A confirmation of the general relativistic prediction of the Lense–Thirring effect

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An important early prediction of Einstein's general relativity[1–3] was the advance of the perihelion of Mercury's orbit, whose measurement provided one of the classical tests of Einstein's theory[4]. The advance of the orbital point-of-closest-approach also applies to a binary pulsar system[5,6] and to an Earth-orbiting satellite[3]. General relativity also predicts that the rotation of a body like Earth will drag the local inertial frames of reference around it[3,7], which will affect the orbit of a satellite[8]. This Lense–Thirring effect has hitherto not been detected with high accuracy[9], but its detection with an error of about 1 per cent is the main goal of Gravity Probe B—an ongoing space mission using orbiting gyroscopes[10]. Here we report a measurement of the Lense–Thirring effect on two Earth satellites: it is 99 ± 5 per cent of the value predicted by general relativity; the uncertainty of this measurement includes all known random and systematic errors, but we allow for a total ±10 per cent uncertainty to include underestimated and unknown sources of error.

Another general relativistic shift of a gyroscope has already been measured to an accuracy of about 0.35% using the Moon's orbit[11,12]: the de Sitter or geodetic precession[3]. This shift arises from the effect of the gravitational field on the velocity of an orbiting gyroscope. But whereas the de Sitter effect is just due to the mass of a central non-rotating body, the Lense–Thirring effect is due to the rotation of a mass and displays the new general relativistic phenomenon that currents of mass generate additional space-time curvature[3]. This phenomenon has been called gravitomagnetism[3,7] for its formal analogy with magnetism in electrodynamics. Though the Lense–Thirring effect may play a basic dynamical role in the accretion disk of a supermassive spinning black hole[13,14] and in the alignment of jets in active galactic nuclei and quasars[7], it is however extremely small on a satellite orbiting the Earth. For example, for a satellite with an orbital semimajor axis of about 12,000 km, the shift of its node (the intersection of the Earth's equatorial plane with the satellite's orbit, see Fig. 1) is only about 33 mas yr^{-1} (mas = millisecond of arc), that is, nearly 1.9 m yr^{-1}. However, using the technique of laser-ranging with retro-reflectors to send back the short laser pulses, to date we are able to measure distances to a point on the Moon with a precision of a few centimetres, and distances to a small artificial satellite with a precision of a few millimetres.

In the present analysis we used the two laser-ranged satellites[15,16] LAGEOS (NASA) and LAGEOS 2 (NASA-ASI). Their instantaneous position can be measured with a precision of a few millimetres and their orbits, with semimajor axes $a_{\mathrm{LAGEOS}} \approx 12{,}270$ km and $a_{\mathrm{LAGEOS2}} \approx 12{,}210$ km, can be predicted, over 15-day periods, with a root-mean-square of the range residuals of a few centimetres. This uncertainty in the calculated orbits of LAGEOS and LAGEOS 2 is due to errors in modelling their orbital perturbations and, in particular, in modelling the deviations from spherical symmetry of the Earth's gravity field, mathematically described by a spherical harmonic expansion of the Earth's potential.

The current decade has been called the 'decade of geopotential research', because we already have two unique missions in orbit, studying and mapping the gravitational field of the Earth and its temporal variations: DLR's CHAMP (Challenging Minisatellite Payload), launched in 2000, and NASA's GRACE (Gravity Recovery and Climate Experiment, see Fig. 1), launched on 17 March 2002, to be joined in orbit in mid-2006 by ESA's GOCE. GRACE consists of two identical spacecraft, very similar in design to CHAMP, orbiting in a polar orbit in tandem, some 200–250 km apart. Each of them carries a very precise accelerometer to measure non-gravitational forces, and they both range to each other via a K-band radar that produces a history of the inter-satellite distance variations to better than one micrometre. With non-gravitational forces adequately measured by the on-board accelerometers, the observed inter-satellite distance variations are used to determine the errors in the current models of the terrestrial gravitational field, thereby leading to improvements in the medium wavelengths of the model describing the field to a resolution of 200–250 km half wavelength. At the same time, tracking of both satellites by GPS (Global Positioning System) provides the information to improve the long-wavelength part of the model, including the zonal coefficients which describe the axially symmetric part of the Earth's potential. In the result presented here, we have used the recent Earth gravity model, EIGEN-GRACE02S, obtained by the GeoForschungsZentrum (GFZ) group using the GRACE satellites[17]. (The EIGEN-GRACE02S gravity field coefficients and their calibrated errors are available at http://op.gfz-potsdam.de/grace/index_GRACE.html; the GRACE mission is also described at http://www.csr.utexas.edu.) EIGEN-GRACE02S is accompanied by a set of calibrated error estimates (although still preliminary in nature), appropriate for the assessment of the final error budget of our result.

In our previous analyses[9], we used the Earth gravity models JGM-3 and EGM96; owing to the limited accuracy associated with their low degree zonal terms, it was therefore necessary to use three observables: the node of LAGEOS and the node and perigee of LAGEOS 2. However, whereas the node is a very stable orbital

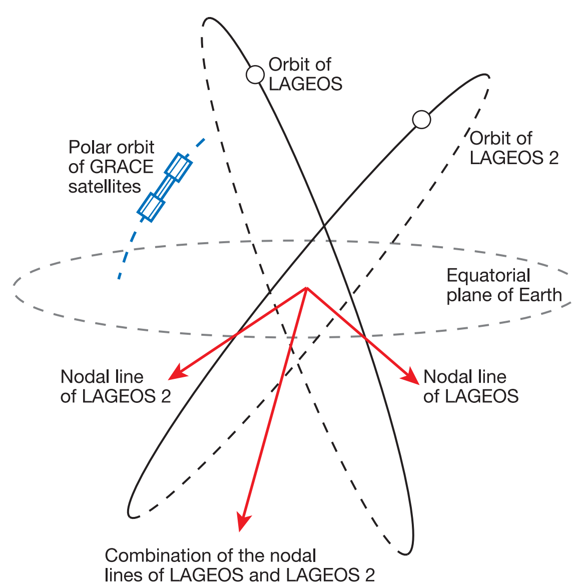

Figure 1 The 'orbital gyroscope' used to measure the Lense–Thirring effect. The 'gyroscope', indicated by the long red arrow, is the combination of the nodal longitudes of the LAGEOS satellites; it is not affected by the huge nodal rate of the LAGEOS satellites because of the Earth's quadrupole moment. This combination is described by equation (1); it is independent of the residual nodal rates due to the error in the Earth quadrupole moment. The blue drawing shows the orbital configuration of the GRACE satellites used to accurately determine the Earth's gravity field.

element under a number of non-gravitational perturbations, the perigee is affected by several perturbations difficult to model[18], implying an accuracy not easy to be confidently assessed. Nevertheless, by analysing the uncertainties in the spherical harmonic coefficients of the recent Earth gravity model EIGEN-GRACE02S, we find that the only relevant uncertainty in the orbit of the LAGEOS satellites, comparing it with the magnitude of the Lense–Thirring effect, is the one, δJ_2, in the Earth's quadrupole moment, J_2, which describes the Earth's oblateness. In the EIGEN-GRACE02S model, the relative uncertainty $\delta J_2/J_2$ is about 10^{-7}. This uncertainty corresponds on the orbits of the LAGEOS satellites to a shift of the node larger than the Lense–Thirring effect. However, the orbital uncertainty due to all the other harmonics is only a few per cent of the general relativity shift. Therefore, in order to eliminate the orbital uncertainty due to δJ_2 and in order to solve for the Lense–Thirring effect, it is necessary and sufficient to use only two observables.

The two orbital observables that we analysed are the two nodes of the LAGEOS satellites (Fig. 1). After modelling all the orbital perturbations, apart from the Lense–Thirring effect, we are able to predict the LAGEOS satellites' orbit with an error (root-mean-square of the residuals) of about 3 cm for a 15-day arc, corresponding to about half a millisecond of arc at the LAGEOS satellites' altitude. The Lense–Thirring effect is, in contrast, $31\,\mathrm{mas\,yr^{-1}}$ on the LAGEOS node, and $31.5\,\mathrm{mas\,yr^{-1}}$ on the LAGEOS 2 node, as calculated by the Lense–Thirring formula: $\dot{\Omega} = 2J_{\oplus}/[a^3(1-e^2)^{3/2}]$, where $\dot{\Omega}$ is the satellite's rate of change of the nodal longitude, $J_{\oplus}$ is the Earth's angular momentum, and a and e are the satellite's semimajor axis and orbital eccentricity, respectively. The residual (calculated minus observed) nodal rate of the LAGEOS satellites, $\delta\dot{\Omega}^{\mathrm{OBS}}$, is therefore: (residual nodal rate) = (nodal rate from δJ_2 error) + (nodal rate from other δJ_{2n} errors) + (Lense–Thirring effect) + (other smaller modelling errors), where the δJ_{2n} are the errors in the Earth's even zonal harmonic coefficients, J_{2n}, of degree $2n \geq 4$. We can then solve for the Lense–Thirring effect[19] the system of the two observed residual nodal rates, $\delta\dot{\Omega}^{\mathrm{OBS}}$, and simultaneously eliminate the error due to the δJ_2 uncertainty (see Methods). We accordingly combined the residuals as: $\delta\dot{\Omega}_{\mathrm{I}}^{\mathrm{OBS}} + k\,\delta\dot{\Omega}_{\mathrm{II}}^{\mathrm{OBS}}$, where the subscripts I and II indicate respectively LAGEOS and LAGEOS 2, and $k = 0.545$ is the ratio of the coefficients, K^2, of J_2 in the nodal rate equations of LAGEOS and LAGEOS 2. In other words, $k = \frac{K_{\mathrm{I}}^2}{K_{\mathrm{II}}^2}$ is that coefficient that makes the combination $\dot{\Omega}_{\mathrm{I}} + k\,\dot{\Omega}_{\mathrm{II}}$ of the nodal rates of LAGEOS and LAGEOS 2 independent of any contribution of J_2. Indeed, we have: $\dot{\Omega}_{\mathrm{I}}(J_2) + k\,\dot{\Omega}_{\mathrm{II}}(J_2) = K_{\mathrm{I}}^2 J_2 + kK_{\mathrm{II}}^2 J_2 = 0$, whereas by combining the nodal rates with the k factor we get for the Lense–Thirring contribution: $\dot{\Omega}_{\mathrm{I}}^{\mathrm{Lense-Thirring}} + k\,\dot{\Omega}_{\mathrm{II}}^{\mathrm{Lense-Thirring}} = (31 + k31.5)\,\mathrm{mas} \approx 48.2\,\mathrm{mas}$. The maximum error in this combination of the residuals due to the δJ_{2n} is 4% of the Lense–Thirring effect (see Methods and Supplementary Information).

We analysed nearly eleven years of laser-ranging data, from January 1993 to December 2003, corresponding to about one million normal points, that is, to about 100 million laser ranging observations from more than 50 ILRS stations distributed all over the world[20]. The total uncertainty of our measurement is, including systematic errors, ±5% of the Lense–Thirring effect and ±10% allowing for underestimated and unknown error sources (until more accurate Earth gravity field models are available, and until additional simulations are completed to exhaustively study the modelling of non-gravitational perturbations and measurement errors). For example, if we consider the time-independent gravitational error (root-sum-square) to be three times larger, we get a corresponding error of 9% and a total uncertainty of less than 10% (see Methods and Supplementary Information).

In Fig. 2 we show the observed residuals of the nodal longitudes of

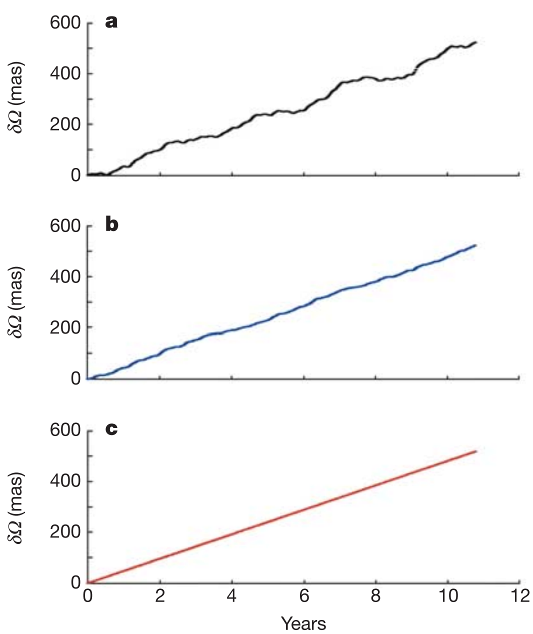

Figure 2 Observed orbital residuals of the LAGEOS satellites. The residual nodal longitudes of the LAGEOS satellites, $\delta\Omega$, were combined according to equation (1). In black (**a**) is the raw, observed, residual nodal longitude of the LAGEOS satellites without removal of any signal, whereas in blue (**b**) is the observed residual nodal longitude after removal of six periodic signals. The best-fit line (13-parameter fit) through these observed residuals has a slope of $47.9\,\mathrm{mas\,yr^{-1}}$. In red (**c**) is the theoretical Lense–Thirring prediction of Einstein's general relativity for the combination (equation (1)) of the nodal longitudes of the LAGEOS satellites; its slope is $48.2\,\mathrm{mas\,yr^{-1}}$.

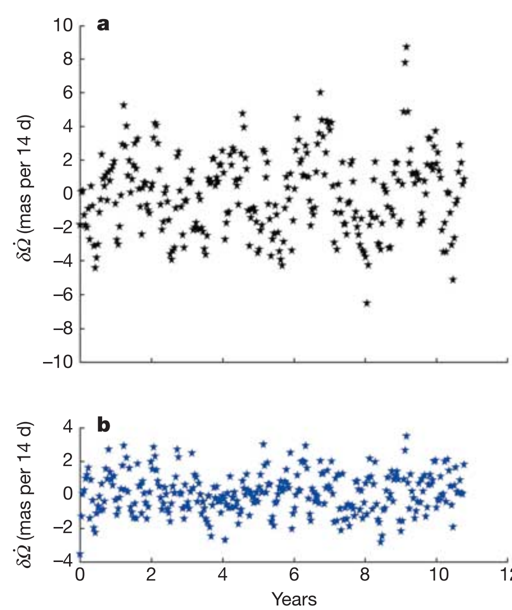

Figure 3 Post-fit orbital residuals of the LAGEOS satellites. These, 14-day, residuals of the nodal rates, $\delta\dot{\Omega}$, combined using equation (1), correspond to the case (**a**) of the fit of a secular trend only (black stars) and to the case (**b**) of a trend plus phase and amplitude of six periodic signals (blue stars) with periods of 1,044, 905, 281, 569, 111 and 284.5 days. We also fitted the residuals with a straight line plus the two LAGEOS nodal frequencies and plus ten signals. The maximum relative variation in the measured value of the Lense–Thirring effect in all the different fits was 2%.

the LAGEOS satellites, $\delta\Omega$, combined according to equation (1) in Methods, that is, we plot the residuals of the nodes of Fig. 1, combined with $k = 0.545$. The best fit line through the raw residuals in black (one-parameter fit) has a slope of 47.4 mas yr^{-1}; the root-mean-square of these post-fit residuals is 15 mas. In blue are the residuals after removal of six main frequencies, corresponding to a 13-parameter fit with a secular trend plus phase and amplitude of six main signals (see Methods). In this case the secular trend is 47.9 mas yr^{-1}, but the root-mean-square of these post-fit residuals is only 6 mas. In red is the Lense–Thirring effect predicted by general relativity for the combination of the LAGEOS nodal longitudes, which amounts to 48.2 mas yr^{-1}. Therefore, corresponding to the 13-parameter fit (in blue) of Figs 2 and 3, the observed Lense–Thirring effect is 47.9 mas yr^{-1}, corresponding to 99% of the general relativistic prediction. In conclusion, this analysis confirms the general relativistic predictions of frame-dragging and the Lense–Thirring effect. □

Methods

Data analysis

The laser-ranging data were processed using NASA's orbital analysis and data reduction software GEODYN II[21], over 15-day periods (with overlap of 1 day), following ref. 22, except for the use of the recent Earth static gravitational model EIGEN-GRACE02S. Solar radiation pressure, Earth albedo and anisotropic thermal thrust effects were modelled according to refs 23–26 using the LOSSAM-2004 model[27] (LAGEOS Spin Axis Model) of the satellites' spin axis, and the Earth tides using the GOT99.2 (Goddard/Grenoble Ocean Tide) model[28]. Lunar, solar and planetary perturbations (JPL ephemerides DE-403) were included in the equations of motion and Earth's rotation was modelled from very long baseline interferometry and GPS. The Lense–Thirring effect was set equal to zero. For every 14-day period we obtained one node residual for LAGEOS and one for LAGEOS 2, which were then combined according to equation (1):

$$\delta\dot{\Omega}_{\mathrm{I}}^{\mathrm{OBS}} + k\delta\dot{\Omega}_{\mathrm{II}}^{\mathrm{OBS}} = \dot{\Omega}_{\mathrm{I}}^{\mathrm{Lense–Thirring}} + k\,\dot{\Omega}_{\mathrm{II}}^{\mathrm{Lense–Thirring}} \pm \sum_{2n\geq 4}\left(K_{\mathrm{I}}^{2n}|\delta J_{2n}| + k\,K_{\mathrm{II}}^{2n}|\delta J_{2n}|\right) \quad (1)$$

where $k = 0.545$ and K^{2n}, well determined functions of the orbital elements, are the coefficients in the nodal rate equations of the even zonal harmonics (even degree and zero order), J_{2n}, of the Earth's gravity field.

Error assessment

The dominant error in modelling the nodal drift of the LAGEOS satellites is due to the errors, δJ_{2n}, in the static part of the J_{2n}. We assessed this error by using the EIGEN-GRACE02S model, with its δJ_{2n} calibrated (that is, including systematic errors) uncertainties, and using our combination, equation (1), to eliminate the largest uncertainty, δJ_2, in the Earth's quadrupole moment. By taking the square root of the sum of the squared errors due to the δJ_{2n} in equation (1), we found a relative error of about 3% of the Lense–Thirring effect on the LAGEOS satellites. However, to get an upper bound to this error (since the covariance matrix was not available to us) we simply added the absolute values of the errors due to the δJ_{2n} in equation (1); we thus obtained an error estimate of about 4% of the Lense–Thirring effect.

To assess the error in modelling the nodal drifts due to all the other orbital perturbations, we used two different methods: (1) an *a priori* detailed analysis of the uncertainties in the perturbations affecting the LAGEOS satellites, based on the extensive scientific literature on this subject[23–27,29–31], and (2) an *a posteriori* analysis of the orbital residuals. Both methods produced the same result. Using the *a priori* error analysis, for the gravitational perturbations (solar and lunar Earth tides, secular trends in the even zonal harmonics of the Earth's field and other periodic variations in the Earth's harmonics), we obtained, over a period of 11 yr, a total relative uncertainty of about 2% of the Lense–Thirring effect[30]. For the non-gravitational perturbations (atmospheric drag, solar radiation pressure, Earth albedo, anisotropic thermal radiation from the satellites, both from solar radiation and from the Earth's infrared radiation heating) we got, over a period of 11 yr, a total relative uncertainty of about 2% of the Lense–Thirring effect[30]. We also included an error of about 2% due to stochastic errors, such as seasonal variations in the Earth's gravity field, and observation biases.

Using the orbital residuals and considering that the main periods of the non-gravitational perturbations and of the tidal effects are well determined[23–31], we performed a number of different fits of the residuals. We fitted the residuals with a secular trend only, or with a trend plus a different number of the main periodic terms (fitting both phase and amplitude). These terms correspond to various linear combinations of the LAGEOS satellites' nodal periods (1,044 and 569 days), the Earth's revolution (365.25 days) and the Moon's nodal period (18.61 yr). In addition, we also performed a Fourier analysis of the residuals and removed the main identified frequencies in our fit. The maximum variation we found in all these different fits was at most a 2% change in the measured value of the Lense–Thirring effect, confirming that the error in the modelling of the periodic perturbations may in the worst case affect our Lense–Thirring determination at a level of about 2% only.

Total uncertainty

Finally, considering all the error sources described above and in agreement with previous error analysis of the LAGEOS III-LARES experiment[29–31] (see also Supplementary Discussion), we obtained a total root-sum-square error of ±5% of the Lense–Thirring effect. This uncertainty refers to all the known errors, however, allowing for unknown and unmodelled error sources we assume a ±10% uncertainty in our measurement. For example, if we double the maximum time-independent gravitational error and the non-gravitational and time-dependent gravitational errors, we get respectively 8%, 4% and 4% errors, and thus a total uncertainty of 10% of the Lense–Thirring effect.

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