Non-Standard Interactions & the study of $\tau \to \eta\pi\nu_\tau$

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Motivation

The decays of $\tau - leptons$ provide a good laboratory for the analysis of various aspects of Particle Physics.
In particular, $\tau - decays$ into hadrons allow us to study hadronization of vector and axial vector currents and thus can be used to study to determine intrinsic properties of hadronic resonances that, together with chiral symmetry, governs the dynamics of these processes.
Consider a general decay

$$\tau(k) \to X(p_x) + \nu_\tau(k')$$

X is any number of hadrons allowed by energy conservation.
The T matrix is

$$T = -\frac{G'}{\sqrt{2}} \langle X | J^\mu | 0 \rangle \overline{u}(k') \gamma_\mu (1 - \gamma_5) u(k)$$

$$J^\mu = V^\mu - A^\mu$$

Example (1):

$$X = \pi^- \pi^0 \pi^+$$

[Ref: D. Gomez Dumm et al. arXiv:0911.4436]
In the iso-spin limit

$$\langle \pi^-(p_1)\pi^0(p_2)\pi^+(p_3)|A^\mu|0\rangle \quad = V_1^\mu F_1 + V_2^\mu F_2 + \theta^\mu F_p$$

$$\begin{aligned} V_1^\mu &= (g^{\mu\nu} - \frac{\theta^\mu \theta^\nu}{\theta^2})(p_1 - p_3)_\nu \\ V_2^\mu &= (g^{\mu\nu} - \frac{\theta^\mu \theta^\nu}{\theta^2})(p_2 - p_3)_\nu \\ \theta^\mu &= p_1^\mu + p_2^\mu + p_3^\mu \end{aligned}$$

F_1 and F_2 derive a $J^p = 1^+$ transition while F_p accounts for $J^p = 0^-$ transitions: very much suppressed, $O(m_{\pi^2})$

Example (2):

$$X = \pi^+\pi^0, \; J^\mu = V^\mu$$

$$\langle\pi^+(k)\pi^0(p)|V^\mu|0\rangle = -\sqrt{2}G_F V_{ud}[f_+(t)(p-k)^\mu - f_-(t)(p+k)^\mu]$$

$$t = (p+k)^2$$

Using iso-spin limit and CVC

$$f_+(t) = F_\pi(t), \quad f_-(t) = 0$$

$F_\pi(t)$ is the pion electromagnetic form factor

$$\begin{aligned} T^\mu &= \langle\pi^+(k)\pi^-(p)|V^\mu_{em}|0\rangle \\ &= (p-k)^\mu F_\pi(t) \end{aligned}$$

$F_\pi(0) = 1$, normalization condition for the pion electric charge

Theoretical Frame work

At low energies $E < M_p(770Mev)$, Chiral symmetry is an approximate symmetry of QCD which then derives the interaction of light pseudoscalar mesons. However, in general this approximation cannot be excited to the intermediate energy range, in which the dynamics of resonant states play a major role.

Example I:

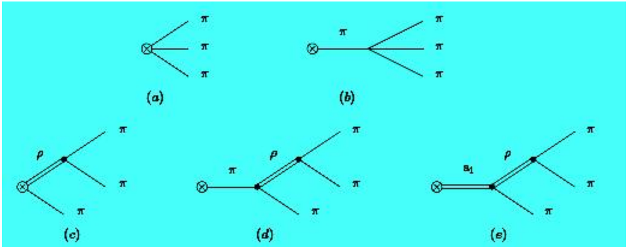


Figure 1: Diagrams contributing to the hadron axial-vector form factors F_i : (a) and (b) contribute to F_1^X, (c) and (d) to F_1^R and (e) to F_1^{RR}.

Example II: For $t \geq 4m_\pi^2$, $t = (p+4)^2$ is time-like
$F_\pi(t)$ being analytically continued to the space-like region, corresponds to

$$\langle \pi^+(k) | V^\mu_{em} | \pi^+(-p) \rangle$$

related to $\langle \pi^+(k)\pi^-(p) | V^\mu_{em} | 0 \rangle$, by assuming crossing symmetry.
Perturbative QCD gives

$$\lim_{t \to -\infty} F_\pi(t) \sim \frac{ds(t)}{t}$$

An important role is played by the dispersion relation.

$$F_\pi(t) = \frac{1}{\pi} \int_{4m\pi^2}^{\infty} dt \frac{\Im F_\pi(t')}{t' - t - ic}$$

The above asymptotic behaviour allows an unsubtracted dispersion relation. Now

$$Abs[T^{\mu}] = \frac{1}{2}\sum_{h}\int d\tau_h \langle \pi^+(k)\pi^-(p)|h\rangle\langle h|V^{\mu}_{em}|0\rangle$$

The states which contribute

$$J^{pc}(I^G) = 1^{--}(1^+) \text{ i.e. } \rho\ ,\ \rho', \ldots$$

$$f(t) \equiv F_{\pi}(t) = \frac{R_{\rho}M_{\rho}^2}{M_{\rho}^2 - t} + \frac{R_{\rho'}M_{\rho'}^2}{M_{\rho'}^2 - t} + \ldots$$

$$\begin{aligned} R_{\rho}M_{\rho}^2 &= F_p g_{\rho\pi\pi}, \\ \langle \rho^0|V^{\mu}_{em}|0\rangle &= f_{\rho}\epsilon^{\mu} \\ \langle \pi^+(k)\pi^-(p)|\rho^0\rangle &= (p-k)_{\nu}\epsilon^{\nu} g_{\rho\pi\pi} \end{aligned}$$

Contributions of Excited States

One possibility is to adopt the "dual resonance model" inspired by Veneziano.
(see for Example:

- *Frampton, Phys. Rev. D1, 3141 (1969);*
- *Urrutia, Phys. Rev. D9, 3213 (1974);*
- *C. A. Dominguez, Phys. Lett. B512, 331 (2001);*
- *Bruch, Eur. Phys. J. C39, 41 (2005)*)

Sum of poles corresponding to radial excitations n (with $n = 0 \equiv \rho$, $n = 1 \equiv \rho', etc.$):

$$F_\pi(t) = \sum_n g_n f_n \frac{1}{M_n^2 - t}$$

with, in our case, $n = 0$ for ρ and $n = 1$ for ρ'.

Ingredients: ρ-meson "universal" linear Regge trajectory

$$\alpha_\rho(t) = 1 + \alpha' \left(t - M_\rho^2\right), \qquad \alpha' \simeq \frac{1}{2M_\rho^2}$$

giving the spectrum

$$M_n^2 = M_0^2 \cdot (1 + 2n)$$

This model would give for the $\rho'(1450)$: $M_{\rho'} = 1.33\ GeV$, a reasonable approximation to the measured mass (10%). The couplings are given by

$$g_n f_n = \frac{(-1)^n \Gamma(\beta - 1/2)}{\alpha' \sqrt{\pi}\, \Gamma(n+1) \Gamma(\beta - n - 1)}$$

with β related to the asymptotic behavior $F(t) \to 1/t^{\beta-1}$ as $t \to -\infty$.

$$F_\pi(t) = \frac{g_{\rho\pi\pi} f_\rho}{M_\rho^2 - t} + \frac{g_{\rho'\pi\pi} f_{\rho'}}{M_{\rho'}^2 - t}$$

$$\frac{g_{\rho'\pi\pi}f_{\rho'}}{g_{\rho\pi\pi}f_{\rho}} = -\frac{\Gamma(\beta-1)}{\Gamma(\beta-2)} = -\left(\beta-2\right)$$

For $\beta \simeq$ 2.2-2.3 as used by Bruch to fit $F_\pi(t)$ in the timelike region, one finds numerically:

$$\frac{g_{\rho\pi\pi}f_{\rho}}{M_\rho^2} \simeq 1.17; \quad \frac{g_{\rho'\pi\pi}f_{\rho'}}{g_{\rho\pi\pi}f_{\rho}} \simeq 0.3, \quad F_\pi(0) \simeq 1.04$$

Therefore: ρ VDM plus corrections, normalization at $t = 0$ missed by only 4% taking into account finite width of resonances.
Final ansatz for $F_\pi(t)$ is:

$$F_\pi(t) = \frac{g_{\rho\pi\pi}f_{\rho}}{M_\rho^2}\left[\frac{M_\rho^2}{M_\rho^2 - t - iM_\rho\Gamma_\rho\left(t\right)} - \left(\beta-2\right)\frac{M_\rho^2}{M_{\rho'}^2 - t - iM_{\rho'}\Gamma_{\rho'}\left(t\right)}\right]$$

We could add as many resonances as we want, the masses and couplings are explicitly determined by the model.
For the widths, in principle we should take them t-dependent, in order not to miss the normalization at $t \to 0$, i.e.:

$$\Gamma_n(t) = \frac{M_n^2}{t} \left(\frac{q(t)}{q(M_n^2)} \right)^3 \Gamma_n,$$

where q is the C.M. momentum.

The Decay $\tau^+ \rightarrow \eta\pi^+\nu_\tau$

Why it is interesting?
We may assign to parts of hadronic weak current, J_λ, quantum numbers, that the strong interactions conserve: Charge conjugation parity, hypercharge, isospin, charge symmetry $U = e^{i\pi \cdot I_2}$ and G-parity. Such an assignment is very important and is guided by selection rules which holds for weak interaction.
Classification w.r.t G.parity

$$G = Ce^{i\pi \cdot I_2} = CU$$

gives

$$GJ_\lambda^0 G^{-1} = \eta\epsilon J_\lambda^0$$

$\eta = \pm$ according as J is V or A, superscript 0 indicates that it is $\Delta Y = 0$ current.
The current is called first or second class, accordingly as $\epsilon = +1$ or -1 under G-parity.

In the Standard model currents are of first class. Thus for $I = 1$ vector current, which is relevant for the above process is V_i^λ, $i = 1, 2, 3$ $G = 1$. On the other hand

$$G(\eta\pi^+) = -1$$

Thus within the Standard Model, the decay is iso-spin and $G-$parity violating and as such is suppressed by small value of $\frac{(m_d - m_u)}{\Lambda_{QCD}}$ or α_{EM}. Various estimates, indicate, its Branching ratio $\mathcal{B}_{\text{expt}} \simeq 10^{-5}$ far below $\mathcal{B}_{\text{expt}} < 1.4 \times 10^{-4}$.
Thus the detection of the decay $\tau \to \eta\pi\nu_\tau$ which is expected in near future, might provide a unique signature for the "**second class currents**". It is worthwhile to point out interesting consequences of various $\mathcal{B}$ values.

We focus on the rate as expected within standard model due to isospin violation.

$$\begin{aligned}
T^\mu &= \langle \pi^+(k)\eta(p)|V^\mu_{1+i2}|0\rangle \\
&= -\sqrt{2}[f_+(t)(p-k)^\mu + f_-(t)(p+k)^\mu] \\
&= -\sqrt{2}\{f_1(t)[(p-k)^\mu - \frac{M_\eta^2 - M_\pi^2}{t}(p+k)^\mu] \\
&\quad +f_0(k)\frac{M_\eta^2 - M_\pi^2}{t}q^\mu\} \\
q &= p+k, \quad q^2 = t \\
\langle \pi^+(k)\eta(p)|i\partial_\mu V^\mu_{1+i2}|0\rangle &= -(-\sqrt{2})[(M_\eta^2 - M_\pi^2)f_+(t) + tf_-(t)] \\
&= \sqrt{2}f_0(t)[M_\eta^2 - M_\pi^2] \\
f_0(t) &= f_+(t) + \frac{t}{M_\eta^2 - M_\pi^2}f_-(t) \\
f_1(t) &= f_+(t) \\
f_1(0) &= f_+(0) = f_0(0)
\end{aligned}$$

Chiral Symmetry Constraints

$$\begin{aligned} i\partial_\mu V^\mu_{1+i2} &= (m_d - m_u)\bar{u}d \\ &= (m_d - m_u)S_{1+i2} \\ &= (m_d - m_u)\left\langle \pi^+\eta \left| S_{1+i2} \right| 0 \right\rangle \\ \sqrt{2}F_\pi \lim_{k\to 0}\left\langle \pi^+\eta \left| S_{1+i2} \right| 0 \right\rangle &= i\left\langle \eta \left| \left[F^5_{1-i2}, S_{1+i2}\right] \right| 0 \right\rangle \end{aligned}$$

Commutator

$$\left[F^5_{1-i2}, S_{1+i2}\right] = i d_{1-i2,1+i2,k} P_k = i(2P)$$

where $(P_j = \bar{q} i\gamma^5 (\lambda_j/2) q,\ \lambda_0 = \sqrt{2/3} I)$

$$\begin{aligned} 2P &= \frac{2}{\sqrt{3}}(P_8 + \sqrt{2}P_0) \\ &= 2\frac{1}{2}\left(\bar{u} i\gamma_5 u + \bar{d} i\gamma_5 d\right) \end{aligned}$$

Finally, the soft-pion low-energy theorem for $k \to 0$ [$M_\pi \to 0$]reads

$$f_0(M_\eta^2) = -\frac{1}{2F_\pi}\left(\frac{m_d - m_u}{M_\eta^2 - mM_\pi^2}\right)\langle \eta(p)\,|2P|\,0\rangle\,.$$

where in the Chiral limit

$$\langle \eta(0)|2P|0\rangle = +\frac{1}{F_8}\frac{2}{\sqrt{3}}\nu$$

$$-\nu = \langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle = \langle 0|\bar{s}s|0\rangle$$

$$F_\pi = F_8$$

In this limit

$$M_\eta^2 = \frac{2\bar{m} + 4m_s}{3F_\pi^2}\nu$$

Thus

$$f_0(M_\eta^2) = \tilde{\epsilon}$$

where

$$\tilde{\epsilon} = \frac{\sqrt{3}(m_d - m_u)}{4(m_s + \frac{\bar{m}}{2})} \simeq \epsilon$$

Limiting to just ρ and ρ'

$$\sqrt{2}f_1(t) = \frac{g_{\rho\eta\pi} f_{\rho^+}}{M_\rho^2} \left[\frac{M_\rho^2}{M_\rho^2 - t - iM_\rho \Gamma_\rho(t)} - \frac{(\beta-2)}{\Gamma_\rho'^2} \frac{M_{\rho'}^2}{M_{\rho'}^2 - t - iM_{\rho'} \Gamma_{\rho'}(t)} \right]$$

$$f_{\rho^+} = -\sqrt{2} f_\rho$$

$g_{\rho\eta\pi}$ is defined as

$$\left\langle \pi^+(p) \eta^0(k) | \rho^+ \right\rangle = g_{\rho\eta\pi}(p-k)_\nu \epsilon^\nu$$

Now

$$\begin{aligned} |\eta\rangle &= -\sin\epsilon|\pi_3\rangle + \cos\epsilon|\pi_8\rangle \\ |\pi^0\rangle &= \cos\epsilon|\pi_3\rangle + \sin\epsilon|\pi_8\rangle \end{aligned}$$

Thus

$$\langle \pi^+(p)\eta^0(k)|\rho^+\rangle = -\sin\epsilon\,\langle \pi^+(p)\pi^0(k)|\rho^+\rangle$$

giving

$$g_{\rho\eta\pi} \simeq \epsilon g_{\rho\pi\pi}$$

where $g_{\rho\pi\pi}$ appears in the pion form factor. Thus

$$\begin{aligned} f_+(t) &= \epsilon F_\pi(t) \\ &= \epsilon \frac{g_{\rho\pi\pi} f_\rho}{M_\rho^2}\left[\frac{M_\rho^2}{M_\rho^2 - t - iM_\rho\Gamma_\rho(t)} - \frac{(\beta-2)}{\Gamma_\rho'^2}\frac{M_{\rho'}^2}{M_{\rho'}^2 - t - iM_{\rho'}\Gamma_{\rho'}(t)}\right] \end{aligned}$$

where

$$\begin{aligned} \frac{g_{\rho\pi\pi} f_\rho}{M_\rho^2} &\simeq 1.17, \qquad \epsilon \approx 10^{-2} \\ q(t) &= \frac{1}{2\sqrt{t}}\sqrt{t-(M_\eta - M_\pi)^2}\sqrt{t-(M_\eta + M_\pi)^2} \end{aligned}$$

Scalar Form Factor $f_0(t)$

Dominated by states with quantum number J^{PC} $(I^G) = 0^{++}(1^+)$.
Restricting to the two resonances a_0 (980 MeV) and a_0' (1450 MeV), call them a_0 and a_1.

$$f_0(t) = \frac{g_0}{M_0^2 - t - iM_0\Gamma_0(t)} + \frac{g_1}{M_1^2 - t - iM_1\Gamma_1(t)}$$

$$\Gamma_n(t) = \frac{M_n^2}{t}\left(\frac{q(t)}{q(M_n^2)}\right)\Gamma_n$$

and

$$g_0 = F_{a_0} g_{a_0\eta\pi}; \quad g_1 = F_{a_0'} g_{a_0'\eta\pi}$$

where F_{a_0} is defined by

$$\left\langle a_0(q) \left| V^{\mu}_{1+i_2} \right| 0 \right\rangle = \sqrt{2} F_{a_0} q^{\mu}$$

Using the constant unit

$$f_0\left(M_\eta^2\right) = \epsilon$$

retaining only a_0

$$f_0\left(t\right) = f_0\left(M_\eta^2\right)\frac{M_0^2 - M_\eta^2 - iM_0\Gamma_0\left(M_\eta^2\right)}{M_0^2 - t - iM_0\Gamma_0\left(t\right)}$$

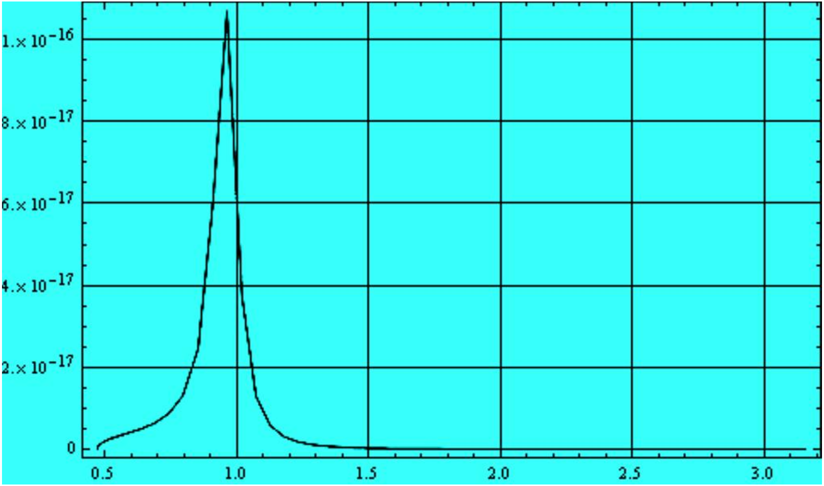
Retaining a_0 and a_0',

$$\begin{aligned} f_0\left(t\right) &= f_0\left(M_\eta^2\right)\frac{M_1^2 - M_\eta^2 - iM_1\Gamma_1\left(M_\eta^2\right)}{M_1^2 - t - iM_1\Gamma_1\left(t\right)} + \frac{g_0}{M_0^2 - M_\eta^2 - iM_0\Gamma_0\left(M_\eta^2\right)} \\ &\times \left[\begin{array}{c} \frac{t - M_\eta^2 - iM_0\left(\Gamma_0\left(M_\eta^2\right) - \Gamma_0(t)\right)}{M_0^2 - t - iM_0\Gamma_0(t)} \\ -\frac{t - M_\eta^2 - iM_1\left(\Gamma_1\left(M_\eta^2\right) - \Gamma_1(t)\right)}{M_1^2 - t - iM_1\Gamma_1(t)} \end{array}\right] \end{aligned}$$

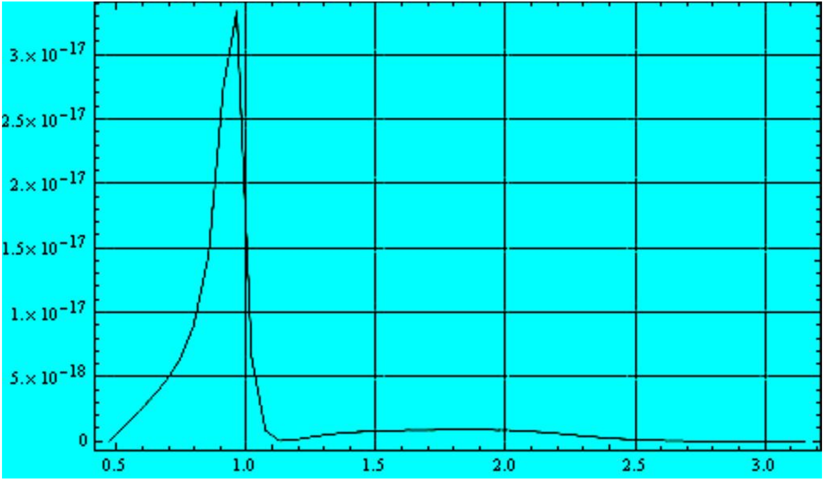
Fix $g_0 = F_{a_0} g_{a_0\eta\pi}$ from exprimental width of a_0 and take [*from QCD Sum Rules*]

$$F_{a_0} = 1.28\ MeV\ = 128\ MeV\ \epsilon$$

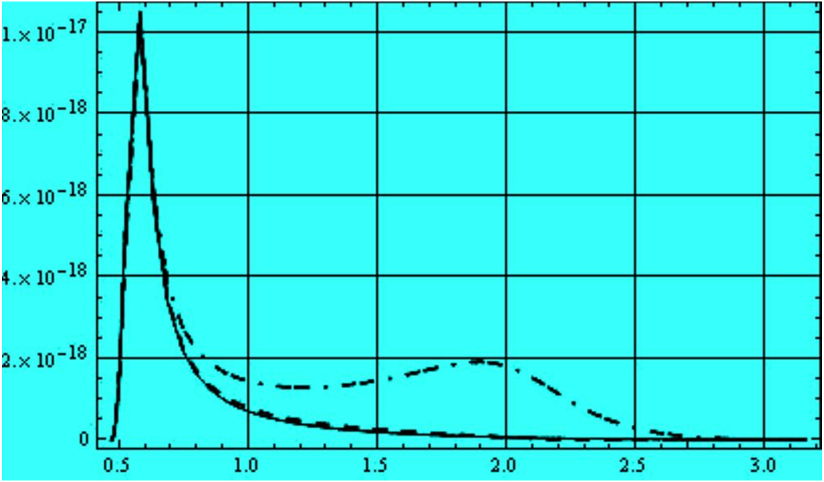
Decay rate corresponding to a_0 verses q^2



Decay rate corresponding to f_0 with two resonances verses q^2



Decay rate corresponding to f_1 with two resonances verses q^2



Total Decay Rate verses q^2: Branching Ratio $= 5.45 \times 10^{-6}$