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A Comparison Study of Several Adaptive Control Strategies for Resilient Flight Control *

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In this paper we decribe our initial development and testing of the framework referred to as Algorithm Design & Validation for Adaptive Nonlinear Control Enhancement (ADVANCE). The key elements of ADVANCE are suitable performance metrics for adaptive control systems, and an automated tuning procedure using our in-house developed Automatic Tuner for non-Linear and Adaptive Systems (ATLAS). The paper describes a comparison study of the state-of-the-art adaptive flight control algorithms on two challenging testbeds.

The development of the ADVANCE framework involved several steps: (i) Development of suitable performance comparison metrics for adaptive control systems; (ii) Development of a "plug-and-play" capability that enables rapid implementation and simulation testing of different advanced adaptive control algorithms; and (iii) Comprehensive simulation studies and performance comparison of the state-of-the-art adaptive control algorithms on a high-fidelity simulation of miniature tail-sitter UAV with significant nonlinearities and uncertainty, and a semi-nonlinear simulation of F/A-18 dynamics.

Results presented in the paper demonstrate the feasibility and potential of the ADVANCE framework, and further development of the algorithms and testing procedures is expected to give rise to a set of recommendations and guidelines regarding the use, tuning and implementation of different adaptive flight control algorithms to different problems in flight control. This will also facilitate flight certification of adaptive flight control algorithms.

I. Introduction

One of the main objectives of the Integrated Resilient Aircraft Control (IRAC) component of the NASA Aviation Safety Program is to streamline the research in advanced flight control system design and implementation, and arrive at approaches that assure stability, maneuverability and safe landing of aircraft under flight-critical upsets and external hazards. Since it has been well recognized that adaptive control appears to be the most suitable approach for the problem of accommodation of critical aircraft failures and damage, design of efficient flight control systems with adjustable parameters that assure system stability under flight-critical faults and upsets is of great interest in practice.

However, due to inherent unfavorable properties of adaptive systems (nonlinear time-varying dynamics, difficulties in selecting the most suitable tuning parameters, and problems associated with predicting transient performance), it is not clear at all as to which of the many available adaptive control techniques should be used in a specific application, and how they should be tuned, implemented and validated.

To address these issues, we have been developing and testing a framework referred to as the ADVANCE (Algorithm Design and Validation for Adaptive Nonlinear Control Enhancement) technology, within which we have been performing a comparison study of the state-of-the-art adaptive flight control algorithms on challenging testbeds. This study has resulted in an initial set of recommendations and guidelines regarding the use, tuning and implementation of different adaptive flight control algorithms to different problems in flight control.

The ADVANCE framework is shown in Figure 1. The trade study within the ADVANCE framework involved several steps: (i) Characterization of uncertainty and constraints; (ii) Development of suitable comparison metrics and tuning procedures; in the Figure, the tuning block is our in-house developed evolutionary optimization based Automatic

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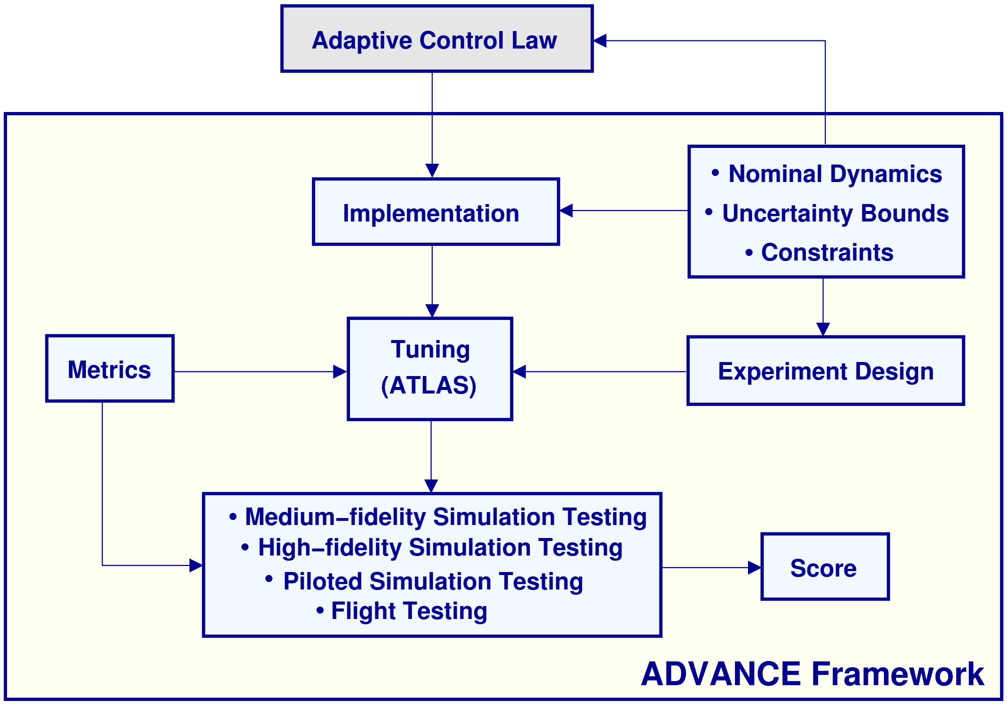

Figure 1. *Diagram of the ADVANCE framework*

Tuner for non-Linear and Adaptive Systems (ATLAS); (iii) Development of a "plug-and-play" capability that enables rapid implementation and simulation testing of different advanced adaptive control algorithms; (iii) Comprehensive simulation studies involving increasing fidelity flight simulators, and (iv) Hardware flight testing. A miniature tailsitter UAV, representing a subscale flight testbed with significant nonlinearities and uncertainty, and F/A-18 aircraft were chosen as testbeds for the study.

Another objective of the project was to develop ADVANCE algorithms and techniques as the most suitable combination of those that represent the state-of-the-art in nonlinear adaptive flight control. For instance, the so-called hybrid architectures were proposed to be tested, involving combinations of direct and indirect adaptive control, combined direct (or indirect) and variable-structure (or sliding mode) control, combined neural network based architectures (direct or indirect) with sliding mode control, etc. The objective of combining these algorithms was to retain the most favorable features of the existing algorithms, while minimizing their disadvantages and unfavorable interactions. This is of great interest in practice since the abundance of the adaptive control algorithms and the lack of studies regarding their inherent features and practical feasibility have lead to increased confusion in the area of implementation of advanced flight control designs.

Relation to the State-Of-The-Art: Over the past two decades, there has been increased interest in the applications of adaptive control to aerospace systems. This is primarily due to the fact that it was long recognized that adaptive control has a potential to solve difficult problems associated with control design to handle significant modeling uncertainties. The major development in adaptive control began with the breakthrough regarding the stability of standard direct and indirect adaptive control systems by Narendra, Lin and Valavani,[1] Morse,[2] and Goodwin and Sin.[3] Since that time, it has also been recognized that adaptive systems need to be made robust by either providing sufficient persistent excitation,[4] or by suitable modifications of the adaptive laws.[4] Throughout this time period, several new adaptive control approaches have also been developed, including adaptive control using multiple model switching and tuning,[10,19] adaptive neural network control,[8,914-,16] and, as of late, $\mathcal{L}_1$ adaptive control.[11]

While each of these individual techniques has been shown to perform well in certain applications, a comprehensive study that would compare their properties has not been performed. Hence performance comparison within the proposed ADVANCE framework effectively addresses this problem by developing a set of design and implementation guidelines for advanced adaptive flight control techniques.

Features of the ADVANCE framework: The main objective of the project whose results are presented in this paper was to carry out initial development and testing of the ADVANCE framework, algorithms and procedures for tuning and implementation of advanced adaptive flight control systems, as well as their performance evaluation and comparison. The design steps included the development of performance metrics for adaptive control systems, automated tuning procedures for adaptive controllers using our in-house developed tool referred to as the Automatic Tuner for non-Linear and Adaptive Systems (ATLAS), implementation in Matlab and C++, and comprehensive testing and performance comparison of the state-of-the-art adaptive control algorithms through computer simulations on challenging testbeds.

The steps completed in the development of the ADVANCE framework are described below.

- **Development of the performance metrics for adaptive control systems**

Since the adaptive control systems are, in general, nonlinear and time-varying, performance measures such as those for linear systems are not applicable. Hence the performance metrics based on the system response was chosen, containing terms such as the integral of square of the tracking and input errors, norm of the tracking and input errors, number of oscillations, etc. A weighted sum of these terms was integrated with the ATLAS tool that uses Genetic Algorithms to minimize the corresponding multi-objective optimization criterion.

- **Selection and implementation of the adaptive control algorithms for the ADVANCE study**

Based on our extensive experience with a variety of adaptive control algorithms, we selected the following ones: direct adaptive control, indirect adaptive control, L1 adaptive control, and combined direct and indirect adaptive control. Several variants of these basic algorithms were also implemented: combined robust and adaptive controllers using output error feedback, and neural network based algorithms where the neural network replaces observers or controllers. All of the above algorithms were implemented in Matlab and C++ to achieve fast testing and comparison. All the algorithms were tuned using the same procedure in ATLAS based on the ACM criterion, and tested under the same scenarios.

- **Algorithm comparison through simulation testing**

Under a related NASA Phase I SBIR project we developed a high-fidelity simulation of a tail-sitter UAV dynamics with significant nonlinearities and uncertainty, based on flight-test data provided by MIT. We also updated our semi-nonlinear simulation of dynamics of F/A-18 aircraft replicate damage, based on the data provided by Boeing Phantom Works. For both simulations, ranges of uncertainty were determined corresponding to the type of failure/damage, cross-coupling effects, and signal time delay. Adaptive control algorithms were tested and compared on both simulations, and their respective performance scores were generated. Combined robust and adaptive algorithms were found to perform the best (combined robust control using output error feedback, and adaptive control) in the indirect adaptive control settings. Direct adaptive control in most of the cases fared worst. In many cases combined direct and indirect adaptive controller with output error feedback outperformed other algorithms. It appears that finding suitable combinations of algorithms can result in designs that achieve excellent overall performance and optimize both transient and steady state performance.

These steps are described in detail in the following sections.

II. Performance Metrics for Adaptive Control Systems

Since adaptive control systems are, in general, nonlinear and time-varying, performance measures such as those for linear systems are not applicable. Hence the performance metrics based on the system response were chosen, containing terms such as the integral of square of the tracking and input errors, norm of the tracking and input errors, number of oscillations, etc. A weighted sum of these terms was integrated with the ATLAS tool that uses Genetic Algorithms to minimize the corresponding multi-objective optimization criterion. The performance metrics for adaptive control systems proposed under this project are referred to as the Adaptive Control Metrics (ACM), and are described below.

The process that we adopted for defining adaptive control system metrics is shown in Figure 2.

It is seen that ACM's integral elements include:

- Characterization of the Nominal Dynamics: Nominal model of plant dynamics, baseline control law, set of admissible commands, desired dynamics (reference model, reference trajectory), etc.

- System constraints: Position & rate limits, sampling rate, Zero Order Hold (ZOH) delay, actuator dynamics, control allocation law, etc.

- Uncertainty Characterization: A region in the parametric space; a range of parameter variations (e.g. time & type of failures and damage levels, etc.); a range of time delays; a range of disturbances; a range of noise parameters; a range

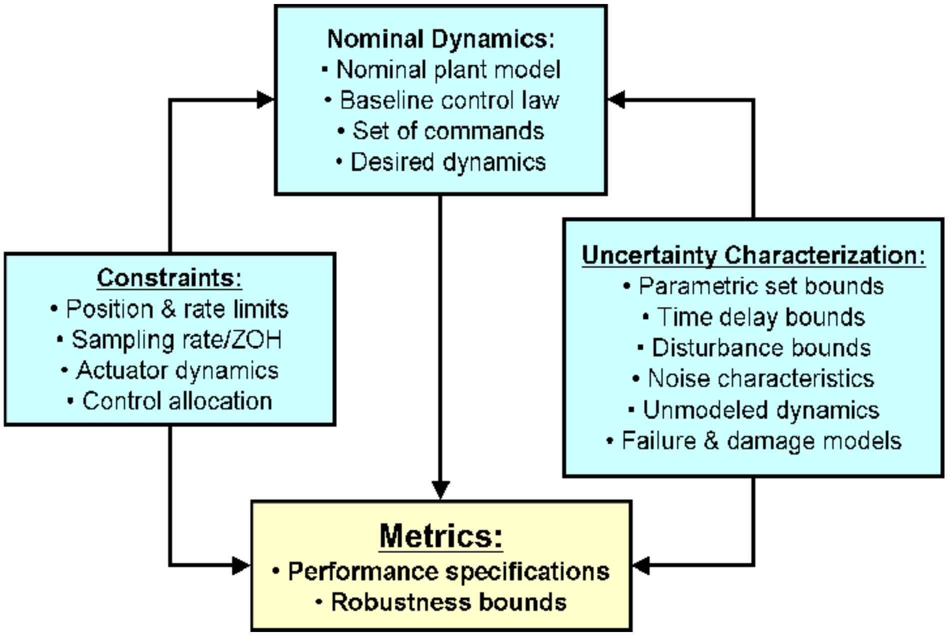

Figure 2. ***Defining Performance Metrics for Adaptive Control Systems***

of unmodeled dynamics; etc.

In this study we proposed the adaptive control metrics (ACM) as a weighted sum of transient performance specification terms such as:

- Transient tracking error (e.g. norm square of the instantaneous tracking error - NSTE, $\|e(t)\|_{\mathcal{L}_\infty}$);
- Average tracking error over an interval (e.g. an integral of the square tracking error - ISTE, $\|e(t)\|_{\mathcal{L}_2}$);
- Transient input error (e.g. norm of the instantaneous input error - NSIE, $\|e_u(t)\|_{\mathcal{L}_\infty}$, where $e_u = u - u^*$, and u^* denotes the ideal control input that would have been applied had the uncertainty been known);
- Average input error over an interval (e.g. an integral of the square input error - ISIE, $\|e_u(t)\|_{\mathcal{L}_2}$); and
- Frequency of signal oscillation.

The above list is by no means exclusive and other performance specification terms are possible such as parametric error measures, PIO indication measure, actuator rates, inter-axes coupling, etc.

The ACM criterion is now defined as:

$$J = w_1 NSTE + w_2 ISTE + w_3 NSIE + w_4 ISIE + w_5 OSCF + ... \tag{1}$$

The constraints arising from problem-dependent performance specifications and safety bounds are of the form:

$$\begin{aligned} NSTE &\leq NSTE_{max} \\ ISTE &\leq ISTE_{max} \\ NSIE &\leq NSIE_{max} \\ ISIE &\leq ISIE_{max} \\ OSCF &\leq OSCF_{max} \\ &... \end{aligned} \tag{2}$$

III. The Adaptive Controller Tuning Problem in the Context of ACM

Based on the above metrics and constraints, we now formulate the tuning problem for adaptive control systems as follows. Let the plant dynamics be described by:

$$\dot{x} = f(x, u, p), \quad x(0) = x_o, \tag{3}$$

where x and u respectively denote state and input vectors and p denotes an uncertain parameter vector. Let the adaptive controller be of the form:

$$u = g(x, x_m, r, \theta), \tag{4}$$

where x_m is desired dynamics vector (e.g. state vector of a stable reference model), r is a vector of reference inputs (commands), and θ is a vector of adjustable controller parameters generated by an adaptive law of the form:

$$\dot{\theta} = h(\gamma, x, x_m, \theta, r), \tag{5}$$

where γ is a vector of *free-design parameters* (FDP) such as adaptation gains, observer gains, filter gains, etc.

The tuning problem is now stated as follows: *Find a set of FDPs such that for given nominal dynamics, constraints, and range of uncertainty, the ACM performance criterion 1 is minimized under constraints 3.*

This is a constrained multi-objective optimization problem for which a feasible solution may not exist for a given adaptive controller.

We have recently developed an evolutionary optimization-based Automatic Tuner for non-Linear & Adaptive Systems (ATLAS). The ATLAS tool arrives at the best set of FDPs to meet the specifications under constraints. This tool was used under the project. The tool uses Genetic Algorithms to run a number of simulations for many of generations of solutions to calculate "the best" gains. The best solution from a previous generation is kept and mutated, and the process is continued. *The overall procedure results in finding a near-optimal solution while avoiding local minima.*

IV. Selection and Implementation of the Adaptive Control Algorithms for the ADVANCE Study

Based on our extensive experience with a variety of adaptive control algorithms, we selected the following ones:

(a) Direct adaptive control: In direct adaptive control, Figure 3, controller parameters are adjusted directly based on the system response to assure asymptotic following of a stable reference model.

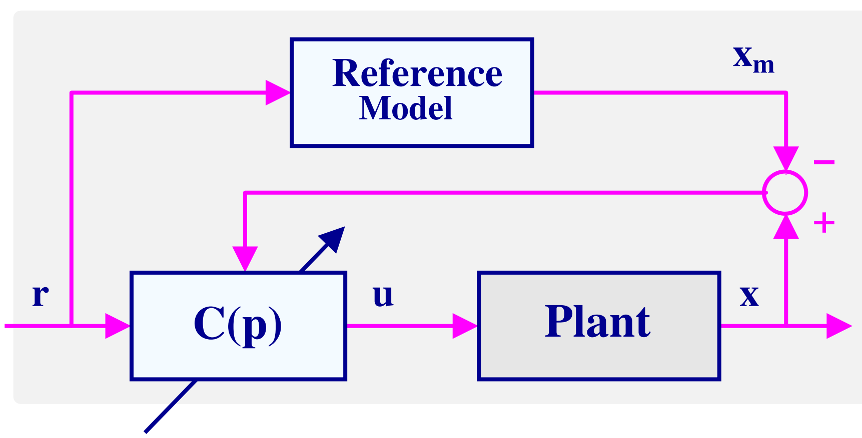

Figure 3. ***Direct Adaptive Control Strategy***

(b) Indirect adaptive control: In Indirect Adaptive Control, Figure 4, plant parameters are estimated using an on-line observer, and these estimates are in turn used in the control law to assure asymptotic model following. Within the framework of indirect adaptive control, gradient adaptive algorithm, as well as adaptive algorithms with normalization were tested.

(c) Combined direct and indirect adaptive control: In this case the plant parameters are estimated using an observer, while the controller parameters are adjusted statically based on closed-loop estimation errors.[4]

(d) $\mathcal{L}_1$ adaptive control: One of the variants of indirect adaptive control is the $\mathcal{L}_1$ control, Figure 5 that is seen to contain an additional filter in series with the controller. The $\mathcal{L}_1$ adaptive control technique[11] is of specific interest in the algorithm comparison as it has recently received considerable attention due to its theoretical properties. In theory the method ensures transient response bounds for the input and output signals of the system. These bounds are inversely

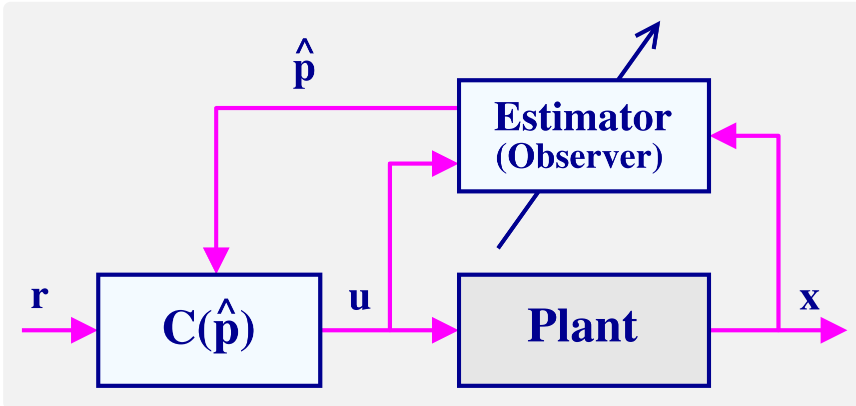

Figure 4. ***Indirect Adaptive Control Strategy***

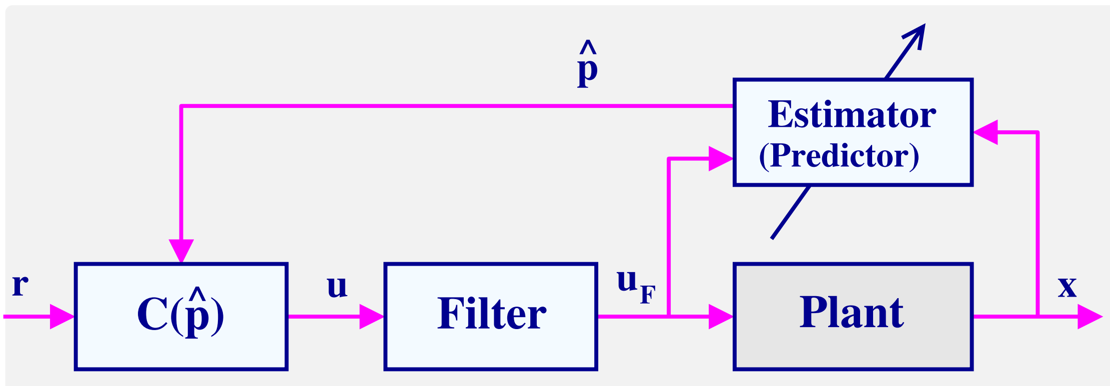

Figure 5. $\mathcal{L}_1$ ***Adaptive Control Strategy***

proportional to the magnitude of the adaptation gain, which in turn is limited by the processing capabilities of the flight control computer. $\mathcal{L}_1$ adaptive control was recently extended to a class of uncertain nonlinear systems in the presence of unmodelled dynamics making it a candidate for the proposed comparison.

Note that all of the given adaptive control approaches where also tested with a robust fixed (non-adaptive) controller referred to as the Output Error Feedback. In addition, the adaptive controllers were also tested with neural networks, where the observers in indirect adaptive control or controllers in direct adaptive control were replaced by on-line neural networks, Figure 7.

(e) Multiple Model Adaptive Control: Multiple model adaptive control is another technique of interest in the proposed comparison study. The concept of Multiple Model Switching and Tuning (MMST) is based on the idea of describing the dynamics of a system using different models for different operating regimes; with each of these models there is an associated on-line observer, also referred to as the identification model. The basic idea is to set up such identification models and corresponding controllers in parallel, Figure 8 (left), and to devise a suitable strategy for switching among the controllers to achieve the desired control objective. While the plant is being controlled using one of these controllers, the identification models are run in parallel to generate performance metrics or a measure of the ”closeness” between the dynamics of the plant and the observers. The system than switches to the controller corresponding to the observer closest to the current plant dynamics. The need for MMST in plants with rapidly changing dynamics can be conveniently motivated using Figure 8 (right) where P_o and P represent the nominal and uncertain plants respectively in the parametric space. The top figure illustrates the case when adaptation using a single model may be too slow to identify the new operating regime. In such a case placing several models in the parametric set, switching to the model closest to the dynamics of the failed plant, and adapting from there can result in fast and accurate system response.

The main feature of the MMST approach is that it results in a stable overall system in which asymptotic convergence

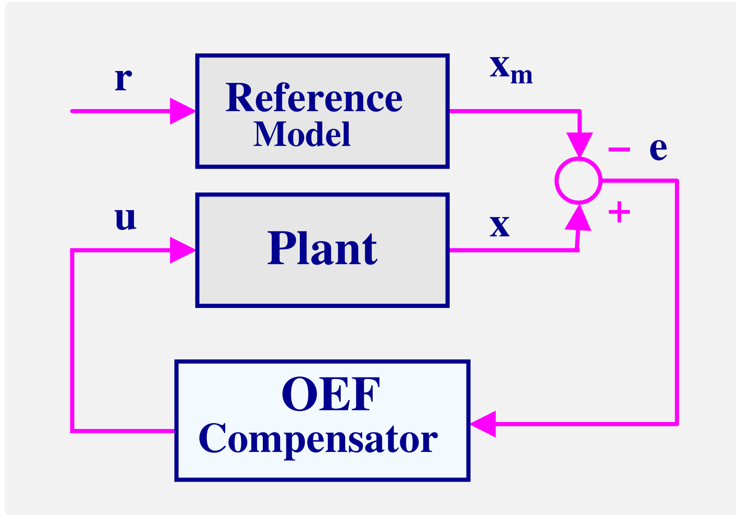

Figure 6. ***Output Error Feedback Strategy***

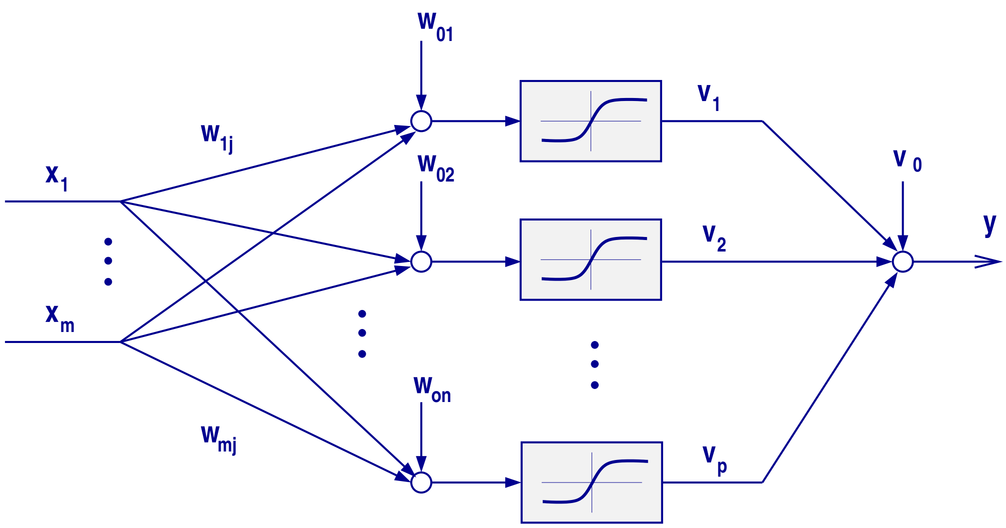

Figure 7. ***Diagram of a single hidden layer sigmoidal neural network structure***

of the output error to zero is guaranteed under relatively mild conditions. As also shown through extensive simulations,[10] the performance of the overall switching system can be dramatically improved as compared to that achieved using a single-model adaptive controller.

All of the above algorithms were implemented in Matlab and C++ to achieve fast testing and comparison. All the algorithms were tuned using the same procedure in ATLAS based on the ACM criterion, and tested under the same scenarios.

V. Algorithm Comparison through Simulation Testing

Under the related project, we developed a high-fidelity simulation of a tail-sitter UAV dynamics with significant nonlinearities and uncertainty, based on flight-test data provided by MIT. We also updated our semi-nonlinear simulation of dynamics of F/A-18 aircraft to represent damage, based on the data provided by Boeing Phantom Works. Mathematical models on which both simulations are based are described in detail in a report.[18] For both simulations, ranges of uncertainty were determined corresponding to the type of failure/damage, cross-coupling effects, and signal time delay. Adaptive control algorithms were tested and compared on both simulations, and their respective perfor-

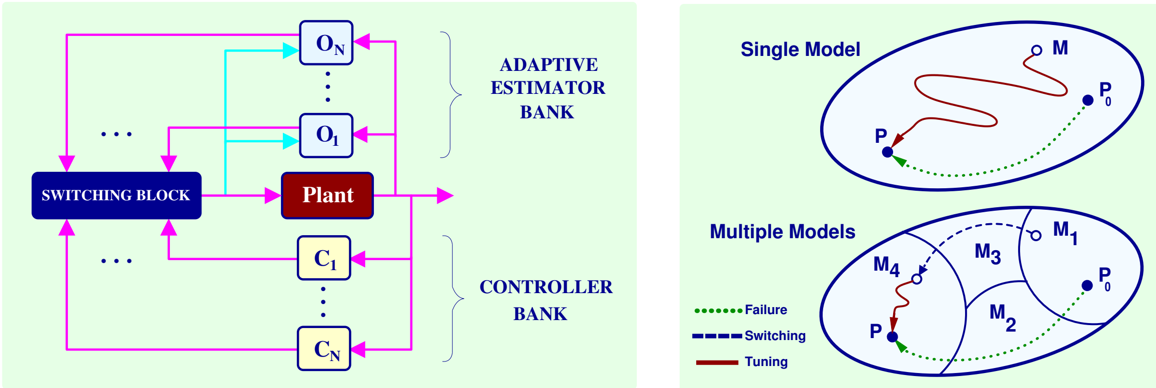

Figure 8. ***Structure of the Multiple Models, Switching and Tuning (MMST) System (Left); Single Model vs Multiple Model Adaptation (Right)***

mance scores were generated.

A. ACM scores for the tail-sitter simulation

In this case, the above 10 algorithms were compared on the tailsitter simulation. The techniques were applied to control attitude under a series of doublet commands. All of the approaches were derived from the same feedback linearizing controller for the nonlinear tailsitter model.

The comparison of these algorithms was performed for four specific cases. The first or baseline case included no time delay ($\theta_\tau = 0$) and no actuator failure. For the second of these cases, a 4 sample pure time delay was introduced with for a delay of 0.08 seconds. In the third case a Loss-Of-Effectiveness (LOE) failure was incorporated where actuator three was made 70% effective while in the last case both the same time delay and LOE failure were applied. For all of the algorithms the same ACM criterion was used. All algorithms were tuned using ATLAS for the specified scenario.

A list of the algorithms used in the tailsitter study is given below:

- *Baseline controller* - Inverse dynamics controller which does not cancel the coupling nonlinearity.
- *Output Error PID (OE PID)* - Inverse dynamics controller with output error PID terms which does not cancel the coupling nonlinearity.
- *Indirect Single Hidden Layer (SHL) Neural Network (NN)* - Indirect SHL NN controller which is identical to the Baseline method but with NN to cancel the coupling nonlinearity. The outer layer NN weights are adapted in an indirect manner while the hidden layer weights are fixed and random.
- *Direct Single Hidden Layer (SHL) Neural Network (NN)* - Direct SHL NN controller which is identical to the Baseline method but with NN to cancel the coupling nonlinearity. The outer layer NN weights are adapted in a direct manner while the hidden layer weights are fixed and random.
- *Combined SHL NN* - Combined indirect and direct SHL NN controller which is identical to the Baseline method but with NN to cancel the coupling nonlinearity. The outer layer NN weights are adapted in a combined indirect/direct manner while the hidden layer weights are fixed and random.
- $\mathcal{L}_1$ *SHL NN* - $\mathcal{L}_1$ SHL NN controller which is identical to the Baseline method but with NN to cancel the coupling nonlinearity. The outer layer NN weights are adapted in an $\mathcal{L}_1$ manner while the hidden layer weights are fixed and random.
- *Indirect SHL NN with OE PID* - Indirect SHL NN controller which is identical to the OE PID method but with NN to cancel the coupling nonlinearity. The outer layer NN weights are adapted in an indirect manner while the hidden layer weights are fixed and random.

- *Direct SHL NN with OE PID* - Direct SHL NN controller which is identical to the OE PID method but with NN to cancel the coupling nonlinearity. The outer layer NN weights are adapted in a direct manner while the hidden layer weights are fixed and random.
- *Combined SHL NN with OE PID* - Combined indirect and direct SHL NN controller which is identical to the OE PID method but with NN to cancel the coupling nonlinearity. The outer layer NN weights are adapted in a combined indirect/direct manner while the hidden layer weights are fixed and random.
- $\mathcal{L}_1$ *SHL NN with OE PID* - $\mathcal{L}_1$ SHL NN controller which is identical to the OE PID method but with NN to cancel the coupling nonlinearity. The outer layer NN weights are adapted in an $\mathcal{L}_1$ manner while the hidden layer weights are fixed and random.

For the first comparison in the baseline case, the algorithms were all tuned with no pure time delay and no failures. Table 1 shows the results of this comparison. *It can be seen that given the fitness weighting indirect SHL NN with OE PID performed best.*

	Fitness	$\|\|e\|\|_{\mathcal{L}_2}$	$\|\|u\|\|_{\mathcal{L}_2}$	n_{osc}
Baseline	5.5320	2.4508	6.6040	22
OE PID	4.4668	1.4298	6.6233	21
Indirect SHL NN	4.3036	1.4294	6.5807	18
Direct SHL NN	4.6092	1.4062	6.8433	23
Combined SHL NN	4.3057	1.4234	6.6076	18
L1 SHL NN	5.0416	2.2085	6.6103	17
Indirect SHL NN with OE PID	4.2947	1.4205	6.5806	18
Direct SHL NN with OE PID	4.4801	1.4502	6.5998	21
Combined SHL NN with OE PID	4.3125	1.4363	6.5876	18
L1 SHL NN with OE PID	5.0398	2.2062	6.6121	17

Table 1. ***Table of tail-sitter results for the all of the algorithms tested with no pure time delay and no failures (OE PID - Output error PID; SHL NN - single hidden layer neural network)***

In the case with time delay and no failure, the algorithms were all tuned with a 4 sample pure time delay of 0.08 seconds and no failures. The results of this comparison are shown in Table 2. It can be seen that for this case as opposed to the previous the Direct SHL NN with OE PID approach outperformed all other methods.

	Fitness	$\|\|e\|\|_{\mathcal{L}_2}$	$\|\|u\|\|_{\mathcal{L}_2}$	n_{osc}
Baseline	8.1617	2.7716	13.1337	29
OE PID	6.2861	2.0494	9.4555	28
Indirect SHL NN	6.0018	2.0962	9.6855	20
Direct SHL NN	7.0548	2.0176	12.6239	25
Combined SHL NN	5.9354	2.1765	8.6963	23
L1 SHL NN	7.4945	2.8771	11.3914	24
Indirect SHL NN with OE PID	5.5704	2.3206	7.4993	20
Direct SHL NN with OE PID	5.5167	2.3695	6.6572	23
Combined SHL NN with OE PID	5.5650	2.4542	6.8696	21
L1 SHL NN with OE PID	6.4318	3.0022	7.7651	22

Table 2. ***Table of tail-sitter results for the all of the algorithms tested with a pure time delay of*** 0.08 ***seconds and no failures (OE PID - Output error PID; SHL NN - single hidden layer neural network)***

In the case of LOE failure and no time delay, the actuator three was made 70% effective. After tuning the methods it was found the combined indirect and direct SHL NN with OE PID performed best. The results for the case are displayed in Table 3.

	Fitness	$\|\|e\|\|_{\mathcal{L}_2}$	$\|\|u\|\|_{\mathcal{L}_2}$	n_{osc}
Baseline	6.9497	4.6440	6.5190	7
OE PID	5.2568	2.0519	7.5166	19
Indirect SHL NN	5.1013	2.5833	6.5601	11
Direct SHL NN	5.4158	2.9694	7.1544	6
Combined SHL NN	5.1078	2.5866	6.5706	11
L1 SHL NN	6.6991	4.5397	6.5314	4
Indirect SHL NN with OE PID	5.1209	2.5401	6.6027	12
Direct SHL NN with OE PID	5.4474	2.2816	6.8861	22
Combined SHL NN with OE PID	5.0778	2.5546	6.5775	11
L1 SHL NN with OE PID	6.7274	4.5739	6.5119	4

Table 3. ***Table of tail-sitter results for the all of the algorithms tested with no pure time delay and a LOE failure (OE PID - Output error PID; SHL NN - single hidden layer neural network)***

In the case with time delay and LOE failure, the indirect approach with output error gave the best results. This can be seen in the Table 4.

	Fitness	$\|\|e\|\|_{\mathcal{L}_2}$	$\|\|u\|\|_{\mathcal{L}_2}$	n_{osc}
Baseline	8.9222	5.0300	11.1406	11
OE PID	6.8685	3.1560	8.3752	24
Indirect SHL NN	6.5646	3.2277	8.4562	16
Direct SHL NN	7.4395	3.0295	10.7001	24
Combined SHL NN	6.6741	3.3271	8.4900	16
L1 SHL NN	8.3978	5.2321	9.2190	8
Indirect SHL NN with OE PID	6.3256	3.4619	7.3788	13
Direct SHL NN with OE PID	6.4825	3.7645	6.7265	14
Combined SHL NN with OE PID	6.4566	3.7745	6.9404	12
L1 SHL NN with OE PID	7.6449	5.0077	7.7907	6

Table 4. ***Table of tail-sitter results for the all of the algorithms tested with pure time delay of*** 0.08 ***seconds and LOE failure (OE PID - Output error PID; SHL NN - single hidden layer neural network)***

In the last case, the best score was achieved by the indirect adaptive controller with a single hidden layer NN and PID-type output error feedback. The corresponding simulation results using this algorithm are shown in Figure 9.

B. ACM scores for the F/A-18 simulation

In the case of F/A-18 simulation, 15 adaptive control algorithms were compared. Comparison with the baseline controller was also made. The algorithms used are described in detail below. Note that for the description the following

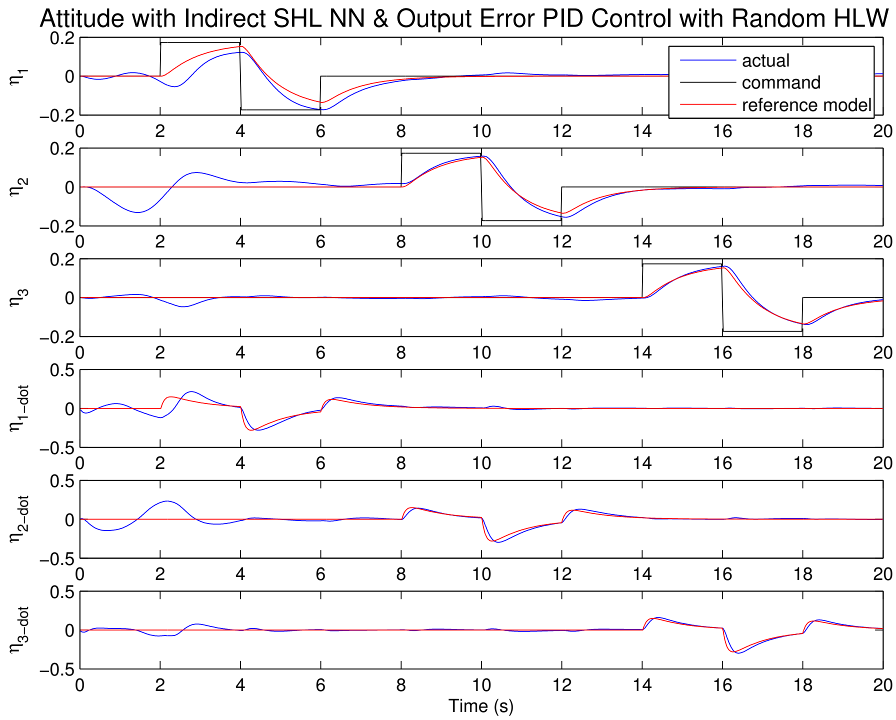

Figure 9. ***Tailsitter simulation response of the SHL NN indirect with output error controller with a pure time delay of*** 0.08 ***seconds and LOE failure.***

definitions are assumed: the matrix

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{6}$$

which is designed to select the relative degree 1 states, system error

$$e = x - x_m, \tag{7}$$

observer error

$$\hat{e} = \hat{x} - x \tag{8}$$

and the matrices $R_{m\times n}$, I_m, $0_{m\times n}$, and $1_{m\times n}$, denoting a random ($R_{i,j} \in [-1, 1]$ for $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$), identity, zeros, and ones matrix with dimensions m and n.

Baseline Control (BASE):

$$\textit{Control Law:} \qquad u_c = B^T C^T \left(CBB^T C^T\right)^{-1} C\left(-Ax + A_m x + B_m r\right) \tag{9}$$

Output Error Feedback Control (OE):

$$\begin{aligned} \textit{Control Law:} \quad & u_c = B^T C^T \left(CBB^T C^T\right)^{-1} C \\ & \left(-Ax + A_m x + B_m r + A_m \left(k_p e + k_i \int e dt + k_d \frac{de}{dt}\right)\right) \\ \textit{Gains:} \quad & k_p, \ \ k_i, \ \ k_d \end{aligned} \tag{10}$$

Indirect Adaptive Control (IND):

$$\text{Control Law:} \quad u_c = \hat{K}^T B^T C^T \left(CB\hat{K}\hat{K}^T B^T C^T\right)^{-1} C\left(-\hat{A}x + A_m x + B_m r\right) \tag{11}$$

$$\text{Observer:} \quad \dot{\hat{x}} = \hat{A}x + B\hat{K}u - \lambda\hat{e} \qquad \hat{x}_0 = x_0 \tag{12}$$

$$\text{Adaptive Law:} \quad \dot{\hat{A}} = \text{Proj}\left(\hat{A}, -\gamma_A\left(x\hat{e}^T + \delta\|\hat{e}\|\hat{A}\right)\right) \qquad \hat{A}_0 = A \tag{13}$$

$$\dot{\hat{k}} = \text{Proj}\left(\hat{k}, -\gamma_k\left(UB^T\hat{e} + \delta\|\hat{e}\|\hat{k}\right)\right) \qquad \hat{k}_0 = 1_{10\times1} \tag{14}$$

$$\text{Gains:} \quad \lambda,\ \gamma_A,\ \gamma_k,\ \delta$$

$$\text{where:} \quad U = \text{diag}(u)$$

$$\hat{K} = \text{diag}(\hat{k})$$

Direct Adaptive Control (DIR):

$$\text{Control Law:} \quad u_c = \hat{K}_x x + \hat{K}_r r \tag{15}$$

$$\text{Adaptive Law:} \quad \dot{\hat{K}}_x = \text{Proj}\left(\hat{K}_x, -\gamma_x \text{Sgn}(K)\left(B^T P e x^T + \delta\|e\|\hat{K}_x\right)\right) \tag{16}$$

$$\hat{K}_{x0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(-A + A_m)$$

$$\dot{\hat{K}}_r = \text{Proj}\left(\hat{K}_r, -\gamma_r \text{Sgn}(K)\left(B^T P e r^T + \delta\|e\|\hat{K}_r\right)\right) \tag{17}$$

$$\hat{K}_{r0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(B_m)$$

$$\text{Gains:} \quad q,\ \gamma_x,\ \gamma_r,\ \delta$$

$$\text{where:} \quad A_m^T P + P A_m = -q I_{10}$$

$$\text{Sgn}(K) = I_{10}$$

$\mathcal{L}_1$ Adaptive Control (L1):

$$\text{Control Law:} \quad \dot{u}_c = -\xi\left(u_c - \hat{K}^T B^T C^T \left(CB\hat{K}\hat{K}^T B^T C^T\right)^{-1} C\left(-\hat{A}x + A_m x + B_m r\right)\right)$$

$$u_0 = B^T C^T \left(CBB^T C^T\right)^{-1} C\left(-\hat{A}x + A_m x + B_m r\right) \tag{18}$$

$$\text{Observer:} \quad \dot{\hat{x}} = \hat{A}x + B\hat{K}u - A_m\hat{e} \qquad \hat{x}_0 = x_0 \tag{19}$$

$$\text{Adaptive Law:} \quad \dot{\hat{A}} = \text{Proj}\left(\hat{A}, -\gamma_A x\hat{e}^T\right) \qquad \hat{A}_0 = A \tag{20}$$

$$\dot{\hat{k}} = \text{Proj}\left(\hat{k}, -\gamma_k U B^T \hat{e}\right) \qquad \hat{k}_0 = 1_{10\times1} \tag{21}$$

$$\text{Gains:} \quad \gamma_A,\ \gamma_k,\ \xi,\ \nu \text{ (see Remark 1)}$$

$$\text{where:} \quad U = \text{diag}(u)$$

$$\hat{K} = \text{diag}(\hat{k})$$

Remark 1 *ν is the number of iterations the discrete implementation of the $\mathcal{L}_1$ controller is run for every time step of the F/A-18 simulation. One should note that, the $\mathcal{L}_1$ approach is based on selecting large adaptation gains. For this approach, the basic strategy is to make the filter gain (ξ) large, such the the reference system is close to the reference model. The adaptation gains (γ_A and γ_k) should then be chosen to be much larger than the filter gain to ensure that the system states remains close to the reference system. See Refs[12] and[13] for the reference system definition and theoretic error signal bounds. One significant downfall of this strategy is that the discrete implementation of large gain dynamics can be grossly inaccurate depending the size of the time step. Thus to accommodate the large gain approach the internal time step for the $\mathcal{L}_1$ controller dynamics was allowed to be reduced by selecting $\nu > 1$. One should note that this does not change the zero-order-hold time for controller inputs and outputs (i.e. sensor measurements used by the controller and applied actuator commands) but it does require additional computation between each time step which can potentially have limitations. Note that $\nu \in \{1, 2, \cdots, 10\}$ and thus the smallest controller time step is constrained to be $dt = 0.001$.*

Combined Indirect and Direct Adaptive Control (CID):

$$\textit{Control Law:} \quad u_c = \hat{K}_x x + \hat{K}_r r \tag{22}$$

$$\textit{Observer:} \quad \dot{\hat{x}} = \hat{A}x + B\hat{K}u - \lambda\hat{e} \qquad \hat{x}_0 = x_0 \tag{23}$$

$$\textit{Adaptive Law:} \quad \dot{\hat{K}}_x = \text{Proj}\left(\hat{K}_x, -\gamma_x \text{Sgn}(K)\left(B^T\Sigma_x + \delta\|e\|\hat{K}_x\right)\right) \tag{24}$$

$$\hat{K}_{x0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(-A + A_m)$$

$$\dot{\hat{K}}_r = \text{Proj}\left(\hat{K}_r, -\gamma_r \text{Sgn}(K)\left(B^T\Sigma_r + \delta\|e\|\hat{K}_r\right)\right) \tag{25}$$

$$\hat{K}_{r0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(B_m)$$

$$\dot{\hat{A}} = \text{Proj}\left(\hat{A}, -\gamma_A\left(x\hat{e}^T + \Sigma_x + \delta\|\hat{e}\|\hat{A}\right)\right) \tag{26}$$

$$\hat{A}_0 = A$$

$$\dot{\hat{G}} = \text{Proj}\left(\hat{G}, -\gamma_G\left(u\hat{e}^T + \Sigma_x\hat{K}_x^T + \Sigma_r\hat{K}_r^T + \delta\|\hat{e}\|\hat{G}\right)\right) \tag{27}$$

$$\hat{G}_0 = B$$

Gains: $\lambda, \ \gamma_x, \ \gamma_r, \ \gamma_A, \ \gamma_G, \ \delta$

where: $\Sigma_x = \hat{G}\hat{K}_x - A_m + \hat{A}$

$\Sigma_r = \hat{G}\hat{K}_r - B_m$

$\text{Sgn}(K) = I_{10}$

Indirect Adaptive Control with Output Error Feedback (IND OE):

$$\textit{Control Law:} \quad u_c = \hat{K}^T B^T C^T \left(CB\hat{K}\hat{K}^T B^T C^T\right)^{-1} C \left(-\hat{A}x + A_m x + B_m r + A_m\left(k_p e + k_i \int e dt + k_d \frac{de}{dt}\right)\right) \tag{28}$$

$$\textit{Observer:} \quad \dot{\hat{x}} = \hat{A}x + B\hat{K}u - \lambda\hat{e} \qquad \hat{x}_0 = x_0 \tag{29}$$

$$\textit{Adaptive Law:} \quad \dot{\hat{A}} = \text{Proj}\left(\hat{A}, -\gamma_A\left(x\hat{e}^T + \delta\|\hat{e}\|\hat{A}\right)\right) \qquad \hat{A}_0 = A \tag{30}$$

$$\dot{\hat{k}} = \text{Proj}\left(\hat{k}, -\gamma_k\left(UB^T\hat{e} + \delta\|\hat{e}\|\hat{k}\right)\right) \qquad \hat{k}_0 = 1_{10\times 1} \tag{31}$$

Gains: $\lambda \in [10^{-4}, 10^4]$

$\gamma_A, \ \gamma_k, \ k_p, \ k_i, \ k_d, \ \delta$

where: $U = \text{diag}(u)$

$\hat{K} = \text{diag}(\hat{k})$

Direct Adaptive Control with Output Error Feedback (DIR OE):

$$\textit{Control Law:} \quad u_c = \hat{K}_x x + \hat{K}_r r + B^T C^T \left(CBB^T C^T\right)^{-1} CA_m\left(k_p e + k_i \int e dt + k_d \frac{de}{dt}\right) \tag{32}$$

$$\textit{Adaptive Law:} \quad \dot{\hat{K}}_x = \text{Proj}\left(\hat{K}_x, -\gamma_x \text{Sgn}(K)\left(B^T Pex^T + \delta\|e\|\hat{K}_x\right)\right) \tag{33}$$

$$\hat{K}_{x0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(-A + A_m)$$

$$\dot{\hat{K}}_r = \text{Proj}\left(\hat{K}_r, -\gamma_r \text{Sgn}(K)\left(B^T Per^T + \delta\|e\|\hat{K}_r\right)\right) \tag{34}$$

$$\hat{K}_{r0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(B_m)$$

Gains: $q, \ \gamma_x, \ \gamma_r, \ k_p, \ k_i, \ k_d, \ \delta$

where: $A_m^T P + PA_m = -qI_{10}$

$\text{Sgn}(K) = I_{10}$

$\mathcal{L}_1$ Adaptive Control with Output Error Feedback (L1 OE):

$$\begin{aligned}
\textit{Control Law:} \quad & \dot{u}_c = -\xi u_c + \xi \hat{K}^T B^T C^T \left(CB\hat{K}\hat{K}^T B^T C^T\right)^{-1} C \\
& \quad \left(-\hat{A}x + A_m x + B_m r + A_m \left(k_p e + k_i \int e dt + k_d \frac{de}{dt}\right)\right) && (35) \\
& u_0 = B^T C^T \left(CBB^T C^T\right)^{-1} C \left(-\hat{A}x + A_m x + B_m r\right) \\
\textit{Observer:} \quad & \dot{\hat{x}} = \hat{A}x + B\hat{K}u - A_m \hat{e} \qquad \hat{x}_0 = x_0 && (36) \\
\textit{Adaptive Law:} \quad & \dot{\hat{A}} = \text{Proj}\left(\hat{A}, -\gamma_A x \hat{e}^T\right) \qquad \hat{A}_0 = A && (37) \\
& \dot{\hat{k}} = \text{Proj}\left(\hat{k}, -\gamma_k U B^T \hat{e}\right) \qquad \hat{k}_0 = 1_{10\times1} && (38) \\
\textit{Gains:} \quad & \gamma_A \in [10^{-4}, 10^7] \\
& \gamma_k, \ \ \xi, \ \ \nu \text{ (see Remark 1)}, \ \ k_p, \ \ k_i, \ \ k_d \\
\textit{where:} \quad & U = \text{diag}(u) \\
& \hat{K} = \text{diag}(\hat{k})
\end{aligned}$$

Combined Indirect and Direct Adaptive Control with Output Error Feedback (CID OE):

$$\begin{aligned}
\textit{Control Law:} \quad & u_c = \hat{K}_x x + \hat{K}_r r && (39) \\
& \quad + B^T C^T \left(CBB^T C^T\right)^{-1} C A_m \left(k_p e + k_i \int e dt + k_d \frac{de}{dt}\right) \\
\textit{Observer:} \quad & \dot{\hat{x}} = \hat{A}x + B\hat{K}u - \lambda \hat{e} \qquad \hat{x}_0 = x_0 && (40) \\
\textit{Adaptive Law:} \quad & \dot{\hat{K}}_x = \text{Proj}\left(\hat{K}_x, -\gamma_x \text{Sgn}(K)\left(B^T \Sigma_x + \delta \|e\| \hat{K}_x\right)\right) && (41) \\
& \hat{K}_{x0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(-A + A_m) \\
& \dot{\hat{K}}_r = \text{Proj}\left(\hat{K}_r, -\gamma_r \text{Sgn}(K)\left(B^T \Sigma_r + \delta \|e\| \hat{K}_r\right)\right) && (42) \\
& \hat{K}_{r0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(B_m) \\
& \dot{\hat{A}} = \text{Proj}\left(\hat{A}, -\gamma_A \left(x\hat{e}^T + \Sigma_x + \delta \|\hat{e}\| \hat{A}\right)\right) && (43) \\
& \hat{A}_0 = A \\
& \dot{\hat{G}} = \text{Proj}\left(\hat{G}, -\gamma_G \left(u\hat{e}^T + \Sigma_x \hat{K}_x^T + \Sigma_r \hat{K}_r^T + \delta \|\hat{e}\| \hat{G}\right)\right) && (44) \\
& \hat{G}_0 = B \\
\textit{Gains:} \quad & \lambda, \ \ \gamma_x, \ \ \gamma_r, \ \ \gamma_A, \ \ \gamma_G, \ \ k_p, \ \ k_i, \ \ k_d, \ \ \delta \\
\textit{where:} \quad & \Sigma_x = \hat{G}\hat{K}_x - A_m + \hat{A} \\
& \Sigma_r = \hat{G}\hat{K}_r - B_m \\
& \text{Sgn}(K) = I_{10}
\end{aligned}$$

Indirect SHL NN Adaptive Control with Output Layer Adaptation Only (IND NNW):

$$\textit{Control Law:} \quad u_c = B^T C^T \left(CBB^T C^T\right)^{-1} C \left(-Ax + A_m x + B_m r - C^T \hat{W}^T \sigma\left(\hat{V}^T \bar{x}\right)\right) \tag{45}$$

$$\textit{Observer:} \quad \dot{\hat{x}} = Ax + BKu + C^T \hat{W}^T \sigma\left(\hat{V}^T \bar{x}\right) - \lambda \hat{e} \qquad \hat{x}_0 = x_0 \tag{46}$$

$$\textit{Adaptive Law:} \quad \dot{\hat{W}} = \text{Proj}\left(\hat{W}, -\Gamma_W \left(\sigma\left(\hat{V}^T \bar{x}\right) \hat{e}^T C^T + \delta \|\hat{e}\| \hat{W}\right)\right) \tag{47}$$

$$\hat{W}_0 = 0_{\mu+1\times 6}$$

$$\hat{V} = \hat{V}_0 \qquad \hat{V}_0 = R_{21\times\mu} \tag{48}$$

$$\textit{Gains:} \quad \lambda,\ \gamma_{W1},\ \gamma_{W2},\ \delta,\ a,\ \mu$$

$$\textit{where:} \quad \sigma(z) = \left[\frac{1}{1+e^{-az_1}},\ \frac{1}{1+e^{-az_2}},\ \cdots,\ \frac{1}{1+e^{-az_\mu}},\ 1\right]^T$$

$$\Gamma_W = \text{diag}(\gamma_{W1},\ \gamma_{W1},\ \cdots,\ \gamma_{W1},\ \gamma_{W2})$$

$$\bar{x} = [x^T,\ u^T,\ 1]^T$$

Direct SHL NN Adaptive Control with Output Layer Adaptation Only (DIR NNW):

$$\textit{Control Law:} \quad u_c = B^T C^T \left(CBB^T C^T\right)^{-1} C \left(-Ax + A_m x + B_m r + C^T \hat{W}^T \sigma\left(\hat{V}^T \bar{x}\right)\right) \tag{49}$$

$$\textit{Adaptive Law:} \quad \dot{\hat{W}} = \text{Proj}\left(\hat{W}, -\Gamma_W \left(\sigma\left(\hat{V}^T \bar{x}\right) e^T P C^T + \delta \|e\| \hat{W}\right)\right) \tag{50}$$

$$\hat{W}_0 = 0_{\mu+1\times 6}$$

$$\hat{V} = \hat{V}_0 \qquad \hat{V}_0 = R_{21\times\mu} \tag{51}$$

$$\textit{Gains:} \quad q,\ \gamma_{W1},\ \gamma_{W2},\ \delta,\ a,\ \mu$$

$$\textit{where:} \quad A_m^T P + P A_m = -q I_{10}$$

$$\sigma(z) = \left[\frac{1}{1+e^{-az_1}},\ \frac{1}{1+e^{-az_2}},\ \cdots,\ \frac{1}{1+e^{-az_\mu}},\ 1\right]^T$$

$$\Gamma_W = \text{diag}(\gamma_{W1},\ \gamma_{W1},\ \cdots,\ \gamma_{W1},\ \gamma_{W2})$$

$$\bar{x} = [x^T,\ u^T,\ 1]^T$$

Indirect SHL NN Adaptive Control (IND NNWV):

$$\text{Control Law:} \quad u_c = B^T C^T \left(CBB^T C^T\right)^{-1} C \left(-Ax + A_m x + B_m r - C^T \hat{W}^T \sigma\left(\hat{V}^T \bar{x}\right)\right) \tag{52}$$

$$\text{Observer:} \quad \dot{\hat{x}} = Ax + BKu + C^T \hat{W}^T \sigma\left(\hat{V}^T \bar{x}\right) - k_r \left(\|\hat{Z}\|_F + \bar{Z}\right)\hat{e} - \lambda \hat{e} \tag{53}$$

$$\hat{x}_0 = x_0$$

$$\text{Adaptive Law:} \quad \dot{\hat{W}} = \text{Proj}\left(\hat{W}, -\Gamma_W \left(\left(\sigma\left(\hat{V}^T \bar{x}\right) - \nabla\sigma \hat{V}^T \bar{x}\right)\hat{e}^T C^T + \delta\|\hat{e}\|\hat{W}\right)\right) \tag{54}$$

$$\hat{W}_0 = 0_{\mu+1\times 6}$$

$$\dot{\hat{V}} = \text{Proj}\left(\hat{V}, -\Gamma_V \left(\bar{x}\hat{e}^T C^T \hat{W}^T \nabla\sigma + \delta\|\hat{e}\|\hat{V}\right)\right) \tag{55}$$

$$\hat{V}_0 = 0_{21\times\mu}$$

$$\text{Gains:} \quad \lambda,\ \gamma_{W1},\ \gamma_{W2},\ \gamma_{V1},\ \gamma_{V2},\ \delta,\ a,\ \mu,\ k_r$$

$$\text{where:} \quad \sigma(z) = \left[\frac{1}{1+e^{-az_1}},\ \frac{1}{1+e^{-az_2}},\ \cdots,\ \frac{1}{1+e^{-az_\mu}},\ 1\right]^T$$

$$\Gamma_W = \text{diag}(\gamma_{W1},\ \gamma_{W1},\ \cdots,\ \gamma_{W1},\ \gamma_{W2})$$

$$\Gamma_V = \text{diag}(\gamma_{V1},\ \gamma_{V1},\ \cdots,\ \gamma_{V1},\ \gamma_{V2})$$

$$\bar{x} = [x^T,\ u^T,\ 1]^T$$

$$\hat{Z} = \begin{bmatrix} \hat{W} & 0 \\ 0 & \hat{V} \end{bmatrix}$$

$$\bar{Z} \geq \|\hat{Z}\|_F$$

Direct SHL NN Adaptive Control (DIR NNWV):

$$\text{Control Law:} \quad u_c = B^T C^T \left(CBB^T C^T\right)^{-1} C \left(-Ax + A_m x + B_m r + C^T \hat{W}^T \sigma\left(\hat{V}^T \bar{x}\right) - k_r \left(\|\hat{Z}\|_F + \bar{Z}\right)\frac{\|e\|Pe}{\|Pe\|}\right) \tag{56}$$

$$\text{Adaptive Law:} \quad \dot{\hat{W}} = \text{Proj}\left(\hat{W}, -\Gamma_W \left(\left(\sigma\left(\hat{V}^T \bar{x}\right) - \nabla\sigma \hat{V}^T \bar{x}\right)e^T PC^T + \delta\|e\|\hat{W}\right)\right) \tag{57}$$

$$\hat{W}_0 = 0_{\mu+1\times 6}$$

$$\dot{\hat{V}} = \text{Proj}\left(\hat{V}, -\Gamma_V \left(\bar{x}e^T PC^T \hat{W}^T \nabla\sigma + \delta\|e\|\hat{V}\right)\right) \tag{58}$$

$$\hat{V}_0 = 0_{21\times\mu}$$

$$\text{Gains:} \quad q,\ \gamma_{W1},\ \gamma_{W2},\ \gamma_{V1},\ \gamma_{V2},\ \delta,\ a,\ \mu,\ k_r$$

$$\text{where:} \quad A_m^T P + PA_m = -qI_{10}$$

$$\sigma(z) = \left[\frac{1}{1+e^{-az_1}},\ \frac{1}{1+e^{-az_2}},\ \cdots,\ \frac{1}{1+e^{-az_\mu}},\ 1\right]^T$$

$$\Gamma_W = \text{diag}(\gamma_{W1},\ \gamma_{W1},\ \cdots,\ \gamma_{W1},\ \gamma_{W2})$$

$$\Gamma_V = \text{diag}(\gamma_{V1},\ \gamma_{V1},\ \cdots,\ \gamma_{V1},\ \gamma_{V2})$$

$$\bar{x} = [x^T,\ u^T,\ 1]^T$$

$$\hat{Z} = \begin{bmatrix} \hat{W} & 0 \\ 0 & \hat{V} \end{bmatrix}$$

$$\bar{Z} \geq \|\hat{Z}\|_F$$

Indirect Adaptive Control with Normalization and Output Error Feedback (IND OE NORM):

$$\textit{Control Law:}\quad u_c = \hat{K}^T B^T C^T \left(CB\hat{K}\hat{K}^T B^T C^T\right)^{-1} C \left(-\hat{A}x + A_m x + B_m r + A_m\left(k_p e + k_i \int e dt + k_d \frac{de}{dt}\right)\right) \tag{59}$$

$$\textit{Observer:}\quad \dot{\hat{x}} = \hat{A}x + B\hat{K}u - \lambda\hat{e} \qquad \hat{x}_0 = x_0 \tag{60}$$

$$\textit{Adaptive Law:}\quad \dot{\hat{A}} = \text{Proj}\left(\hat{A}, -\gamma_A\left(\frac{x\hat{e}^T}{\epsilon + x^T x + u^T u} + \delta\|\hat{e}\|\hat{A}\right)\right) \qquad \hat{A}_0 = A \tag{61}$$

$$\dot{\hat{k}} = \text{Proj}\left(\hat{k}, -\gamma_k\left(\frac{UB^T\hat{e}}{\epsilon + x^T x + u^T u} + \delta\|\hat{e}\|\hat{k}\right)\right) \qquad \hat{k}_0 = 1_{10\times 1} \tag{62}$$

Gains: $\lambda,\ \gamma_A,\ \gamma_k,\ k_p,\ k_i,\ k_d,\ \epsilon,\ \delta$

where: $U = \text{diag}(u)$

$\hat{K} = \text{diag}(\hat{k})$

Combined Indirect and Direct with Normalization and Output Error Feedback (CID OE NORM):

$$\textit{Control Law:}\quad u_c = \hat{K}_x x + \hat{K}_r r + B^T C^T \left(CBB^T C^T\right)^{-1} CA_m\left(k_p e + k_i \int e dt + k_d \frac{de}{dt}\right) \tag{63}$$

$$\textit{Observer:}\quad \dot{\hat{x}} = \hat{A}x + B\hat{K}u - \lambda\hat{e} \qquad \hat{x}_0 = x_0 \tag{64}$$

$$\textit{Adaptive Law:}\quad \dot{\hat{K}}_x = \text{Proj}\left(\hat{K}_x, -\gamma_x \text{Sgn}(K)\left(B^T\Sigma_x + \delta\|e\|\hat{K}_x\right)\right) \tag{65}$$

$$\hat{K}_{x0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(-A + A_m)$$

$$\dot{\hat{K}}_r = \text{Proj}\left(\hat{K}_r, -\gamma_r \text{Sgn}(K)\left(B^T\Sigma_r + \delta\|e\|\hat{K}_r\right)\right) \tag{66}$$

$$\hat{K}_{r0} = B^T C^T \left(CBB^T C^T\right)^{-1} C(B_m)$$

$$\dot{\hat{A}} = \text{Proj}\left(\hat{A}, -\gamma_A\left(\frac{x\hat{e}^T}{\epsilon + x^T x + u^T u} + \Sigma_x + \delta\|\hat{e}\|\hat{A}\right)\right) \tag{67}$$

$$\hat{A}_0 = A$$

$$\dot{\hat{G}} = \text{Proj}\left(\hat{G}, -\gamma_g\left(\frac{u\hat{e}^T}{\epsilon + x^T x + u^T u} + \Sigma_x\hat{K}_x^T + \Sigma_r\hat{K}_r^T + \delta\|\hat{e}\|\hat{G}\right)\right) \tag{68}$$

$$\hat{G}_0 = B$$

Gains: $\lambda,\ \gamma_x,\ \gamma_r,\ \gamma_A,\ \gamma_G,\ k_p,\ k_i,\ k_d,\ \epsilon,\ \delta$

where: $\Sigma_x = \hat{G}\hat{K}_x - A_m + \hat{A}$

$\Sigma_r = \hat{G}\hat{K}_r - B_m$

$\text{Sgn}(K) = I_{10}$

Simulation Testing: Under the F/A-18 study three failure cases were tested, which included a combination of stabilator loss of effectiveness, uncertain nonlinearities, and time delay during powered approach.

In the first scenario, the simulated upset was non-severe damage (loss of effectiveness) of 90% in the left stabilator at 4.0 seconds, and mild nonlinear coupling. All of the algorithms were simultaneously tuned and evaluated with ATLAS. The results for this test are summarized in Table 5.

For this case, several trends can be seen when observing Table 5. First, one should note that in the non-severe case most approaches outperform the baseline controller. The controllers that do not are direct adaptive techniques. In general we can observe that direct approaches are found to perform worse than indirect algorithms. Another apparent trend is that the addition of output error feedback in general improves the response of adaptive systems. This is manifested in almost all methods. Normalization, in the case of indirect adaptive control was also shown to improve performance. As for the the neural network controllers, another interesting result can be seen. For both SHL direct and indirect methods, tuning the inner layer (with backpropogation) did not improve the overall score over fixed random inner layer weights. This result is interesting as it contradicts the notion that better performance can be achieved with nonlinearly parameterized neural network control (i.e. SHL NN control with inner layer weight adaptation).[14,15]

	Fitness	$\|\|W_e e\|\|_{\mathcal{L}_2}$	$\|\|W_{e_u} e_u\|\|_{\mathcal{L}_2}$	$\|\|W_e e\|\|_{\mathcal{L}_\infty}$	$\|\|W_{e_u} e_u\|\|_{\mathcal{L}_\infty}$	n_{osc}	$\|e_h\|$
BASE	9.7071	7.0540	3.3451	0.1410	0.1307	22	6.5203
OE	3.4536	1.3406	6.0048	0.0417	0.4632	22	0.2036
IND	3.5600	1.8047	2.7423	0.0369	0.1354	18	1.5638
DIR	9.8186	7.4187	3.3835	0.1485	0.1309	17	7.9325
L1	3.0338	1.2271	2.9078	0.0333	0.1706	20	0.4043
CID	2.9562	0.8083	2.4956	0.0327	0.1301	25	0.0352
IND OE	2.5786	0.6994	3.1001	0.0257	0.1593	21	0.2738
DIR OE	3.4075	1.3021	6.0947	0.0414	0.4769	22	0.0802
L1 OE	2.7151	0.8163	2.8723	0.0260	0.1853	21	0.5753
CID OE	2.7041	0.8109	3.4724	0.0299	0.1585	21	0.2063
IND NNW	2.4740	0.9008	2.3162	0.0235	0.1226	17	0.8257
DIR NNW	9.6661	7.3554	3.3357	0.1473	0.1304	17	7.0698
IND NNWV	2.6365	0.9013	2.3127	0.0238	0.1243	19	0.8453
DIR NNWV	9.3531	7.0924	3.3439	0.1418	0.1307	17	6.5932
IND OE NORM	2.2271	0.5372	2.5275	0.0201	0.1580	19	0.2953
CID OE NORM	3.2086	0.8074	2.7944	0.0271	0.1488	28	0.0411

Table 5. ***Table of F/A-18 results for LOE of 0.9 and mild coupling (BASE - Baseline controller; OE - output error term; IND - Indirect adaptive controller; DIR - Direct adaptive controller; L1 - $\mathcal{L}_1$ adaptive controller; CID - combined indirect and direct adaptive controller; NNW - single hidden layer neural network with only output layer weights adjustment; NNWV - single hidden layer neural network with both input and input layer weights adjustment; NORM - adaptive algorithm with normalization)***

In the second scenario, a similar experiment was performed but with more severe damage. In this case, left stabilator loss of effectiveness was applied at 99% along with more significant nonlinear coupling.

As with the previous case the ATLAS tuner was employed to select gains and evaluate each approach. Table 6 gives the results for this case.

For this case, Table 6 shows similar trends to those found in the first case. It can be seen in this case that direct adaptive control is outperformed by indirect methods. Also, output error feedback control and normalization in general improve performance when applied. The same trend is also found with the neural network based algorithms, despite the substantial increase in nonlinear coupling. It can be observed that no improvement is made by inner layer weight adaptation.

In the third case, the same damage scenario simulated in second case was investigated, but a pure time delay was applied to the actuator command signal. The controller commands were buffered and applied at a 5 sample delay which amounts to a time delay of 0.05 seconds. For this case the same free design parameters were used as in the second case (i.e. the gains were not tuned specifically for this scenario). This test was intended to study the robustness of the adaptive controllers to time delay. Results of this robustness test are summarized in Table 7.

In this case similar results to the second case were obtained for a group of methods, while other approaches are shown to fail when subject to the time delay. These methods are namely, standard output error feedback control (OE), direct adaptive control (DIR), direct adaptive control with output error feedback (DIR OE), both direct SHL NN adaptive controllers (DIR NNW and DIR NNWV), indirect SHL NN adaptive control (IND NNWV), and indirect adaptive control with normalization and output error feedback (IND OE NORM). One should note that these methods may possibly be tuned to handle the applied time delay. However, given the optimal gains for the second case, these controllers were shown not to be robust to the 5 sample delay.

It is seen that the algorithm that scored best in the third case is the indirect adaptive controller with a NN where only the output layer weights were adjusted. Simulation results obtained using this algorithm are shown in Figure 10

	Fitness	$\|\|W_e e\|\|_{\mathcal{L}_2}$	$\|\|W_{e_u} e_u\|\|_{\mathcal{L}_2}$	$\|\|W_e e\|\|_{\mathcal{L}_\infty}$	$\|\|W_{e_u} e_u\|\|_{\mathcal{L}_\infty}$	n_{osc}	$\|e_h\|$
BASE	44.9062	36.2602	8.6089	1.0641	0.1776	20	60.7899
OE	10.9181	6.3056	13.4302	0.1710	0.6831	42	4.7841
IND	3.4721	1.1998	2.8931	0.0279	0.1793	25	1.0918
DIR	30.6418	26.4320	39.6693	0.6982	2.6362	22	0.5126
L1	3.1817	1.0735	3.2612	0.0353	0.1890	22	1.6269
CID	3.3410	1.2607	3.4172	0.0455	0.1500	23	0.4290
IND OE	3.2486	1.0374	3.7034	0.0337	0.1955	25	0.0432
DIR OE	11.4387	6.9640	14.9775	0.1836	0.6913	33	9.7668
L1 OE	3.2112	0.9859	3.1038	0.0415	0.1935	23	2.0458
CID OE	3.3547	1.2723	4.1347	0.0352	0.1756	23	0.1361
IND NNW	2.8301	0.9035	2.9273	0.0239	0.1746	21	0.8398
DIR NNW	44.6440	36.3072	8.6159	1.0658	0.1776	16	60.8858
IND NNWV	2.8383	0.9073	3.0559	0.0238	0.1606	21	0.8235
DIR NNWV	NaN	NaN	NaN	NaN	NaN	NaN	NaN
IND OE NORM	2.6651	0.5395	3.2338	0.0210	0.1910	24	0.2859
CID OE NORM	4.9031	2.2859	5.5072	0.0646	0.1836	25	3.0498

Table 6. ***Table of F/A-18 results for LOE of 0.99 and significant coupling (BASE - Baseline controller; OE - output error term; IND - Indirect adaptive controller; DIR - Direct adaptive controller; L1 - $\mathcal{L}_1$ adaptive controller; CID - combined indirect and direct adaptive controller; NNW - single hidden layer neural network with only output layer weights adjustment; NNWV - single hidden layer neural network with both input and input layer weights adjustment; NORM - adaptive algorithm with normalization)***

VI. Conclusions

Results presented in the paper demonstrate the feasibility and potential of the ADVANCE framework, and further development of the algorithms and testing procedures is expected to give rise to a set of recommendations and guidelines regarding the use, tuning and implementation of different adaptive flight control algorithms to different problems in flight control. This will also facilitate flight certification of adaptive flight control algorithms.

However, to make the ADVANCE framework more practical, the following issues need to be addressed: (i) How to choose the relative weighting between the terms in the ACM criterion? (ii) How to minimize the number of terms and remove the redundant ones? (iii) How to design an effective criterion for the frequency of signal oscillations; (iv) How to incorporate the transient and steady-state parametric errors in the criterion, and which terms should these replace? (v) How to incorporate a term that penalizes unfavorable pilot-aircraft interaction and minimizes the probability of PIO? and (vi) How to minimize the number of test cases over ranges of uncertainty, times of failure, reference inputs and time delays? Hence, further study is needed in order to arrive at truly effective performance metrics that will facilitate flight certification of adaptive flight control algorithms.

Implementation of a large number of advanced adaptive control algorithms in Matlab and C++ was a formidable undertaking. Each algorithm was carefully implemented and first tested in Matlab. Following its transfer to C++, it was compared with the Matlab one to assure that the implementations are identical. Following that, the adaptive controllers were tuned using the proposed ACM criterion under ATLAS by running a large number of simulations. The resulting tuning parameters resulted in good performance in most of the cases. Hence this is a feasible, unbiased, and effective procedure for tuning and comparing adaptive controllers.

Combined robust and adaptive algorithms performed the best (combined robust control using output error feedback, and adaptive control) in the indirect adaptive control settings. Direct adaptive control in most of the cases fared worst. In many cases combined direct and indirect adaptive controller with output error feedback outperformed other algorithms. It appears that finding suitable combinations of algorithms can result in designs that achieve excellent overall performance and optimize both transient and steady state performance. This will be the focus of our future work.

Acknowledgement

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	Fitness	$\|\|W_e e\|\|_{\mathcal{L}_2}$	$\|\|W_{e_u} e_u\|\|_{\mathcal{L}_2}$	$\|\|W_e e\|\|_{\mathcal{L}_\infty}$	$\|\|W_{e_u} e_u\|\|_{\mathcal{L}_\infty}$	n_{osc}	$\|e_h\|$
BASE	1887376.3999	9098.5883	36417116.0245	445.9235	2281724.1253	82	1493.8547
OE	371721.4903	3368.4622	7100933.3415	155.2840	525114.7523	86	939.7021
IND	4.6304	1.3743	2.9777	0.0310	0.1760	37	1.2732
DIR	1876411.0654	9101.3043	36205092.1819	444.9525	2267035.6115	86	1499.0553
L1	4.3574	1.2390	3.3622	0.0349	0.1816	35	1.2829
CID	4.1568	1.3426	3.4596	0.0477	0.1539	32	0.5347
IND OE	4.1173	1.0934	3.8969	0.0357	0.2022	35	0.0612
DIR OE	1094040.6213	5849.3728	20906387.9489	273.9075	1704034.5784	86	1271.5278
L1 OE	4.2392	1.0710	3.2392	0.0394	0.1873	35	1.8189
CID OE	4.1796	1.2950	4.1902	0.0363	0.1730	33	0.1263
IND NNW	3.9297	1.0146	3.2638	0.0264	0.1809	33	0.9416
DIR NNW	NaN	NaN	NaN	NaN	NaN	NaN	NaN
IND NNWV	98209.7222	2199.0497	1858439.7045	103.7385	117910.3367	93	816.1957
DIR NNWV	NaN	NaN	NaN	NaN	NaN	NaN	NaN
IND OE NORM	116777.1270	2216.2459	2186421.6161	103.1104	204518.2107	92	679.2981
CID OE NORM	5.3768	2.2651	5.7579	0.0660	0.2844	31	3.0369

Table 7. /em Table of F/A-18 results for LOE of 0.99 and significant coupling with 0.05 sec time delay (BASE - Baseline controller; OE - output error term; IND - Indirect adaptive controller; DIR - Direct adaptive controller; L1 - $\mathcal{L}_1$ adaptive controller; CID - combined indirect and direct adaptive controller; NNW - single hidden layer neural network with only output layer weights adjustment; NNWV - single hidden layer neural network with both input and input layer weights adjustment; NORM - adaptive algorithm with normalization)

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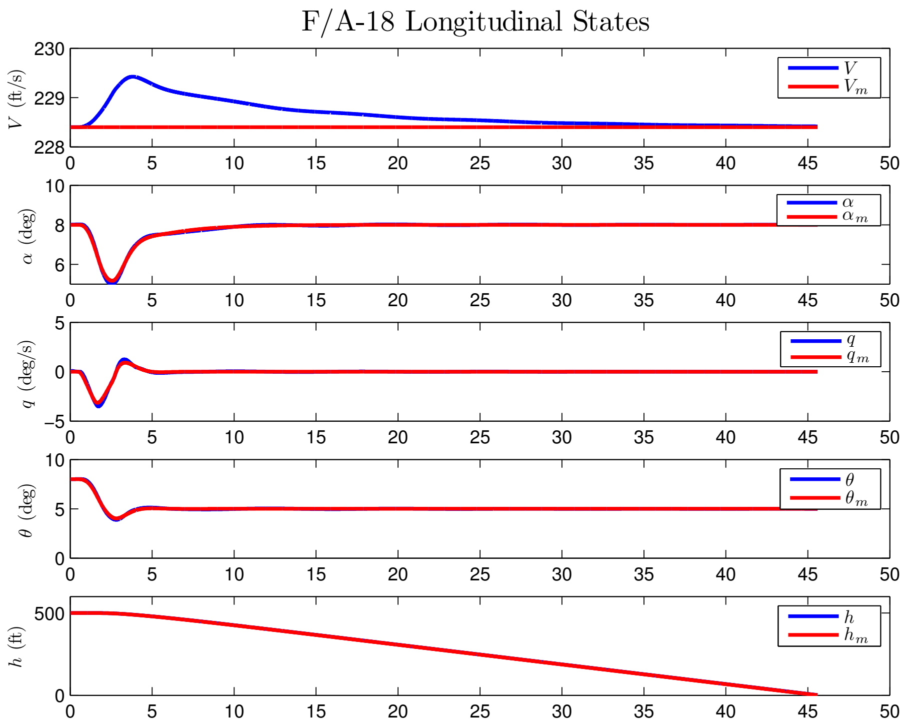
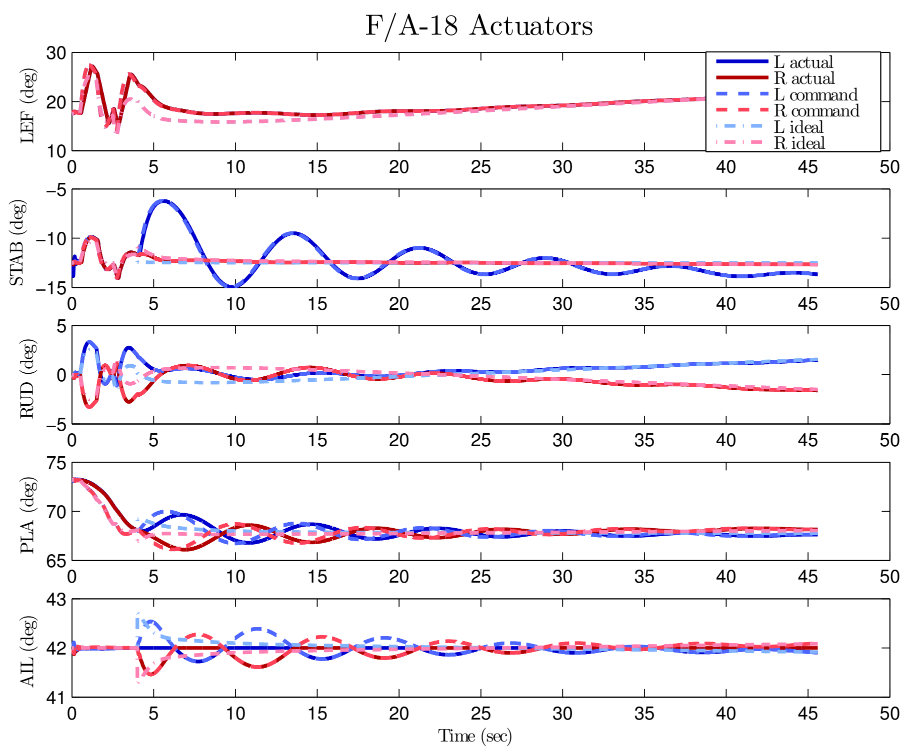


(a)



(b)

Figure 10. ***Indirect SHL NN Adaptive Control with Output Layer Adaptation Only (IND NNW) for F/A-18 control with at stabilator LOE failure. The controller was tuned with ATLAS for the failure and then additional time delay was added in the above simulation to test for robustness.***