What Makes a Whip Crack?

(revised 9/11)

What makes a whip crack is the tip exceeding the speed of sound. The crack is a miniature sonic boom just like the one an airplane creates when it exceeds the speed of sound. The speed of sound at sea level is approximately 760 miles per hour or about 340 meters per second (it varies a few miles per hour depending on temperature and air pressure) Some people have trouble believing that physics would allow a person - even a very strong person - to throw anything at 760 miles per hour. Since it's easily demonstrable that even a child can crack a whip, these people believe something else must be making the sound—perhaps the whip is hitting itself. Contrary to this belief, physics shows that a sonic boom is exactly what happens.

I will use three equations, some simple algebra and the concept of ratios to show this.

Kinetic Energy

Kinetic energy is the energy of motion. The kinetic energy of an object is the energy it possesses because of its motion. Kinetic energy is an expression of the fact that a moving object can do work on anything it hits; it quantifies the amount of work the object could do as a result of its motion. The kinetic energy of a point mass *m* is given by:

equation 1

$$E = \tfrac{1}{2} m\left(V^2\right)$$

Algebraic transformation gives the equivalent equation solved for velocity:

equation 4b

$$V = \frac{\sqrt{2E}}{\sqrt{m}}$$

Where:

E = energy in Joules

m = mass in grams

V = velocity in meters per second

Ratios

We all know that 1/2 is more than 1/100, and that is the key to understanding what happens when a whip cracks. When a whip is thrown, the part that's moving, the part that 'holds' the kinetic energy shrinks, but the energy remains about the same. This is represented in equation 4b by the *m* in the bottom-right of the equation. As *m* gets smaller, the number represented by the equation, *V*, gets bigger. This manifests as a dramatic increase in the speed of the tip as a whip approaches full extension.

What Makes a Whip Crack?

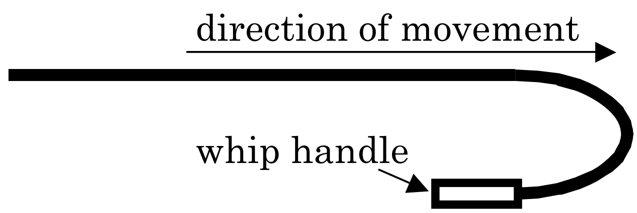
(revised 9/11)

This is perhaps intimidating, but you're past the worst of it, and I did say I was going to use equations and ratios. I promise to make it as easy as I can. I have put the algebraic translation of the above equation at the end so mathphobes will not be too bothered.

What really happens:

There are at least three modes of motion that will cause the tip of a whip to exceed the speed of sound. Here, I use the example of a single moving section of the whip and a single stationary section of the whip with one direction reversal - as in an overhand throw.

In a simple overhand whip throw, the whip starts with most of its length - it's mass - behind the person throwing it. The whip thrower brings his hand forward, imparting some velocity to the whip - let's say 25 miles per hour, or about 11.5 meters per second - then his hand stops when his arm reaches full extension. This gives us a situation where the whip handle is stationary and most of the whip is moving forward, as shown in drawing 1.



Drawing 1

The whip handle is being held by the person throwing the whip, and is more-or-less stationary. The part of the whip body represented by the upper section of the line is moving.

Given that professional baseball players routinely throw baseballs at over 90 miles per hour, 25 miles per hour is a reasonable assumption for how fast a whip can be thrown.

For this discussion, let us assume the part of the whip that is moving has a mass of 6.2oz, or about 175 grams.

If we put these numbers into equation 1, we arrive at:

equation 1 $$E = \tfrac{1}{2} m\left(V^2\right)$$

$$E = \tfrac{1}{2}175\left(11.5^2\right) = 87.5 \times 132 = 11{,}550 \, Joules$$

This is the kinetic energy in the moving part of the whip.

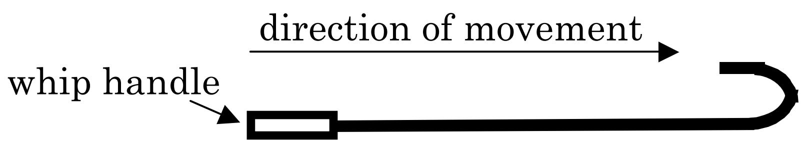
What Makes a Whip Crack?

(revised 9/11)

In the real world, some energy will be lost to internal friction as the bend moves down the whip and to resistance by the air to the moving whip. For this discussion, let us assume that 15% of the energy is lost and the other 85% of the energy is transferred to, and concentrated in, the moving part of the whip. Taking our energy number from above, we have:

$$11{,}550 \times 85\% = 9{,}815\,Joules$$

As the upper part of the whip and the curve in the body of the whip move, more and more of the whip is stationary because it is stretched out from the stationary handle, and less and less is in motion, as shown in drawing 2.



Drawing 2

Much less of the whip body is now moving, and more is stationary.

Let us assume the part of the whip that is moving is just the cracker, and it has a mass of five one-thousandths (.005) of an ounce, or about 0.15 grams.

If we put these numbers into equation 4b, we arrive at:

equation 4b

$$V = \frac{\sqrt{2E}}{\sqrt{m}}$$

$$V = \frac{\sqrt{2 \times 9{,}815}}{\sqrt{.15}} = \frac{140}{.387} = 362\,meters/\mathrm{sec}$$

362 meters per second works out to about 810 miles per hour - well above the 760 mph required to create a sonic boom.

In practice, as the bend in the whip approaches the tip, the mass of the moving part of the whip approaches zero, so we could pick any small number for the mass of the tip that makes our equations work, and simply wait a fraction of a second until that was the only part of the whip left moving.

The conclusion that whips exceed the speed of sound is born out by shadowgraphs—photographs of the shadows caused by the refraction of light by the shock waves of a whip's sonic booms. The first of these were taken in 1927, although the correct interpretation was not applied until 1958 in a paper by B. Bernstein, D. A. Hall, and H. M. Trent of the U.S. Navel Laboratory. Shadowgraphs from whip cracks can be found several places on the Internet.

Addendum - September 2011
Modes of Motion

From the above, it should be apparent that the bend in the whip - or reversal of direction - is, for many types of whip throws, a significant feature in creating a whip crack. It can be thought of as a 'moving kinetic barrier' through which the kinetic energy in the whip does not pass. When kinetic energy meets a reversal of direction, it 'concentrates' behind it.

It should also be apparent that the end of the whip acts in a similar, but significantly different way. The most relevant differences are the result of the free movement allowed by the un-restrained tip and the decrease in the mass-per-length ratio.

The moment when the end of the whip meets a bend in the whip is also significant. Here the kinetic energy becomes so 'concentrated'—as speed—that it accelerates the moving part of the whip to supersonic speeds. This changes the way energy is removed from the whip as well as causing a transition in the way the whip is moving.

One of the implications of this is that more of the length of lighter "thin" crackers will exceed the speed of sound as compared to heaver "fat" crackers.

Another implication, for the overhand throw and others like it, is that if the person throwing the whip pulls the handle back toward themself, they will effectively cause more of the whip to become stationary faster and this will cause the whip to crack before some of the energy that would have otherwise been lost, is gone.

When I originally wrote this in 2005, I was looking for and thinking about the simplest example I could find for a cracking whip–a half-wave motion. This was done primarily as an attempt to make my explanation easy to understand.

Since then, I have recognized and considered more complex modes of motion that concentrate enough kinetic energy in the tip of a whip to cause it to exceed the speed of sound. I have identified two other modes, and suspect there may be more.

An example of the second mode of motion I identified occurs when the whip is held with the handle horizontal, and the handle is moved up and down vigorously a short distance - no more than ten inches. This causes a wave shape or 'S-curve' to form and move down the whip carrying the energy from the handle to the tip. When the wave reaches the tip, the physics is the same as with the simple overhand throw. This is a 'ringmaster's crack' done with restrained movement. The energy arrives at the cracker as a full-wave motion rather than linear motion as in the overhand throw.

This throw can be done in any plane of rotation as long as the movement of the handle is perpendicular to the body of the whip. Done correctly, the whip will crack twice in rapid succession—once for each reversal of direction.

The third mode of motion I have identified occurs when a whip is thrown in a way that forms a loop that moves down the whip carrying energy from the handle to the tip. When the loop reaches the tip of the whip, it unrolls 180° and assumes the half-wave configuration discussed above, and the same physics apply as in that mode of motion. I do not yet have simple instructions for producing an example of this mode.

The energy arrives at the cracker as both rotary and liner motion.

Although Professor A. Goriely does not mention modes of motion in his paper, I believe this is the mode he was looking at in his first analysis of whip cracking. This mode may be easier to achieve with longer whips.

What Makes a Whip Crack?

(revised 9/11)

Solving for Velocity (optional reading)

For this discussion, we need to solve for velocity as well as energy, so we need to translate the energy equation to a velocity equation. If your eyes glaze over at the sight of algebraic translations, just assume I did it right and accept the discussion on pages 2 through 4. I only include my work in the name of rigor.

First multiply both sides of the equal sign by 2:

equation 2a
$$2E = \cancel{2}\frac{1}{\cancel{2}}m\left(V^2\right)$$

The upper and lower 2s on the right cancel giving us:

equation 2b
$$2E = m\left(V^2\right)$$

Then we divide both sides of the equal sign by *m*

equation 3a
$$\frac{2E}{m} = \frac{\cancel{m}\left(V^2\right)}{\cancel{m}}$$

The *m*'s on the right cancel giving us:

equation 3b
$$\frac{2E}{m} = V^2$$

Finally, we take the square root of both sides giving us:

equation 4a
$$\sqrt{\frac{2E}{m}} = V$$

Or the equivalent equation:

equation 4b
$$V = \frac{\sqrt{2E}}{\sqrt{m}}$$

The key when looking at equation 4b is that the term *m* for mass is on the bottom of the fraction on the right side and the term *V* for velocity is on the top on the left. This gives them an inverse relationship such that when one approaches zero, the other approaches infinity.

What Makes a Whip Crack?

(revised 9/11)

Some More Optional Stuff

The astute reader will note that it's possible to combine equation 1 and equation 4b and our percentage of loss calculation into one equation with four variables:

equation 5
$$V_2 = \sqrt{\frac{m_1 \times 0.85}{m_2}} \times V_1$$

Where:
m_1 = the mass of the whip body in grams
m_2 = the mass of the whip cracker in grams
V_1 = the velocity the whip is thrown in meters/sec
V_2 = terminal velocity of the cracker in meters/sec

I leave it as an exercise for the reader to convert these numbers to other units if they desire to. The conversion factors are all readily available to anyone with a computer.

It can be amusing and enlightening to create a spreadsheet and play with the variables in this formula. It will rapidly become clear why longer and/or heaver whips are easier to crack, and why thin crackers with few strands are easier to crack than fat, fluffy crackers with many strands.

Bottom line is: long heavy whips with thin crackers will get more of their length above the speed of sound and make louder cracks or crack with less effort or less skill.

What is perhaps not so clear from these calculations is how these variables effect accuracy and what happens when something other than 'perfect' throws are made.