THE PRACTICAL RANGE OF SMALL ARMS

Many factors control how far a rifle can be fired effectively

By G. L. M. KJELLGREN

	Muzzle velocity (m/sec.)	(f.p.s.)	Bullet weight (grams)	(grs.)	Cartridge weight (grams)	(grs.)	Rifle Weight* kg.	(lbs.)
9 mm/UZI	435**	(1430)	8.0	(123)	11.5	(177)	3.5	(7.7)
7.62x39/AK-47	710	(2330)	7.9	(122)	16.5	(255)	3.8	(8.4)
5.56 mm/M16	990	(3250)	3.6	(55)	11.9	(184)	3.0	(6.6)
7.62 NATO/G-3	840	(2760)	9.45	(145)	24.0	(370)	4.3	(9.5)

* Less magazine.
** Maximum performance.

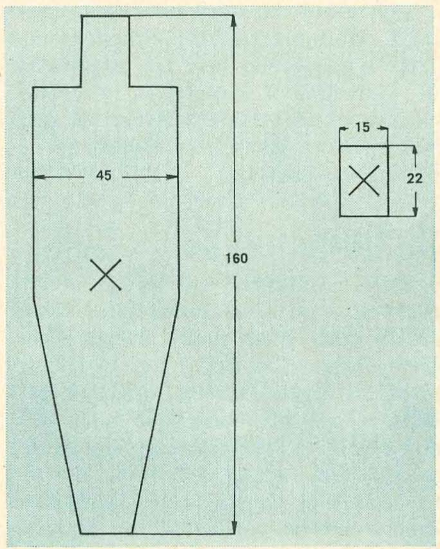

Fig. 1. The International silhouette target and its head (measurements in centimeters). X is point of aim in all cases discussed in the text.

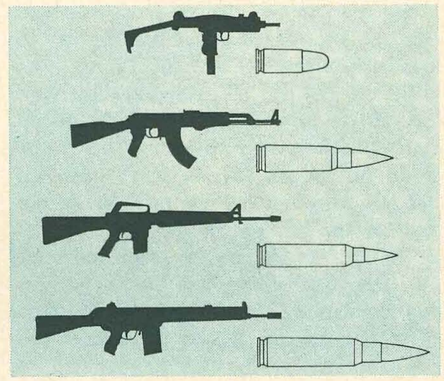

Fig. 2. Weapons and cartridges, the two groups shown in comparative sizes respectively (From top): UZI/9 mm., AK-47/7.62x39, M16/5.56 mm., and G-3/7.62 NATO.

THE distance at which a military rifle can be used depends on many factors, including the target, dispersion, errors in estimates of wind and range, hit percentage required, and to a great extent the shooter.

The bolt-action rifles used until the end of World War II usually were provided with sights graduated up to 2000 meters. Today's rifles are sighted to 600 meters, except for the Russian which have settings up to 800 or 1000 meters. This decrease in sighting range is mostly a result of changed tactics.

I will here deal with the problems of shooting against visible, non-moving targets, represented by the International silhouette target and its head (Fig. 1). Shooting against invisible targets in a given area is practically affected by only terminal ballistics and common statistics, and therefore is not discussed here.

Because of their worldwide distribution, and their characteristics which are representative of their types, I will discuss these 4 cartridge/weapon combinations: 9 mm/UZI, 7.62x39/AK-47, 5.56 mm./M16, and 7.62 NATO/G-3. The table shows their elementary characteristics.

Naturally the ballistic properties of the cartridge are more important than the model of the arm where range is concerned. Most rifles are designed for a certain cartridge, not the other way round.

To get an accurate expression for the practical range and to illustrate the effect of the different disturbances on the trajectory, each factor must be isolated and discussed separately. This method of comparison has the advantage that errors derived from different shooters are eliminated.

The maximum range of a small-arms projectile is attained at about 30° elevation. These 4 cartridges have about the following maximum ranges:

9 mm.	1500 meters	1600 yds.
7.62x39	2300 meters	2500 yds.
5.56 mm.	2700 meters	3000 yds.
7.62 NATO	4000 meters	4400 yds.

This maximum range is of only safety interest, not for firing. Some 7.62 NATO machine guns have sight settings up to 1500 or 2200 meters, but fire is very seldom extended to more than 1200 meters, this always on a tripod.

ABOUT THE AUTHOR

Goran L. M. Kjellgren, a native of Sweden, is a professional engineer who has been in the research department of the Husqvarna Arms Factory. After a tour of duty in the Swedish Navy, he studied mathematics and made an informal study of ballistics in Germany. He is an NRA Member.

Dispersion

Dispersion of the arm could be expressed by the zones containing 50% of the shots (H_{50}, B_{50}), by the circle containing 50% (D_{50}), or as the size of the 10-shot group. For calculations the H_{50} and B_{50} are the most useful.

The dispersion is almost directly proportional to the distance. The dispersion caused by each component can be isolated by studying the groups produced by firing by the following different methods:

Stage	Method	Dispersion Component	Total Dispersion
1	Ammunition test in test barrel	a	a
2	Rifle in machine rest	b	$\sqrt{a^2 + b^2}$
3	Rifle with telescope, bench rest	c	$\sqrt{a^2 + b^2 + c^2}$
4	Rifle with iron sights, bench rest	d	$\sqrt{a^2 + b^2 + c^2 + d^2}$
5	Shooting under field conditions	e	$e\sqrt{a^2 + b^2 + c^2 + d^2}$

Component a is caused by unavoidable variations in ammunition manufacture. Bullet unbalance is one of the most important factors within component a.

Component b is of very complicated nature, including general design of the weapon, barrel bedding, and quality of manufacture. The dispersion of the machine rest can be ignored.

Component c is very small and is mainly caused by differences in recoil absorption by the shooter. The sighting errors from the telescopic sight can be ignored.

Component d is caused by sighting errors from the iron sights. This error is inversely proportional, almost exactly, to the sight radius. "d" is easily determined by sighting experiments. A 10-shot group fired at Stage 4 with these arms will have the following average size at 100 meters:

UZI 25 cm. (9.8")	**AK-47** 15 cm. (5.9")
M16 11 cm. (4.3")	**G-3** 11 cm. (4.3")

These figures thus represent the smallest groups statistically attainable by a shooter using the standard sights.

Component e represents the skill and training level of the rifleman. This factor naturally varies with the combat situation, shooting position, and light condition. For a fair rifleman in prone position, good light, and not too much combat stress, I have chosen the factor e = 2.5. The numerical value of e is not too important for this discussion; the more important thing is that the 4 cases be fully comparable. Fig. 3 shows the dispersion at Stage 5 against the stated targets.

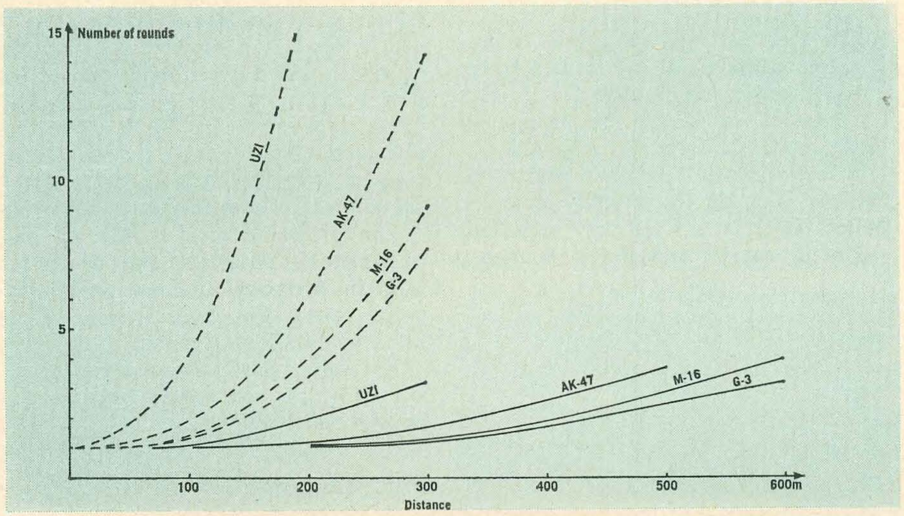

Fig. 3. Number of rounds required to obtain one hit, firing with dispersion $e\sqrt{a^2+b^2+c^2+d^2}$ in field shooting at known distance with no wind. Dotted curves are for the head target. (Compare with Fig. 11.)

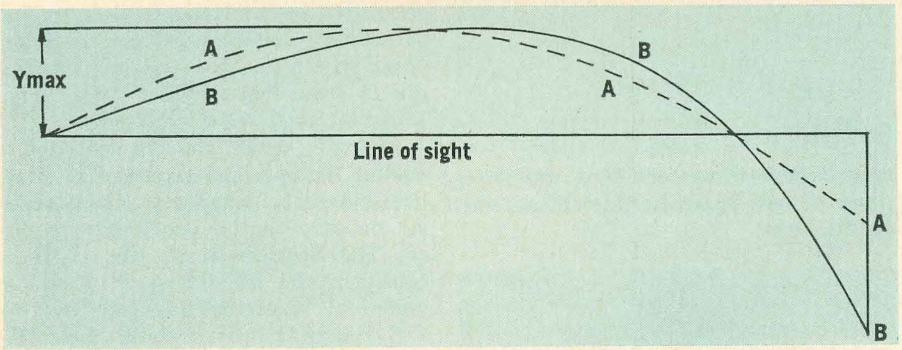

Fig. 4. Despite equal maximum height, trajectories can differ considerably. Trajectory B, given by high muzzle velocity and a light bullet, evidently is much more sensitive to errors in range estimation.

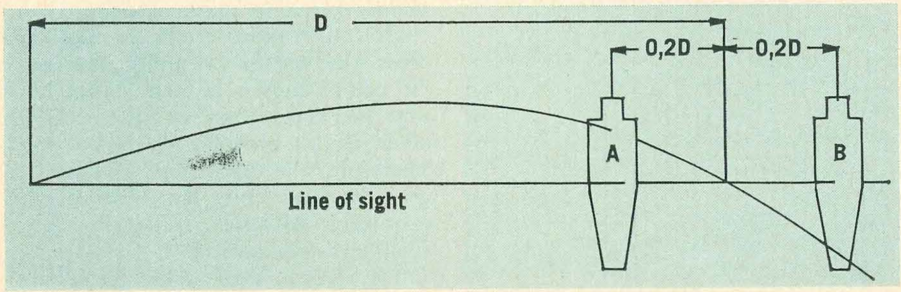

Fig. 5. D is estimated range, with targets A and B shown at actual ranges. A short estimate causes less height error than an equal long, estimate.

Distance to target

As the projectile's flight is curved downward by gravity, the sights must be graduated for different ranges. On modern rifles the adjustment usually is in 100-meter steps from 200 to 600 meters. At ranges between the 100-meter steps (with most rifles) the shooter must change his aim according to the trajectory. Consequently, he needs to determine the distance to his target, and this grows more important as the range increases. The most common way to determine the range is plain estimation. It is evident that an error in range estimation results in a height displacement of the shot. It is also evident that a flat trajectory is less affected by errors in range estimation.

Flatness of the trajectory can be expressed as its maximum height above the sight line (y_{max}). The y_{max}, however, will not tell anything about the look of the descending part of the trajectory, where the target is. One trajectory can have a smaller angle of fall in spite of a higher y_{max}, compared to another trajectory (Fig. 4). In some cases of machine gun fire, the effect on the range can be of interest. With an y_{max} of 1.6 meters, as determined by a target of that height, the danger zone will be:

9 mm.	**7.62x39**	**5.56 mm.**	**7.62 NATO**
350 meters	530 meters	650 meters	690 meters

(Note: All trajectory calculations were done by the Siacci method, with drop approximation according to McShane).

But the practical effect of trajectory flatness at the target is in height displacement of the shot, caused by error in estimating the range.

Take the greater error

The range may be estimated too long or too short, which will displace the shot upward or downward. At the same percentage error in range estimation, the short estimate will cause the greater height displacement (Fig. 5). Consequently only this case will be further discussed.

Without any instruments or special maps, a fair rifleman should be able to estimate the distance to his target with an error of less than 20%. In Fig. 6 the estimated distance is "D", while the actual distance is 1.20 D; the curves then show the height displacement of the cartridges discussed.

Wind deflection

A cross wind will displace the shot to the side. Consequently the shooter must determine the cross wind and change his aim according to the deflection. A fair rifleman should be able to estimate the wind with an error less than 2 meters per second. When the wind is not at right angle to the range, the 2 m/sec. represents the cross component. Fig. 7 shows the wind deflection of the bullets discussed, in a cross wind of 2 m/sec.

By the way, it is easy to show that the ranges determined by a certain deflection (horizontal lines in Fig. 7) follow the expression $k\sqrt{v_0 c}$ with an error of only 3% for small arms projectiles. As seen by comparing Figs. 6 and 7, *the wind deflection, even in very low winds, will have much more serious effect on the practical range of the rifle than the height displacement caused by errors in range estimation.*

Terminal ballistics

The terminal ballistics which a projectile must have are determined by its intended target. Small arms are primarily intended for direct fire against almost unprotected enemy personnel.

It is extremely difficult to find a numerical value for the lethality of a rifle bullet, but 2 factors are very important —kinetic energy and bullet diameter. Small caliber cartridges like the 5.56 mm. can be improved by a very high muzzle velocity causing "hydrostatic shock" at short ranges, or by bullets tumbling on impact. It has been reported that the change from 14" to 12" rifling twist in the M16 rifle reduced lethality by 40%; other reports state a lesser reduction.

The most used expression for lethality is the kinetic energy of the bullet (Fig. 8). The minimum energy required to put a man out of action is generally believed to be 15 kilogram-meters (108 ft.-lbs.). The distance at which this is the projectile's remaining energy thus is a good expression of the useful maximum range, as far as lethality is concerned. In the 4 projectiles discussed it is:

9 mm	7.62x39	5.56 mm.	7.62 NATO
630 meters	1500 meters	900 meters	2600 meters

The projectile also should be able to penetrate obstacles, and protection worn by personnel (helmets and bullet-proof vests). Fig. 9 illustrates the relative penetrations in metal.

Naturally, good terminal and exterior ballistics are joined together at normal bullet weights.

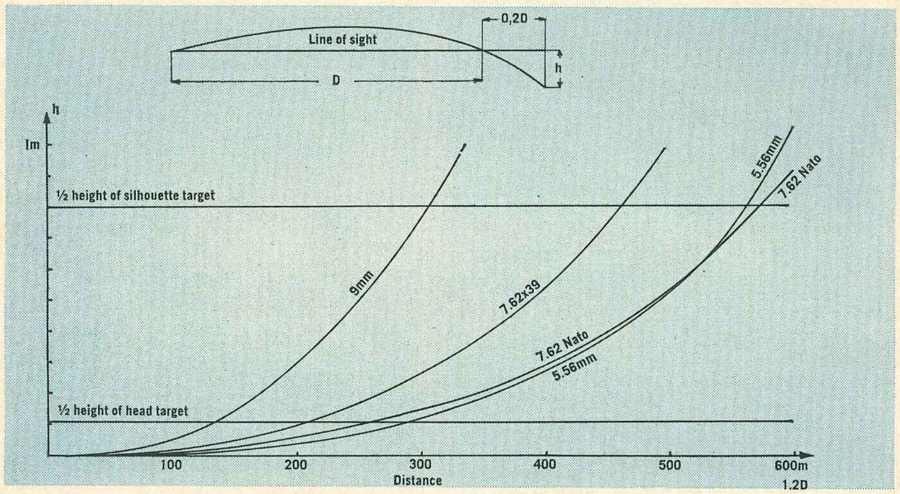

Fig. 6. Height error (h) resulting from 20% error in estimating range (D is estimated range, 1.2 D the actual range). Horizontal lines at half target height indicate the range at which the shot is displaced outside the target.

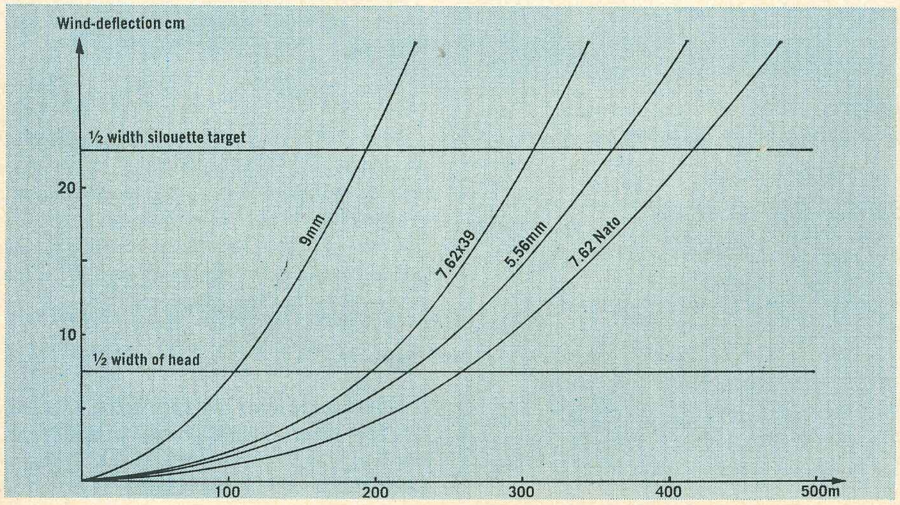

Fig. 7. Deflection caused by 2 m/sec. cross wind. Lines at half target width indicate the range at which the shot is displaced outside the target.

Practical range

If the components of dispersion, height and side displacement which have been discussed are put together, the result will be a displaced group of shots of certain characteristics (Fig. 10). Other disturbances such as differences in air pressure and temperature, are so small that they can be included in the factor "e" of the dispersion.

Fig. 11 shows the hit probability against the specified targets. The practical range of a weapon is now dependent on only the hit probability required. The Swedish Army, for instance, considers one hit out of 7 to be the minimum required; this will give its G-3 rifle a range of more than 500 meters. The West German Army, on the other hand, states the range of exactly the same rifle to be only 300 meters. These differences in opinion are mainly caused by different tactics.

Of course other errors in range and cross wind estimation and in the factor "e" can be chosen. If these values, however, are kept within realistic limits, the effect on hit probability will be small. Therefore, the distance-probability diagram of Fig. 11 is the most accurate expression for the practical range of small arms yet devised.

The relative sizes of the range-determining components are also of interest (Fig. 12), expecially when calculating the effect of a telescopic sight or when

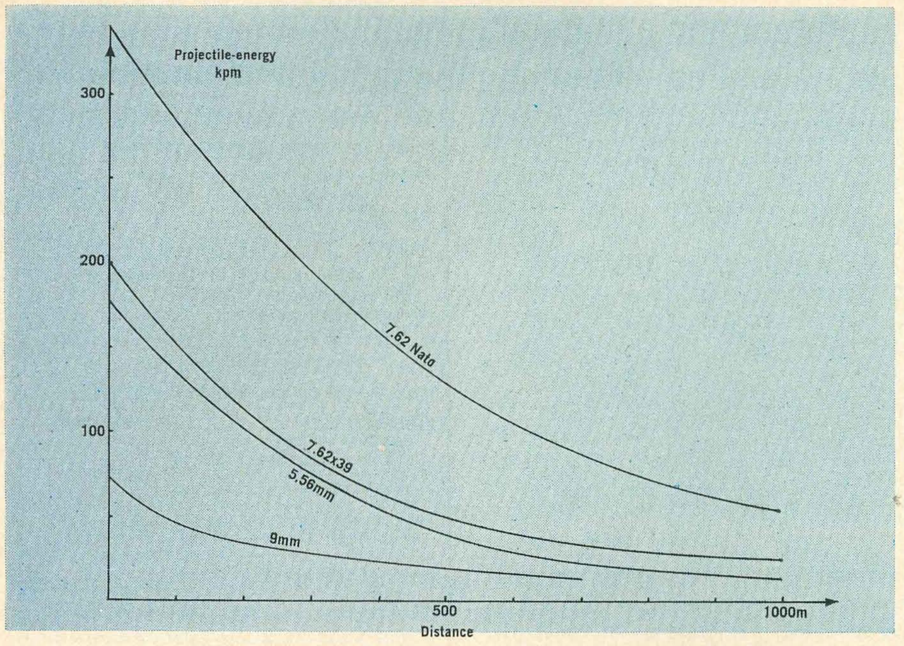

Fig. 8. Kinetic energy of the cartridges discussed. Superiority of the full power 7.62 NATO over the intermediate 7.62 x 39 and 5.56 mm. is evident.

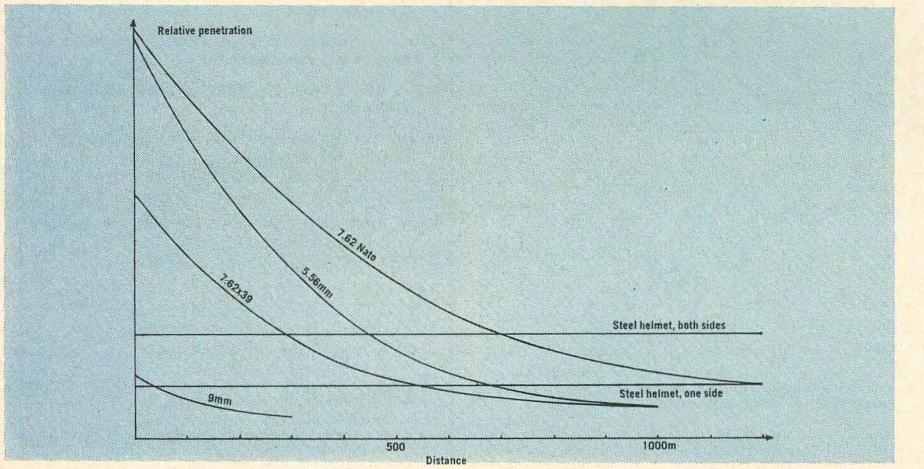

Fig. 9. Relative penetration (ball projectiles, not AP). Lines indicate a normal steel helmet. Great penetration of the 5.56 mm. is due to its small caliber.

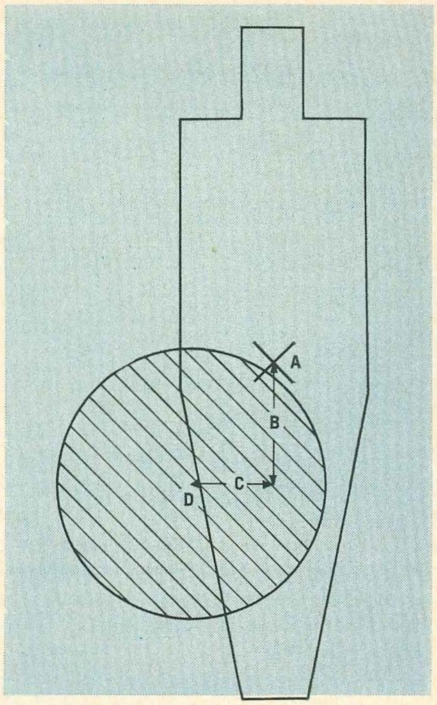

Fig. 10. The result of a G-3 shot at 400 m. A is point of aim, B height error (see Fig. 6), C side error (see Fig. 7), D center of D_{50} circle. This illustrates the principle on which Fig. 11 is calculated.

designing new cartridges. As seen from the diagram, the most important range-shortener, besides the shooter, is the cross wind deflection. At close ranges the dispersion has a considerable relative effect, but as the other components rapidly increase with the distance, the dispersion loses in importance. When the group center is displaced a certain amount outside the target, a large dispersion would even improve the hit probability, but this usually occurs outside the practical range of the weapon.

To get a complete view, it is necessary to study the hit probability in relation to ammunition weight. Fig. 13 shows the weight of ammunition of each type needed to obtain one hit in the silhouette target. It is interesting to note that the 5.56 mm. is superior to the others, except at very close range where it is beaten by the 9 mm., and at long range where the 7.62 NATO takes over. It is also interesting to see that the 7.62x39 is only slightly better than the NATO cartridge and beyond 250 meters is completely inferior.

If a fair comparison of the overall combat efficiency is to be made, it is important to note that the 7.62 NATO is the only one of these cartridges that is capable not only of ordinary rifle use, but also capable of long range machine gun and sniping fire. The 5.56 mm. is too wind sensitive and lacks the terminal ballistics for this role. The trajectory of the 7.62x39 is too curved and wind sensitive for long range precision fire, but it may be used in machine guns up to 1000 meters.

As previously mentioned, the limiting range set by the wind deflection can be accurately expressed by: $k\sqrt{v_0c}$. It is easy to prove that the practical range (Fig. 11) also follows $\sqrt{v_0c}$, not with the same accuracy but tolerable for this use.

Consequently, for military small arms purposes the ballistic coefficient is as important as the muzzle velocity.

It should also be noted that it is much easier to improve the ballistic coefficient than the muzzle velocity. If, for instance, the 5.56 mm. case is lengthened by 5 mm. the improvement in muzzle velocity will be about 2.5%, but if the same length is added to the projectile head, the ballistic coefficient increases by almost 40%! The increase in practical range will be approximately half that percentage.

This relationship between case and head length has not always been fully understood. Examples of well designed cartridges in this respect are the British .280 and the 7.62x39. Not so well designed are the 5.56 mm. and Czech M52. For reasons of stability and friction, however, the projectile head length is normally limited to about 3 calibers.

As the muzzle energy of a rifle cartridge stays almost constant in spite of changes in bullet weight, and the important property of a cartridge is the product v_0c, where c is directly proportional to the bullet weight, the practical range will be increased by a lower muzzle velocity and heavier bullet. Of course there is a minimum v_0, of about 700 m/sec., for necessary flatness of trajectory.

It is my belief that these points will grow in importance as the military rifle cartridge tends to shrink and so necessitates an optimum design. ■

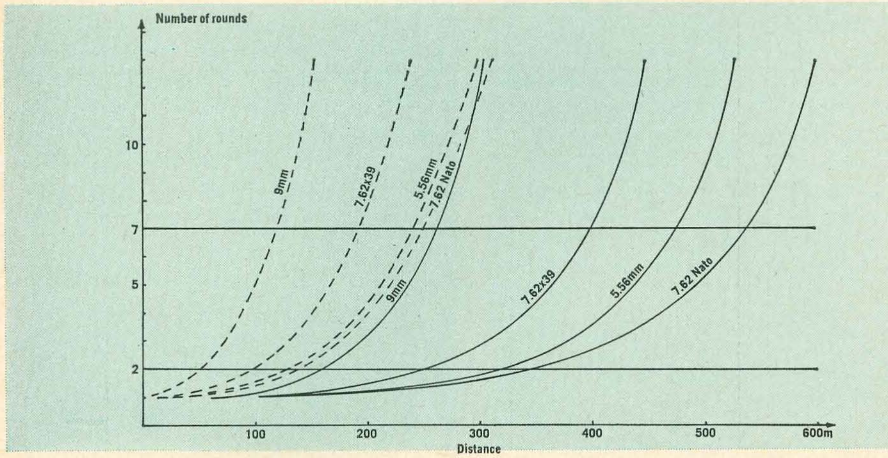

Fig. 11. Number of rounds required for one hit, under conditions of 20% range error, 2 m/sec. cross wind error, and dispersion from Fig. 3. This accurately expresses the practical range. Line at 7 rounds represents the minimum hit probability as chosen by the Swedish Army; line at 2 rounds indicates the distance at which the hit chance is 50%.

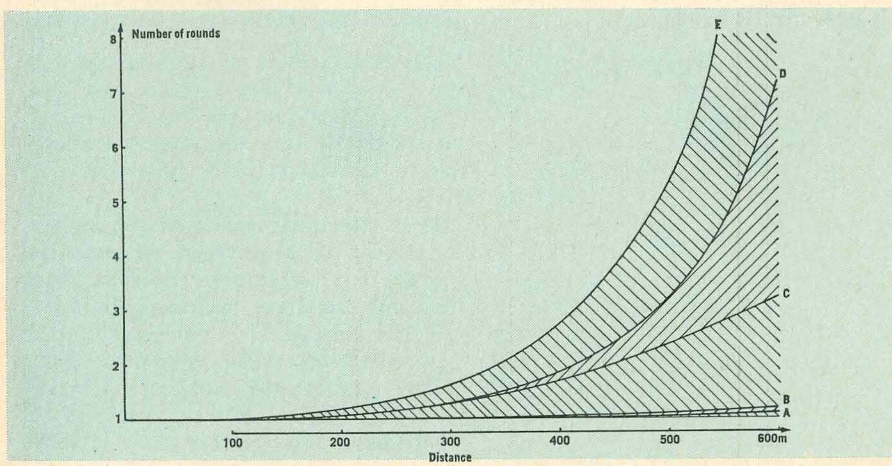

Fig. 12. Relative size of range-determining components, for G-3 rifle and 7.62 NATO cartridge. Conditions: A—machine rest; B—bench rest, iron sights; C—field conditions, "e" = 2.5; D—with 20% range estimation error; E—with 2 m/sec. wind estimation error. The relative sizes of D and E will be larger for the other cartridges discussed.

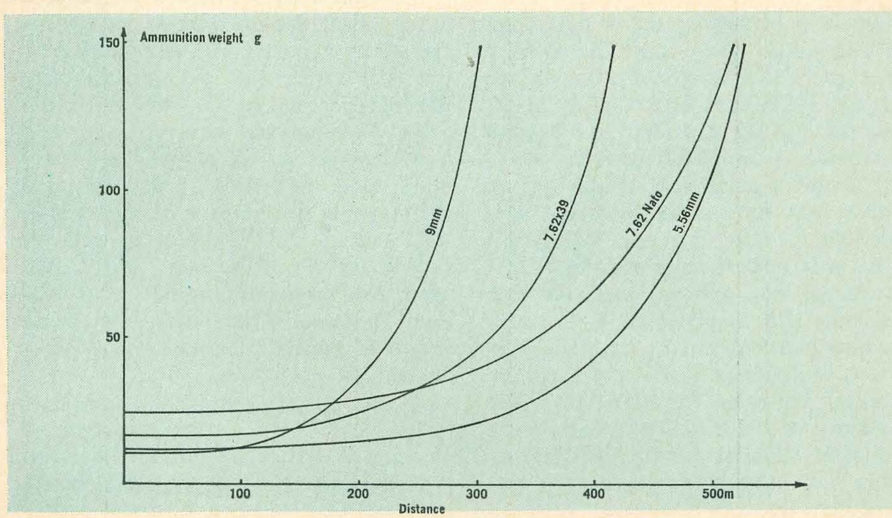

Fig. 13. Weight of ammunition required for one hit. The curves for 5.56 mm. and 7.62 NATO will meet at 535 m. range, beyond which the 5.56 mm. will require the greater weight of ammunition for a hit.

Camp Perry, Ohio
SCHEDULE OF EVENTS

July

Tues. 28—Billeting of Police School Students

Wed. 29—Police Instructor School

Thur. 30—Police Instructor School

Fri. 31—Police Instructor School

PISTOL

August

Sat. 1—Billeting of Pistol Competitors
Police Instructor School
Pistol Practice
Pistol Competitor Briefing

Sun. 2—Warm-up Matches

Mon. 3—.22 Caliber Matches

Tues. 4—
Center-Fire Matches
Mayleigh Cup Team Match

Wed. 5—.45 Caliber Matches

Thur. 6—National Trophy Pistol Individual and Team Match

SMALLBORE RIFLE

Fri. 7—
Billeting of Smallbore Rifle Competitors
Smallbore Competitor Briefing

Sat. 8—
Smallbore Rifle (Prone) Team Matches
Billeting for NRA Instructor School

Sun. 9—Smallbore Rifle (Prone) Matches
NRA Instructor School

Mon. 10—Smallbore Rifle (Prone) Matches
NRA Instructor School

Tues. 11—Smallbore Rifle (Prone) Matches
Randle Team Match
NRA Instructor School
Billeting for NRA Junior School

Wed. 12—Smallbore Rifle (Prone) Matches
Int'l Dewar Team Match
NRA Instructor Junior School

Thur. 13—
Smallbore Rifle (Position) Individual and Team Matches
NRA Instructor Junior School

Fri. 14—
Smallbore Rifle (Position) Individual and Team Matches
NRA Instructor Junior School

HIGH POWER RIFLE

Sun. 16—
Billeting for High Power Competitors
Practice
Competitor Briefing

Mon. 17—
Member's Trophy Match
Scott Trophy Match
Phase 1 and 2 President's Match

Tues. 18—
Coast Artillery Match
Army Cup Match
Phase 3 and 4 President's Match

Wed. 19—
Navy Cup Match
Coast Guard Trophy Match
Phase 1 and 2 Nevada Trophy Match

Thur. 20—
Marine Corps Cup Match
Air Force Cup Match
Phase 3 and 4 Nevada Trophy Match

Fri. 21—
Leech Cup Match
Wimbledon Trophy Match

Sat. 22—
National Trophy Individual Rifle Match

Sun. 23—
NRA Team Matches (Rumbold Trophy, Enlisted Men's Trophy Matches)

Mon. 24—
National Trophy Team Rifle Match