical so that the techniques of the calculus of variations are employed to generate a volume of expected trajectories for specific vehicles and missions. Numerical curve fitting methods are employed to obtain satisfactory series solutions for the guidance and cut-off commands.

A functional diagram for the Saturn guidance system is shown in Fig. 111-6. During flight the thrust acceleration magnitude is computed approximately once per second by differentiating the outputs of the integrating accelerometers and taking the square root of the sum of the squares of the resulting derivatives. Guidance and cut-off commands are computed as polynomial functions of position, velocity, thrust acceleration and time at intervals of approximately one second. During the burning of the first stage of Saturn, the guidance program is obtained as a polynomial expansion in time only because of structural and control problems.

The chief difficulty with the Adaptive Guidance Mode is determining the best method of representing the volume of expected trajectories which provide minimum fuel consumption. The required number of terms in the polynomials to obtain acceptable accuracy has been found to vary from 40 to 60 depending on the mission.

#### VELOCITY-TO-BE-GAINED METHODS

Conic orbits can be exploited to advantage in solving many guidance problems. For those major orbital transfer maneuvers which can be accomplished conceptually by a single impulsive velocity change, an instantaneous velocity-to-be-gained vector based on conic orbits can often be defined and the vehicle steered to null this vector.

Refer to Fig. 111-7 and let a vector  $\underline{v}_r$  be defined, corresponding to the present vehicle location  $\underline{r}$ , as the instantaneous velocity required to satisfy a set of stated mission objectives. The velocity difference  $\underline{v}_g$  between  $\underline{v}_r$  and the present vehicle velocity  $\underline{v}$  is then the instantaneous velocity-to-be-gained,

Two convenient guidance laws are immediately apparent which will assure that all three components of the vector  $\underline{v}_g$  are simultaneously driven to zero. First, we may orient the vehicle to align the thrust acceleration vector  $\underline{a}_T$  with the direction of the velocity-to-be-gained vector. Alternatively, since a convenient expression can be developed for the time rate of change of the  $\underline{v}_g$  vector, we may direct the vector  $\underline{a}_T$  to cause the vector  $\dot{\underline{v}}_g$  to be parallel to  $\underline{v}_g$  and oppositely directed. If the thrust acceleration magnitude is not sufficiently large it may not be possible to align the vector  $\underline{v}_g$  with its derivative. However, with typical chemical rockets for which the burning time is relatively short, no difficulty has been encountered with this guidance logic,

A combination of these two techniques leads to a highly efficient steering law which compares favorably with calculus of variations optimum solutions.<sup>(9)</sup> The scalar mixing parameter  $\gamma$  is chosen empirically to maximize fuel economy during the maneuver. A constant value of  $\gamma$  is usually sufficient for a particular mission phase; however, if required, it may be allowed to vary as a function of some convenient system variable.

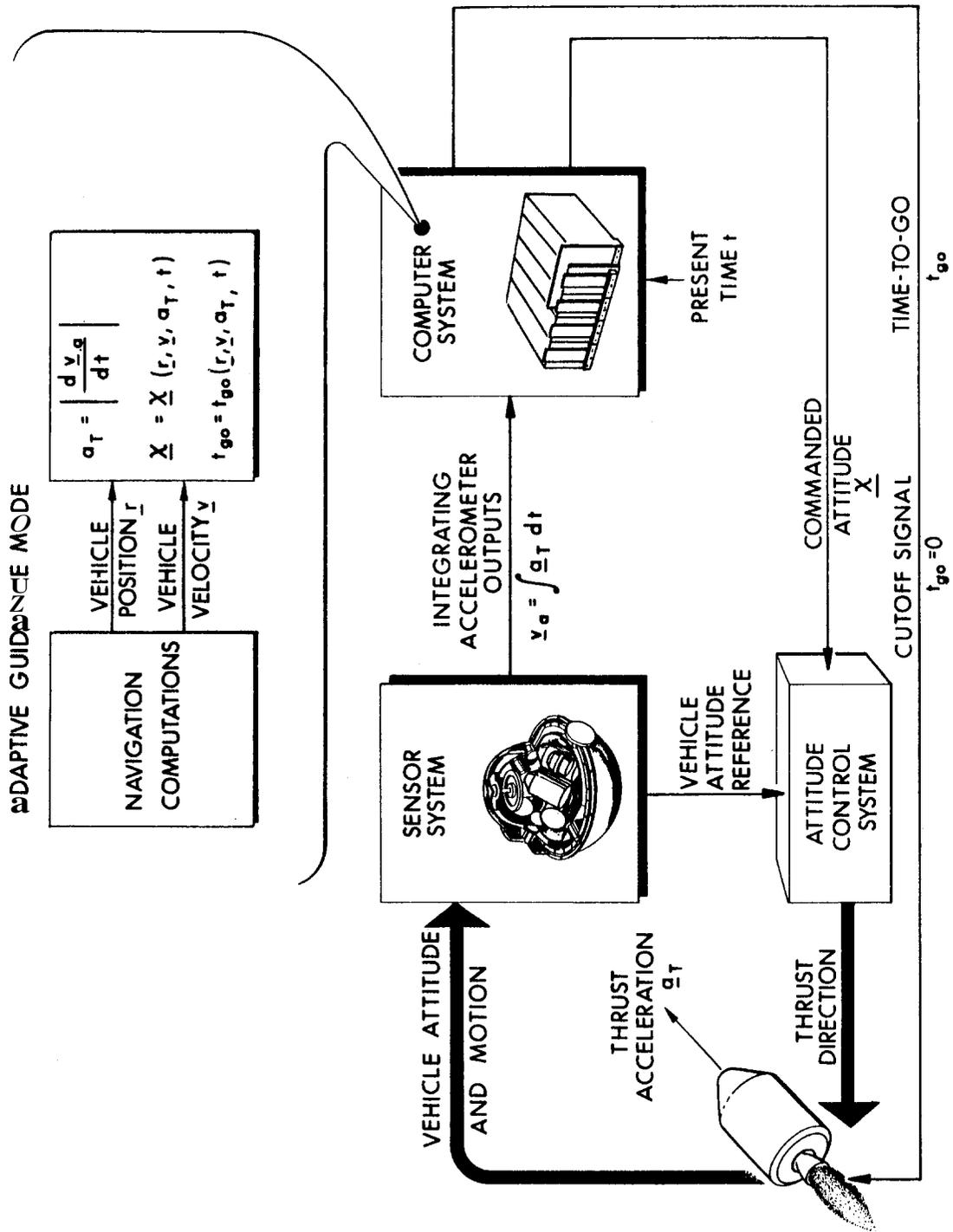
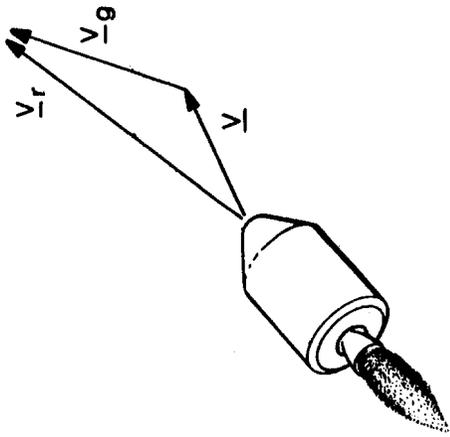


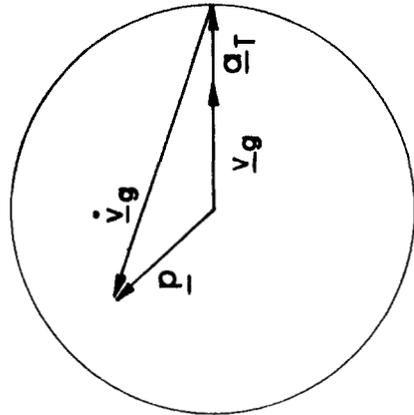
Fig. III-6 Powered Flight Guidance System



- $\underline{v}_r$  : REQUIRED IMPULSIVE VELOCITY
- $\underline{v}$  : ACTUAL VEHICLE VELOCITY
- $\underline{v}_g = \underline{v}_r - \underline{v}$  : VELOCITY-TO-BE-GAINED
- $\underline{g}$  : LOCAL GRAVITY VECTOR
- $\underline{a}_T$  : THRUST ACCELERATION VECTOR

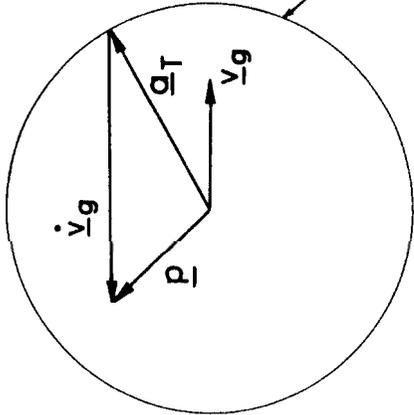
$$\dot{\underline{v}}_g = \dot{\underline{v}}_r - \dot{\underline{g}} - \underline{a}_T = \underline{p} - \underline{a}_T$$

I.  $\gamma = 0$



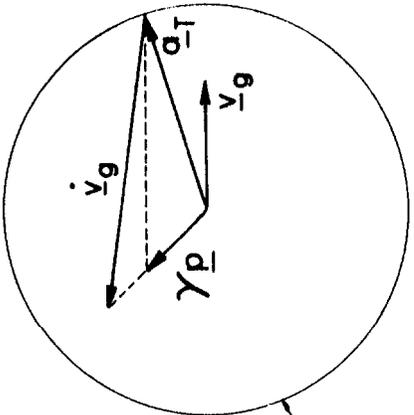
$$\underline{a}_T \times \underline{v}_g = 0$$

II.  $\chi = 1$



$$\underline{v}_g \times \dot{\underline{v}}_g = 0$$

III.  $0 \leq \gamma \leq 1$



$$\underline{v}_g \times (\gamma \underline{p} - \underline{a}_T) = 0$$

LOCUS OF  $\underline{a}_T$

Fig. III-7 Velocity-to-be-Gained Methods

A functional diagram illustrating the computation of the error signal required for control purposes is shown in Fig. III-8. The position, velocity and gravitation vectors are computed from the outputs of integrating accelerometers as described earlier in the section on navigation. The required impulsive velocity needed to achieve mission objectives is determined as a function of the position vector and used to calculate the velocity-to-be-gained. Numerical differentiation of the required velocity vector and the accelerometer outputs, using values stored from the previous sample time, provides two important ingredients of the error signals. When properly scaled, the system output is a vector rate of command whose magnitude is proportional to the small angular differences between the actual and commanded thrust acceleration vectors and whose direction defines the direction of vehicle rotation required to null the error. Near the end of the maneuver, when the velocity-to-be-gained is small, cross-product steering is terminated, the vehicle holds a constant attitude and engine cut-off is made on the basis of the magnitude of the  $\underline{v}_g$  vector.

This guidance technique is being considered for steering the Apollo Command Module during the following mission phases: (1) translunar injection which refers to the process of transfer from earth parking orbit to a trajectory linking earth and moon; (2) transfer from a hyperbolic approach trajectory to a circular orbit of the moon; and (3) transearth injection or transfer from a lunar orbit to an earth-bound trajectory.

For each of these maneuvers, the required impulsive velocity is as follows:

#### A. Translunar Injection

The required velocity for translunar injection is defined as that velocity, at the present position, that will place the vehicle on a conic passing through a specified time. Specifically, this velocity vector  $\underline{v}_r$  is calculated from

$$\underline{v}_r = \sqrt{\frac{\mu_E}{2}} [A(\underline{i}_c + \underline{i}_r) + B(\underline{i}_c - \underline{i}_r)] \quad (111-33)$$

where

$$A = \pm \sqrt{\frac{1}{s-c} - \frac{1}{2a}}$$

$$B = \text{sgn}(t_m - t) \sqrt{\frac{1}{s} - \frac{1}{2a}}$$

In these formulae,  $c$  is the linear distance from the present position  $\underline{r}$  to the target position  $\underline{r}_T$ ;  $s$  is the semi-perimeter of the triangle formed by the vectors  $\underline{r}$  and  $\underline{r}_T$ ;  $a$  is the semimajor axis of the conic;  $t$  is the time of flight; and  $t_m$  is the time to fly the minimum energy path from  $\underline{r}$  to  $\underline{r}_T$ . The choice

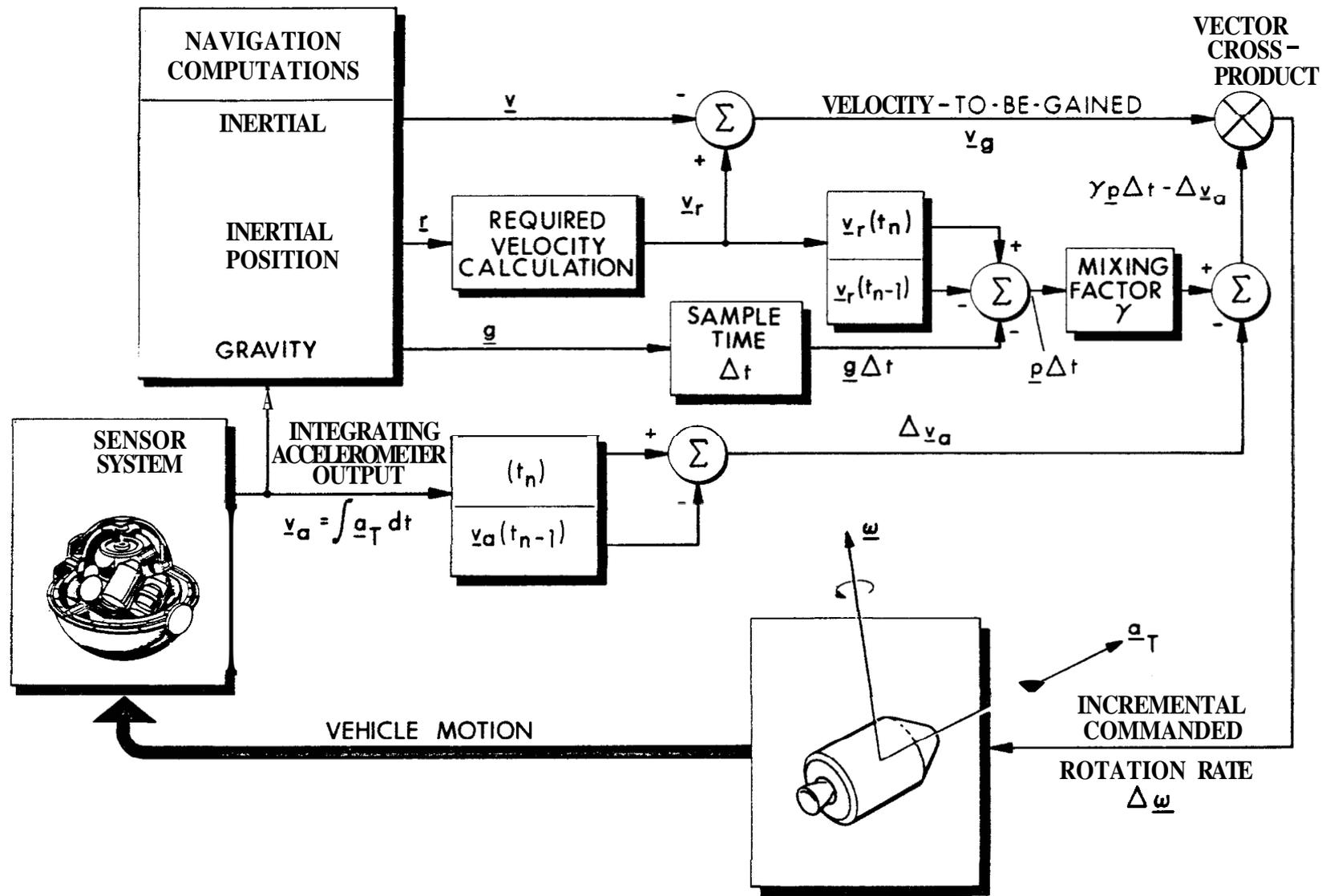


Fig. 111-8 Velocity-to-beGained Steering

of upper or lower sign in the expression for A is made according as the transfer angle is less than or greater than  $180^\circ$ , respectively.

The target point is actually offset by a calibrated amount from the desired position to account for gravitational perturbations. To simplify the computational load the fixed time requirement is readily approximated by holding constant the semimajor axis of the conic at a pre-determined value.

#### B. Circular Orbit Insertion

To guide a vehicle into a circular orbit of the moon by a rocket braking maneuver initiated on an approach trajectory, the vector  $\underline{v}_r$  may be defined as that velocity impulse required at the present position to circularize the orbit in a specified plane. If  $\underline{r}$  is the position vector of the vehicle relative to the moon and  $\underline{i}_n$  is the unit normal to the desired orbit plane, then

$$\underline{v}_r = \sqrt{\frac{\mu M}{r}} (\underline{i}_r \times \underline{i}_n) \quad (\text{III-34})$$

The shape and orientation of the final orbit is controlled by this means but direct control of the orbital radius is not possible. However, there is an empirical relationship between the final radius and the pericenter of the approach trajectory, so that a desired radius can be established by an appropriate selection of the approach orbit.

#### C. Transearth Injection

In the vicinity of the moon the spacecraft trajectory is very nearly hyperbolic. Therefore, the required velocity for transearth injection from lunar orbit may be conveniently defined by the magnitude  $v_\infty$  and direction  $\underline{i}_\infty$  of the asymptotic velocity  $\underline{v}_\infty$ . Thus

$$\underline{v}_r = \frac{v_\infty}{2} [(D + 1) \underline{i}_\infty + (D - 1) \underline{i}_r] \quad (\text{III-35})$$

where

$$D = \sqrt{1 + \frac{4\mu M}{r v_\infty^2 (1 + \underline{i}_r \cdot \underline{i}_\infty)}}$$

The direction in space that the thrust vector should be oriented at the beginning of a powered flight maneuver is determined from the equation

$$\underline{a}_T = \gamma \underline{p} + (q - \underline{i}_g \cdot \gamma \underline{p}) \underline{i}_g \quad (\text{III-36})$$

where  $\underline{i}_g$  is a unit vector in the direction of the  $\underline{v}_g$  vector and

$$q = \sqrt{a_T^2 - (\gamma p)^2 + (\underline{i}_g \cdot \gamma \underline{p})^2}$$

The quantities  $\underline{v}_g$  and  $\underline{p}$  are both continuous functions through the ignition point and, thus, their computation can be started to align the vehicle initially prior to the firing of the engine.

#### TERMINAL STATE VECTOR. CONTROL

The explicit technique just discussed is workable if it is possible, at thrust termination, to define the required velocity as a function of position and thereby eliminate the need for position control. On the other hand, when burn-out position and velocity are independently specified, an alternate guidance method, based on an explicit solution of the powered flight dynamics, is frequently applicable<sup>(10)</sup>. As examples, consider the problems of insertion of a vehicle into a circular orbit at a pre-specified altitude which lies in a prescribed plane, soft-landing a vehicle on the surface of the moon and orbital rendezvous.

The guidance computations, needed for solving explicitly the more general boundary value problem, involve a determination of the time remaining before thrust termination. For fixed thrust rocket problems, the termination time is calculated cyclically by an iteration process in such a manner as to control the final velocity along one coordinate axis. As a part of the calculation, the effective exhaust velocity of the rocket engine, based on a mathematical model of the engine performance, is needed.

When the vehicle is propelled by a controllable thrust engine, the magnitude of the thrust acceleration can be commanded to cause burn-out to occur at a pre-specified terminal time. In this case, the time-to-go is a trivial calculation. Prior to thrust initiation, the thrust termination time is chosen according to criteria which depend on the particular guidance problem. For orbital rendezvous, the time and desired terminal position and velocity are chosen from a knowledge of the target vehicle ephemeris. For a lunar landing, the terminal time is selected to maximize the initial thrust acceleration.

The development of an explicit steering equation for a controllable thrust engine, which will guide a vehicle to a desired set of terminal conditions, is based on the solution to the following simple variational problem. Let it be required to find the acceleration program  $\underline{a}(t)$ , which will minimize the functional

$$J = \int_t^{t_D} a(\tau)^2 d\tau \quad (\text{III-37})$$

where  $t$  is present time and  $t_D$  is the desired terminal time. If  $\underline{a}(t)$  is the total acceleration influencing the vehicle motion, then

$$\frac{d\underline{r}}{dt} = \underline{v} \quad \frac{d\underline{v}}{dt} = \underline{a}(t) \quad (111-38)$$

subject to the boundary conditions

$$\begin{aligned} \underline{r}(t) &= \underline{r} & \underline{r}(t_D) &= \underline{r}_D \\ \underline{v}(t) &= \underline{v} & \underline{v}(t_D) &= \underline{v}_D \end{aligned} \quad (111-39)$$

This minimization problem is readily solved using the calculus of variations. By introducing the two vector Lagrange multipliers  $\underline{\lambda}$  and  $\underline{\eta}$  we may combine Eqs. (111-37) and (111-38) in the form

$$J = \int_t^{t_D} \left\{ \underline{a}(\tau)^T \underline{a}(\tau) + \underline{\lambda}^T \left( \frac{d\underline{r}}{dt} - \underline{v} \right) + \underline{\eta}^T \left[ \frac{d\underline{v}}{dt} - \underline{a}(\tau) \right] \right\} d\tau$$

for which the Euler-Lagrange equations are found to be

$$\begin{aligned} \frac{d}{dt} \underline{\lambda}^T &= 0 \\ \frac{d}{dt} \underline{\eta}^T + \underline{\lambda}^T &= 0 \\ -2\underline{a}(t)^T + \underline{\eta}^T &= 0 \end{aligned} \quad (111-40)$$

The solution of Eqs. (111-40) yields

$$\underline{a}(t) = \underline{e}_1 + \underline{e}_2 t$$

and the constants of integration  $\underline{c}_1$  and  $\underline{c}_2$  are chosen to satisfy the boundary conditions (111-39). The final result is simply

$$\underline{a}(t) = \frac{4}{t_{go}} (\underline{v}_D - \underline{v}) + \frac{6}{t_{go}^2} [\underline{r}_D - (\underline{r} + \underline{v}_D t_{go})]$$

where

$$t_{go} = t_D - t$$

is the time-to-go before termination.

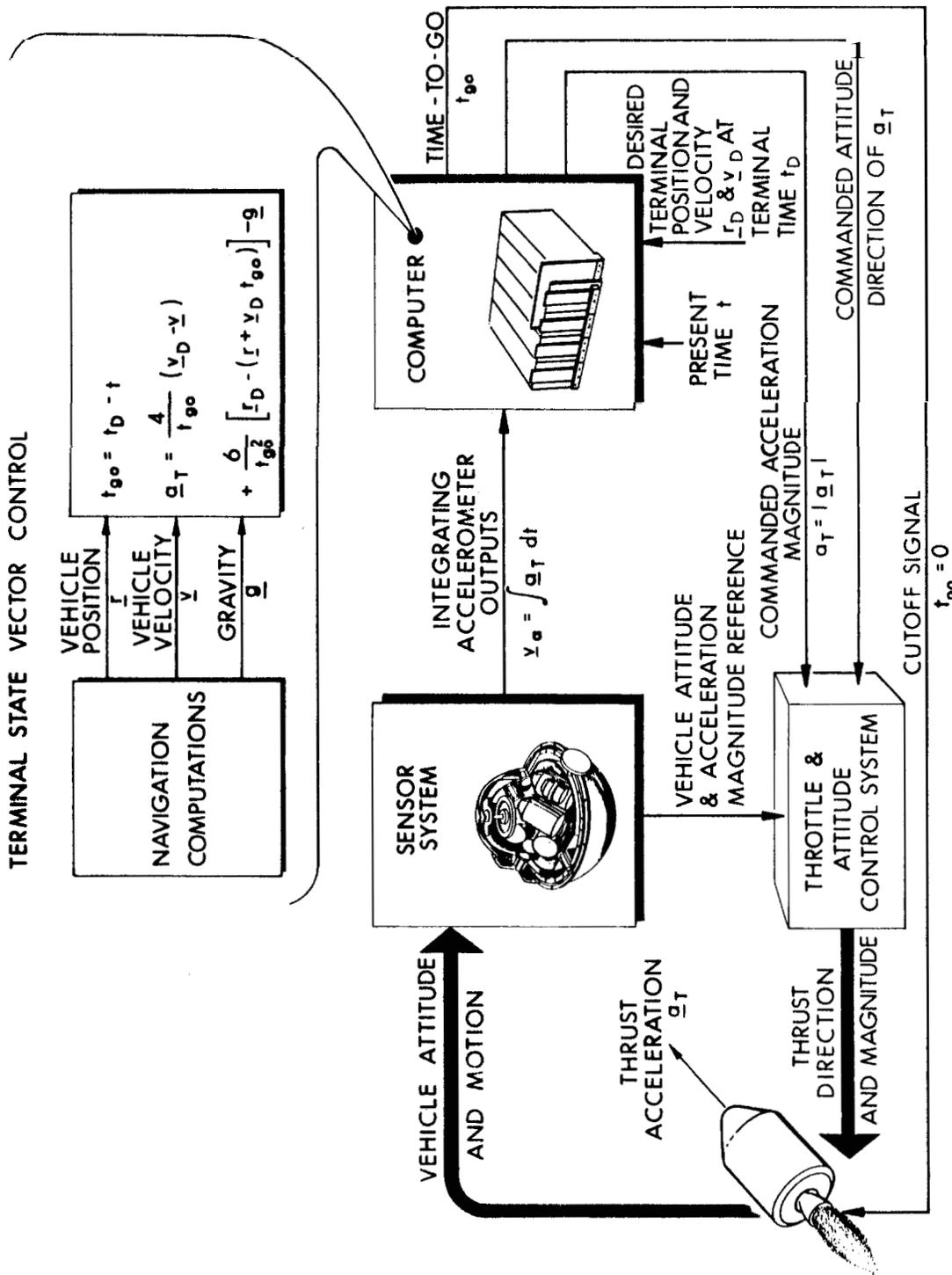


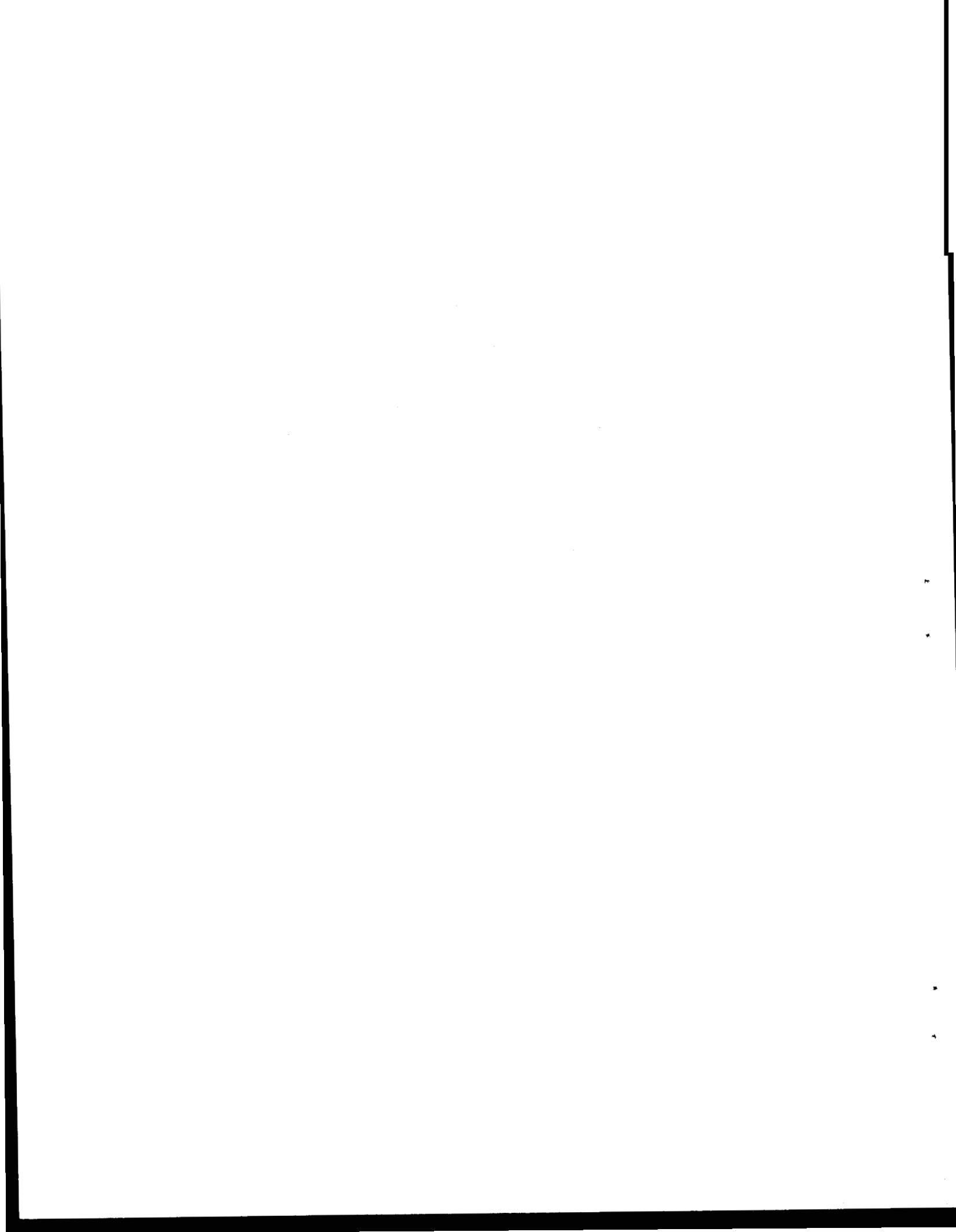
Fig. III-9 Variable Thrust Guidance System

In a guidance maneuver the total acceleration  $\underline{a}(t)$  is the sum of thrust acceleration  $\underline{a}_T(t)$  and gravity  $\underline{g}(r)$ . If the gravity vector were a constant, then the exact solution to the guidance problem would be

$$\underline{a}_T = \frac{4}{t_{go}} (\underline{v}_D - \underline{v}) + \frac{6}{t_{go}^2} [\underline{r}_D - (\underline{r} + \underline{v}_D t_{go})] - \underline{g} \quad (\text{III-41})$$

In problems of practical interest, the vector  $\underline{g}$  is not constant and the integral-square criterion of Eq. (III-37) is not appropriate for fuel minimization. However, it happens that Eq. (III-41) does provide a nearly optimum steer law for a wide variety of problems.

Figure III-9 illustrates the guidance computations required to achieve a given terminal position and velocity at a specified time with a throttleable rocket engine. From the navigation system the present position, velocity, gravitational acceleration and time are determined and the direction and magnitude of the thrust acceleration to be commanded are calculated. As the terminal conditions are approached, time-to-go approaches zero and the computation clearly becomes unstable. The singularity is readily avoided, with only slight loss in potential performance, by holding time-to-go constant in the guidance expression when it is less than a pre-assigned amount. Engine cut-off can then be commanded when the actual time-to-go reaches zero.



## Chapter 111-4

### MID-COURSE GUIDANCE

#### LINEARIZED GUIDANCE THEORY

Techniques of guidance and navigation of a spacecraft in interplanetary or cislunar space are often based on the method of linearized perturbations. The approach is to linearize the equations of motion by a series expansion about a nominal or reference orbit in which only first-order terms are retained. The resulting equations are far simpler and superposition techniques, as well as all of the powerful tools of linear analysis, may be exploited to obtain solutions to a wide variety of navigation and guidance problem.

Consider, for example, the guidance problem illustrated in Fig. III-10. A vehicle is launched into orbit at time  $t_L$  and moves under the influence of one or more gravity fields to reach a target point at time  $t_A$ . Let  $\underline{r}_0(t_n)$  and  $\underline{v}_0(t_n)$  be the position and velocity vectors at time  $t_n$  for a vehicle traveling along a reference path connecting the initial and final points. Because of errors, the true position and velocity vectors  $\underline{r}(t_n)$  and  $\underline{v}(t_n)$  will deviate from the associated reference quantities. If the deviations from the reference path are always small, so that linearization techniques are applicable, the velocity correction  $\Delta \underline{v}_n^*$  may be computed as a linear combination of the position and velocity deviations. The three-dimensional matrix  $C_n^*$  is the matrix of partial derivatives, with respect to the components of  $\underline{r}$ , of the components of the velocity vector  $\underline{v}^*$  required to reach the target from position  $\underline{r}$ .

For these calculations to remain valid it is, of course, necessary to restrict the magnitude of the deviations from the corresponding nominal values. Another disadvantage of the method is that all possible times of velocity corrections must be anticipated and associated values of the  $C^*$  matrix stored in the guidance computer. Also to provide an adequate launch window, a family of reference trajectories is mandatory and the guidance computer storage requirements rapidly become excessive using this approach.

#### EXPLICIT TECHNIQUES

The quantity of stored data required for mid-course guidance maneuvers can be markedly reduced if explicit techniques are employed using conic arcs suitably modified

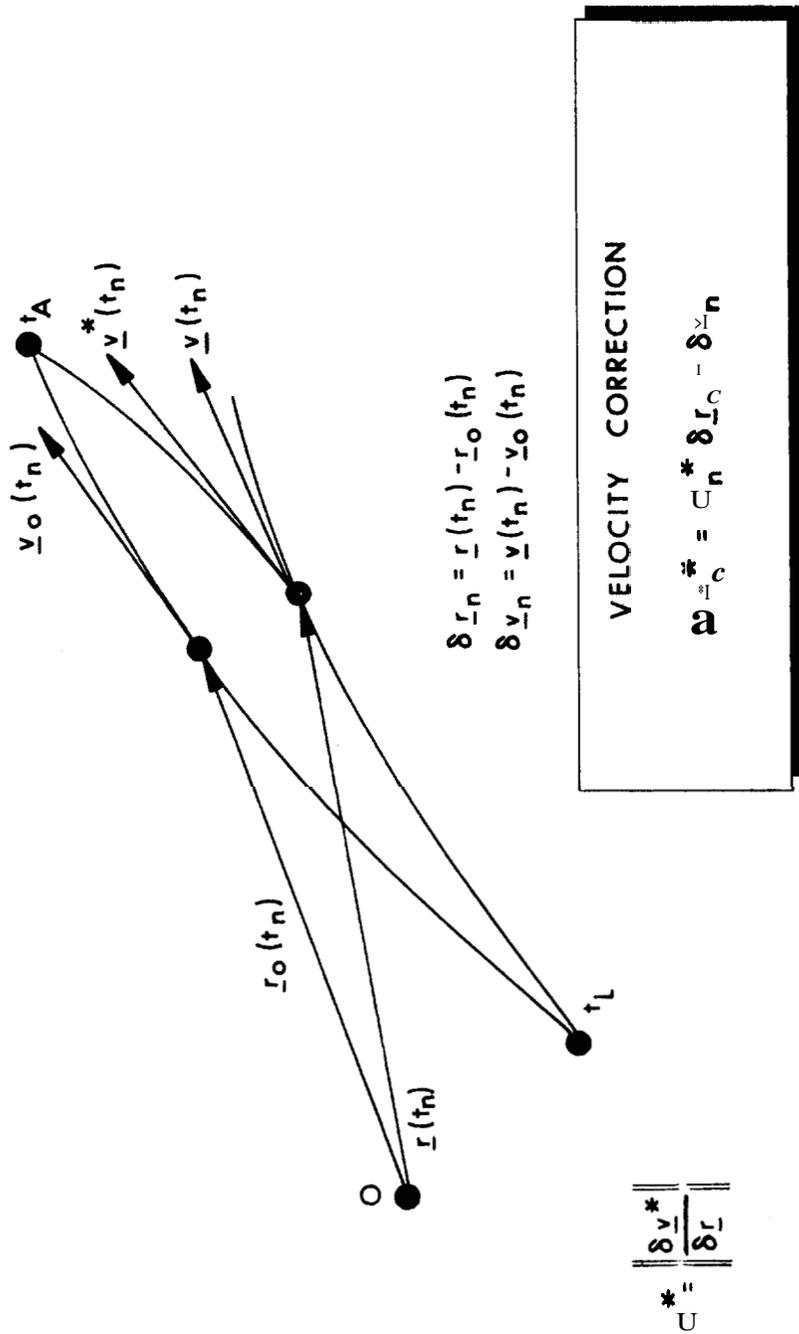


Fig. III-10 Linearized Guidance Theory

to account for small non-central force field effects. Both fixed and variable -time-of-arrival velocity corrections can be calculated and the procedures will be illustrated by two specific examples.

Fixed-Time-of-Arrival Guidance - Because of initial errors arising from a failure to inject the spacecraft in an appropriate trajectory to the moon, a velocity correction is frequently required after a few hours of coasting flight. During the post-injection phase, an accurate determination of the vehicle's orbit is made using navigation techniques as previously discussed. An intermediate target point  $\underline{r}_T$  is selected as the position on the lunar sphere of influence through which the reference vehicle would pass at the reference time.

Refer to Fig. III-11 and let  $\underline{r}$  and  $\underline{v}$  be the position and velocity estimates of the vehicle at the time a correction is to be made. Using the trajectory integration routine, which is a part of the coast phase navigation program, the position of the vehicle is extrapolated to determine the point  $\underline{r}_{T'}$  at which the spacecraft would be found at the target reference time if no corrective action were taken. By calculating the conic arc connecting the position vectors  $\underline{r}$  and  $\underline{r}_{T'}$  in the same time interval, the conic velocity vector  $\underline{v}_{c1}$  at  $\underline{r}$  is determined. The difference between the conic velocity and the vehicle's actual velocity is a good measure of the effect of lunar and solar perturbations. A second conic arc connecting the vehicle position vector  $\underline{r}$  and the desired target point  $\underline{r}_T$  produces the conic velocity vector  $\underline{v}_{c2}$ . If this velocity is corrected for the effect of perturbations, the velocity necessary to reach the desired target from position  $\underline{r}$  is obtained. Thus, an excellent approximation to the required velocity correction is just the difference between the two conic velocities. The computation may, of course, be repeated iteratively to achieve any desired degree of convergence. However, in practice, one computation cycle is usually sufficient.

Fixed-time-of-arrival guidance may be summarized as follows:

A. The conic velocity required at  $\underline{r}$  to arrive at  $\underline{r}_T$  is calculated from

$$\underline{v}_{c1} = \text{sgn}(\pi^2 - x) \sqrt{\frac{\mu_E [2 - xC(x)]}{4s}} (\underline{i}_c - \underline{i}_r) \pm \sqrt{\frac{-yC(y)}{4(s-c)}} (\underline{i}_c + \underline{i}_r) \quad (\text{III-42})$$

where  $\mu_E$  is the gravitational constant of the earth,  $\underline{i}_r$  is a unit vector in the direction of  $\underline{r}$ ,  $\underline{i}_c$  is a unit vector in the direction of  $\underline{r}_{T'} - \underline{r}$ , and  $s$  is the semi-perimeter of the triangle formed by  $\underline{r}$  and  $\underline{r}_{T'}$ . The quantities  $x$  and  $y$  are determined as the roots of the equations

1. INTEGRATE FORWARD TO DETERMINE  $\underline{r}'_T$
2. FIT CONIC FROM  $\underline{r}$  TO  $\underline{r}'_T$  ( $\underline{v}_{C1}$ )
3. FIT CONIC FROM  $\underline{r}$  TO  $\underline{r}_T$  ( $\underline{v}_{C2}$ )

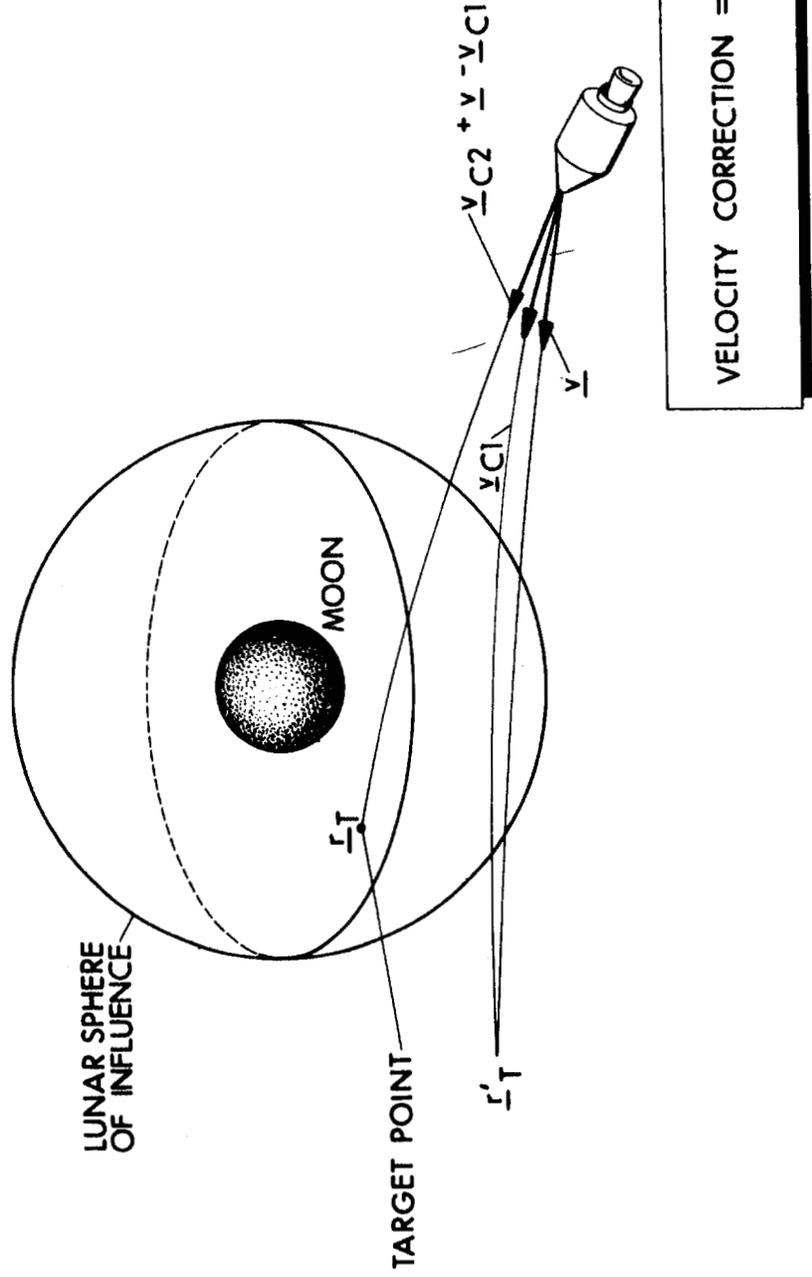


Fig. III-11 Fixed-Time-of-Arrival Midcourse Correction

$$\sqrt{\mu_E} \Delta t = \left[ \frac{s}{C(x)} \right]^{3/2} S(x) \pm \left[ \frac{s-c}{C(y)} \right]^{3/2} S(y) \quad (\text{III-43})$$

$$sC(y) = (s - c)x C(x)$$

where  $\Delta t$  is the time difference between the reference time of arrival and present time. The special transcendental functions  $S$  and  $C$  are defined in Eqs. (III-11). The choice of the upper and lower signs in Eqs. (111-42) and (III-43) is made according to whether the angle between  $\underline{r}$  and  $\underline{r}_T'$  is less than or greater than 180 degrees, respectively.

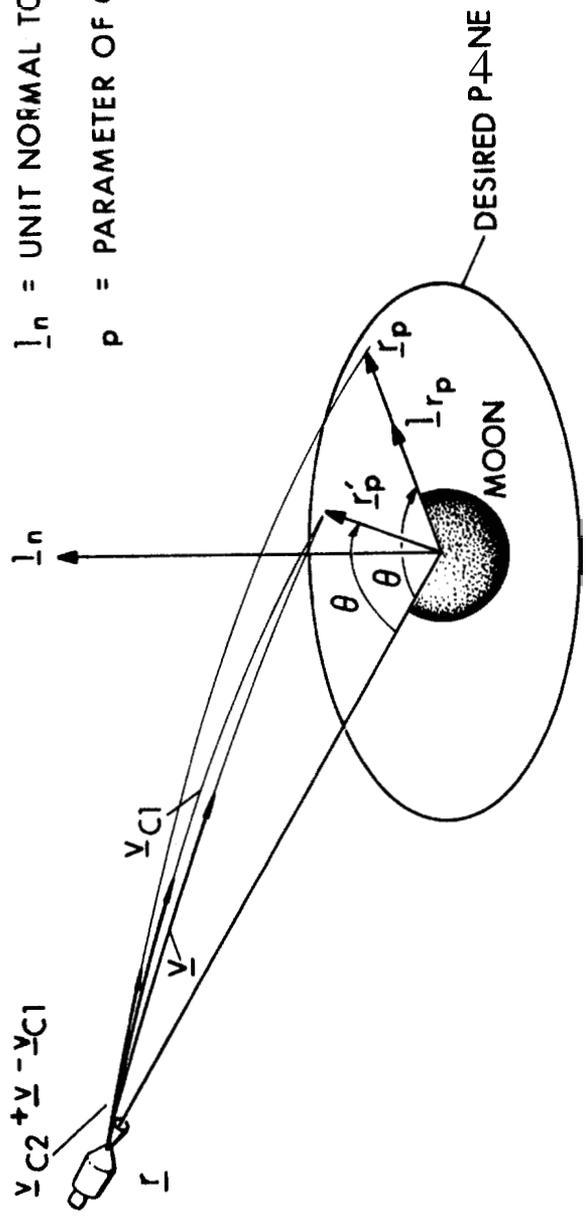
- B. The conic velocity  $\underline{v}_{c2}$  for attaining  $\underline{r}_T'$  is computed by repeating step A with  $\underline{r}_T'$  substituted for  $\underline{r}_T$ .
- C. The estimated velocity correction is then given by

$$\Delta \underline{v} = \underline{v}_{c2} - \underline{v}_{c1} \quad (\text{III-44})$$

Variable-Time-of-Arrival Guidance - When a velocity correction is made in the vicinity of the moon, the arrival time at perilune may be allowed to vary thereby reducing substantially the required velocity correction as well as the terminal velocity deviation from its nominal value. Specifically, let the desired terminal conditions at the moon be a specified altitude at perilune and a fixed plane in which the perilune vector is to lie. Again, as shown in Fig. 111-12, the trajectory is extrapolated forward in time to locate the perilune vector  $\underline{r}_p'$  which would result in the absence of a velocity correction. A conic arc with  $\underline{r}_p'$  as perilune and connecting the position vector  $\underline{r}$  is then determined to obtain a measure of the gravitational perturbation. The desired perilune vector  $\underline{r}_p$  is calculated from  $\underline{r}_p'$  by scaling its length to correspond to the required perilune distance and then rotating it into the required plane while keeping the central angle  $\theta$  fixed. A second conic arc with  $\underline{r}_p$  as perilune is calculated and the difference between the two conic velocities again provides an excellent approximation to the necessary velocity correction.

Theoretically, the desired plane should not be fixed in space, but should rotate with the moon. However, the change in perilune arrival time combined with the moon's own rotation leads to terminal deviations which are smaller than the navigation uncertainties. Hence, it is sufficiently accurate to aim for a fixed plane when approaching perilune.

$r_p$  = DESIRED PERILUNE ALTITUDE  
 $\hat{l}_n$  = UNIT NORMAL TO DESIRED PLANE  
 $p$  = PARAMETER OF CONIC ORBIT



INSENSIVITY DIRECTION

$$\hat{l}_D = \text{UNIT} \left[ (1 - \cos \theta)^2 \hat{l}_{r_p} + \left(1 - \cos \theta + \frac{p}{r_p}\right) \sin \theta \hat{l}_{r_p} \times \hat{l}_n \right]$$

VELOCITY CORRECTION =  $(v_{C2} - v_{C1}) - \hat{l}_D \cdot (v_{C2} - v_{C1}) \hat{l}_D$

Fig. III-12 Variable-Time-of-Arrival Midcourse Correction

Perilune guidance may be summarized as follows:

- A. The conic velocity  $\underline{v}_{c1}$  required at  $\underline{r}$  to attain pericenter at  $\underline{r}_p'$  is computed from

$$\underline{v}_{c1} = \frac{\sqrt{\mu_M p}}{r r_p' \sin \theta} \left\{ \underline{r}_p' - \left[ 1 - \frac{r_p}{p} (1 - \cos \theta) \right] \underline{r} \right\} \quad (\text{III-45})$$

where  $\theta$  is the angle between  $\underline{r}$  and  $\underline{r}_p'$ , and  $p$ , the parameter of the conic, is given by

$$p = \frac{r r_p' (1 - \cos \theta)}{r_p' - r \cos \theta} \quad (\text{III-46})$$

- B. The pericenter vector  $\underline{r}_p'$  is rotated into the desired plane and scaled to the desired length  $r_p$  by means of

$$\underline{r}_p = r_p \left[ \sqrt{1 + \beta \cos \theta} \text{UNIT}(\underline{i}_n \times \underline{r}_p') + \beta \underline{i}_n \times (\underline{i}_n \times \underline{r}_p') \right] \quad (\text{III-47})$$

where

$$\beta = - \frac{\cos \theta}{1 - (\underline{i}_n \cdot \underline{r}_p')^2}$$

The unit vector  $\underline{i}_n$  is normal to the desired plane in the direction of the angular momentum vector.

- C. The conic velocity  $\underline{v}_{c2}$ , required to attain pericenter at  $\underline{r}_p$ , is then calculated by repeating step A with  $\underline{r}_p$  in place of  $\underline{r}_p'$ .
- D. The magnitude of the required velocity correction may be further reduced by noting that there is a direction along which a velocity change may be made without altering the altitude at pericenter. If the component of velocity correction along this insensitive direction is deducted from the total correction, the effect will be simply a small rotation of the perilune vector  $\underline{r}_p$ . This insensitive direction is computed from

$$\underline{i}_D = \text{UNIT} \left[ - (1 - \cos \theta)^2 \underline{i}_r + \sin \theta \left( 1 - \cos \theta + \frac{p}{r} \right) \underline{i}_r \times \underline{i}_n \right] \quad (\text{III-48})$$

where  $\underline{i}_r$  is a unit vector in the direction of  $\underline{r}_p$ .

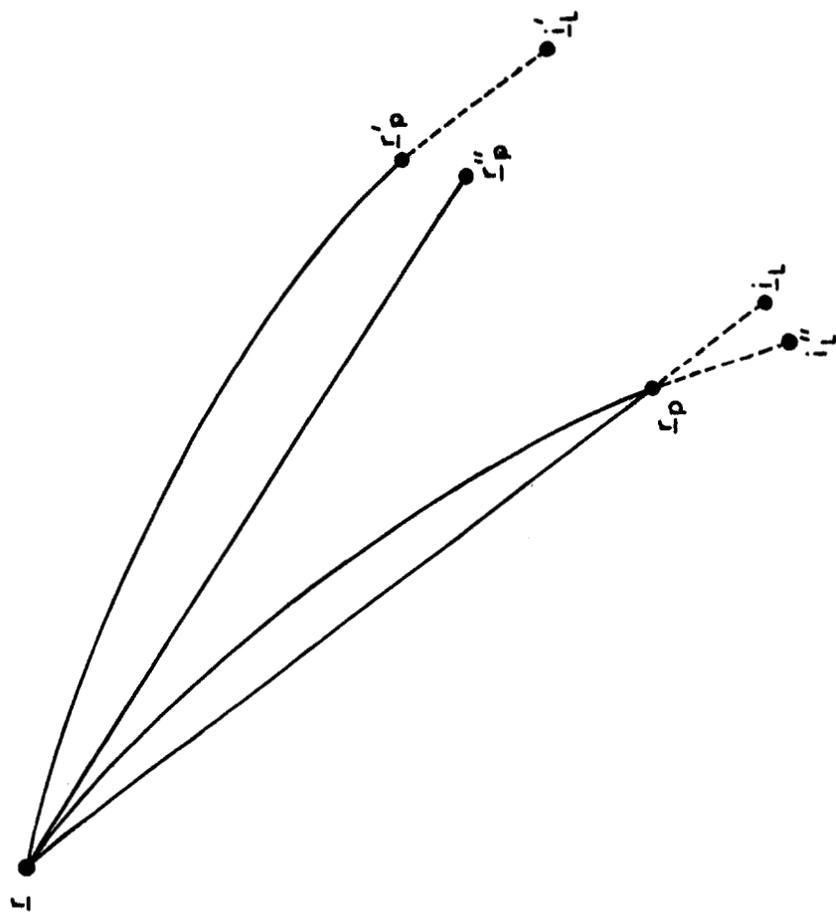


Fig. III-13 Geometry of Perigee Guidance

E. The estimated velocity correction is then given by the component of the vector  $\underline{v}_{c2} - \underline{v}_{c1}$  in the plane perpendicular to  $\underline{i}_D$  and is calculated from

$$\Delta \underline{v} = (\underline{v}_{c2} - \underline{v}_{c1}) - \underline{i}_D \cdot (\underline{v}_{c2} - \underline{v}_{c1}) \underline{i}_D \quad (\text{III-49})$$

During transearth flight it is not sufficient to aim for a fixed plane when making a velocity correction to the vicinity of the earth. The desired terminal conditions are a vacuum perigee distance (which is equivalent to an entry angle) and a landing site fixed to the earth. This type of velocity correction, called perigee guidance, is an extension of perilune guidance with the plane determined so that the spacecraft will be directed to the desired landing site.

Inherent in perigee guidance is a timing problem which necessitates taking into account the correction to be made in estimating the time of arrival at perigee. The change in perigee time due to a correction that alters the perigee distance from  $r_P'$  to  $r_P$  is given by the empirically determined formula  $kr(r_P' - r_P)$ , where  $k$  has been found experimentally to be  $16 \times 10^{-10}$  hr/mi<sup>2</sup>. A simple calculation shows that a velocity correction, which is made at the lunar sphere of influence (about 200,000 miles from the earth) and which changes the perigee distance by 500 miles, alters the perigee arrival time by nearly ten minutes.

Let  $\delta t_P$  be the estimated deviation in perigee arrival time in hours,  $\underline{i}_{L_0}$  a unit vector in the direction of the landing site at the nominal time of arrival, and  $\alpha_0$  the nominal angle from perigee to  $\underline{i}_{L_0}$ . Assuming that the spacecraft travels on the average at circular orbital speed during entry, the deviation in the angle through which the earth rotates is given by

$$\delta A = \frac{\pi}{12} \delta t_P + \frac{\alpha - \alpha_0}{16} \quad (\text{III-50})$$

where  $\alpha$  is the actual angle from perigee to the landing site. When the earth rotates through an angle  $\delta A$ , the landing site changes from  $\underline{i}_{L_0}$  to  $\underline{i}_L$  according to

$$\underline{i}_L = \begin{pmatrix} \cos \delta A & -\sin \delta A & 0 \\ \sin \delta A & \cos \delta A & 0 \\ 0 & 0 & 1 \end{pmatrix} \underline{i}_{L_0} \quad (\text{III-51})$$

The angle  $\alpha$  satisfies

$$\alpha = \begin{cases} \cos^{-1}(\underline{i}_r \cdot \underline{i}_L) - \theta \\ 2a - \cos^{-1}(\underline{i}_r \cdot \underline{i}_L) - e \end{cases} \quad (\text{III-52})$$

where the choice of the first or second equation is made according to whether the angle between  $\underline{i}_r$  and  $\underline{i}_L$  is greater or less than 180 degrees, respectively. The desired plane is then determined by the initial position  $\underline{r}$  and the landing site vector  $\underline{r}_L$  calculated from Eqs. (III-50), (III-51) and (III-52).

If the spacecraft trajectory was, indeed planar, the steps outlined in the discussion of perilune guidance would be adequate for calculating the velocity correction. Unfortunately, the non-planar characteristics are sufficiently pronounced that an additional step is required before the perilune guidance technique can be applied.

In Fig. III-13 is represented, schematically, and edge-wise view of the trajectory problem in which a planar path appears as a straight line. The unmodified perilune guidance method would cause the vehicle to head for a landing site at  $\underline{r}_L''$  instead of  $\underline{r}_L'$ . To counteract this effect, the vector  $\underline{r}_p'$  is projected ahead to  $\underline{i}_L'$ , the position the spacecraft would achieve on an uncorrected trajectory at the time the target landing site is at  $\underline{i}_L$ . Then a false perigee position  $\underline{r}_p''$ , in the plane determined by  $\underline{r}$  and  $\underline{i}_L'$ , is used in place of  $\underline{r}_p'$ .

Perigee guidance may be summarized as follows:

- A. The estimated change in time of arrival at perigee is calculated from

$$\delta t_P = \delta t_P' + kr(r_P - r_P') \quad (III-53)$$

where  $\delta t_P'$  is the deviation in perigee time obtained in the extrapolation of  $\underline{r}$  and  $\underline{v}$  to perigee  $\underline{r}_p'$ .

- B. The position of the desired landing site is found by solving the transcendental equations (III-50), (III-51) and (III-52) for  $\underline{i}_L$  and  $\alpha$ .
- C. The unit vector  $\underline{i}_L'$  is calculated from

$$\cos \alpha + i \sin \alpha \quad (111-54)$$

where  $\underline{i}_r$  and  $\underline{i}_v$  are unit vectors in the directions of the position and velocity vectors at  $\underline{r}_p'$ .

- D. The false perigee vector is located using Eq. (111-47) with the substitutions

$$\begin{aligned} \pm \text{UNIT}(\underline{i}_r \times \underline{i}_L') &\rightarrow \underline{i}_n \\ r_p' &\rightarrow r_p \\ \underline{r}_p' &\rightarrow \underline{r}_p \end{aligned}$$

E. The unit vector normal to the desired plane is computed from

$$\underline{i}_n = \pm \text{UNIT}(\underline{i}_r \times \underline{i}_L)$$

F. The remaining calculations are exactly as outlined in steps A through E for perilune guidance. In the determination of the vector  $\underline{i}_n$  in steps D and E above, the upper or lower sign is selected according to whether the angle between  $\underline{i}_r$  and either  $\underline{i}_L'$  or  $\underline{i}_L$ , whichever is relevant, is greater or less than 180 degrees, respectively.

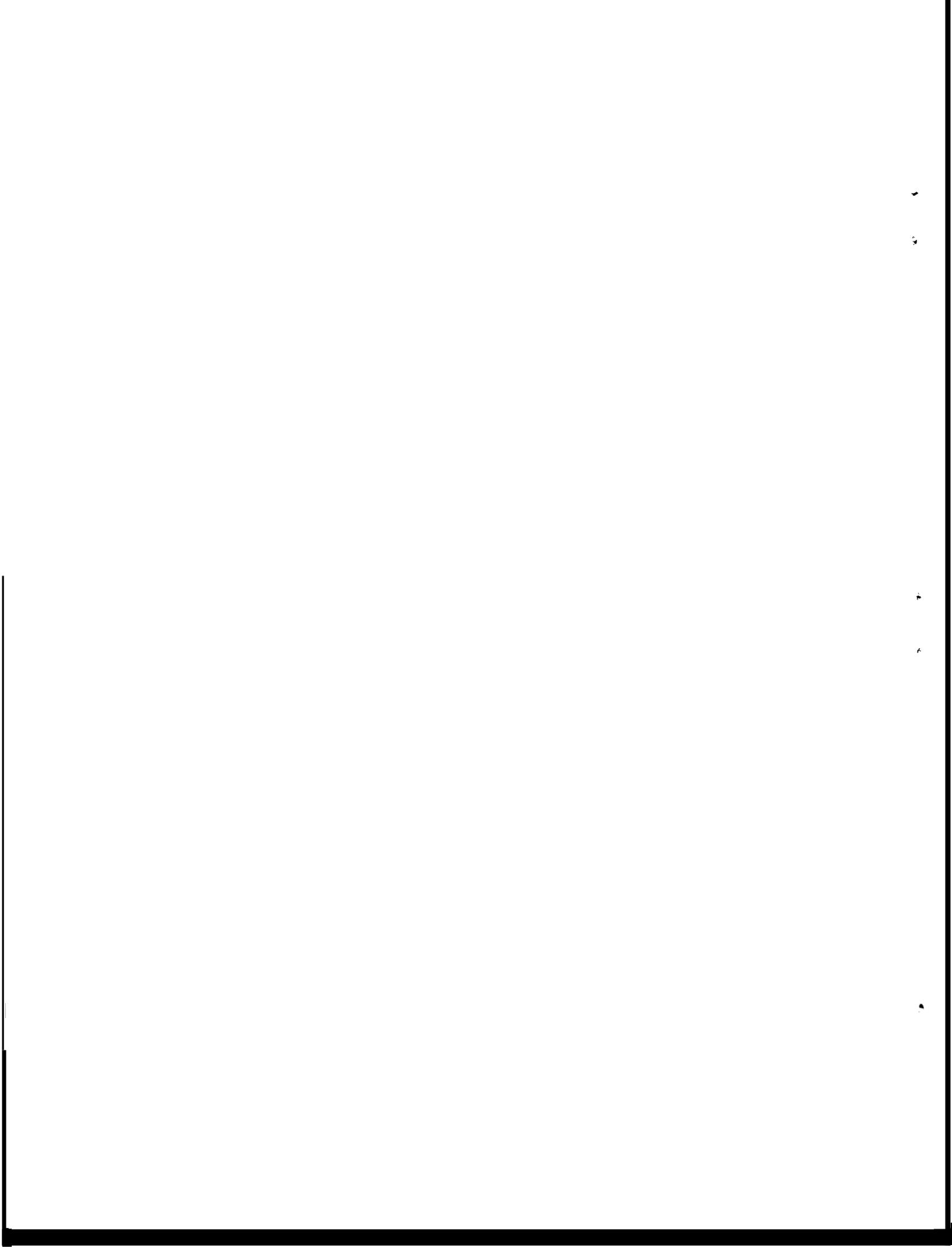
#### OPTIMUM GUIDANCE POLICIES

In an effort to compensate for initial errors by means of a mid-course velocity correction, new errors will inevitably be made which must again be corrected. The problem of determining when and how to perform impulsive corrections to a spacecraft orbit can be classified as a multistage decision process. Various guidance policies have been proposed and the new mathematical techniques of dynamic programming and steepest ascent optimization theory have been used with some success in an attempt to formulate an optimum policy. Several useful guidance policies are described and compared by Curkendall and Pfeiffer<sup>(11)</sup> in a recent paper.

As with all applications of dynamic programming techniques, the computational requirements are extensive and rapidly become impractical as the dimension of the problem increases. The results obtained by Arcon<sup>(12)</sup> and Orford<sup>(13)</sup> using this approach have been rather limited and numerical examples are restricted to problems of only one or two dimensions rather than six.

Denham and Speyer<sup>(7)</sup> have formulated a method of improving a velocity correction schedule iteratively using steepest ascent techniques in a manner similar to that mentioned earlier for optimizing measurement schedules. No numerical results are yet available.

Unfortunately, the Monte Carlo approach to the determination of velocity-correction times remains the most practical method. The lack of mathematical elegance and an over-abundance of computer usage time, which characterize this technique, are, nevertheless, balanced by the capability of utilizing a realistic, rather than an oversimplified, mathematical model.



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