$1/f$ Noise, Telegraph Noise

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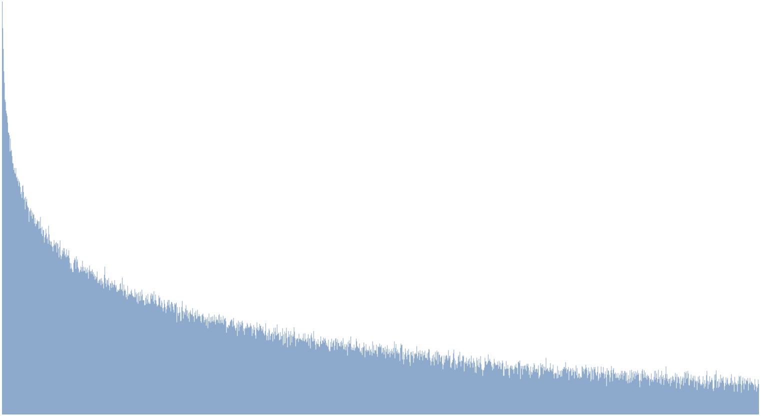
Content

1/f Noise



What are we going to talk about?

- In 1925, J. B. Johnson examined fluctuations in electron emission from a heated filament
- For decreasing frequencies in low frequency regime, the fluctuation strength increased
- So far only white noise had been observed (W. Schottky, 1918)
- This low frequency noise obeys a power law (noise intensity $\sim 1/f^{\gamma}, \gamma \geq 0$)

1/f Noise as Part of Total Noise

Classification:

- Spectral density function of power law noise can be decomposed into different parts:

$$S(f) = c_0 f^0 + c_{-1} f^{-1} + c_{-2} f^{-2} + \dots$$

- This is not an approximation; every term represents different type of noise!
- Most important terms are $f^0,\ f^{-1}$ and f^{-2}
- (there are systems, where also $f^{-\frac{1}{2}}$ or $f^{-\frac{3}{2}}$ appear)

Names and Terminology

Other common names for 1/f noise in literature:

- 1/f type noise, $S(f) \sim f^{\gamma}$, with $\gamma \approx -1$
- low frequency noise
- pink noise (analogy to optic spectra)
- excess noise
- flicker noise

What do we mean by "low frequencies"?

1/f type noise is typically dominant below corner frequency $f_c \lesssim 10^2 ... 10^6$ Hz.

Occurrence of 1/f Noise

Where does 1/f noise occur? What quantities are affected? – Answer: Almost everything you can imagine!

Examples:

- measured quantities of electric circuits and components (I, U, R)
- frequency of quartz crystal oscillators; affects time measurement precision
- rate of traffic flow on highways
- astronomy: number of sunspots apart from regular cycles, light intensity of stellar objects (e.g. quasars)
- loudness and pitch of music and speech
- economic and financial data
- biological systems
- and many, many more...

Statistical Properties of 1/f Noise

Paradox of infinite power:

- variance = total power contained in fluctuations of x

$$\sigma_x^2(f_l, f_h) = \int_{f_l}^{f_h} S_x(f) df \sim \ln \frac{f_h}{f_l}$$

- $\Rightarrow$ total power diverges at both frequency limits $f_l \rightarrow 0$ and $f_h \rightarrow \infty$
- this paradox has not yet been resolved!
- upper limit not a problem, since f_h never accessible to measurement due to dominant white noise
- for lower limit, no cutoff frequency was ever observed; analyses have shown no deviations down to $10^{-6.3}$Hz in operational amplifiers [Caloyannides, 1974]

Correlation Function and Memory of the System

Consider measurement with finite frequency bandwidth between f_1 and f_2, $f_2 \gg f_1$:

Correlation function is then

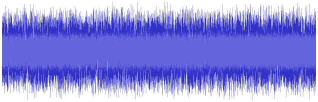
$$\frac{\psi_x(t)}{\sigma_x^2} \approx 1 - \frac{1}{\ln f_2/f_1}[C + \ln(2\pi f_2 t)], \qquad C = \text{ constant}$$

$\rightarrow \psi_x(t)$ is very slowly (logarithmically) decaying with time

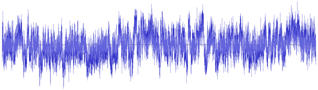
system has a very long memory $\Leftrightarrow$ present state strongly dependent on the past

Comparison to Other Types of Noise

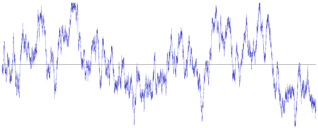
Comparison of white noise, pink noise and red noise:
Let's look at and listen to some signals
(from http://whitenoisemp3s.com, length: 1 second)



- White noise ($\gamma = 0$);
  constant variance, correlation function is $\sim \delta(t)$



- Pink noise ($\gamma = -1$);
  variance increases like $1 + \ln(t/\tau)$, correlation function decreases only slowly

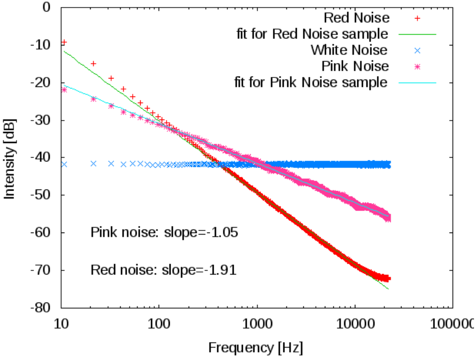


- Red noise ($\gamma = -2$, "Random Walk"); variance increases linearly with time, constant correlation function

Spectra of Different Noises

Spectral density functions of above noise samples:

- notice log-scale on both axes → slope gives exponent γ
- different power law behavior of noise samples is apparent
- numerical analysis confirms exponents that are suggested by creator



Origin of 1/f Noise

What causes 1/f noise?

- fundamentally different systems exhibit 1/f noise
  $\Rightarrow$ highly improbable that identical mechanism causes noise in all of them
- however, mechanism gives rise to similar or identical mathematical properties
- for most systems, origin of 1/f noise is completely unknown or at least subject to (controversal) debate
- for some systems, there exist theories; none of them capable of explaining all details

Resistance 1/f Noise in Electric Circuits

Any resistance shows fluctuations, often with 1/f

Is this due to temperature fluctuations?

$$S_R(f) \stackrel{?}{=} S_T(f) \cdot \left(\frac{\partial R}{\partial T}\right)^2$$

- A model by [Voss & Clarke, 1976] for temperature fluctuations as source of resistance 1/f noise was investigated in wide temperature range (100 – 600K) [Eberhard & Horn, 1977] but finally refuted
- Others, too, [Scofield, 1981] found no dependence of voltage 1/f noise on temperature fluctuations in thermally coupled thin metal films
- ⇒ temperature fluctuations are unlike to cause 1/f noise!

Consider fluctuations of resistance itself, then. . .

Example: Conductivity of Semiconductors

The Hooge parameter α

In an effort to systematically collect data on 1/f noise, F. H. Hooge introduced empirical relation:

$$\frac{S_R}{R^2} = \frac{\alpha}{fN},$$

α: normalized measure for relative noise; N: number of conductance electrons. First estimates gave $\alpha \approx 2 \cdot 10^{-3}$.

Since noise heavily depends on sample preparation (growing, doping, surface properties, contacting), α is only meaningful, if samples are somehow similar!

Example: Conductivity of Semiconductors

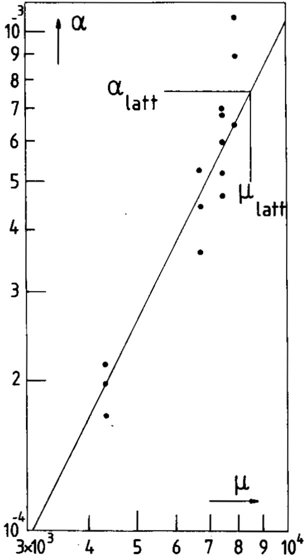
A few comments are necessary:

- noise is bulk effect, not surface effect
- specimen should be rather homogeneous
- $\rightarrow$ relation between α and electron mobility μ was found

(remember mobility: $\mu\vec{E} = \vec{v_D}$)

Why are we interested in μ?
The conductivity can be written as

$$\sigma = q \cdot n \cdot \mu$$



[Hooge, 1994]

Results

Does resistance fluctuation come from electron number or mobility?

Again, several experiments were performed. Analyses of the effect of Δn and $\Delta \mu$ on α showed that experimental results were much better described by mobility fluctuations.

Noise obeying the Hooge relation is also called α noise. Further investigation showed that it is caused by lattice scattering.

Since electron mobility is linked to lattice vibrations through scattering, one could finally interpret 1/f-like conductivity noise as phonon number fluctuations.

Applications of 1/f Noise

Is there anything useful that we can do with 1/f noise?
Yes, there is:

- remember: 1/f noise power $\sim \ln \frac{f_h}{f_l}$
- for constant frequency ratio we get constant power
  $\Leftrightarrow$ every decade contains same amount of power
- used for calibration of high fidelity audio equipment (too heavy load for high frequency speakers with white noise)

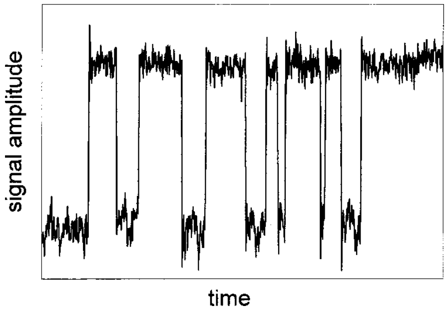
Random Telegraph Noise

image from http://www.trueller-snacks.de

Do you like Popcorn?

What is random telegraph noise?

- commonly used to describe resistance fluctuations that show random switching between several, often only two, discrete values
- in literature you will often find: "Random Telegraph Noise" (RTN) or "Random Telegraph Signal" (RTS)
- also termed "burst noise" or "popcorn noise"
- signal resembles telegraph signals with two different "states" – ON and OFF



[Yuzhelevski, 2000]

Noise in Small Samples

RTN often occurs in very small specimen of:

- semiconductor components, e.g. MOSFETs, p-n junctions and resistors (electrical quantities: U, I, R)
- metal contacts, e.g. nanobridges, metal-insulator-metal tunnel junctions
- quantum dots (light intensity)

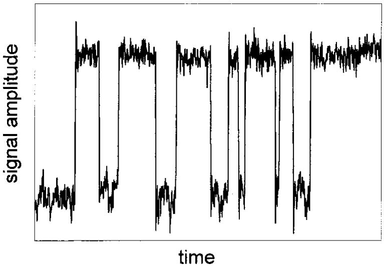
How small is "small"?

Examples of some experiments:

- MOSFETs: length scales $0.1 \ldots 1\mu m$ [Ralls et al., 1984]
- Cu nanobridges: volume $V = 40 \ldots 8000 nm^3$, width 3...40nm [Ralls & Buhrmann, 1988]
- tunnel junctions: active cross-section $A = 0.03 \ldots 2\mu m^2$, thickness $d \sim 1nm$ [Farmer, 1987]

Some general properties

Let's look at the temporal behaviour!



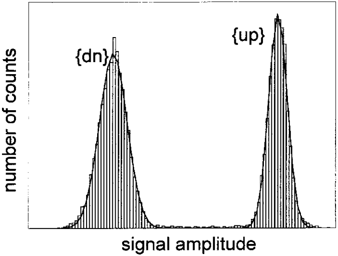
- time between switching processes is random but signal values are time-independent
- time scale of actual switching process much shorter than time interval during which system remains in one state
- future state of system only depends on present state, not on history ⇔ system has no "long-term memory"

Statistical Properties

Consider a two level fluctuation of quantity x:

w_1, w_2: probability of finding system in state (1) or (2);

τ^{-1} = total rate of transitions between (1) and (2)



[Yuzhelevski, 2000]

- correlation function:

$$\psi_x(t) = \Theta(t) w_1 w_2 (x_1 - x_2)^2 e^{-t/\tau}$$

- spectral density function (Lorentzian shape):

$$S_x(f) = 4 w_1 w_2 (x_1 - x_2)^2 \frac{\tau}{1 + (2\pi f)^2 \tau^2}$$

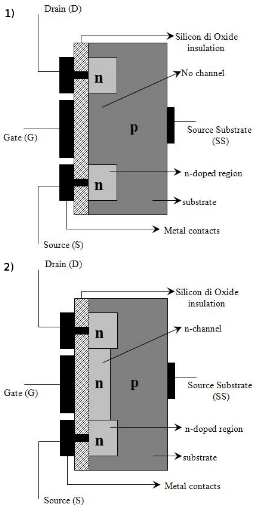
- mathematical tool to describe RTN: discrete Markov processes

What causes RTN?

In general, different mechanisms → let's look at an example: MOSFETs

1)

2)



What is a MOSFET?

- Metal-Oxide-Semiconductor Field Effect Transistor
- left: MOSFET schematically, 1) without and 2) with applied voltage G–SS
- gate voltage leads to bending of band structure, crossing of Fermi level
- ⇒ channel at interface with free charge carriers
- source–drain current can be controlled by voltage at gate, not by current!

RTN in MOSFETs

- fluctuations between constant discrete values $\Rightarrow$ assume single electron processes
- electrons are captured and released again by "traps" (e.g. impurities, lattice defects) in nearby oxide layer, within few Å from interface
- trapping and releasing causes conduction electron number N to change by 1
- (electrostatic field of trapped electrons changes mobility of other electrons in addition to number fluctuations, magnitude difficult to estimate)

What causes RTN?

Consequences of number fluctuations ΔN

- number fluctuations ΔN directly affect conductivity $\sigma = nq\mu$
- result: current through structure changes
- amplitude of fluctuation: $\Delta R/R = \Delta N/N$

MOSFETs [Ralls et al., 1984]: $\Delta R/R \sim 10^{-3} \Rightarrow N \lesssim 1000.$

Ratio RTN vs. background noise: $\Delta R/R_{backgr} = 3 \ldots 100$

In larger systems, RTN disappears and often 1/f noise arises – is it caused by number fluctuations after all?

Summary

Summary

1/f Noise

- occurs almost everywhere, universal type of noise
- not well understood or explained, unresolved fundamental problems
- 1/f type noise in resistance of semiconductors described by empirical relation and can be attributed to mobility fluctuations

RTN

- random switching between discrete values
- observed in very small structures
- interesting statistics, system has no long-term memory
- single electron trapping processes likely to cause RTN in MOSFETs

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