# **Math Course Catalogue**

# Updated 10/01/2012

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# Math 105 - Data, Functions, and Graphs

Prerequisites: 3-4 years HS math including trigonometry

Frequency: Fall (I), Winter (II), Summer (IIIb)

Student Body: First-year students

Credit: 4 Credits. No credit granted for those who have completed any Math course numbered 110 or

higher. 2 credits granted to those who have completed 103.

Recent Texts: Functions Modeling Change: A Preparation for Calculus (3rd edition) by Connally,

Hughes-Hallett, Gleason et. al.

# Background and Goals:

Math 105 serves both as a preparatory class to the calculus sequences and as a class for students who are interested in strengthening their math skills. Students who successfully complete 105 are fully prepared for Math 115.

#### Content:

This course presents the concepts of precalculus from four points of view: geometric (graphs), numeric (tables), symbolic (formulas), and written (verbal descriptions). The emphasis is on the mathematical modeling of real-life problems using linear, polynomial, exponential, logarithmic, and trigonometric functions. Students develop their reading, writing, and questioning skills in an interactive classroom setting.

#### Alternatives:

None

#### Subsequent Courses:

The course prepares students for Math 115 (Calculus I).

# Math 107 - Mathematics for the Information Age

**Prerequisites:** 3-4 years HS math **Frequency:** Sporadically

Student Body: First-year students (non-mathematics concentrators) who are not necessarily required to

take calculus

Credit: 3 Credits.

Recent Texts:

#### Background and Goals:

The course will investigate topics relevant to the information age in which we live. An investigation of cryptography and coding methods, including prime numbers, randomness, and data compression will lead to the mathematics of the Web. Use of interactive web sites and web data are an integral part of the course. The course will emphasize the representation of mathematical data in graphical, tabular, and symbolic forms and investigate the inferences that can be drawn from these models. Emphasis will be placed on the development of estimation skills, the ability to determine reasonableness of answers, and the ability to find alternative approaches to a problem.

#### Content:

Typical topics include cryptography, coding, politics, biological data, populations, chaos, and game theory. Topics will be presented as modules, generally a week or two in length.

# Alternatives:

None.

# Subsequent Courses:

Math 127 or Math 128 could be taken after Math 107.

## Math 110 - Pre Calculus (Self-Paced)

**Prerequisites:** 3-4 years HS math **Frequency:** Fall (I), Winter (II) **Student Body:** First-year students

Credit: 2 Credits. No credit granted for those who have completed a pre-calculus course

Recent Texts: Functions Modeling Change: A Preparation for Calculus (3rd edition) by Connally,

Hughes-Hallett, Gleason et. al.

#### Background and Goals:

Math 110 is a condensed, half-term version of Math 105 designed specifically to prepare students for Math 115. It is open only to students who have enrolled in Math 115 and whose performance on the first uniform examination indicates that they will have difficulty completing that course successfully. This self-study course begins shortly after the first uniform examination in Math 115, and is completed under the guidance of an instructor without regular classroom meetings. Students must receive permission from the Math 115 Course Director or other designated representative to enroll in the course, and they must visit the Math Lab as soon as possible after enrolling to receive printed course information. Enrollment opens the day after the first Math 115 uniform examination, and enrollment must be completed by the Friday of the following week.

#### Content:

The course is a condensed, half-term version of Math 105 designed for students who appear to be prepared to handle calculus but are not able to successfully complete Math 115. Students may enroll in Math 110 only on the recommendation of a mathematics instructor after the third week of classes in the Fall and must visit the Math Lab to complete paperwork and receive course materials. The course covers data analysis by means of functions and graphs.

#### Alternatives:

Math 105 (Data, Functions and Graphs) covers the same material in a traditional classroom setting.

#### Subsequent Courses:

The course prepares students for Math 115 (Calculus I).

# Math 115 - Calculus I

**Prerequisites:** 3-4 years HS math including trigonometry **Frequency:** Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

Student Body: First-year students

Credit: 4 Credits. No credit after Math 116, 185, 215, or 216

Recent Texts: Calculus: Single Variable (5th edition) by Hughes-Hallett, et. al.

#### Background and Goals:

The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof. All sections are given two uniform midterms and a final exam.

#### Content:

The course presents the concepts of calculus from four points of view: geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing, and questioning skills, as well as their ability to work cooperatively. Topics include functions and graphs, derivatives and their applications to real-life problems in various fields, and an introduction to integration. The classroom atmosphere is interactive and cooperative. Both individual and team homework is assigned.

#### Alternatives:

Math 185 (Honors Calculus I) is a more theoretical course which covers some of the same material. Math 175 (Intro to Cryptology) is a non-calculus alternative for students with a good command of first-semester calculus. Math 295 (Honors Mathematics I) is a much more intensive and rigorous course. A student whose preparation is insufficient for Math 115 should take Math 105 (Data, Functions and Graphs).

# Subsequent Courses:

Math 116 (Calculus II) is the natural sequel. A student who has done very well in this course could enter the honors sequence at this point by taking Math 156 (Applied Honors Calculus II).

# Math 116 - Calculus II

Prerequisites: Math 115 or AP credit

Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

**Student Body:** First-year students

Credit: 4 Credits. No credit after Math 156, 176, 186, 215, 216, 255, 256, 285, or 286.

Recent Texts: Calculus: Single Variable (5th edition) by Hughes-Hallett, et. al.

## Background and Goals:

The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof. All sections are given two uniform midterms and a final exam.

#### Content:

The course presents the concepts of calculus from four points of view: geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing and questioning skills, as well as their ability to work cooperatively. Topics include techniques of integration, applications of integration, Taylor series, an introduction to differential equations, and infinite series. The classroom atmosphere is interactive and cooperative. Both individual and team homework is assigned.

#### Alternatives:

Math 186 (Honors Calculus II) is a somewhat more theoretical course which covers much of the same material. Math 156 (Applied Honors Calculus II) also covers much of the same material using MAPLE and emphasizing applications to science and engineering.

# Subsequent Courses:

Math 215 (Calculus III) is the natural sequel. A student who has done very well in this course could enter the honors sequence at this point by taking Math 255 (Applied Honors Calculus III) or Math 285 (Honors Calculus III).

## Math 127 - Geometry and the Imagination

Prerequisites: 3 years HS math

Frequency: Winter (II)

Student Body: First-year students and sophomores

*Credit:* 4 Credits. No credit after completing any 200+ level math course, except 385, 489, and 497. *Recent Texts:* Beyond the Third Dimension: Geometry, Computer Graphics and Higher Dimensions by

Banchoff

## Background and Goals:

This course introduces students to the ideas and some of the basic results in Euclidean and non-Euclidean geometry. Beginning with geometry in ancient Greece, the course includes the construction of new geometric objects from old ones by projecting and by taking slices. The course is intended for students who want an introduction to mathematical ideas and culture. Emphasis is on conceptual thinking — students will do hands-on experimentation with geometric shapes, patterns, and ideas.

#### Content:

The course begins with the independence of Euclid's Fifth Postulate and with the construction of spherical and hyperbolic geometries in which the Fifth Postulate fails; we discuss how spherical and hyperbolic geometry differ from Euclidean geometry. We then study the geometry of higher dimensions: coordinization — the mathematician's tool for studying higher dimensions; construction of higher-dimension analogues of some familiar objects like spheres and cubes; discussion of the proper higher-dimensional analogues of some geometric notions (length, angle, orthogonality, etc.).

#### Alternatives:

None

#### Subsequent Courses:

This course does not provide preparation for any further study of mathematics.

# Math 128 - Explorations in Number Theory

Prerequisites: 3 years HS math

Frequency: Fall (I)

Student Body: First-year students and sophomores

Credit: 4 Credits. No credit after completing any 200+ level math course, except 385, 485, 489, and 497.

Recent Texts: Coursepack
Area: Number Theory

# **Background and Goals:**

This course is intended for students who want to engage in mathematical reasoning without having to take calculus first. It is particularly well-suited for non-science concentrators or those who are thoroughly undecided. Students will make use of software to conduct numerical experiments and to make empirical discoveries. Students will formulate precise conjectures and in many cases prove them. Thus the students will, as a group, generate a logical development of the subject.

#### Content:

After studying factorizations and greatest common divisors, emphasis will shift to the patterns that emerge when the integers are classified according to the remainder produced upon division by some fixed number (congruences). Once some basic tools have been established, applications will be made in several directions. For example, students may derive a precise parameterization of Pythagorean triples.

#### Alternatives:

None

#### Subsequent Courses:

This course does not provide preparation for any further study of mathematics.

# Math 145 - Houghton Scholars Calculus Workshop I

Prerequisites: Concurrent enrollment in Math 115 and permission of instructor

Frequency: Fall (I)

Student Body: First year students

Credit: 2 Credits.
Recent Texts: None

# Background and Goals:

This course is an intensive supplemental workshop experience for students in the Douglas Houghton Scholars Program. In a small-class setting, students work in small groups on problems more challenging than those in the regular section of Math 115. The goal is to develop the students' problem-solving capabilities and promote their interest in challenging problems.

#### Content:

The workshop follows the structure of Math 115 (Calculus I). Content includes geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing, and questioning skills, as well as their ability to work cooperatively. Topics include functions and graphs, derivatives and their applications to real-life problems in various fields, and an introduction to integration.

#### Alternatives:

None

# Subsequent Courses:

Math 146 (Houghton Scholars Calculus Workshop II)

# Math 146 - Houghton Scholars Calculus Workshop II

Prerequisites: Concurrent enrollment in Math 116 and permission of instructor

Frequency: Winter (II)

Student Body: First year students

Credit: 2 Credits.
Recent Texts: None

## Background and Goals:

This course is an intensive supplemental workshop experience for students in the Douglas Houghton Scholars Program. In a small-class setting, students work in small groups on problems more challenging than those in the regular section of Math 116. The goal is to develop the students' problem-solving capabilities and promote their interest in challenging problems.

#### Content:

The workshop follows the structure of Math 116 (Calculus II). Content includes geometric (graphs), numeric (tables), symbolic (formulas), and verbal descriptions. Students will develop their reading, writing and questioning skills, as well as their ability to work cooperatively. Topics include techniques of integration, applications of integration, Taylor series, an introduction to differential equations, and infinite series.

#### Alternatives:

None

#### Subsequent Courses:

Math 215 (Calculus III), Math 216 (Intro. to Differential Equations)

#### Math 147 - Introduction to Interest Theory

Prerequisites: Math 115

Frequency: Fall (I), Winter (II)

Student Body: First and second year students

Credit: 3 Credits. No credit after completing any 200+ level math course, except 385, 489, or 497.

Recent Texts: Mathematics of Finance (6th edition) by Zima and Brown

Area: Actuarial & Financial

## Background and Goals:

This course is designed for students who seek an introduction to the mathematical concepts and techniques employed by financial institutions such as banks, insurance companies, and pension funds. Actuarial students, and other mathematics concentrators, should elect Math 424 which covers the same topics but on a more rigorous basis requiring considerable use of calculus. The course is not part of a sequence. Students should possess financial calculators.

#### Content:

Topics covered include: various rates of simple and compound interest, present and accumulated values based on these; annuity functions and their application to amortization, sinking funds, and bond values; depreciation methods; introduction to life tables, life annuity, and life insurance values.

#### Alternatives:

Math 424 (Compound Interest and Life Ins) covers the same material in greater depth and with a higher level of mathematical content.

# Subsequent Courses:

None

## Math 156 - Applied Honors Calculus II

Prerequisites: Score of 4 or 5 on Advanced Placement AB or BC Calculus exam

Frequency: Fall (I)

**Student Body:** First-year students

*Credit:* 4 Credits. Credit is granted for either 116 or 156, but not both. *Recent Texts:* Single Variable Calculus (UM edition) by Stewart

## Background and Goals:

Math 156 is part of the applied honors calculus sequence for engineering and science concentrators. The course is an alternative to Math 116 for students with strong mathematical ability. Applications and concepts receive equal treatment. Theorems are stated precisely and are derived, but technical details are omitted. Examples are given to illustrate the theory. Critical thinking and class participation are encouraged. The goal is to provide students with the solid background needed for subsequent courses in mathematics, engineering, and science.

#### Content:

Riemann sums, definite integral, fundamental theorem of calculus, applications of integral calculus (e.g. arclength, surface area, work, center of mass, probability density functions), improper integrals, infinite sequences and series, geometric series, alternating series, power series, Taylor series, differential equations, complex numbers. Students are introduced to MAPLE.

#### Alternatives:

Math 116 (Calculus II).

# Subsequent Courses:

Math 255 (Applied Honors Calculus III) is the natural sequel.

## Math 174 - Proofs in Geometry

Prerequisites: Permission of honors advisor.

Frequency: Fall (I)

**Student Body:** First-year seminar

Credit: 4 Credits. No credit granted to those who have completed a 200-level or higher Mathematics

course.

Recent Texts: Geometry Revisited by Coxeter and Greitzer

## Background and Goals:

The course will be very interactive, eliciting suggestions towards proof from the students so that all the problems are eventually solved by a joint effort between the students and the instructor. This format has worked well in the past for honors courses. To enhance the visualization, we plan to develop software for two-dimensional geometric constructions. This software will be able to produce multi-color pictures of geometric configurations. In the long run, such software will save us time in creating problem sets, handouts and perhaps slides. Additional topics may be added depending on the interest and abilities of the students.

#### Content:

As a precursor to the mathematics, the course will use familiar games such as the old game Mastermind where player A has a code which player B has to use. Students will pair off and play the game, with the important additional feature that the guesser must write down what (s)he knows and can deduce after each guess, and therefore motivate his/her next guess. This should help set the mood and instill the idea of analyzing the facts at hand and making logical deductions. After this, the course will develop some basic theorems of Euclidean geometry. An example of such a theorem is that the angle bisectors (or medians, or altitudes, or perpendicular bisectors) of a triangle are concurrent. These results are fairly straightforward but exemplify the spirit of the course by providing a good introduction to rigorous proofs, Then we move to some more difficult, but beautiful, theorems from geometry such as Ceva's theorem, the Euler line, the nine-point circle theorem, Ptolemy's theorem, and Morley's theorem.

#### Alternatives:

none

# Subsequent Courses:

none

## Math 175 - Introduction to Cryptology

Prerequisites: Permission of honors advisor

Frequency: Fall (I)

Student Body: First-year students

Credit: 4 Credits. No credit granted to those who have completed a 200-level or higher Mathematics

course.

Recent Texts: None

## Background and Goals:

This course is an alternative to Math 185 as an entry to the honors calculus sequence. The course stresses discovery as a vehicle for learning.

#### Content:

This course gives a historical introduction to Cryptology, from ancient times up to modern public key encryption, particularly RSA, and introduces a number of mathematical ideas involved in the development and analysis of codes. Mathematical topics include some enumeration, probability, and statistics, but the bulk of the course is devoted to elementary number theory. Students also work throughout the course on effectively communicating mathematics, both written and orally. Moreover, students will develop rigorous mathematical proof writing skills, and a primary goal of the course is to not only understand how various cryptosystems work, but why.

#### Alternatives:

Math 115 (Calculus I), Math 185 (Honors Calculus I), or Math 295 (Honors Mathematics I).

# Subsequent Courses:

Math 176 (Dynamical Systems and Calculus), or Math 116 (Calculus II).

# Math 176 - Explorations in Topology and Analysis

Prerequisites: Math 175 or permission of instructor

Frequency: Winter (II)

Student Body: First-year students

Credit: 4 Credits. Credit is granted for either Math 176 or 186, but not both

Recent Texts: None

## Background and Goals:

The goal of this course is to explore the underpinnings of single- variable calculus, and to develop some fundamental notions of topology in Euclidean space. It is intended for students who have had some exposure to calculus (AB or BC in high school), and would like to deepen their understanding of the subject. It can be seen as a continuation of Math 175, but more broadly it is a course for anyone interested to gain a deeper understanding of single-variable calculus.

#### Content:

The course will move towards proving theorems that are among the crowning jewels of single-variable calculus and topology in Euclidean space: The fundamental theorems of calculus and the Brauer fixed-point theorem. The course will be taught in an IBL manner, and it will follow roughly the same format as Math 175.

#### Alternatives:

Math 116 (Calculus I) or Math 186 (Honors Calculus II)

# Subsequent Courses:

Math 285 (Honors Calculus III)

# Math 185 - Honors Calculus I

**Prerequisites:** Permission of honors advisor during orientation

Frequency: Fall (I)

**Student Body:** First-year students

Credit: 4 Credits.

Recent Texts: Calculus (4th ed) by Michael Spivak, Calculus and Linear Algebra (UM edition) by Wilfred

Kaplan

## Background and Goals:

Most students take calculus in high school, and it may seem that there isn't much new to learn. The goal of this course is to develop the familiar concepts of calculus using a more rigorous and theoretical approach. In particular, with its emphasis on how to use appropriate mathematical language, this course lays a solid foundation for future math courses, and is suitable for students intending to pursue a concentration in mathematics, science, or engineering who desire a more complete understanding of the underpinnings of calculus. This sequence is not restricted to students enrolled in the LSA Honors Program.

#### Content:

Topics covered include functions, graphs, continuity, limits, derivatives, and integrals. Tuesday meetings are usually devoted to introducing linear algebra.

#### Alternatives:

Math 115 (Calculus I) is a less theoretical course which covers much of the same material. Math 295 (Honors Mathematics I) gives a much more theoretical treatment of much of the same material.

# Subsequent Courses:

Math 186 (Honors Calculus II) is the natural sequel.

#### Math 186 - Honors Calculus II

Prerequisites: Permission of honors advisor or Math 185

Frequency: Winter (II)

**Student Body:** First-year students

Credit: 4 Credits. Credit is granted for either Math 176 or 186, but not both.

Recent Texts: Calculus (4th ed) by Michael Spivak

## Background and Goals:

Most students take calculus in high school, and it may seem that there isn't much new to learn. The goal of this course is to develop the familiar concepts of calculus using a more rigorous and theoretical approach. In particular, with its emphasis on how to use appropriate mathematical language, this course lays a solid foundation for future math courses, and is suitable for students intending to pursue a concentration in mathematics, science, or engineering who desire a more complete understanding of the underpinnings of calculus. This sequence is not restricted to students enrolled in the LSA Honors Program. This course is a continuation of Math 185.

#### Content:

Topics include integral calculus, transcendental functions, infinite sequences and series (including Taylor's series), and – time permitting – some simple applications to elementary differential equations. Tuesdays are mostly devoted to an introduction to linear algebra.

#### Alternatives:

Math 116 (Calculus II) is a less theoretical course which covers much of the same material. Math 156 (Applied Honors Calculus II) is more application based, but covers much of the same material.

# Subsequent Courses:

Math 285 (Honors Calculus III) is the natural sequel.

## Math 214 - Linear Algebra

**Prerequisites:** Math 115 and 116 **Frequency:** Fall (I), Winter (II)

Student Body: Engineering students, particularly in Industrial and Operations Engineering

Credit: 4 Credits. No credit granted to those who have completed or are enrolled in Math 217, 417, 419,

or 420.

**Recent Texts:** Linear Algebra with Applications (4th edition) by Bretscher

## Background and Goals:

An introduction to matrices and linear algebra. This course covers the basics needed to understand a wide variety of applications that use the ideas of linear algebra, from linear programming to mathematical economics. The emphasis is on concepts and problem solving. The sequence 214-215 is not for math concentrators. It is designed as an alternate to the sequence 215-216 for engineering students who need more linear algebra and less differential equations background.

#### Content:

An introduction to the main concepts of linear algebra... matrix operations, echelon form, solution of systems of linear equations, Euclidean vector spaces, linear combinations, independence and spans of sets of vectors in Euclidean space, eigenvectors and eigenvalues, similarity theory. There are applications to discrete Markov processes, linear programming, and solutions of linear differential equations with constant coefficients.

#### Alternatives:

Math 217 (Linear Algebra) is a more theoretical course which covers much of the material of Math 214 at a deeper level. Mathematics concentrators are required to take Math 217.

#### Subsequent Courses:

Math 561 (Linear Programming I), Math 462 (Mathematical Models ), Math 571 (Numer. Meth. for Sci. Comput. I).

# Math 215 - Calculus III

Prerequisites: Math 116, 156, or 186

Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

Student Body: Sophomores

Credit: 4 Credits. Credit is granted for only one course among Math 215, 255, and 285.

Recent Texts: Multivariable Calculus (6th edition) by Stewart

# Background and Goals:

The sequence Math 115-116-215 is the standard complete introduction to the concepts and methods of calculus. It is taken by the majority of students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields. The emphasis is on concepts and solving problems rather than theory and proof.

#### Content:

Topics include vector algebra and vector functions; analytic geometry of planes, surfaces, and solids; functions of several variables and partial differentiation; line, surface, and volume integrals and applications; vector fields and integration; Green's Theorem, Stokes' Theorem, and Gauss' Theorem. There is a weekly computer lab using MAPLE.

#### Alternatives:

Math 285 (Honors Calculus III) is a somewhat more theoretical course which covers the same material. Math 255 (Applied Honors Calculus III) is also an alternative.

#### Subsequent Courses:

For students intending to concentrate in mathematics or who have some interest in the theory of mathematics as well as its applications, the appropriate sequel is Math 217 (Linear Algebra). Students who intend to take only one further mathematics course and need differential equations (respectively, linear algebra) should take Math 216 (respectively, Math 214).

## Math 216 - Introduction to Differential Equations

Prerequisites: Math 116, 156, 176, 186, or 296

Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

**Student Body:** Sophomore engineering students

Credit: 4 Credits. 2 credits granted to those who have completed or are enrolled in Math 214. Credit is

granted for only one course among Math 216, 256, 286, and 316.

Recent Texts: Differential Equations: Computing and Modeling (4th edition) by Edwards and Penney

## Background and Goals:

For a student who has completed the calculus sequence, there are two sequences which deal with linear algebra and differential equations: 216&417 (or 419) and 217&316. The sequence 216&417 emphasizes problem-solving and applications and is intended for students of engineering and the sciences. Mathematics concentrators and other students who have some interest in the theory of mathematics should elect the sequence 217&316.

#### Content:

Math 216 is a basic course on differential equations, intended for engineers and other scientists who need to apply the techniques in their work. The lectures are accompanied by a computer lab and recitation section where students have the opportunity to discuss problems and work through computer experiments to further develop their understanding of the concepts of the class. Topics covered include some material on complex numbers and matrix algebra, first and second order linear and non-linear systems with applications, introductory numerical methods, and elementary Laplace transform techniques.

# Alternatives:

Math 286 (Honors Differential Equations) covers much of the same material, but with much more emphasis on theory. The sequence Math 217&316 covers all of this material and substantially more at greater depth and with greater emphasis on the theory. Math 256 (Applied Honors Calculus IV) is also an alternative.

#### Subsequent Courses:

Math 404 (Intermediate Diff. Eq.) covers further material on differential equations. Math 217 (Linear Algebra) and Math 417 (Matrix Algebra I) cover further material on linear algebra. Math 371 (Engin. 303—Numerical Methods) and Math 471/472 (Intro. to Numerical Methods) cover additional material on numerical methods.

# Math 217 - Linear Algebra

Prerequisites: Math 215, 255, or 285

Frequency: Fall (I), Winter (II), Spring (IIIa)

Student Body: Sophomore prospective mathematics concentrators

Credit: 4 Credits. No credit granted to those who have completed or are enrolled in Math 419 or 420. 2

credits granted to those who have completed Math 214 or 417.

**Recent Texts:** Linear Algebra and Its Applications (3rd updated edition) by Lay

## Background and Goals:

For a student who has completed the calculus sequence, there are two sequences which deal with linear algebra and differential equations: 216&417 (or 419) and 217&316. The sequence 216&417 emphasizes problem-solving and applications and is intended for students of Engineering and the sciences. Mathematics concentrators and other students who have some interest in the theory of mathematics should elect the sequence 217&316. These courses are explicitly designed to introduce the student to both the concepts and applications of their subjects and to the methods by which the results are proved.

#### Content:

The topics covered include: systems of linear equations; matrix algebra; vectors, vector spaces, and subspaces; geometry of  $\mathbf{R}^n$ ; linear dependence, bases, and dimension; linear transformations; Eigenvalues and Eigenvectors; diagonalization; inner products. Throughout there will be an emphasis on the concepts, logic, and methods of theoretical mathematics.

#### Alternatives:

Math 214, 417, and 419 cover similar material with more emphasis on computation and applications and less emphasis on proofs.

#### Subsequent Courses:

The intended course to follow Math 217 is Math 316 (Differential Equations). Math 217 is also prerequisite for Math 312 (Applied Modern Algebra), Math 412 (Introduction to Modern Algebra), and all of the more advanced courses in mathematics.

## Math 255 - Applied Honors Calculus III

Prerequisites: Math 156, or permission of instructor

Frequency: Winter (II)

**Student Body:** First-year students

Credit: 4 Credits. Credit is granted for only one course among Math 215, 255, and 285.

Recent Texts: Multivariable Calculus (5th edition) by Stewart

# Background and Goals:

Math 255 is part of the applied honors calculus sequence for engineering and science concentrators. Applications and concepts receive equal treatment. Theorems are stated precisely and are derived, but technical details are omitted. Examples are given to illustrate the theory. Critical thinking and class participation are encouraged. The goal is to provide students with the solid background needed for subsequent courses in mathematics, engineering, or science.

#### Content:

Analytic geometry of lines and planes using vector notation, parametric representation of curves and surfaces, multivariable calculus, line surface and volume integrals, vector fields, Green's theorem, Stokes' theorem, divergence theorem, applications (e.g. electromagnetic fields, fluid dynamics). MAPLE will be used throughout.

#### Alternatives:

Math 215 (Calculus III) or Math 285 (Honors Calculus III).

# Subsequent Courses:

Math 256 (Applied Honors Calculus IV) is the natural sequel.

## Math 256 - Applied Honors Calculus IV

Prerequisites: Math 255 Frequency: Fall (I)

Student Body: Sophomores

Credit: 4 Credits. Credit is granted for only one course among Math 216, 256, 286, and 316.

Recent Texts: Elementary Differential Equations and Boundary Value Problems (9th edition) by Boyce

and DiPrima

## Background and Goals:

Math 256 is part of the applied honors calculus sequence for engineering and science concentrators. Applications and concepts receive equal treatment. Theorems are stated precisely and are derived, but technical details are omitted. Examples are given to illustrate the theory. Critical thinking and class participation are encouraged. The goal is to provide students with the solid background needed for subsequent courses in mathematics, engineering, or science.

#### Content:

Linear algebra, matrices, systems of differential equations, initial value problems, qualitative theory of dynamical systems (e.g. equilibria, phase space, stability, bifurcations), nonlinear equations, numerical methods. Optional: boundary value problems. Students will learn to use MATLAB for computer simulations.

#### Alternatives:

Math 216 (Intro. to Differential Equations) or Math 286 (Honors Differential Equations).

# Subsequent Courses:

Many upper-level courses

# Math 285 - Honors Calculus III

Prerequisites: Math 156, 176, 186, or permission of instructor

Frequency: Fall (I)

Student Body: Sophomores and first-year students with suitable background

Credit: 4 Credits. Credit is granted for only one course among Math 215, 255, and 285.

Recent Texts: Multivariable Calculus (6th edition) by Stewart

# Background and Goals:

The sequence Math 185-186-285-286 is an introduction to calculus at the honors level. It is taken by students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields who want a somewhat more theoretical approach. Although much attention is paid to concepts and solving problems, the underlying theory and proofs of important results are also included. This sequence is not restricted to students enrolled in the LS&A Honors Program.

#### Content:

Topics include vector algebra and vector functions; analytic geometry of planes, surfaces, and solids; functions of several variables and partial differentiation, maximum-minimum problems; line, surface, and volume integrals and applications; vector fields and integration; curl, divergence, and gradient; Green's Theorem and Stokes' Theorem. Additional topics may be added at the discretion of the instructor.

#### Alternatives:

Math 215 (Calculus III) is a less theoretical course which covers the same material. Math 255 (Applied Honors Calc. III) is an applications-oriented honors course which covers much of the same material.

# Subsequent Courses:

Math 216 (Intro. to Differential Equations), Math 286 (Honors Differential Equations), or Math 217 (Linear Algebra).

## Math 286 - Honors Differential Equations

Prerequisites: Math 285 or permission of instructor

Frequency: Winter (II)
Student Body: Sophomores

Credit: 3 Credits. Credit is granted for only one course among Math 216, 256, 286, and 316.

Recent Texts: Elementary Differential Equations and Boundary Value Problems (9th edition) by Boyce

and DiPrima

## Background and Goals:

The sequence Math 185-186-285-286 is an introduction to calculus at the honors level. It is taken by students intending to concentrate in mathematics, science, or engineering as well as students heading for many other fields who want a somewhat more theoretical approach. Although much attention is paid to concepts and solving problems, the underlying theory and proofs of important results are also included. This sequence is not restricted to students enrolled in the LS&A Honors Program.

#### Content:

Topics include first-order differential equations, higher-order linear differential equations with constant coefficients, an introduction to linear algebra, linear systems, the Laplace Transform, series solutions, and other numerical methods (Euler, Runge-Kutta). If time permits, Picard's Theorem will be proved.

#### Alternatives:

Math 216 (Intro. to Differential Equations) and Math 316 (Differential Equations) cover much of the same material. Math 256 (Applied Honors Calculus IV) is also an alternative.

# Subsequent Courses:

Math 471 (Intro. to Numerical Methods) and/or Math 572 (Numer. Meth. for Sci. Comput. II) are natural sequels in the area of differential equations, but Math 286 is also preparation for more theoretical courses such as Math 451 (Advanced Calculus I).

#### Math 289 - Problem Solving

Prerequisites: Permission

Frequency: Fall (I), Winter (II)

Student Body: Undergraduate students insterested in learning to solve problems.

Credit: 1 Credit. May be repeated for credit

Recent Texts: None

## Background and Goals:

One of the better ways to develop mathematical abilities is by solving problems using a variety of methods. Familiarity with numerous methods is a great asset to the developing student of mathematics. Methods learned in attacking a specific problem frequently find application in many other areas of mathematics. In many instances an interest in and appreciation of mathematics is better developed by solving problems than by hearing formal lectures on specific topics. This course is intended for superior students who have exhibited both ability and interest in doing mathematics, but it is not restricted to honors students. This course is excellent preparation for the Putnam competition.

#### Content:

Students and one or more faculty and graduate student assistants will meet in small groups to explore problems in many different areas of mathematics. Problems will be selected according to the interests and background of the students.

# Alternatives:

None

# Subsequent Courses:

This course may be repeated for credit.

# Math 295 - Honors Mathematics I

Prerequisites: Permission of honors advisor

Frequency: Fall (I)

**Student Body:** First-year students

Credit: 4 Credits.

Recent Texts: Calculus (4th edition) by M. Spivak

## Background and Goals:

Math 295-296-395-396 is the most theoretical and demanding honors math sequence. The emphasis is on concepts, problem solving, as well as the underlying theory and proofs of important results. It provides an excellent background for advanced courses in mathematics. The expected background is high school trigonometry and algebra (previous calculus is not required, but is helpful.) This sequence is **not** restricted to students enrolled in the LS&A Honors program. Math 295 and 296 may be substituted for any Math 451 requirement. Math 296 and 395 may be substituted for any Math 217 requirement.

#### Content:

Axioms of the real numbers, completeness and connectedness in the real line. Functions of a real variable, limits and continuity, uniform continuity, extreme and intermediate value theorems, differentiation, integration, the fundamental theorem of calculus, Taylor's theorem with remainder.

#### Alternatives:

Math 156 (Applied Honors Calc. II), Math 175 (Intro to Cryptology), and Math 185 (Honors Calculus I) are alternative honors courses.

#### Subsequent Courses:

Math 296 (Honors Mathematics II)

# Math 296 - Honors Mathematics II

Prerequisites: Math 295
Frequency: Winter (II)

Student Body: First-year students

Credit: 4 Credits.

Recent Texts: Linear Algebra (2nd edition) by Kunze & Hoffman

# Background and Goals:

Math 295-296-395-396 is the most theoretical and demanding honors calculus sequence. The emphasis is on concepts, problem solving, as well as the underlying theory and proofs of important results. It provides an excellent background for advanced courses in mathematics. The expected background is high school trigonometry and algebra (previous calculus is not required, but is helpful.) This sequence is **not** restricted to students enrolled in the LS&A Honors program. Math 295 and 296 may be substituted for any Math 451 requirement. Math 296 and 395 may be substituted for any Math 217 requirement.

#### Content:

Sequences and series of functions, power series, uniform convergence, real analytic functions. Vector spaces, bases, linear transformations, dual spaces, determinants, traces, eigenvalues, inner-product spaces, spectral theory. Limits and continuity in Euclidean space, derivative as a linear map, Chain Rule, inverse/implicit function theorems.

#### Alternatives:

None

## Subsequent Courses:

Math 395 (Honors Analysis I)

# Math 310 - Elementary Topics: Choice and Chance

**Prerequisites:** Sophomore standing and one previous university math course

Frequency: Winter (II)

Student Body: Credit: 3 Credits. Recent Texts: None

# Background and Goals:

The Elementary Topics course may focus on any one of several topics. The material is presented at a level appropriate for sophomores and juniors without extensive coursework in math. The current offering of the course focuses on game theory.

#### Content:

Every day the media showers us with news, analysis, and op-eds, which use and misuse numbers to arrive at various far-reaching conclusions. The objective of the course is to help students to acquire some basic mathematical skills to navigate this sea of numbers. Often, this boils down to understanding a few fundamental, ancient, and deep concepts: randomness, fairness, coincidence, and bias. We will study what "probability", "events", and "independence" mean, how to compute some basic probabilities and why it can be costly to assume that events are independent when in fact they are not, as illustrated by recent and not so recent events in the insurance industry and the stock market. We will also discuss why randomized strategies in games can be quite helpful.

Alternatives:

None

Subsequent Courses:

None

## Math 312 - Applied Modern Algebra

Prerequisites: Math 217
Frequency: Fall (I)

**Student Body:** Sophomore and Junior mathematics and computer science concentrators **Credit:** 3 Credits. No credit for those who have completed or are enrolled in Math 412. **Recent Texts:** A Concrete Introduction to Higher Algebra (3rd edition) by Childs

# Background and Goals:

One of the main goals of the course (along with every course in the algebra sequence) is to expose students to rigorous, proof-oriented mathematics. Students are required to have taken Math 217, which should provide a first exposure to this style of mathematics. A distinguishing feature of this course is that the abstract concepts are

not studied in isolation. Instead, each topic is studied with the ultimate goal being a real-world application.

#### Content:

Sets and functions, relations and graphs, rings, Boolean algebras, semigroups, groups, and lattices. Applications from areas such as switching, automata, and coding theory, and may include finite and minimal state machines, algebraic decompositions of logic circuits, semigroup machines, binary codes, and series and parallel decomposition of machines.

#### Alternatives:

Math 412 (Introduction to Modern Algebra) is a more abstract and proof-oriented course with less emphasis on applications and is better preparation for most subsequent mathematics courses. EECS 203 (Discrete Structures) covers some of the same topics with a more applied approach.

#### Subsequent Courses:

Math 312 is one of the alternative prerequisites for Math 416 (Theory of Algorithms), and several advanced EECS courses make substantial use of the material of Math 312. Another good follow-up course is Math 475 (Elementary Number Theory).

# Math 316 - Differential Equations

Prerequisites: Math 215, 255, or 285; and 217

Frequency: Fall (I), Winter (II)

**Student Body:** A mix of undergraduate mathematics, science, and economics concentrators **Credit:** 3 Credits. Credit is granted for only one course among Math 216, 256, 286, and 316.

Recent Texts: Elementary Differential Equations and Boundary Value Problems (9th edition) by Boyce

and DiPrima

## Background and Goals:

This is an introduction to differential equations for students who have studied linear algebra (Math 217). It treats techniques of solution (exact and approximate), existence and uniqueness theorems, some qualitative theory, and many applications. Proofs are given in class; homework problems include both computational and more conceptually oriented problems.

#### Content:

First-order equations: solutions, existence and uniqueness, and numerical techniques; linear systems: eigenvector-eigenvalue solutions of constant coefficient systems, fundamental matrix solutions, nonhomogeneous systems; higher-order equations, reduction of order, variation of parameters, series solutions; qualitative behavior of systems, equilibrium points, stability. Applications to physical problems are considered throughout.

#### Alternatives:

Math 216 covers somewhat less material without presupposing linear algebra and with less emphasis on theory. Math 286 (Honors Differential Equations) is the honors version of Math 316.

# Subsequent Courses:

Math 471 (Intro. to Numerical Methods) and/or Math 572 (Numer. Meth. for Sci. Comput. III) are natural sequels in the area of differential equations, but Math 316 is also preparation for more theoretical courses such as Math 451 (Advanced Calculus I).

# Math 327 - Evolution of Mathematical Concepts

Prerequisites: Math 116 or Math 186

Frequency: Sporadically

Student Body: Juniors and Seniors interested in mathematics and the history of science

Credit: 3 credits.
Recent Texts: None

## Background and Goals:

This course examines the evolution of major mathematical concepts from mathematical and historical points of view. The course's goal is to throw light on contemporary mathematics by retracing the history of some of the major mathematical discoveries.

#### Content:

This course follows the evolution of three fundamental mathematical ideas in geometry, analysis and algebra. Typical choices of subject are: Euclid's parallel postulate and the development of non-Euclidean geometries, the notions of limit and infinitesimals, and the development of the theory of equations culminating with Galois theory.

#### Alternatives:

None

# Subsequent Courses:

None

# Math 351 - Principles of Analysis

Prerequisites: Math 215 and 217 or permission of instructor.

Frequency: Fall (I), Winter (II)

**Student Body:** Sophomores and Juniors

Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 451.

Recent Texts: Understanding Analysis by S. Abbott

Area: Analysis

## Background and Goals:

The course content is similar to that of Math 451, but Math 351 assumes less background. This course covers topics that might be of greater use to students considering a Mathematical Sciences concentration or a minor in Mathematics.

#### Content:

Analysis of the real line, rational and irrational numbers, infinity— limits, convergence, infinite sequences and series, continuous functions, power series, and differentiation.

# Alternatives:

Math 451 (Advanced Calculus I) covers similar topics while assuming more background than 351.

# Subsequent Courses:

None

## Math 354 - Fourier Analysis and its Applications

Prerequisites: Math 216, 256, 286, or 316

Frequency: Fall (I), Winter (II)

Student Body: Junior and Senior math and non-math concentrators

Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 450 or 454.

Recent Texts: Fourier Series and Orthogonal Polynomials by Jackson

Area: Analysis

## Background and Goals:

This course is an introduction to Fourier analysis with emphasis on applications. The course also can be viewed as a way of deepening one's understanding of the 100-and 200-level material by applying it in interesting ways.

#### Content:

This is an introduction to Fourier analysis at an elementary level, emphasizing applications. The main topics are Fourier series, discrete Fourier transforms, and continuous Fourier transforms. A substantial portion of the time is spent on both scientific/technological applications (e.g. signal processing, Fourier optics), and applications in other branches of mathematics (e.g. partial differential equations, probability theory, number theory). Students will do some computer work, using MATLAB, an interactive programming tool that is easy to use, but no previous experience with computers is necessary.

#### Alternatives:

Math 454 (Bound Val. Probs. for Part. Diff. Eq.) covers some of the same material with more emphasis on partial differential equations.

# Subsequent Courses:

This course is good preparation for Math 451 (Advanced Calculus I), which covers the theory of calculus in a mathematically rigorous way.

## Math 371 (Engin 371) - Numerical Methods

Prerequisites: Engin 101; and one of Math 216, 256, 286, or 316; and one of Math 214, 217, 417, or

419.

Frequency: Fall (I), Winter (II)

Student Body: Sophomore, Junior, and Senior engineering students

Credit: 3 Credits. No credit after Math 471.

Recent Texts: A Friendly Introduction to Numerical Analysis by B. Bradie

Area: Applied/NA

#### Background and Goals:

This is a survey course of the basic numerical methods which are used to solve practical scientific problems. Important concepts such as accuracy, stability, and efficiency are discussed. The course provides an introduction to MATLAB, an interactive program for numerical linear algebra, and may provide practice in FORTRAN programming and the use of software library subroutines. Convergence theorems are discussed and applied, but the proofs are not emphasized.

#### Content:

Floating point arithmetic, Gaussian elimination, polynomial interpolation, spline approximations, numerical integration and differentiation, solutions to non-linear equations, ordinary differential equations, polynomial approximations. Other topics may include discrete Fourier transforms, two-point boundary-value problems, and Monte-Carlo methods.

#### Alternatives:

Math 471 (Numerical Analysis) provides a more in-depth study of the same topics, with a greater emphasis on analyzing the accuracy and stability of the numerical methods. Math 571 (Numerical Linear Algebra) is a detailed study of the solution of systems of linear equations and eigenvalue problems, with some emphasis on large-scale problems. Math 572 (Numerical Methods for Differential Equations) covers numerical methods for both ordinary and partial differential equations. (Math 571 and 572 can be taken in either order).

#### Subsequent Courses:

This course is basic for many later courses in science and engineering. It is good background for 571-572.

# Math 385 - Math for Elementary School Teachers

**Prerequisites:** One year each of HS algebra and geometry

Frequency: Fall (I)

Student Body: Undergraduate concentrators in the Teaching Certificate Program

*Credit:* 3 Credits. No credit granted to those who have taken or are enrolled in Math 485. *Recent Texts:* Mathematics for Elementary School Teachers (4th edition) by Bassarear

Area: Teaching

# Background and Goals:

This course, together with its sequel Math 489, provides a coherent overview of the mathematics underlying the elementary and middle school curriculum. It is required of all students intending to earn an elementary teaching certificate and is taken almost exclusively by such students. Concepts are heavily emphasized with some attention given to calculation and proof. The course is conducted using a discussion format. Class participation is expected and constitutes a significant part of the course grade. Enrollment is limited to 30 students per section. Although only two years of high school mathematics are required, a more complete background including pre-calculus or calculus is desirable.

### Content:

Topics covered include problem solving, sets and functions, numeration systems, whole numbers (including some number theory), and integers. Each number system is examined in terms of its algorithms, its applications, and its mathematical structure.

### Alternatives:

None

# Subsequent Courses:

Math 489 (Math for Elem. and Middle School Teachers) is the sequel.

# Math 389 - Explorations in Mathematics

**Prerequisites**: None formally; see instructor beforehand.

Frequency: Winter (II)
Student Body: Undergraduates

Credit: 3 Credits.
Recent Texts: None

# Background and Goals:

The course is designed to show you how new mathematics is actually created: how to take a problem, make models and experiment with them, and search for underlying structure. The format involves little formal lecturing, much laboratory work, and student presentations discussing partial results and approaches. Course website: <a href="http://www.math.lsa.umich.edu/courses/389/">http://www.math.lsa.umich.edu/courses/389/</a>

### Content:

Problems for projects are drawn from a wide variety of mathematical areas, pure and applied. Problems are chosen to be accessible to undergraduates.

### Alternatives:

None

# Subsequent Courses:

After this course students should be ready for a variety of courses and research experiences.

# Math 395 - Honors Analysis I

Prerequisites: Math 296
Frequency: Fall (I)

Student Body: First-year students and sophomores

Credit: 4 Credits.

Recent Texts: Analysis on Manifolds by Munkres

# Background and Goals:

This course is a continuation of the sequence Math 295-296 and has the same theoretical emphasis. Students are expected to understand and construct proofs.

### Content:

Inverse/implicit function theorems, immersion/submersion theorems. Quotient and dual spaces, inner product spaces, spectral theory. Metric spaces, basic point-set topology. Integration in Euclidean space, Fubini's theorem, change of variables formula. Topics in linear algebra: tensor products, exterior and symmetric powers, Jordan and rational canonical forms.

# Alternatives:

None

# Subsequent Courses:

Math 396 (Honors Analysis II)

# Math 396 - Honors Analysis II

Prerequisites: Math 395
Frequency: Winter (II)
Student Body: Sophomores

Credit: 4 Credits.

Recent Texts: Comprehensive Introduction to Differential Geometry (3rd ed) by Spivak

# Background and Goals:

This course is a continuation of Math 395 and has the same theoretical emphasis. Students are expected to understand and construct proofs.

#### Content:

Submanifolds (with or without corners) of Euclidean space, abstract manifolds, tangent and cotangent spaces, immersion/submersion theorems. Partitions of unity, vector fields and differential forms on manifolds, exterior differentiation, integration of differential forms. Stokes' theorem. deRham cohomology, Riemannian metrics, Hodge star operator and the standard vector calculus versions of Stokes' theorem.

### Alternatives:

None

# Subsequent Courses:

Students who have successfully completed the sequence Math 295-396 are generally prepared to take a range of advanced undergraduate and graduate courses such as Math 493 (Algebraic Structures), Math 594 (Introduction to Linear Algebra), Math 525 (Probability Theory), Math 590 (Intro. to Topology), and many others.

# Math 404 - Intermediate Differential Equations

Prerequisites: Math 216, 256, 286, or 316

Frequency: Fall (I), Winter (II)

Student Body: Undergraduate and graduate students from engineering and LS&A

Credit: 3 Credits.

Recent Texts: Nonlinear Dynamics and Chaos: With Applications in Physics, Biology, Chemistry, and

Engineering by Strogatz

Area: Applied/NA

## Background and Goals:

This course is an introduction to the modern qualitative theory of ordinary differential equations with emphasis on geometric techniques and visualization. Much of the motivation for this approach comes from applications. Examples of applications of differential equations to science and engineering are a significant part of the course. There are relatively few proofs.

#### Content:

Geometric representation of solutions, autonomous systems, flows and evolution, linear systems and phase portraits, nonlinear systems, local and global behavior, linearization, stability, conservation laws, periodic orbits. Applications: free and forced oscillations, resonance, relaxation oscillations, competing species, Zeeman's models of heartbeat and nerve impulse, chaotic orbits, strange attractors.

### Alternatives:

Math 558 (Ordinary Differential Equations) covers some of the same material at a faster pace and includes additional topics.

# Subsequent Courses:

Math 454 (Boundary Value Problems for Partial Differential Equations) is a natural sequel.

# Math 412 - Introduction to Modern Algebra

Prerequisites: Math 215, 255, or 285; and Math 217

Frequency: Fall (I), Winter (II)

Student Body: Mainly undergraduate mathematics concentrators with some graduate students from

other departments

Credit: 3 Credits. 1 credit after Math 312

Recent Texts: Introduction to Abstract Algebra (7th edition) by McCoy and Janusz

Area: Algebra

# Background and Goals:

This course is designed to serve as an introduction to the methods and concepts of abstract mathematics. A typical student entering this course has substantial experience in using complex mathematical (calculus) calculations to solve physical or geometrical problems, but is inexperienced at analyzing carefully the content of definitions and the logical flow of ideas which underlie and justify these calculations. Although the topics discussed here are quite distinct from those of calculus, an important goal of the course is to introduce the student to this type of analysis. Much of the reading, homework exercises, and exams consists of theorems (propositions, lemmas, etc.) and their proofs. Math 217, or equivalent, required as background.

### Content:

The initial topics include ones common to every branch of mathematics: sets, functions (mappings), relations, and the common number systems (integers, rational numbers, real numbers, complex numbers). These are then applied to the study of particular types of mathematical structures: groups, rings, and fields. These structures are presented as abstractions from many examples such as the common number systems together with the operations of addition or multiplication, permutations of finite and infinite sets with function composition, sets of motions of geometric figures, and polynomials. Notions such as generator, subgroup, direct product, isomorphism, and homomorphism are defined and studied.

### Alternatives:

Math 312 (Applied Modern Algebra) is a less abstract course which replaces some of the material on rings and fields of Math 412 with additional applications to areas such as switching and coding theory.

### Subsequent Courses:

A student who successfully completes this course will be prepared to take a number of other courses in abstract mathematics: Math 416 (Theory of Algorithms), Math 451 (Advanced Calculus I), Math 475 (Elementary Number Theory), Math 575 (Intro. to Theory of Numbers), Math 493-494, Math 481(Intro. to Mathematical Logic), and Math 582 (Intro. To Set Theory). All of these courses will extend and deepen the student's grasp of modern abstract mathematics.

# Math 416 - Theory of Algorithms

Prerequisites: Math 312, 412; or EECS 280 and Math 465; or permission of instructor

Frequency: Sporadically

Student Body: Largely computer science concentrators with a few graduate students from other fields

Credit: 3 Credits.

Recent Texts: Algorithm Design by Kleinberg and Tardos

# Background and Goals:

Many common problems from mathematics and computer science may be solved by applying one or more algorithms — well-defined procedures that accept input data specifying a particular instance of the problem and produce a solution. Students entering Math 416 typically have encountered some of these problems and their algorithmic solutions in a programming course. The goal here is to develop the mathematical tools necessary to analyze such algorithms with respect to their efficiency (running time) and correctness. Different instructors will put varying degrees of emphasis on mathematical proofs and computer implementation of these ideas.

### Content:

Typical problems considered are: sorting, searching, matrix multiplication, graph problems (flows, travelling salesman), and primality and pseudo-primality testing (in connection with coding questions). Algorithm types such as divide-and-conquer, backtracking, greedy, and dynamic programming are analyzed using mathematical tools such as generating functions, recurrence relations, induction and recursion, graphs and trees, and permutations. The course often includes a section on abstract complexity theory including NP completeness.

### Alternatives:

This course has substantial overlap with EECS 586 (Design and Analysis of Algorithms) — more or less depending on the instructors. In general, Math 416 will put more emphasis on the analysis aspect in contrast to the design of algorithms aspect.

### Subsequent Courses:

EECS 574 (Computational Complexity) and 575 (Advanced Cryptography) include some topics which follow those of this course.

# Math 417 - Matrix Algebra I

**Prerequisites:** Three mathematics courses beyond Math 110 **Frequency:** Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

Student Body: Largely engineering and science students, both undergraduate and graduate

Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 214, 217, 419,

or 420.

Recent Texts: Linear Algebra with Applications (4th edition) by Bretscher

Area: Algebra

## Background and Goals:

Many problems in science, engineering, and mathematics are best formulated in terms of matrices — rectangular arrays of numbers. This course is an introduction to the properties of and operations on matrices with a wide variety of applications. The main emphasis is on concepts and problem-solving, but students are responsible for some of the underlying theory. Diversity rather than depth of applications is stressed. This course is not intended for mathematics concentrators, who should elect Math 217, and/or 493-494 if pursuing the honors concentration.

### Content:

Topics include matrix operations, echelon form, general solutions of systems of linear equations, vector spaces and subspaces, linear independence and bases, linear transformations, determinants, orthogonality, characteristic polynomials, eigenvalues and eigenvectors, and similarity theory. Applications include linear networks, least squares method (regression), discrete Markov processes, linear programming, and differential equations.

### Alternatives:

Math 419 (Lin. Spaces and Matrix Thy.) is an enriched version of Math 417 with a somewhat more theoretical emphasis. Math 217 (Linear Algebra) is also a more theoretical course which covers much of the material of 417 at a deeper level (despite its lower number).

### Subsequent Courses:

This course serves as prerequisite to several courses: Math 452 (Advanced Calculus II), Math 462 (Mathematical Models), Math 420 (Advanced Linear Algebra), and Math 571 (Numer. Meth for Sci. Comput. I)

# Math 419 - Linear Spaces and Matrix Theory

Prerequisites: 4 courses beyond Math 110

Frequency: Fall (I), Winter (II)

Student Body: Largely engineering graduate students and undergraduates; some mathematics

undergraduates

Credit: 3 Credits. Credit is granted for only one course among Math 214, 217, 417, and 419. No credit

granted to those who have completed or are enrolled in Math 420.

Recent Texts: Linear Algebra and its Applications (4th ed) by Strang

Area: Algebra

## Background and Goals:

Math 419 covers much of the same ground as Math 417 (Matrix Algebra I) but presents the material in a somewhat more abstract way in terms of vector spaces and linear transformations instead of matrices. There is a mix of proofs, calculations, and applications with the emphasis depending somewhat on the instructor. A previous proof-oriented course is helpful but by no means necessary.

### Content:

Basic notions of vector spaces and linear transformations: spanning, linear independence, bases, dimension, matrix representation of linear transformations; determinants; eigenvalues, eigenvectors, Jordan canonical form, inner-product spaces; unitary, self-adjoint, and orthogonal operators and matrices, applications to differential and difference equations.

### Alternatives:

Math 417 (Matrix Algebra I) is less rigorous and theoretical and more oriented to applications. Math 217 (Linear Algebra) is similar to Math 419 but more proof-oriented. Math 420 (Advanced Linear Algebra) is much more abstract and sophisticated.

# Subsequent Courses:

This course serves as prerequisite to several courses: Math 420 (Advanced Linear Algebra), Math 452 (Advanced Calculus II), Math 462 (Mathematical Models), Math 561(Linear Programming I), and Math 571 (Numer. Meth. for Sci. Comp. I)

# Math 420 - Advanced Linear Algebra

Prerequisites: Math 214, 217, 417, or 419 and one of 296, 412, or 451

**Frequency:** Winter term each year.

**Student Body:** Graduate students who pursue, or may pursue, the Mathematics masters program and undergraduate mathematics concentrators. This is a required course for MS degree in mathematics.

Credit: 3 Credits.

Recent Texts: Linear Algebra (2nd Edition), Kenneth M Hoffman (Author), Ray Kunze (Author)

Area: Algebra

# **Background and Goals:**

This is an introduction to the formal theory of abstract vector spaces and linear transformations. It is expected that students have complete at least one prior linear algebra course. The emphasis is on concepts and proofs with some calculations to illustrate the theory. Students should have significant mathematical maturity, at the level of Math 412 or 451. In particular, students should expect to work with and be tested on formal proofs.

### Content:

Topics are selected from: vector spaces over arbitrary fields (including finite fields); linear transformations, bases, and matrices; inner product spaces, duals and spaces of linear transformations, theory of determinants, eigenvalues and eigenvectors; applications to linear differential equations; bilinear and quadratic forms; spectral theorem; Jordan Canonical Form, least squares, singular value theory.

### Alternatives:

None.

# **Subsequent Courses:**

The natural sequel to Math 420 is Math 593 (Algebra I).

# Math 422 (BE 440) - Risk Management and Insurance

Prerequisites: Math 115, Junior standing, and permission of instructor

Frequency: Winter (II)

Student Body: Junior and Senior mathematics concentrators; some business undergraduates

Credit: 3 Credits. Satisfies the Upper Level Writing Requirement (URLW).

Recent Texts: Introduction to Risk Management and Insurance (9th edition) by Dorfman

Area: Actuarial & Financial

# Background and Goals:

This course is designed to allow students to explore the insurance mechanism as a means of replacing uncertainty with certainty. A main goal of the course is to explain, using mathematical models from the theory of interest, risk theory, credibility theory, and ruin theory, how mathematics underlies many important individual and societal problems.

### Content:

We will explore how much insurance affects the lives of students (automobile insurance, social security, health insurance, theft insurance) as well as the lives of other family members (retirements, life insurance, group insurance). While the mathematical models are important, an ability to articulate why the insurance options exist and how they satisfy the consumer's needs are equally important. In addition, there are different options available (e.g., in social insurance programs) that offer the opportunity of discussing alternative approaches. This course may be used to satisfy the LS&A upper-level writing requirement.

#### Alternatives:

None

# Subsequent Courses:

None

# Math 423 - Mathematics of Finance

Prerequisites: Math 217, Math 425, and EECS 183 or equivalents

Frequency: Fall (I), Winter (II), Spring (IIIa)

Student Body: Junior and Senior mathematics concentrators; some business undergraduates

Credit: 3 Credits.

Recent Texts: Mathematics for Finance: An Introduction to Financial Engineering by Capinksi and

Zastawniak

Area: Actuarial & Financial

# Background and Goals:

This course is an introduction to the mathematical models used in finance and economics with particular emphasis on models for pricing derivative instruments such as options and futures. The goal is to understand how the models derive from basic principles of economics and to provide the necessary mathematical tools for their analysis. A solid background in basic probability theory is necessary.

### Content:

Topics include risk and return theory, portfolio theory, the capital asset pricing model, the random walk model, stochastic processes, Black-Scholes Analysis, numerical methods, and interest rate models.

### Alternatives:

None

# Subsequent Courses:

Math 523 (Risk Theory) and Math 623 (Computational Finance).

## Math 424 - Compound Interest and Life Insurance

Prerequisites: Math 215, 255, or 285

Frequency: Fall (I), Winter (II), Spring (IIIa)

Student Body: Undergraduate and graduate students in the Actuarial Mathematics Program, or students

interested in exploring the concepts underlying the theory of interest.

Credit: 3 Credits.

Recent Texts: Mathematical Interest Theory (2nd edition) by Daniel and Vaaler

Area: Actuarial & Financial

## Background and Goals:

This course explores the concepts underlying the theory of interest and then applies them to concrete problems. The course also includes applications of spreadsheet software. The course is a prerequisite to advanced actuarial courses. It also helps students prepare for some of the professional actuarial exams.

### Content:

The course covers compound interest (growth) theory and its application to valuation of monetary deposits, annuities, and bonds. Problems are approached both analytically (using algebra) and geometrically (using pictorial representations). Techniques are applied to real-life situations: bank accounts, bond prices, etc. The text is used as a guide because it is prescribed for the professional examinations; the material covered will depend somewhat on the instructor.

### Alternatives:

Math 424 is required for students concentrating in actuarial mathematics; others may take Math 147 (Introduction to Interest Theory), which deals with the same techniques but with less emphasis on continuous growth situations.

# Subsequent Courses:

Math 520 (Life Contingencies I) applies the concepts of Math 424 and probability theory to the valuation of life contingencies (death benefits and pensions).

# Math 425 (Stats 425) - Introduction to Probability

Prerequisites: Math 215, 255, or 285

Frequency: Fall (I), Winter (II), Spring (IIIa), Summer (IIIb)

Student Body: About 80% undergraduate mathematics, engineering, and computer science

concentrators with a few graduate students

Credit: 3 Credits.

Recent Texts: A First Course in Probability (8th edition) by Ross

Area: Analysis; Actuarial & Financial

# Background and Goals:

This course introduces students to both useful and interesting ideas from the mathematical theory of probability and to a number of applications of probability to a variety of fields including genetics, economics, geology, business, and engineering. The theory developed together with other mathematical tools such as combinatorics and calculus are applied to everyday problems. Concepts, calculations, and derivations are emphasized. The course will make essential use of the material of Math 116 and 215.

### Content:

Topics include the basic results and methods of both discrete and continuous probability theory: conditional probability, independent events, random variables, joint distributions, expectations, variances, and covariances. The culminating results are the Law of Large Numbers and the Central Limit Theorem. Beyond this, different instructors may add additional topics of interest.

#### Alternatives:

Math 525 (Probability Theory) is a similar course at a faster pace and with deeper coverage. A stronger mathematical background is helpful for Math 525.

## Subsequent Courses:

Stats 426 (Introduction to Theoretical Statistics) is a natural sequel for students. Math 423 (Mathematics of Finance) and Math 523 (Risk Theory) include many applications of probability theory.

# Math 427 - Retirement Plans and Other Employee Benefits

Prerequisites: Math 115, Junior standing or permission of instructor

Frequency: Fall (I)

Student Body: Mainly Actuarial Mathematics students, but also some non-mathematics students

Credit: 3 Credits. Satisfies the Upper Level Writing Requirement (URLW).

Recent Texts: Pension Planning: Pensions, Profit-Sharing, and Other Deferred Compensation Plans by

Allen et. al.

Area: Actuarial & Financial

### Background and Goals:

An overview of the range of employee benefit plans, the considerations (actuarial and others) which influence plan design and implementation practices, and the role of actuaries and other benefit plan professionals and their relation to decision makers in management and unions. This course is certified for satisfaction of the upper-level writing requirement.

### Content:

Particular attention will be given to government programs which provide the framework, and establish requirements, for privately operated benefit plans. Relevant mathematical techniques will be reviewed, but are not the exclusive focus of the course.

#### Alternatives:

None

# Subsequent Courses:

Math 521 (Life Contingencies II) and/or Math 522 (Act. Theory of Pensions and Soc. Sec.) (which can be taken independently of each other) provide more in depth examination of the actuarial techniques used in employee benefit plans.

# Math 429 - Internship

**Prerequisites:** Concentration in Mathematics

Frequency: Winter (II)

Student Body: International Mathematics Students with CTP

requirements for internships

Credit: 1 Credit (Credit/No Credit grading scheme)

Recent Texts: None

# Background and Goals:

Credit is granted for a full-time internship of at least eight weeks that is used to enrich a student's academic experience and/or allows the student to explore careers related to his/her academic studies. Internship credit is not retroactive and must be prearranged.

### Content:

Course content is determined by student's internship.

# Alternatives:

None

# Subsequent Courses:

None

# Math 431 - Topics in Geometry for Teachers

Prerequisites: MATH 215, 255, or 285

Frequency: Fall (I)

Student Body: Mainly teaching certificate candidates; a few other mathematics and engineering students

Credit: 3 Credits.

Recent Texts: Euclidean and Non-Euclidean Geometry (4th edition) by Greenberg

Area: Teaching

# Background and Goals:

This course is a study of the axiomatic foundations of Euclidean and non-Euclidean geometry. Concepts and proofs are emphasized; students must be able to follow as well as construct clear logical arguments. For most students this is an introduction to proofs. A subsidiary goal is the development of enrichment and problem materials suitable for secondary geometry classes.

#### Content:

Topics selected depend heavily on the instructor but may include classification of isometries of the Euclidean plane; similarities; rosette, frieze, and wallpaper symmetry groups; tessellations; triangle groups; finite, hyperbolic, and taxicab non-Euclidean geometries.

### Alternatives:

An alternative geometry course at this level is Math 433 (Intro to Differential Geometry).

# Subsequent Courses:

None

# Math 433 - Introduction to Differential Geometry

Prerequisites: Math 215, 255, or 285; and Math 217

Frequency: Fall (I)

Student Body: Half undergraduate mathematics concentrators, half graduate students from EECS and

physics

Credit: 3 Credits.

Recent Texts: Elements of Differential Geometry by Millman and Parker

Area: Geometry/Topology

## Background and Goals:

This course is about the analysis of curves and surfaces in 2- and 3-space using the tools of calculus and linear algebra. There will be many examples discussed, including some which arise in engineering and physics applications. Emphasis will be placed on developing intuition and learning to use calculations to verify and prove theorems. Students need a good background in multivariable calculus (215) and linear algebra (preferably 217). Some exposure to differential equations (216 or 316) is helpful but not absolutely necessary.

#### Content:

Curves and surfaces in three-space using calculus. Curvature and torsion of curves. Curvature, covariant differentiation, parallelism, isometry, geodesics, and area on surfaces. Gauss-Bonnet Theorem. Minimal surfaces.

#### Alternatives:

Math 437 is a substantially more advanced course which requires a strong background in topology (590), linear algebra, and advanced multivariable calculus (452). It treats some of the same material from a more abstract and topological perspective and introduces more general notions of curvature and covariant derivative for spaces of any dimension.

### Subsequent Courses:

Math 635 (Differential Geometry) and Math 636 (Topics in Differential Geometry) further study Riemannian manifolds and their topological and analytic properties. Physics courses in general relativity and gauge theory will use some of the material of this course.

# Math 437 - Introduction to Differentiable Manifolds

Prerequisites: Math 590 and 420

Frequency: Fall (I)

Student Body: Mainly graduate students in mathematics

Credit: 3 Credits.

Recent Texts: Differential Topology by Guillemin and Pollack; Differential Topology by Hirsch

Area: Geometry/Topology

# Background and Goals:

This course is an introduction to the theory of smooth manifolds. The prerequisites for this course are a basic knowledge of analysis, algebra, and topology.

# Content:

The following topics will be discussed: smooth manifolds and maps, tangent spaces, submanifolds, vector fields and flows, basic Lie group theory, group actions on manifolds, differential forms, de Rham cohomology, orientation and manifolds with boundary, integration of differential forms, Stokes' theorem.

# Alternatives:

Math 433 (Intro. to Differential Geometry) is an undergraduate version which covers less material in a less sophisticated way.

# Subsequent Courses:

Math 635 (Differential Geometry)

# Math 450 - Advanced Mathematics for Engineers I

Prerequisites: Math 215, 255, or 285 and Math 216, 256, 286, or 316

Frequency: Fall (I), Winter (II), Summer (IIIb)

Student Body: Mathematics and engineering students; engineering graduate students

Credit: 4 Credits. No credit after Math 354 or Math 454.

Recent Texts: Advanced Engineering Mathematics (2nd edition) by Greenberg

Area: Analysis

# Background and Goals:

This course is an introduction to some of the main mathematical techniques in engineering and physics. It is intended to provide some background for courses in those disciplines with a mathematical requirement that goes beyond calculus. Model problems in mathematical physics are studied in detail. Applications are emphasized throughout.

#### Content:

Topics covered include: Fourier series and integrals; the classical partial differential equations (the heat, wave and Laplace's equations) solved by separation of variables; an introduction to complex variables and conformal mapping with applications to potential theory. A review of series and series solutions of ODEs will be included as needed. A variety of basic diffusion, oscillation, and fluid flow problems will be discussed.

### Alternatives:

This course overlaps with 454 and, to a much lesser extent, with 555. The coverage of PDEs in 450 is not as in-depth as 454; for example, in 450 coverage of special functions is reduced to the simplest Bessel functions. Those students needing a more thorough discussion of PDEs and boundary-value problems should take 454. On the other hand, 450 provides a broader introduction to applied methods.

### Subsequent Courses:

Math 555 (Complex Variables) and Math 556 (Methods of Applied Math I) are graduate-level courses that further develop both the theory and applications covered in 450.

### Math 451 - Advanced Calculus I

Prerequisites: A thorough understanding of Calculus and one of 217, 312, 412, or permission of

instructor

**Frequency:** Fall (I), Winter (II), Spring (IIIa)

Student Body: Half undergraduate mathematics concentrators, half science and engineering graduate

students

Credit: 3 Credits. No credit after 351.

Recent Texts: Elementary Analysis: The Theory of Calculus by K. Ross

Area: Analysis

## Background and Goals:

This course has two complementary goals: (1) a rigorous development of the fundamental ideas of Calculus, and (2) a further development of the student's ability to deal with abstract mathematics and mathematical proofs. The key words here are "rigor" and "proof;" almost all of the material of the course is geared toward understanding and constructing definitions, theorems (propositions, lemmas, etc.), and proofs. This is considered one of the more difficult among the undergraduate mathematics courses, and students should be prepared to make a strong commitment to the course. In particular, it is strongly recommended that some course which requires proofs (such as Math 412) be taken before Math 451.

#### Content:

Topics covered include: logic and techniques of proofs; sets, functions, and relations; cardinality; the real number system and its topology; infinite sequences, limits, and continuity; differentiation; integration, the Fundamental Theorem of Calculus, infinite series; sequences and series of functions.

### Alternatives:

There is really no other course which covers the material of Math 451. Although Math 450 is an alternative prerequisite for some courses, the emphasis of the two courses is quite distinct. Math 351 covers similar topics with much less rigor.

# Subsequent Courses:

The natural sequel to Math 451 is 452, which extends the ideas considered here to functions of several variables. In a sense, Math 451 treats the theory behind Math 115-116, while Math 452 does the same for Math 215 and a part of Math 216. Math 451 is also a prerequisite for several other courses: Math 575, Math 590, Math 596, and Math 597.

# Math 452 - Advanced Calculus II

Prerequisites: Math 217, 417, 419 or 420 (may be concurrent) and Math 451

Frequency: Sporadically

Student Body: Mathematics undergraduates with some non-mathematics graduate students

Credit: 3 Credits.

Recent Texts: Advanced Calculus of Several Variables by Edwards

Area: Analysis

# Background and Goals:

This course gives a rigorous development of multivariable calculus and elementary function theory with some view towards generalizations. Concepts and proofs are stressed. This is a relatively difficult course, but the stated prerequisites provide adequate preparation.

#### Content:

Topics include: (1) partial derivatives and differentiability; (2) gradients, directional derivatives, and the chain rule; (3) implicit function theorem; (4) surfaces, tangent planes; (5) max-min theory; (6) multiple integration, change of variable, etc.; (7) Greens' and Stokes' theorems, differential forms, exterior derivatives.

### Alternatives:

None

# Subsequent Courses:

Math 452 is good general background for any of the more advanced courses in analysis (Math 596, 597) or differential geometry or topology (Math 537, 635).

# Math 454 - Boundary Value Problems for Partial Differential Equations

**Prerequisites:** Math 215, 255, or 285; and Math 216, 256, 286, or 316

Frequency: Fall (I), Winter (II), Spring (IIIa)

Student Body: Some mathematics undergraduates, but more non-mathematics graduate students

Credit: 3 Credits. 1 credit after Math 354. No credit granted to those who have completed or are enrolled

in Math 450.

Recent Texts: Partial Differential Equations for Scientists by Farlow

Area: Applied/Numerical Analysis

# Background and Goals:

This course is devoted to the use of Fourier series and other orthogonal expansions in the solution of initial-value and boundary-value problems for second-order linear partial differential equations. Emphasis is on concepts and calculation. The official prerequisite is ample preparation.

#### Content:

Classical representation and convergence theorems for Fourier series; method of separation of variables for the solution of the one-dimensional heat and wave equation; the heat and wave equations in higher dimensions; eigenfunction expansions; spherical and cylindrical Bessel functions; Legendre polynomials; methods for evaluating asymptotic integrals (Laplace's method, steepest descent); Laplace's equation and harmonic functions, including the maximum principle. As time permits, additional topics will be selected from: Fourier and Laplace transforms; applications to linear input-output systems, analysis of data smoothing and filtering, signal processing, time-series analysis, and spectral analysis; dispersive wave equations; the method of stationary phase; the method of characteristics.

### Alternatives:

None

# Subsequent Courses:

Math 454 is prerequisite to Math 571 (Numer. Meth. For Sci. Comput. I) and Math 572 (Numer. Meth. For Sci. Comput. II). Although it is not a formal prerequisite, it is good background for Math 556 (Methods of Applied Math I).

### Math 462 - Mathematical Models

Prerequisites: Math 216, 256, 286, or 316; and Math 217, 417, or 419

Frequency: Winter (II)

Student Body: Mainly Junior and Senior mathematics concentrators; students from engineering, biology,

physics, and medicine

Credit: 3 Credits. 1-3 credits after 463 depending on overlap

Recent Texts: Mathematical Models in Biology (2nd edition) by Edelstein-Keshet

Area: Applied/Numerical Analysis

# Background and Goals:

The focus of this course is the application of a variety of mathematical techniques to solve real-world problems. Students will learn how to model a problem in mathematical terms and use mathematics to gain insight and eventually solve the problem. Concepts and calculations, using applied analysis and numerical simulations, are emphasized.

### Content:

Construction and analysis of mathematical models in physics, engineering, economics, biology, medicine, and social sciences. Content varies considerably with instructor. Recent versions: Use and theory of dynamical systems (chaotic dynamics, ecological and biological models, classical mechanics), and mathematical models in physiology and population biology.

# Alternatives:

Students who are particularly interested in biology should consider Math 463 (Math Modeling in Biology).

# Subsequent Courses:

Any higher-level course in differential equations.

# Math 463 - Math Modeling in Biology

Prerequisites: Bioinf 463 or Biophys 463; and Math 214, 217, 417, or 419; and 216, 256, 286, or 316

Frequency: Fall (I)

Student Body: Juniors, Seniors, and first year graduate students (half engineering and half LS&A)

Credit: 3 Credits.

Recent Texts: Mathematical Models in Biology (5th ed) by L. Edelstein-Keshet; Mathematical Biology I (3rd ed) by

J.D. Murray

Area: Applied/Interdisciplinary

#### Background and Goals:

The complexities of the biological sciences make interdisciplinary involvement essential and the increasing use of mathematics in biology is inevitable as biology becomes more quantitative. Mathematical biology is a fast growing and exciting modern application of mathematics that has gained worldwide recognition. In this course, mathematical models that suggest possible mechanisms that may underlie specific biological processes are developed and analyzed. Another major emphasis of the course is illustrating how these models can be used to predict what may follow under currently untested conditions. The course moves from classical to contemporary models at the population, organ, cellular, and molecular levels. The goals of this course are: (i) Critical understanding of the use of differential equation methods in mathematical biology and (ii) Exposure to specialized mathematical and computational techniques which are required to study ordinary differential equations that arise in mathematical biology. By the end of this course students will be able to derive, interpret, solve, understand, discuss, and critique discrete and differential equation models of biological systems.

#### Content:

This course provides an introduction to the use of continuous and discrete differential equations in the biological sciences. Biological topics may include single species and interacting population dynamics, modeling infectious and dynamic diseases, regulation of cell function, molecular interactions and receptor-ligand binding, biological oscillators, and an introduction to biological pattern formation. There will also be discussions of current topics of interest such as Tumor Growth and Angiogenesis, HIV and AIDS, and Control of the Mitotic Clock. Mathematical tools such as phase portraits, bifurcation diagrams, perturbation theory, and parameter estimation techniques that are necessary to analyze and interpret biological models will also be covered. Approximately one class period each week will be held in the mathematics computer laboratory where numerical techniques for finding and visualizing solutions of differential and discrete systems will be discussed.

### Alternatives:

None

Subsequent Courses:

Math 563

### Math 464 - Inverse Problems

**Prerequisites:** Math 217, 417, or 419 and Math 216, 256, 286, or 316

Frequency: Sporadically

Student Body: Upper-level undergraduates and graduate students in mathematics, science, and

engineering

Credit: 3 Credits.

Recent Texts: None

Area: Applied/Numerical Analysis

## Background and Goals:

Solution of an inverse problem is a central component of fields such as medical tomography, geophysics, non-destructive testing, and control theory. The solution of any practical inverse problem is an interdisciplinary task. Each such problem requires a blending of mathematical constructs and physical realities. Thus, each problem has its own unique components; on the other hand, there is a common mathematical framework for these problems and their solutions. This framework is the primary content of the course. This course will allow students interested in the above-named fields to have an opportunity to study mathematical tools related to the mathematical foundations of said fields.

### Content:

The course content is motivated by a particular inverse problem from a field such as medical tomography (transmission, emission), geophysics (remote sensing, inverse scattering, tomography), or non-destructive testing. Mathematical topics include ill-posedness (existence, uniqueness, stability), regularization (e.g., Tikhonov, least squares, modified least squares, variation, mollification), pseudoinverses, transforms (e.g., k-plane, Radon, X-ray, Hilbert), special functions, and singular-value decomposition. Physical aspects of particular inverse problems will be introduced as needed, but the emphasis of the course is investigation of the mathematical concepts related to analysis and solution of inverse problems.

### Alternatives:

None

# Subsequent Courses:

None

# Math 465 - Introduction to Combinatorics

Prerequisites: Linear Algebra (one of Math 214, 217, 296, 417, or 419) or permission of instructor

Frequency: Fall (I)

Student Body: Upper-level undergraduates and graduate students in mathematics, science, and

engineering

Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 565 or 566.

Recent Texts: Introductory Combinatorics (4th edition) by R. Brualdi

Area: Discrete Mathematics

# Background and Goals:

Combinatorics is the study of finite mathematical objects, including their enumeration, structural properties, design, and optimization. Combinatorics plays an increasingly important role in various branches of mathematics and in numerous applications, including computer science, statistics and statistical physics, operations research, bioinformatics, and electrical engineering. This course provides an elementary introduction to the fundamental notions, techniques, and theorems of enumerative combinatorics and graph theory.

### Content:

An introduction to combinatorics, covering basic counting techniques (inclusion-exclusion, permutations and combinations, generating functions) and fundamentals of graph theory (paths and cycles, trees, graph coloring). Additional topics may include partially ordered sets, recurrence relations, partitions, matching theory, and combinatorial algorithms.

#### Alternatives:

565 (offered in the Fall) is significantly more demanding and proof-oriented. Math 566 is even more advanced.

### Subsequent Courses:

Math 565 and 566.

# Math 466 (EEB 466) - Mathematical Ecology

Prerequisites: Math 217, 417, or 419; Math 256, 286, or 316; and Math 450 or 451

Frequency: Sporadically

Student Body: The course is intended for graduate students and advanced undergraduates interested in

the mathematical analysis and modeling of ecological systems.

Credit: 3 Credits.

Recent Texts: none

Area: Applied

# Background and Goals:

This course gives an overview of mathematical approaches to questions in the science of ecology. Topics include: formulation of deterministic and stochastic population models; dynamics of single-species populations; and dynamics of interacting populations (predation, competition, and mutualism), structured populations, and epidemiology. Emphasis is placed on model formulation and techniques of analysis.

### Content:

Why do some diseases become pandemic? Why does the introduction of certain species result in widespread invasions? Why do some populations grow while others decline and still others cycle rhythmically? How are all of these phenomena influenced by climate change? These and many other fundamental questions in the science of ecology are intrinsically quantitative and concern highly complex systems. To answer them, ecologists formulate and study mathematical models. This course is intended to provide an overview of the principal ecological models and the mathematical techniques available for their analysis. Emphasis is placed on model formulation and techniques of analysis. Although the focus is on ecological dynamics, the methods we discuss are readily applicable across the sciences. The course presumes mathematical maturity at the level of advanced calculus with prior exposure to ordinary differential equations, linear algebra, and probability.

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none

# Subsequent Courses:

none

# Math 471 - Introduction to Numerical Methods

Prerequisites: Math 216, 256, 286, or 316; Math 214, 217, 417, or 419; and a working knowledge of one

high-level computer language

Frequency: Fall (I), Winter (II), Summer (IIIb)

**Student Body:** Juniors, Seniors, and Master's level (half Engineering, half LS&A)

Credit: 3 Credits. No credit granted to those who have completed or are enrolled in Math 371 or 472.

Recent Texts: A Friendly Introduction to Numerical Analysis by Bradie

Area: Applied/Numerical Analysis

## Background and Goals:

This is a survey of the basic numerical methods which are used to solve scientific problems. The emphasis is evenly divided between the analysis of the methods and their practical applications. Some convergence theorems and error bounds are proved. The course also provides an introduction to MATLAB, an interactive program for numerical linear algebra, as well as practice in computer programming. One goal of the course is to show how calculus and linear algebra are used in numerical analysis.

#### Content:

Topics may include computer arithmetic, Newton's method for non-linear equations, polynomial interpolation, numerical integration, systems of linear equations, initial value problems for ordinary differential equations, quadrature, partial pivoting, spline approximations, partial differential equations, Monte Carlo methods, 2-point boundary value problems, Dirichlet problem for the Laplace equation.

### Alternatives:

Math 371/Engin. 303 (Numerical Methods) is a less sophisticated version intended principally for Sophomore and Junior engineering students; the sequence Math 571—572 (Numer. Meth. for Sci Comput. I & II) is mainly taken by graduate students, but should be considered by strong undergraduates.

### Subsequent Courses:

Math 471 is good preparation for Math 571 and Math 572 (Numer. Meth. for Sci. Comput. I & II), although it is not prerequisite for these courses.

# Math 472 - Numerical Methods with Financial Applications

Prerequisites: Math 216, 256, 286, or 316; Math 214, 217, 417, or 419; and a working knowledge of one

high-level computer language. Math 425 is recommended.

Frequency: Fall (I)

**Student Body:** Concentrators in the Actuarial Mathematics and Financial Mathematics programs. **Credit:** 3 Credits. No credit granted to those who have completed or are enrolled in Math 371 or 471.

Recent Texts: Numerical Analysis by T. Sauer

Area: Actuarial & Financial

# Background and Goals:

This is a survey of the basic numerical methods which are used to solve scientific problems. The goals of the course are similar to those of Math 471, but the applications are chosen to be of interest to students in the Actuarial Mathematics and Financial Mathematics programs.

### Content:

Topics may include: Newton's method for non-linear equations, systems of linear equations, numerical integration, interpolation and polynomial approximation, ordinary differential equations, partial differential equations—in particular the Black-Scholes equation, Monte Carlo simulation, and numerical modeling.

### Alternatives:

Math 371/Engin 303 (Numerical Methods) is a less sophisticated version intended principally for Sophomore and Junior engineering students.

## Subsequent Courses:

None

### Math 475 - Elementary Number Theory

Prerequisites: No specific prerequisite

Frequency: Fall (I)

Student Body: Mainly mathematics undergraduates; some non-mathematics undergraduates and graduate

students

Credit: 3 Credits.

Recent Texts: An Introduction to the Theory of Numbers (5th edition) by Niven, Zuckerman, and Montgomery

Area: Number Theory

### Background and Goals:

This is an elementary introduction to number theory, especially congruence arithmetic. Number Theory is one of the few areas of mathematics in which problems easily describable to a layman (is every even number the sum of two primes?) have remained unsolved for centuries. Recently some of these fascinating but seemingly useless questions have come to be of central importance in the design of codes and ciphers. In addition to strictly number-theoretic questions, concrete examples of structures such as rings and fields from abstract algebra are discussed. Concepts and proofs are emphasized, but there is some discussion of algorithms which permit efficient calculation. Students are expected to do simple proofs and may be asked to perform computer experiments. Although there are no special prerequisites and the course is essentially self-contained, most students have some experience in abstract mathematics and problem solving and are interested in learning proofs. At least three semesters of college mathematics are recommended. A Computational Laboratory (Math 476, 1 credit) will usually be offered as an optional supplement to this course.

#### Content:

Topics usually include the Euclidean algorithm, primes and unique factorization, congruences, Chinese Remainder Theorem, Hensel's Lemma, Diophantine equations, arithmetic in polynomial rings, primitive roots, quadratic reciprocity, and quadratic fields. This material corresponds to Chapters 1-3 and selected parts of Chapter 5 of Niven, Zuckerman, and Montgomery.

## Alternatives:

Math 575 (Intro. to Theory of Numbers) moves much faster, covers more material, and requires more difficult exercises. There is some overlap with Math 412 (Introduction to Modern Algebra).

## Subsequent Courses:

Math 475 may be followed by Math 575 (Intro. to Theory of Numbers) and is good preparation for Math 412 (Introduction to Modern Algebra). All of the advanced number theory courses, Math 675, 676, 677, 678, and 679, presuppose the material of Math 575, although a good student may get by with Math 475.

# Math 476 - Computational Laboratory in Number Theory

Prerequisites: Prior or concurrent enrollment in Math 475 or 575

Frequency: Fall (I) sporadically

Student Body: Undergraduate mathematics concentrators

Credit: 1 Credit.

**Recent Texts:** Coursepack **Area:** Number Theory

# Background and Goals:

Intended as a companion course to Math 475 or 575. Participation should boost the student's performance in either of those classes. Students in the Lab will see mathematics as an exploratory science (as mathematicians do).

### Content:

Students will be provided with software with which to conduct numerical explorations. No programming necessary, but students interested in programming will have the opportunity to embark on their own projects. Students will gain a knowledge of algorithms which have been developed for number theoretic purposes, e.g., for factoring.

### Alternatives:

None

# Subsequent Courses:

None

### Math 481 - Introduction to Mathematical Logic

Prerequisites: Math 412 or 451 or equivalent experience with abstract mathematics

Frequency: Fall (I)

Student Body: Undergraduate mathematics, philosophy, and computer science concentrators; a few non-math

graduate students *Credit:* 3 Credits.

Recent Texts: A Mathematical Introduction to Logic by Enderton

Area: Logic

### Background and Goals:

All of modern mathematics involves logical relationships among mathematical concepts. In this course we focus on these relationships themselves rather than the ideas they relate. Inevitably this leads to a study of the (formal) languages suitable for expressing mathematical ideas. The explicit goal of the course is the study of propositional and first-order logic; the implicit goal is an improved understanding of the logical structure of mathematics. Students should have some previous experience with abstract mathematics and proofs, both because the course is largely concerned with theorems and proofs and because the formal logical concepts will be much more meaningful to a student who has already encountered these concepts informally. No previous course in logic is prerequisite.

#### Content:

In the first third of the course the notion of a formal language is introduced and propositional connectives ('and,' 'or,' 'not,' 'implies'), tautologies, and tautological consequence are studied. The heart of the course is the study of first-order predicate languages and their models. The new elements here are quantifiers ('there exists' and 'for all'). The study of the notions of truth, logical consequence, and probability leads to the completeness and compactness theorems. The final topics include some applications of these theorems, usually including non-standard analysis. This material corresponds to Chapter 1 and sections 2.0-2.5 and 2.8 of Enderton.

### Alternatives:

Math 681, the graduate introductory logic course, also has no specific logic prerequisite but does presuppose a much higher general level of mathematical sophistication. Philosophy 414 may cover much of the same material with a less mathematical orientation.

## Subsequent Courses:

Math 481 is not explicitly prerequisite for any later course, but the ideas developed have application to every branch of mathematics.

# Math 486 - Concepts Basic to Secondary School Math

Prerequisites: Math 215, 255, or 285

Frequency: Winter (II)

Student Body: Undergraduate concentrators in the Teaching Certificate Program and "minors" in other

teaching programs *Credit:* 3 Credits.

Recent Texts: Mathematics for High School Teachers: An Advanced Perspective by Peressini, Usiskin,

Marchisotto, and Stanley

Area: Teaching

# Background and Goals:

This course is designed for students who intend to teach junior high or high school mathematics. It is advised that the course be taken relatively early in the program to help the student decide whether or not this is an appropriate goal. Concepts and proofs are emphasized over calculation. The course is intended to give students a strong theoretical background in high school-level topics such as introductory calculus and algebra, giving them the deeper insight necessary to teach these subjects to others.

### Content:

Topics covered have included number systems and their axiomatics; number theory, particularly a study of divisibility, primes, and prime factorizations; the abstract theory of sets, operators, and functions; and the epsilon-delta underpinnings of limits and derivatives.

### Alternatives:

There is no real alternative, but the Teaching Certificate requirement of Math 486 may be waived for strong students who intend to do graduate work in mathematics.

### Subsequent Courses:

Prior completion of Math 486 may be of use for some students planning to take Math 312, 412, or 425.

# Math 489 - Math for Elem. and Middle School Teachers

**Prerequisites:** Math 385 or permission of instructor

Frequency: Winter (II)

Student Body: Undergraduates in the Elementary Teaching Certificate Program

Credit: 3 Credits.

Recent Texts: Mathematics for Elementary School Teachers by Bassarear

Area: Teaching

# Background and Goals:

This course, together with its predecessor, Math 385, provides a coherent overview of the mathematics underlying the elementary and middle school curriculum. It is required of all students intending to earn an elementary teaching certificate and is taken almost exclusively by such students. Concepts are heavily emphasized with some attention given to calculation and proof. The course is conducted using a discussion format. Class participation is expected and constitutes a significant part of the course grade. Enrollment is limited to 30 students per section. Although only two years of high school mathematics is required, a more complete background including pre-calculus or calculus is desirable.

### Content:

Topics covered include fractions and rational numbers, decimals and real numbers, probability and statistics, geometric figures, and measurement. Algebraic techniques and problem-solving strategies are used throughout the course.

### Alternatives:

There is no alternative course.

# Subsequent Courses:

Math 497 (Topics in Elementary Mathematics).

# Math 490 - Introduction to Topology

Prerequisites: Math 451 or equivalent experience with abstract mathematics

Frequency: Fall (I)

Student Body: Mathematics, math-education, science and

engineering

Credit: 3 Credits.

Recent Texts: None

Area: Geometry/Topology

## Background and Goals:

Topology is the study of a class of interesting spaces, geometric examples of which are knots and surfaces. We focus on those properties of such spaces which don't change if the space is deformed. Much of the course is devoted to understanding particular spaces, such as Moebius strips and Klein bottles. The material in this course has a wide range of applications. Most of the material is theoretical, but it is well-suited for developing intuition and giving convincing proofs which are pictorial or geometric rather than completely rigorous.

#### Content:

Knots, orientable and non-orientable surfaces, Euler characteristic, open sets, connectedness, compactness, metric spaces. The topics covered are fairly constant but the presentation and emphasis will vary significantly with the instructor.

#### Alternatives:

Math 590 (Intro. to Topology) is a deeper and more difficult presentation of much of the same material. Math 433 (Intro. to Differential Geometry) is a related course at about the same level.

### Subsequent Courses:

Math 490 is not prerequisite for any later course but provides good background for Math 591 (General and Differential Topology) or any of the other courses in geometry or topology.

### Math 493 - Honors Algebra I

Prerequisites: Math 296, 412, 451, or permission of instructor

Frequency: Fall (I)

Student Body: Mainly undergraduate mathematics concentrators with a few graduate students from other fields

Credit: 3 Credits.

Recent Texts: Algebra by Artin

Area: Algebra

## Background and Goals:

Math 493-494 is one of the more abstract and difficult sequences in the undergraduate program. It is frequently elected by students who have completed the 295-296 sequence. Its goal is to introduce students to the basic structures of modern abstract algebra (groups, rings, fields, and modules) in a rigorous way.

Emphasis is on concepts and proofs; calculations are used to illustrate the general theory. Exercises tend to be quite challenging. Students must have some previous exposure to rigorous proof-oriented mathematics and be prepared to work hard.

Students from the Math 185-286 and Math 156-256 sequences are strongly advised to take both Math 217 and some 400-500 level course (for examples, Math 451) prior to attempting Math 493.

#### Content:

The course covers basic definitions and properties of groups, fields, and vector spaces including homomorphisms, isomorphisms, subgroups, and bilinear forms.

Further topics are selected from: Sylow theorems; structure theorem for finitely-generated abelian groups; permutation representation; the symmetric and alternating groups; vector spaces over arbitrary fields; spectral theorem; and linear groups.

#### Alternatives:

Math 412 (Introduction to Modern Algebra) is a substantially lower level course which covers about half of the material of Math 493. The sequence Math 593-594 covers about twice as much Group and Field Theory as well as several other topics and presupposes that students have had a previous introduction to these concepts at least at the level of Math 412.

#### Subsequent Courses:

Math 494 (Honors Algebra II)

### Math 494 - Honors Algebra II

Prerequisites: Math 493
Frequency: Winter (II)

Student Body: Mainly undergraduate mathematics concentrators with a few graduate students from

other fields.

Credit: 3 Credits.

Recent Texts: Algebra by Artin

Area: Algebra

# Background and Goals:

Math 493-494 is one of the more abstract and difficult sequences in the undergraduate program. It is frequently elected by students who have completed the 295-296 sequence. Its goal is to introduce students to the basic structures of modern abstract algebra (groups, rings, fields, and modules) in a rigorous way.

Emphasis is on concepts and proofs; calculations are used to illustrate the general theory. Exercises tend to be quite challenging. Students must have some previous exposure to rigorous proof-oriented mathematics and be prepared to work hard.

#### Content:

This course is a continuation of Math 493. It covers basic definitions and properties of rings and modules including quotients, ideals, factorization, and field extensions.

Further topics are selected from: representation theory; structure theory of modules over a PID; Jordan canonical form; Galois theory, Nullstellensatz; finite fields; Euclidean, Principal ideal, and unique factorization domains; polynomial rings in one and several variables; and algebraic varieties.

#### Alternatives:

None

#### Subsequent Courses:

The natural sequel to Math 494 is Math 593.

## Math 497 - Topics in Elementary Mathematics

Prerequisites: Math 489 or permission of instructor

Frequency: Fall (I)

Student Body: Undergraduates in the Elementary Teaching Certificate Program

Credit: 3 Credits.

Recent Texts: None

Area: Teaching

# Background and Goals:

This is a required course for elementary teaching certificate candidates that extends and deepens the coverage of mathematics begun in the required two-course pair Math 385&489. Topics are chosen from geometry, algebra, computer programming, logic, and combinatorics. Applications and problem-solving are emphasized. The class meets three times per week in recitation sections. Grades are based on class participation, two one-hour exams, and a final exam.

#### Content:

Selected topics in geometry, algebra, computer programming, logic, and combinatorics for prospective and in-service elementary, middle, or junior-high school teachers. Content will vary from term to term.

#### Alternatives:

None

# Subsequent Courses:

# Math 498 - Topics in Modern Mathematics

Prerequisites: Junior or Senior standing

Frequency: Sporadically

Student Body: Junior and Senior students from mathematics and other fields

Credit: 3 Credits.

Recent Texts: Mathematics for High School Teachers : An Advanced Perspective by Zalman Usiskin

# Background and Goals:

As a topics course, this course will vary greatly from term to term. In one recent offering, the aim of the course was to introduce, at an elementary level, the basic concepts of the theory of dynamical systems.

### Content:

varies

### Alternatives:

None

# Subsequent Courses:

No specific sequels

# Math 520 - Life Contingencies I

Prerequisites: Math 424 and 425, both with a minimum grade of C-, and permission of instructor

Frequency: Fall (I)

**Student Body:** Undergraduate students of actuarial mathematics

Credit: 3 Credits.

Recent Texts: Actuarial Mathematics (2nd ed) by N.L. Bowers et al.

Area: Actuarial & Financial

### Background and Goals:

Quantifying the financial impact of uncertain events is the central challenge of actuarial mathematics. The goal of this course is to teach the basic actuarial theory of mathematical models for financial uncertainties, mainly the time of death. In addition to actuarial students, this course is appropriate for anyone interested in mathematical modeling outside of the sciences.

#### Content:

The main topics are the development of (1) probability distributions for the future lifetime random variable; (2) probabilistic methods for financial payments on death or survival; and (3) mathematical models of actuarial reserving.

#### Alternatives:

Math 523 (Risk Theory) is a complementary course covering the application of stochastic process models.

### Subsequent Courses:

Math 520 is prerequisite to all succeeding actuarial courses. Math 521 (Life Contingencies II) extends the single decrement and single life ideas of 520 to multi-decrement and multiple-life applications directly related to life insurance and pensions. The sequence 520-521 covers the syllabus of the Course 3 examination of the Casualty Actuarial Society and covers the syllabus of the Exam MLC examination of the Society of Actuaries. Math 522 (Act. Theory of Pensions and Soc. Sec.) applies the models of 520 to funding concepts of retirement benefits such as social insurance, private pensions, retiree medical costs, etc.

# Math 521 - Life Contingencies II

Prerequisites: Math 520, with a minimum grade of C-, is the enforced prerequisite

Frequency: Winter (II)

Student Body: Undergraduate students of actuarial mathematics

Credit: 3 Credits.

Recent Texts: Actuarial Mathematics (2nd edition) by Bowers et al.

Area: Actuarial & Financial

### Background and Goals:

This course extends the single decrement and single life ideas of Math 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520-521 covers the syllabus of the Course 3 examination of the Casualty Actuarial Society and covers the syllabus of the Exam MLC examination of the Society of Actuaries. Concepts and calculation are emphasized over proof.

#### Content:

Topics include multiple life models—joint life, last survivor, contingent insurance; multiple decrement models—disability, withdrawal, retirement, etc.; and reserving models for life insurance. This corresponds to chapters 7-10, 14, and 15 of Bowers et. al.

#### Alternatives:

Math 522 (Act. Theory of Pensions and Soc. Sec.) is a parallel course covering mathematical models for prefunded retirement benefit programs.

# Subsequent Courses:

## Math 522 - Actuarial Theory of Pensions and Social Security

Prerequisites: Math 520, with a minimum grade of C-, or permission

Frequency: Sporadically

Student Body: Undergraduate students of actuarial mathematics

Credit: 3 Credits.

Recent Texts: Fundamentals of Pension Mathematics by B. Berin; Pension Mathematics by A.

Anderson (as a reference)

\*\*Area: Actuarial & Financial

## Background and Goals:

This course develops the mathematical models for pre-funded retirement benefit plans. Concepts and calculation are emphasized over proofs.

#### Content:

Mathematical models for (1) retirement income, (2) retiree medical benefits, (3) disability benefits, and (4) survivor benefits. There is some coverage of how accounting theory and practice can be explained by these models and of the U.S. laws and regulations that give rise to the models used in practice.

#### Alternatives:

Math 521 (Life Contingencies II) is a parallel course covering models for insurance rather than retirement benefits.

### Subsequent Courses:

# Math 523 - Risk Theory

**Prerequisites:** Math 425 or equivalent **Frequency:** Fall (I), Winter (II)

Student Body: Undergraduate students of financial and actuarial mathematics

Credit: 3 Credits.

Recent Texts: Loss Models: From Data to Decisions (3rd edition) by Klugman, Panjer, and Willmot

Area: Actuarial & Financial

### Background and Goals:

Risk management is of major concern to all financial institutions and is an active area of modern finance. This course is relevant for students with interests in finance, risk management, or insurance, and it provides background for the professional examinations in Risk Theory offered by the Society of Actuaries and the Casualty Actuary Society. Students should have a basic knowledge of common probability distributions (Poisson, exponential, gamma, binomial, etc.) and have at least Junior standing. Two major problems will be considered: (1) modeling of payouts of a financial intermediary when the amount and timing vary stochastically over time, and (2) modeling of the ongoing solvency of a financial intermediary subject to stochastically varying capital flow. These topics will be treated historically beginning with classical approaches and proceeding to more dynamic models.

#### Content:

Classical approaches to risk including the insurance principle and the risk-reward tradeoff. Review of probability. Compound Poisson process. Modeling of individual losses that arise in a loss aggregation process, modeling the frequency of losses, and credibility theory.

#### Alternatives:

None

#### Subsequent Courses:

# Math 525 (Stats 525) - Probability Theory

Prerequisites: Math 451 (strongly recommended). Math 425/Stats 425 would be helpful.

Frequency: Fall (I), Winter (II)

Student Body: A mix of undergraduate and graduate students, drawn largely from mathematics,

statistics, and engineering

Credit: 3 Credits.

Recent Texts: Introduction to Probability Models (10th edition) by Ross

Area: Analysis

### Background and Goals:

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

#### Content:

The following topics will be covered: sample space and events, random variables, concept and definition of probability and expectation, conditional probability and expectation, independence, moment generating functions, Law of large numbers, Central limit theorem, Markov chains, Poisson process and exponential distribution.

#### Alternatives:

EECS 501 also covers some of the same material with less emphasis on mathematical rigor. Math/Stats 425 (Intro. To Probability) treats similar topics, but is accessible with less mathematical background.

#### Subsequent Courses:

Math 526 (Discr. State Stoch. Proc.), Stats 426 (Intro. to Math Stats), and the sequence Stats 510-511 (Mathematical Statistics I & II) are natural sequels.

### Math 526 (Stats 526) - Discrete State Stochastic Processes

**Prerequisites:** Math 525 or EECS 501 **Frequency:** Fall (I), Winter (II)

Student Body: Half undergraduates, half graduate students from mathematics and many other fields

Credit: 3 Credits.

Recent Texts: Introduction to Probability Models (10th edition) by Ross

Area: Analysis

### Background and Goals:

This is a course on the theory and applications of stochastic processes, mostly on discrete state spaces. It is a second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications.

#### Content:

The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include generating functions; recurrent events and the renewal theorem; random walks; Markov chains; branching processes; limit theorems; Markov chains in continuous time with emphasis on birth and death processes and queueing theory; an introduction to Brownian motion; stationary processes and martingales.

#### Alternatives:

This course is similar to EECS 502 and IOE 515, although the latter course tends to be somewhat more oriented to applications. It is cross-listed as Statistics 526.

## Subsequent Courses:

The next courses in probability are Math 625 and 626, which presuppose substantial additional background (e.g., Math 597).

### Math 528 - Topics in Casualty Insurance

Prerequisites: Math 217, 417, 419, or permission of instructor

**Frequency:** Winter (II), generally in even years

Student Body: Undergraduate students of actuarial mathematics and insurance concentrators in

business

Credit: 3 Credits.

Recent Texts: Foundations of Casualty Actuarial Science (4th ed)

Area: Actuarial & Financial

# Background and Goals:

Historically the Actuarial Program has emphasized life, health, and pension topics. This course will provide background in casualty topics for the many students who take employment in this field. Guest lecturers from the industry will provide some of the instruction. Students are encouraged to take the Casualty Actuarial Society's Course 3 examination at the completion of the course.

#### Content:

The insurance policy is a contract describing the services and protection which the insurance company provides to the insured. This course will develop an understanding of the nature of the coverages provided and the bases of exposure and principles of the underwriting function, how products are designed and modified, and the different marketing systems. It will also look at how claims are settled, since this determines losses which are key components for insurance ratemaking and reserving. Finally, the course will explore basic ratemaking principles and concepts of loss reserving.

#### Alternatives:

None

#### Subsequent Courses:

## Math 547 - Biological Sequence Analysis

**Prerequisites:** Flexible. Basic probability (level of Math/Stats 425) or molecular biology (level of Biology 427) or biochemistry (level of Chem/BioChem 451) or basic programming skills desirable; or permission of instructor.

Frequency: Sporadically

**Student Body:** Interdisciplinary: mainly mathematics, statistics, biostatistics, and bioinformatics students; also biology, biomedical and engineering students.

Credit: 3 Credits.

Recent Texts: Biological Sequence Analysis by R. Durbin, et. al.; Computational Molecular Biology by

Clote and Backofen *Area:* Analysis

# Background and Goals:

### Content:

Probabilistic models of proteins and nucleic acids. Analysis of DNA/RNA and protein sequence data. Algorithms for sequence alignment, statistical analysis of similarity scores, hidden markov models, neural networks, training, gene finding, protein family profiles, multiple sequence alignment, sequence comparison, and structure prediction. Analysis of expression array data.

#### Alternatives:

**Bioinformatics 526** 

## Subsequent Courses:

Bioinformatics 551 (Proteome Informatics)

## Math 550 (CMPLXSYS 510) - Introduction to Adaptive Systems

**Prerequisites:** Math 215, 255, or 285; Math 217; and Math 425

Frequency: Sporadically

Student Body: Graduate and undergraduate students from many disciplines

Credit: 3 Credits.

Recent Texts: Essential Mathematical Biology by N. Britton

## Background and Goals:

This course centers on the construction and use of agent-based adaptive models to study phenomena which are prototypical in the social, biological, and decision sciences. These models are "agent-based" or "bottom-up" in that the structure is placed at the level of the individuals as basic components; they are "adaptive" in that individuals often adapt to their environment through evolution or learning. The goal of these models is to understand how the structure at the individual or micro level leads to emergent behavior at the aggregate or macro level. Often the individuals are grouped into sub-populations or interesting hierarchies, and the researcher may want to understand how the structure or development of these populations affects macroscopic outcomes.

#### Content:

The course will start with classical differential equation and game theory approaches. It will then focus on the theory and application of particular models of adaptive systems such as models of neural systems, genetic algorithms, classifier systems, and cellular automata. Time permitting, we will discuss more recent developments such as sugarscape and echo.

#### Alternatives:

Cross-listed as Complex Systems 510.

#### Subsequent Courses:

### Math 555 - Introduction to Complex Variables

Prerequisites: Math 450 or 451. Students who had 450 (or equivalent) but not 451 are encouraged to

take 451 simultaneously with 555. *Frequency:* Fall (I), Winter (II)

Student Body: Graduate students in mathematics, science, and engineering, and some advanced

mathematics, science, and engineering undergraduates

Credit: 3 Credits.

Recent Texts: Complex Variables and Applications (8th edition) by Churchill and Brown

Area: Applied/Numerical Analysis

## Background and Goals:

This course is an introduction to the analysis of complex-valued functions of a complex variable with substantial attention to applications in science and engineering. Concepts, calculations, and the ability to apply principles to problems are emphasized rather than the most general proofs of the fundamental theorems, but arguments are rigorous. The prerequisite of a course in advanced calculus is absolutely essential.

#### Content:

Differentiation and integration of complex-valued functions of a complex variable, series, mappings, residues, and applications including evaluation of improper real integrals and fluid mechanics. These topics correspond to the material in Chapters 1-9 of Churchill & Brown.

#### Alternatives:

Math 596 (Analysis I (Complex)) covers the theoretical material of Math 555 with an emphasis on proofs rather than applications.

## Subsequent Courses:

Math 555 is prerequisite to many advanced courses in applied mathematics, including the sequence Math 556 and 557 (Methods of Applied Mathematics I and II), and in science and engineering.

# Math 556 - Methods of Applied Mathematics I

Prerequisites: Math 217, 419, or 420; Math 451; and Math 555

Frequency: Fall (I)

Student Body: Graduate students in mathematics, science, and engineering

Credit: 3 Credits.

Recent Texts: Applied Functional Analysis by Griffel

Area: Applied/Numerical Analysis

### Background and Goals:

This is an introduction to methods of applied analysis with emphasis on Fourier analysis for differential equations. Initial and boundary value problems are covered. Students are expected to master both the proofs and applications of major results. The prerequisites include linear algebra, advanced calculus, and complex variables.

#### Content:

Topics may vary with the instructor but often include Fourier series, separation of variables for partial differential equations, heat conduction, wave motion, electrostatic fields, Sturm-Liouville problems, Fourier transform, signal analysis, sampling theorem, Green's functions, distributions, Hilbert space, complete orthonormal sets, integral operators, spectral theory for compact self-adjoint operators.

#### Alternatives:

Math 454 (Bound Val. Probs. for Part. Diff. Eq.) is an undergraduate course on the same topics.

### Subsequent Courses:

Math 557 (Methods of Applied Math II), Math 558 (Ordinary Differential Equations), Math 656 (Partial Differential Equations), and Math 658 (Ordinary Differential Equations).

## Math 557 - Methods of Applied Mathematics II

Prerequisites: Math 217, 419, or 420; Math 451; and Math 555

Frequency: Winter (II)

Student Body: Graduate students in mathematics, science, and engineering

Credit: 3 Credits.

Recent Texts: Asymptotic Analysis by Murray

Area: Applied/Numerical Analysis

### Background and Goals:

This is an introduction to methods of asymptotic analysis including asymptotic expansions for integrals and solutions of ordinary and partial differential equations. The prerequisites include linear algebra, advanced calculus, and complex variables. Math 556 is not a prerequisite.

#### Content:

Topics include stationary phase, steepest descent, characterization of singularities in terms of the Fourier transform, regular and singular perturbation problems, boundary layers, multiple scales, WKB method. Additional topics depend on the instructor but may include non-linear stability theory, bifurcations, applications in fluid dynamics (Rayleigh-Benard convection), combustion (flame speed).

#### Alternatives:

None

## Subsequent Courses:

Math 656 (Partial Differential Equations) and 658 (Ordinary Differential Equations).

## Math 558 - Ordinary Differential Equations

**Prerequisites:** Math 451 **Frequency:** Fall (I)

Student Body: Graduate students in mathematics, science, and engineering

Credit: 3 Credits.

Recent Texts: Differential Equations, Dynamical Systems, and an Introduction to Chaos (2nd edition) by

Hirsh, Smale, and Davaney

Area: Applied/Numerical Analysis

### Background and Goals:

This course is an introduction to Ordinary Differential Equations and Dynamical Systems with emphasis on qualitative analysis.

#### Content:

The basic results on qualitative behavior, centered on themes of stability and phase plane analysis will be presented in a context that includes applications to a variety of classic examples. The proofs of the fundamental facts will be presented, along with discussions of examples.

#### Alternatives:

Math 404 (Intermediate Differential Equations) is an undergraduate course on similar topics.

# Subsequent Courses:

Math 658 (Ordinary Differential Equations)

# Math 559 - Topics in Applied Mathematics

**Prerequisites:** Vary by topic, check with instructor

Frequency: Sporadically

Student Body: Undergraduate and graduate students in mathematics or science

Credit: 3 Credits.

Recent Texts: Varies

Area: Applied/NA

# Background and Goals:

This is an advanced topics course intended for students with strong interests in the intersection of mathematics and the sciences, but not necessarily experience with both applied mathematics and the application field. Mathematical concepts, as well as intuitions arising from the field of application, will be stressed.

#### Content:

This course will focus on particular topics in emerging areas of applied mathematics for which the application field has been strongly influenced by mathematical ideas. It is intended for students with interests in mathematical, computational, and/or modeling aspects of interdisciplinary science, and the course will develop the intuitions of the field of application as well as the mathematical concepts. The applications considered will vary with the instructor and may come from physics, biology, economics, electrical engineering, and other fields. Recent examples have been: Nonlinear Waves, Mathematical Ecology, and Computational Neuroscience.

#### Alternatives:

None

## **Subsequent Courses:**

Other courses in applied mathematics.

## Math 561 (OMS 518, IOE 510) - Linear Programming I

Prerequisites: Math 214, 217, 417, or 419

Frequency: Fall (I), Winter (II)

Student Body: Graduate and undergraduate students from many fields

Credit: 3 Credits.

Recent Texts: Numerical Linear Algebra by L.N. Trefethen et. al.

#### Background and Goals:

The allocation of constrained resources such as funds among investment possibilities or personnel among production facilities is a fundamental problem which is very well-suited to mathematical analysis. Each such problem has as its goal the maximization of some positive objective such as investment return or the minimization of some negative objective such as cost or risk. Such problems are called Optimization Problems. Linear Programming deals with optimization problems in which both the objective and constraint functions are linear (the word "programming" means "planning" rather than computer programming). In practice, such problems involve thousands of decision variables and constraints, so a primary focus is the development and implementation of efficient algorithms. However, the subject also has deep connections with higher-dimensional convex geometry. A recent survey showed that most Fortune 500 companies regularly use linear programming in their decision making. This course will present both the classical and modern approaches to the subject and discuss numerous applications of current interest.

#### Content:

Formulation of problems from the private and public sectors using the mathematical model of linear programming. Development of the simplex algorithm; duality theory and economic interpretations. Postoptimality (sensitivity) analysis; algorithmic complexity; the ellipsoid method; scaling algorithms; applications and interpretations. Introduction to transportation and assignment problems; special purpose algorithms and advanced computational techniques. Students have opportunities to formulate and solve models developed from more complex case studies and to use various computer programs.

#### Subsequent Courses:

IOE 610 (Linear Programming II) and IOE 611 (Nonlinear Programming)

## Math 562 (IOE 511) - Continuous Optimization Methods

Prerequisites: Math 214, 217, 417, or 419

Frequency: Fall (I)

Student Body: Graduate students in engineering and LSA. Occasionaly it attracts an undergraduate

Math or Engineering student.

Credit: 3 Credits.
Recent Texts:

# Background and Goals:

Optimization is widely used in engineering and science models. The goal of this course is to give a rigorous background to the field. Examples are drawn from engineering and science.

#### Content:

Survey of continuous optimization problems. Unconstrained optimization problems: unidirectional search techniques, gradient, conjugate direction, quasi-Newtonian methods; introduction to constrained optimization using techniques of unconstrained optimization through penalty transformation, augmented Lagrangians, and others; discussion of computer programs for various algorithms.

#### Alternatives:

Cross-listed as IOE 511.

### Subsequent Courses:

This is not a prerequisite for any other course.

### Math 563 - Advanced Mathematical Methods for the Biological Sciences

Prerequisites: Math 217, 417, or 419 and Math 450 or 454

Frequency: Winter (II)

Student Body: Graduate students in mathematics, science, engineering, and medicine

Credit: 3 Credits.

Recent Texts: Mathematical Biology (3rd ed) by J.D. Murray

# Background and Goals:

Natural systems behave in a way that reflects an underlying spatial pattern. This course focuses on discovering the way in which spatial variation influences the motion, dispersion, and persistence of species. The concepts underlying spatially dependent processes and the partial differential equations which model them will be discussed in a general manner with specific applications taken from molecular, cellular, and population biology. This course is centered on modeling in three major areas i) Models of Motion: Diffusion, Convection, Chemotaxis, and Haptotaxis; ii) Biological Pattern Formation; and iii) Delay-differential Equations and Age-structured Models.

#### Content:

This course will introduce and explore partial differential equation modeling in biological settings. Students should have some experience with solution techniques for partial differential equations as well as an interest in biomedical applications. There will also be a brief introduction to delay differential equations and age-structured models; however, no previous background in these areas is required. Mathematical topics covered include derivation of relevant PDEs from first principles; reduction of PDEs to ODEs under steady state, quasi-steady state, and traveling wave assumptions; solution techniques for PDEs and concepts of spatial stability and instability. These concepts will be introduced within the setting of classical and current problems in biology and the biomedical sciences such as cell motion, transport of biological substances, and biological pattern formation. Above all, this course aims to enhance the interdisciplinary training of advanced undergraduate and graduate students from mathematics and other disciplines by introducing fundamental properties of partial differential equations in the context of interesting biological phenomena. Grades will be based on the completion of a research project and weekly (or biweekly) homework assignments, computer lab assignments, and in class presentations.

#### Alternatives:

None

# Subsequent Courses:

### Math 564 - Topics in Mathematical Biology

Prerequisites: Variable, permission of instructor

Frequency: Winter (II), not every year

Student Body: Juniors, Seniors, but mainly graduate students

Credit: 3 credits

Area: Applied/Numerical Analysis

Recent Texts: None

# Background and Goals:

This is an advanced course on further topics in mathematical biology. Topic will vary according to the instructor. Possible topics include modeling infectious diseases, cancer modeling, mathematical neurosciences or biological oscillators, among others. The sample description below is for a course in biological oscillators from Winter 2006.

#### Content:

From sleeping patterns, heartbeats, locomotion, and firefly flashing to the treatment of cancer, diabetes, and neurological disorders, oscillations are of great importance in biology and medicine. Mathematical modeling and analysis are needed to understand what causes these oscillations to emerge, properties of their period and amplitude, and how they synchronize to signals from other oscillators or from the external world. The goal of this course will be to teach students how to take real biological data, convert it to a system of equations and simulate and/or analyze these equations. Models will typically use ordinary differential equations. Mathematical techniques introduced in this course include 1) the method of averaging, 2) harmonic balance, 3) Fourier techniques, 4) entrainment and coupling of oscillators, 4) phase plane analysis, and 5) various techniques from the theory of dynamical systems. Emphasis will be placed on primary sources (papers from the literature) particularly those in the biological sciences. Consideration will be given in the problem sets and course project to interdisciplinary student backgrounds. Teamwork will be encouraged.

#### Alternatives:

None

## Subsequent Courses:

### Math 565 - Combinatorics and Graph Theory

Prerequisites: Math 412, 451, or equivalent experience with abstract mathematics

Frequency: Fall (I)

Student Body: Largely mathematics and EECS graduate students with a few mathematics

undergraduates *Credit:* 3 Credits

Recent Texts: A Course in Combinatorics (2nd edition) by J.H. van Lint and R.M. Wilson

Area: Combinatorics

# Background and Goals:

This course has two somewhat distinct halves devoted to (1) graph theory and (2) topics in the theory of finite partially ordered sets. Students should have taken at least one proof-oriented course.

#### Content:

The first part of this course will be devoted to graph theory. A graph (in the combinatorial sense) is a finite set of points and a specification of which pairs of these points are deemed "adjacent." Despite the simplicity of the concept, it leads to numerous interesting theorems, problems, and applications. Topics in the graph theory part of the course include (if time permits) trees, k-connectivity, Eulerian and Hamiltonian graphs, tournaments, graph coloring, planar graphs, Euler's formula, the 5-Color theorem, Kuratowski's theorem, and the matrix-tree theorem.

The second part of the course will deal with topics in the theory of finite partially ordered sets. This will include material about Möbius functions, lattices, simplicial complexes, and matroids.

## Alternatives:

There is small overlap with Math 566 (Combinatorial Theory). Math 416 (Theory of Algorithms) is somewhat related but much more concerned with algorithms.

#### Subsequent Courses:

Math 566 (Combinatorial Theory)

### Math 566 - Combinatorial Theory

Prerequisites: Math 412, 451, or equivalent experience with abstract mathematics

Frequency: Winter (II)

Student Body: Undergraduates and graduate students from mathematics, statistics, engineering, and

other natural and social sciences

Credit: 3 Credits.

Recent Texts: None

Area: Combinatorics

# Background and Goals:

This course is a rigorous introduction to classical combinatorial theory. Concepts and proofs are the foundation, but there are copious applications to modern industrial problem-solving.

# Content:

Permutations, combinations, generating functions, and recurrence relations. The existence and enumeration of finite discrete configurations. Systems of representatives, Ramsey's Theorem, and extremal problems. Construction of combinatorial designs.

## Alternatives:

There is no real alternative, although there is some overlap with Math 565 (Combinatorics and Graph Theory).

## Subsequent Courses:

Sequels are Math 664-665 and Math 669.

### Math 567 - Introduction to Coding Theory

Prerequisites: Math 217, 417, 419, or 420

Frequency: Fall (I)

Student Body: Undergraduate mathematics concentrators and EECS graduate students

Credit: 3 Credits.

**Recent Texts:** Introduction to Coding Theory by Roth

Area: Algebra

### Background and Goals:

This course is designed to introduce mathematics concentrators to an important area of applications in the communications industry. Using linear algebra it will cover the foundations of the theory of error-correcting codes and prepare a student to take further EECS courses or gain employment in this area. For EECS students it will provide a mathematical setting for their study of communications technology.

#### Content:

Introduction to coding theory focusing on the mathematical background for error-correcting codes. Topic include: Shannon's Theorem and channel capacity; review of tools from linear algebra and an introduction to abstract algebra and finite fields; basic examples of codes such and Hamming, BCH, cyclic, Melas, Reed-Muller, and Reed-Solomon; introduction to decoding starting with syndrome decoding and covering weight enumerator polynomials and the Mac-Williams Sloane identity. Further topics range from asymptotic parameters and bounds to a discussion of algebraic geometric codes in their simplest form.

#### Alternatives:

None

# Subsequent Courses:

Math 565 (Combinatorics and Graph Theory) and Math 556 (Methods of Applied Math I) are natural sequels or predecessors. This course also complements Math 312 (Applied Modern Algebra) by presenting direct applications of finite fields and linear algebra.

### Math 571 - Numerical Methods for Scientific Computing I

Prerequisites: Math 214, 217, 417, 419, or 420 and one of Math 450, 451, or 454; or permission of

instructor

Frequency: Fall (I), Winter (II)

Student Body: Mathematics and engineering graduate students, strong undergraduates

Credit: 3 Credits.

Recent Texts: Numerical Linear Algebra by Trefethen and Bau

Area: Applied/NA

# Background and Goals:

This course is an introduction to numerical linear algebra, which is at the foundation of much of scientific computing. Numerical linear algebra deals with (1) the solution of linear systems of equations, (2) computation of eigenvalues and eigenvectors, and (3) least squares problems. We will study accurate, efficient, and stable algorithms for matrices that could be dense, or large and sparse, or even highly ill-conditioned. The course will emphasize both theory and practical implementation.

#### Content:

Topics: (1) background, orthogonal matrices, vector and matrix norms, singular value decomposition; (2) least squares problems, QR factorization, normal equations, projection matrices, Gram-Schmidt orthogonalization, Householder triangularization; (3) stability, condition number, floating point arithmetic, backward error analysis; (4) iterative methods, classical iterative methods (Jacobi, Gauss-Seidel, SOR), conjugate gradient method, Lanczos iteration, Krylov subspace methods, Arnoldi iteration, GMRES, preconditioning; (5) direct methods, Gaussian elimination, LU factorization, pivoting, Cholesky factorization; (6) eigenvalues and eigenvectors, Schur factorization, reduction to Hessenberg and tridiagonal form, power method, QR algorithm.

#### Alternatives:

Math 471 (Intro. to Numerical Methods) is a survey course in numerical methods at a more elementary level

#### Subsequent Courses:

Math 572 (Number. Meth. for Sci. Comput. II) covers initial value problems for ordinary and partial differential equations. Math 571 and 572 may be taken in either order. Math 671 (Analysis of Numerical Methods I) is an advanced course in numerical analysis with varying topics chosen by the instructor.

### Math 572 - Numerical Methods for Scientific Computing II

Prerequisites: Math 214, 217, 417, 419, or 420 and one of Math 450, 451, or 454; or permission of

instructor

Frequency: Winter (II)

Student Body: Mathematics and engineering graduate students, strong undergraduates

Credit: 3 Credits.

Recent Texts: Finite Difference Methods for Ordinary and Differential Equations by LeVeque

Area: Applied/NA

### Background and Goals:

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. Graduate students from engineering and science departments and strong undergraduates are also welcome. The course is an introduction to numerical methods for solving ordinary differential equations and hyperbolic and parabolic partial differential equations. Fundamental concepts and methods of analysis are emphasized. Students should have a strong background in linear algebra and analysis, and some experience with computer programming.

#### Content:

Content varies somewhat with the instructor. Numerical methods for ordinary differential equations; Lax's equivalence theorem; finite difference and spectral methods for linear time dependent PDEs: diffusion equations, scalar first order hyperbolic equations, symmetric hyperbolic systems.

#### Alternatives:

There is no real alternative; Math 471 (Intro. to Numerical Methods) covers a small part of the same material at a lower level. Math 571 and 572 may be taken in either order.

# Subsequent Courses:

Math 671 (Analysis of Numerical Methods I) is an advanced course in numerical analysis with varying topics chosen by the instructor.

### Math 575 - Introduction to Theory of Numbers

Prerequisites: Math 451 and one of Math 420 or 494, or permission of instructor

Frequency: Fall (I)

Student Body: Roughly half honors mathematics undergraduates and half graduate students

Credit: 3 Credits. 1 credit after Math 475

Recent Texts: A Classical Introduction to Modern Number Theory by Ireland and Rosen

Area: Number Theory

# Background and Goals:

Many of the results of algebra and analysis were invented to solve problems in number theory. This field has long been admired for its beauty and elegance and recently has turned out to be extremely applicable to coding problems. This course is a survey of the basic techniques and results of elementary number theory. Students should have significant experience in writing proofs at the level of Math 451 and should have a basic understanding of groups, rings, and fields, at least at the level of Math 412 and preferably Math 493. Proofs are emphasized, but they are often pleasantly short.

#### Content:

This is a first course in number theory. Topics covered include divisibility and prime numbers, congruences, quadratic reciprocity, quadratic forms, arithmetic functions, and Diophantine equations. Other topics may be covered as time permits or by request.

#### Alternatives:

Math 475 (Elementary Number Theory) is a version of Math 575 which puts much more emphasis on computation and less on proof. Only the standard topics above are covered, the pace is slower, and the exercises are easier.

# Subsequent Courses:

All of the advanced number theory courses Math 675, 676, 677, 678, and 679 presuppose the material of Math 575. Each of these is devoted to a special subarea of number theory.

### Math 582 - Introduction to Set Theory

Prerequisites: Math 412 or 451 or equivalent experience with abstract mathematics

Frequency: Winter (II)

Student Body: Undergraduate mathematics (often honors) concentrators and some graduate students

Credit: 3 Credits.

Recent Texts: Discovering Modern Set Theory I: The Basics (8th edition) by Just and Weese

Area: Logic

### Background and Goals:

One of the great discoveries of modern mathematics was that essentially every mathematical concept may be defined in terms of sets and membership. Thus Set Theory plays a special role as a foundation for the whole of mathematics. One of the goals of this course is to develop some understanding of how Set Theory plays this role. The analysis of common mathematical concepts (e.g., function, ordering, infinity) in set-theoretic terms leads to a deeper understanding of these concepts. At the same time, the student will be introduced to many new concepts (e.g., transfinite ordinal and cardinal numbers, the Axiom of Choice) which play a major role in many branches of mathematics. The development of Set Theory will be largely axiomatic with the emphasis on proving the main results from the axioms. Students should have substantial experience with theorem-proof mathematics; the listed prerequisites are minimal and stronger preparation is recommended. No course in mathematical logic is presupposed.

#### Content:

The main topics covered are set algebra (union, intersection), relations and functions, orderings (partial, linear, well), the natural numbers, finite and denumerable sets, the Axiom of Choice, and ordinal and cardinal numbers.

#### Alternatives:

Some elementary set theory is typically covered in a number of advanced courses, but Math 582 is the only course which presents a thorough development of the subject.

#### Subsequent Courses:

Math 582 is not an explicit prerequisite for any later course, but it is excellent background for many of the advanced courses numbered 590 and above.

### Math 590 - An Introduction to Topology

Prerequisites: Math 451 Frequency: Fall (I)

Student Body: Mathematics graduate students, some non-math graduate students, and mathematics

undergraduates *Credit:* 3 Credits.

Recent Texts: Topology (2nd edition) by Munkres

Area: Geometry/Topology

# Background and Goals:

The purpose of this course is to introduce basic concepts of topology. Most of the course will be devoted to the fundamentals of general (point set) topology.

#### Content:

Topics include metric spaces, topological spaces, continuous functions and homeomorphisms, separation axioms, quotient and product topology, compactness, and connectedness. We will also cover a bit of algebraic topology (e.g., fundamental groups) as time permits.

#### Alternatives:

Math 490 (Introduction to Topology) is a more gentle introduction that is more concrete, somewhat less rigorous, and covers parts of both Math 591 (General and Differential Topology) and Math 592 (Intro. to Algebraic Topology). Math 591 (General and Differential Topology) is a more rigorous course covering much of this material and more. Combinatorial and algebraic aspects of the subject are emphasized over the geometrical.

# Subsequent Courses:

Students who take Math 590 will be well prepared to continue with Math 591 (General and Differential Topology) and/or Math 437 (Differentiable Manifolds).

## Math 591 - General and Differential Topology

Prerequisites: Math 451 Frequency: Fall (I)

Student Body: Mainly mathematics graduate students, a few mathematics undergraduates and non-

math graduate students

Credit: 3 Credits.

Recent Texts: Topology (2nd edition) by Munkres; Differential Topology by Guillemin and Pollack

Area: Geometry/Topology

# Background and Goals:

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

#### Content:

Topological and metric spaces, continuity, subspaces, products and quotient topology, compactness and connectedness, extension theorems, topological groups, topological and differentiable manifolds, tangent spaces, vector fields, submanifolds, inverse function theorem, immersions, submersions, partitions of unity, Sard's theorem, embedding theorems, transversality, classification of surfaces.

#### Alternatives:

None

# Subsequent Courses:

Math 592 (An Introduction to Algebraic Topology) is the natural sequel.

# Math 592 - An Introduction to Algebraic Topology

**Prerequisites:** Math 591 **Frequency:** Winter (II)

Student Body: Largely mathematics graduate students

Credit: 3 Credits.

Recent Texts: Elements of Algebraic Topology by Munkres, Algebraic Topology by Hatcher

Area: Geometry/Topology

# Background and Goals:

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

## Content:

Fundamental group, covering spaces, simplicial complexes, graphs and trees, applications to group theory, singular and simplicial homology, Eilenberg-Steenrod axioms, Brouwer's and Lefschetz' fixed-point theorems, and other topics.

## Alternatives:

None

# Subsequent Courses:

Math 695 (Algebraic Topology I).

## Math 593 - Algebra I

Prerequisites: Math 494; or Math 412, 420, and 451

Frequency: Fall (I)

**Student Body:** Largely mathematics graduate students

Credit: 3 Credits.

Recent Texts: Algebra (an approach via module theory) by Adkins and Weintraub

Area: Algebra

### Background and Goals:

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs. This course, together with Math 594, offer excellent preparation for the PhD Qualifying exam in algebra. Students should have had a previous course equivalent to Math 493 (Honors Algebra I).

#### Content:

Topics include basics about rings and modules, including Euclidean rings, PIDs, UFDs. The structure theory of modules over a PID will be an important topic, with applications to the classification of finite abelian groups and to Jordan and rational canonical forms of matrices. The course will also cover tensor, symmetric, and exterior algebras, and the classification of bilinear forms with some emphasis on the field case.

#### Alternatives:

None

### Subsequent Courses:

Math 594 (Algebra II) and Math 614 (Commutative Algebra I).

## Math 594 - Algebra II

Prerequisites: Math 593
Frequency: Winter (II)

Student Body: Largely mathematics graduate students

Credit: 3 Credits.

Recent Texts: Algebra, A Graduate Course by Isaacs

Area: Algebra

# Background and Goals:

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

## Content:

Topics include group theory, permutation representations, simplicity of alternating groups for n>4, Sylow theorems, series in groups, solvable and nilpotent groups, Jordan-Hölder Theorem for groups with operators, free groups and presentations, fields and field extensions, norm and trace, algebraic closure, Galois theory, and transcendence degree.

#### Alternatives:

None

# Subsequent Courses:

Math 612 (Algebra III), Math 613 (Homological Algebra), Math 614 (Commutative Algebra I), and various topics courses in algebra.

# Math 596 - Analysis I (Complex)

Prerequisites: Math 451 Frequency: Fall (I)

Student Body: Largely mathematics graduate students

Credit: 3 Credits. 2 credits after Math 555

Recent Texts: Complex Analysis (3rd edition) by Ahlfors

Area: Analysis

# Background and Goals:

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

#### Content:

Review of analysis in **R**<sup>2</sup> including metric spaces, differentiable maps, Jacobians; analytic functions, Cauchy-Riemann equations, conformal mappings, linear fractional transformations; Cauchy's theorem, Cauchy integral formula; power series and Laurent expansions, residue theorem and applications, maximum modulus theorem, argument principle; harmonic functions; global properties of analytic functions; analytic continuation; normal families, Riemann mapping theorem.

#### Alternatives:

Math 555 (Intro. to Complex Variables) covers some of the same material with greater emphasis on applications and less attention to proofs.

## Subsequent Courses:

Math 597 (Analysis II (Real)), Math 604 (Complex Analysis II), and Math 605 (Several Complex Variables).

## Math 597 - Analysis II (Real)

Prerequisites: Math 451 and 420

Frequency: Winter (II)

Student Body: Largely mathematics graduate students

Credit: 3 Credits.

Recent Texts: Real Analysis: Modern Techniques and Their Applications (2nd edition) by Folland

Area: Analysis

### Background and Goals:

This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. The approach is theoretical and rigorous and emphasizes abstract concepts and proofs.

## Content:

Topics include: Lebesgue measure on the real line; measurable functions and integration on  $\mathbf{R}$ ; differentiation theory, fundamental theorem of calculus; function spaces,  $\mathsf{L}^\mathsf{P}(\mathbf{R})$ ,  $\mathsf{C}(\mathsf{K})$ , Hölder and Minkowski inequalities, duality; general measure spaces, product measures, Fubini's Theorem; Radon-Nikodym Theorem, conditional expectation, signed measures, introduction to Fourier transforms.

#### Alternatives:

None

# Subsequent Courses:

Math 602 (Real Analysis II).