Reliability of CDE Aluminum Electrolytic Capacitors

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Abstract— All design engineers who consider using aluminum electrolytic capacitors want to know how long they will last and how many they can expect to fail. Many engineers do not realize that these are actually two different but related questions. In this paper we define life and reliability in a manner that will hopefully make the distinction clear, and we compare, contrast, and combine life and reliability models in a way that will allow design engineers to predict from their application conditions not only how long before the capacitors begin to wear out, but also what the expected failure rate is during the useful life of the capacitors.

Review of wearout life

Lifetime of aluminum electrolytic capacitors is generally specified as the time under certain conditions of applied DC voltage, ripple current, and ambient conditions (temperature, airflow, heatsinking) at which the capacitor's electrical parameters have drifted out of some specified limits. The ESR is the first to go, and perhaps it has drifted so high that soon the capacitor will either run so hot that it suddenly shorts out or that it ruptures its safety vent and begins to dry out and drift open circuit. This lifetime is also known as Wearout Life, Expected Life, Operating Life (Lop), or Useful Life. The wearout process is rarely driven by evaporation and escape of the electrolyte unless the safety vent is compromised due to high leakage current and pressure buildup. True dryout has an effect similar to wearout but usually occurs in capacitors smaller than 20 cm^3 or in capacitors of older design with poor seal technology.

Our published wearout lifetime model follows from the results of life tests. The effect of temperature may also be derived from the Arrhenius equation. This derivation and a comparison of voltage derating equations are presented in our paper "Deriving Life Multipliers for Aluminum Electrolytic Capacitors." The life model is

$$L = Lb \times Mv \times 2^{((Tm-Tc)/10)} \qquad (1)$$

Here, Lb is the base life at an elevated core temperature Tm. See Table 1 below. Mv is a voltage multiplier, usually equal to unity at the full rated DC voltage, and greater than one at lower DC volt-

Type	Lb	Tm
101C	8000 h	108 °C
300/301	6000 h	108 °C
325	3000 h	128 °C
330	12000 h	108 °C
380L/LX	5000 h	95°C
380LQ	4000 h	95 °C
381L/LX	5000 h	110 °C
381LQ	4000 h	110 °C
381LR	5000 h	115 °C
400C	7500 h	98 °C
401C	8000 h	108 °C
420C	12000 h	103 °C
450C	15000 h	108 °C
4CMC	5000 h	95 °C
500C	7500 h	98 °C
520C	12000 h	103 °C
550C	15000 h	108 °C
DCMC	5000 h	95 °C
MLP up to 250V	12000 h	88 °C
MLP 300V & up	5000 h	88 °C
MLS	4000 h	125 °C

Table 1: CDE Lb and Tm values used in life and reliability models

ages. Cornell Dubilier generally uses

$$Mv = 4.3 - 3.3Va/Vr \quad (2)$$

where Va is the applied DC voltage and Vr is the rated DC voltage. The rated DC voltage is defined such that the peak applied voltage should not exceed this value.

One complication arises because the electrolyte resistance Ro is a function of the actual core temperature Tc, the core temperature is a function of the power loss, and the power loss is a function of Ro. We use an iterative loop in a Java applet to model the core temperature and the life since the ESR varies with temperature and ripple current.

This leads to the question, What do we mean by the "end of life," anyway? How many capacitors are allowed to fail, and what constitutes a failure? For example, one manufacturer might make a naive and untenable statement such as "None of our capacitors will fail before their rated life expires." Other capacitor manufacturers might state that no more than 10% of their capacitors will have failed by the end of their rated life. Still others might add a confidence interval on a similar claim.

It is important that the method of performing the life test and of specifying the allowable degradation be comparable among capacitor manufacturers. We use EIA standard IS-749 which specifies capacitor mounting, maximum airflow, inter-capacitor spacing, end-of-life definitions. It also specifies the lifetime as that time at which 10% of the capacitors have failed due to parametric failure (ESR more than twice the initial limit), and no more than 10% due to open or short circuit.

The CDE model for wearout lifetime assumes that the lifetime is limited by wearout mechanisms (parametric drift), not by random failures which might suddenly occur during the life of the capacitor. Wearout tends to occur on a group of capacitors as an ensemble process, not uniformly distributed over the capacitor life. See Figure 1. The time to wearout of a capacitor bank is fairly insensitive to the number of capacitors in the bank.

We have found that the wearout lifetime distribution typically has a bell shape, and we model this as a normal distribution with a mean of 1.2L and a standard deviation of 0.1L. We are striving to be able to state that the mean is greater than 1.2L with 90% confidence. Stated another way, our goal is to say with 90% confidence that no more than 10% of the capacitors will have failed from wearout before t=L. Due to the expected distribution, this would be about the same as saying with 90% confidence that no more than 1% will have failed from wearout before t=0.9L.

Improvements in capacitor lifetime

If one looks at the history of lifetime ratings of large aluminum electrolytic capacitors, one would find that these have progressed from 1,000 hours at 65 ºC 40 years ago to up to 15,000 hours at 105 ºC today. This is a factor of 240 in the life, while providing more capacitance and higher ripple current handling in the same package. Several factors

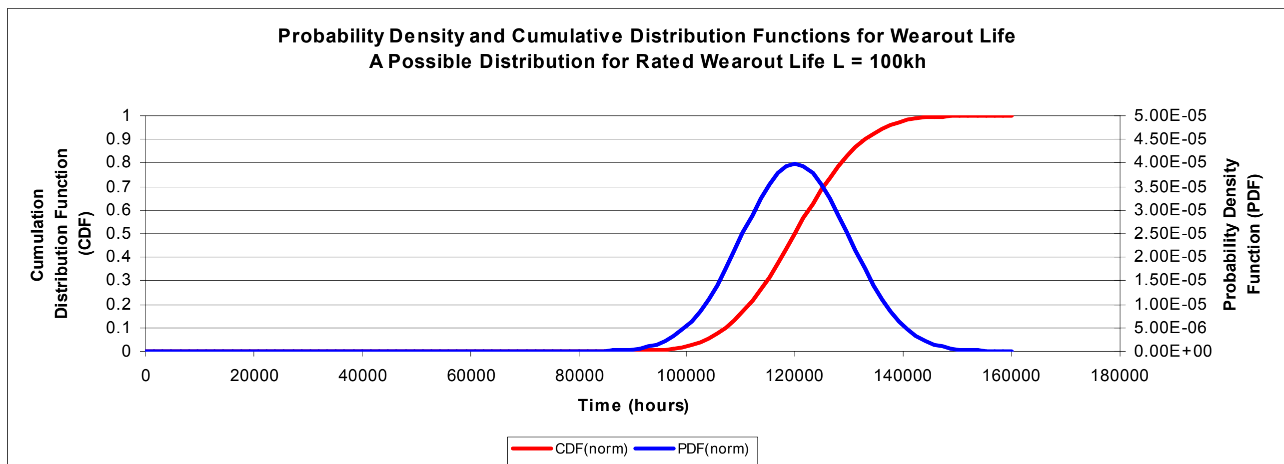

Figure 1: Probability density and cumulative distribution functions for wearout lifetime

have contributed to this accomplishment. These include improvements in purity and stability of all of the capacitor materials and components, especially the electrolyte. Also improved seal technology must receive some credit.

Hence the life model that has been presented is expressed independently of and is unaffected by the capacitance and the voltage rating, and is relatively insensitive to the number of capacitors in a capacitor bank.

Distinguishing lifetime from reliability

Capacitor lifetime is limited by electrochemical degradation that proceeds in a fairly predictable fashion, accelerated by temperature and voltage stress. Along the way, however, random failures are bound to occur if the population is large enough. These are generally unrelated to wearout, and are instead linked to some latent weakness, usually in the paper, foil, or connections. These failures are most often short-circuits, and they may occur suddenly and without warning, although occasionally capacitors may begin to draw excessive leakage current and generate sufficient hydrogen gas pressure to rupture the safety vent then subsequently dry out and fail open circuit.

During the manufacture of the capacitor, rated voltage and temperature are applied in the aging process. Capacitance, leakage current, and ESR are tested on a 100% basis. Usually we employ additional screening techniques to attempt to weed out infant mortalities. Such methods include burn-in, surge voltage test, and hot DC leakage test. At CDE at the present time, these screening methods may weed out an additional 0.1-0.5% of weak capacitors which may fail early in the field. The yield of large high-voltage capacitors has increased from 92% in 1990 to over 98% today. If the burn-in or other high--stress-screening processes are repeated, a small percentage (0.02-0.2%) may be expected to fail each time.

Bank considerations

In the field, it is often the case that several or many capacitors are connected in series and in parallel. Due to adverse effects of fuses (resistance, cost, size, inductance), it is usually only practical to do this in a manner that unfortunately creates the situation that when one capacitor fails short-circuit, the bank and system cease to function. It is the case, therefore, that the failure rate of the bank is approximately equal to the number of capacitors in the bank multiplied by the failure rate of each capacitor.

It is generally true that smaller capacitors (CV rating) are more reliable than larger capacitors. But when a large CV is needed, the highest reliability is usually achieved by using a smaller number of physically larger capacitors. This is true for two reasons. First, the lower number of terminals, welds, connections, potential failure sites of the fewer, large capacitors is an inherent reliability advantage. Second, large capacitors are generally designed, constructed, and screened differently from small capacitors to withstand the application of a large amount of energy connected with miniscule impedance, which is the situation of a capacitor bank. That said, the capacitor design engineer can tailor the design of a large or small capacitor to the application of his capacitor in a large capacitor bank. But the larger capacitor has an inherent advantage in this regard.

Wearout lifetime ratings for large and small capacitors are about the same. So, in summary, a price is paid for high-energy banks with regards to reliability, but the wearout lifetime is essentially energy-inelastic.

Reliability modeling

Even when manufacturing processes and materials and screening methods are state-of-the-art, random failures will still occur in the field. Classical methods to predict reliability of wet aluminum electrolytic capacitors are MIL-HDBK-217 and Bellcore. We are familiar with MIL-HDBK-217 and use it often, but we consider the magnitudes of the FIT rates it predicts at moderate temperatures to be obsolete, as are life models of aluminum electrolytic capacitors from the era when the handbook was compiled and written. Interestingly, the factors of improvement in life and reliability

are about the same.

The MIL Handbook basically starts from the maximum rated conditions then states that the incidence of random failures diminishes by half for each 20% drop in applied DC voltage and for each 20 ºC drop in core temperature.

What we have found is that the predicted failure rate at elevated temperature for small capacitors is somewhat high, usually by a factor of 2-10, but for large capacitors the MIL Handbook is too large by more than a factor of 10. This is in part because of incorrect scaling factors. For example, the failure rate of a capacitor is in actuality related strongly to its energy storage, which is proportional to the square of the rated voltage, and yet in the MIL Handbook there is no reliability factor for the rated voltage, only for the capacitance. This capacitance-only factor ($C^{0.18}$) is not correct because it ignores the effect of voltage rating.

We have also found that reliability approximately doubles every 10 ºC, not every 20 ºC. We do not disagree with the voltage derating factor, but we choose to use a cube-law instead.

What all this means is that for a typical 45-65 ºC application of large electrolytics, the MIL Handbook may indicate a failure rate that is over 100 times larger than for actual CDE capacitors. We believe the reasons are possibly due to excessive conservatism in the military handbook's temperature and voltage factors, along with real improvements in the reliability of our aluminum electrolytic capacitor performance over the past 30 years, associated with some of the same advances in lifetime performance that we have already discussed. One might argue that production yield is related to field reliability, and in that case we note particularly the yield improvements in large capacitors (greater than 300 joules) with voltage ratings of 400VDC and higher.

Reliability data

We rely on customer field failure rate data because from an economic and practical standpoint, it would be impossible to acquire sufficient up-to-date reliability information from our laboratory to assess our present levels of reliability performance in the typical derated conditions that our customers' equipment experiences in the field. This is because billions of unit-hours would be needed, requiring about a million units and a staff of 50 people. Instead, we have had detailed discussions with some of our major customers regarding their applications (bank quantity, temperature, ripple current, applied voltage) and field-return history. In some cases, it was impossible to tell if some of the failures were caused by a capacitor failing, or whether there was another cause, such as an IGBT shorting, causing the capacitor to be destroyed. In those cases, we assumed 50% liability for purposes of reliability estimation.

This effort resulted in our acquiring tens of billions of unit hours data in a variety of applications to give us a good baseline confidence in the 45-75 ºC core temperature range where we most needed data. Most of the field data experienced FIT rates of 0.5-20 from 2"-3" diameter high-voltage capacitors in multiple-capacitor (2-24 caps) banks.

We combined this wealth of field data with observations from our lab regarding the effects of capacitor size, design criteria (basically rated life and temperature), core temperature, and voltage derating. Our combined QA and Engineering labs contain about 60 ovens and 170 power supplies, of which usually 80% or more are in use. We also took into account some published prior studies and the small number of published reliability models for aluminum electrolytic capacitors. We used all of these sources of information together to develop an empirical best-estimate reliability model of our present capacitor reliability.

Scalability issues

What we found with regard to performance of various capacitor types and sizes was that: 1. Premium grade capacitors are more reliable, 2. Equal energy storage results in equal reliability.

The second result above has further implications, viz.: 3. For a given capacitance, lower voltage ratings are more reliable, 4. For a given voltage, lower capacitance is more reliable, 5. For a given CV rating, lower voltage is more reliable. Since capacitor size (volume) is approximately proportional

to $CV^{1.5}$, we can also observe that: 6. For a given size, lower voltage ratings are more reliable.

As already mentioned, we have also observed that 7. For a given bank stored energy, fewer capacitors give higher reliability, 8. The rate of random (pre-wearout) failures of a bank of capacitors is equal to the number of capacitors times the failure rate of a single capacitor.

Lastly, as we have stated, we believe that: 9. Failure rate doubles every 10 ºC hotter the capacitor core is operated, and 10. Failure rate is proportional to the cube of the ratio of the applied to rated voltage.

CDE's reliability model

We have developed a semi-empirical model for the FIT rate λ for pre-wearout random failures that satisfies the 10 criteria outlined in the preceding section.

$$\lambda = 400{,}000 N Va^3 C^{0.5} 2^{(Tc-Tm)/10} / L_B Vr^2 \quad (3)$$

This basically means that for a given capacitor family (type, such as CDE 550C), temperature, and voltage derating, the failure rate is proportional to the product of the rated voltage and the square root of the capacitance. All of these variables have been covered in the life equation except for the nominal capacitance C (farads) and the number of capacitors in the bank, N. The FIT rate λ can then be used as the parameter in the Exponential Distribution, which we will discuss in the next section.

Reliability model for pre-wearout mortality

The manner in which the occurrence of failures are distributed in time determines the appropriate failure distribution. There are many standard distributions, some of which are just special cases of others. The two distributions we will be using are the Exponential Distribution with the λ parameter just discussed for the random failures and the Normal Distribution for wearout failures.

The shape of a probability distributions depends on whether one is looking at the normalized instantaneous failure rate of the initial population, known as the Probability Density Function (PDF) f(t), the total area under which is normalized to equal unity; the proportion of units from the initial population that have failed, which is the integral from time t=0 to the plotted time coordinate of the PDF, in other words the area under the PDF curve, known as the Cumulative Distribution Function (CDF) F(t); or the instantaneous failure rate of the surviving population, known as the Hazard Function (HF) h(t). For any distribution,

$$h(t) \equiv f(t)/[1-F(t)] \quad (4)$$

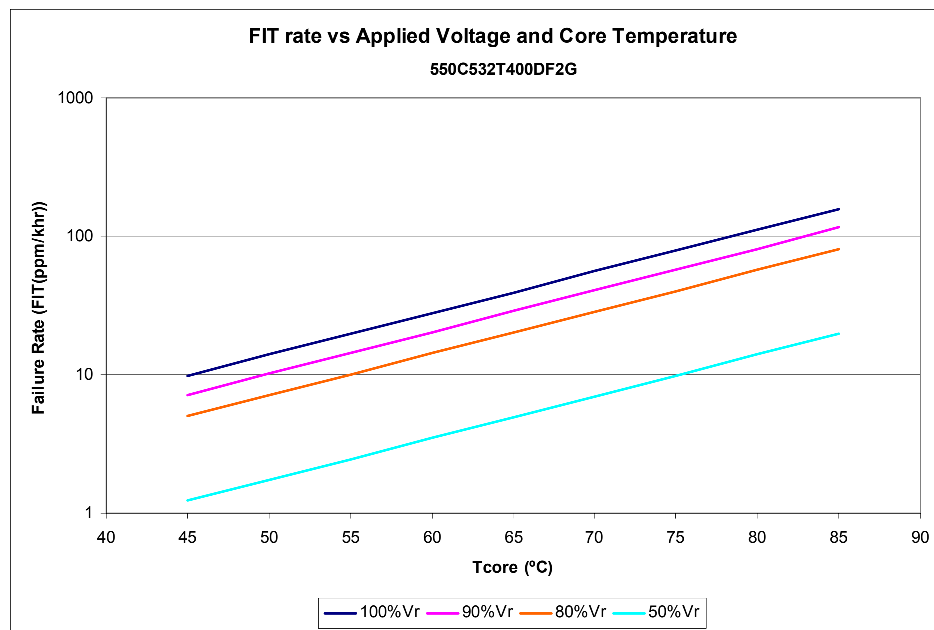

Figure 2: Failure rate model predictions for various applied voltages and core temperatures

The nature of the Exponential Distribution is that its hazard rate has a constant value of λ. This means that the initial failure rate is λ and even after units fail, the failure rate of the remaining units retains the same λ value.

Thus we have for the pre-wearout failures a probability density function

$$f_{exp}(t) = \lambda e^{-\lambda t} \quad (5)$$

and a cumulative distribution function

$$F_{exp}(t) = 1 - e^{-\lambda t} \quad (6)$$

leading to a hazard function

$$h_{exp}(t) = \lambda \quad = \text{FIT Rate (ppm/kh)} \quad (7)$$

Examples of FIT rate and MTBF

The Mean Time Between Failures is the reciprocal of the FIT rate,

$$\text{MTBF} = 1 / \lambda = 1/\text{FIT} \quad \text{(Gh)} \quad (8)$$

and it is straightforward to deduce its units and meaning. See Table 2 for a summary of common units. The units of FIT are ppm/kh, which is failures per billion unit hours, or nanofailures per unit hour. When we reciprocate this unit, we obtain the units of gigahours. The main way to keep the dimensions straight is to use the units if they are given, and if they are given in FIT, remember that the FIT is expressed in ppm/kh so its reciprocal is billions of unit hours.

Reliability and MTBF Unit Conversions			
λ FIT (ppm/kh)	λ f / 10^6	λ % / kh	MTBF = 1 / λ kh
0.1	0.0001	0.00001	10,000,000
1.0	0.001	0.0001	1,000,000
10	0.01	0.001	100,000
100	0.1	0.01	10,000
1,000	1.0	0.1	1,000
10,000	10	1.0	100

Table 2: Summary of common dimensions of failure rate (λ) and MTBF

As an example, suppose we have a capacitor operating at λ=200 FIT in a bank of N=10 capacitors, operating 4,300 hours per year.

(A) What is the MTBF of a single capacitor and of the bank? For the capacitor, from equation (8) it is MTBF = 1/(200FIT) = 1 Gh/200 = 5 million hours. For the bank, we divide the MTBF of a single capacitor by the number of capacitors in the bank to obtain 500 kh.

(B) At what time do we expect to have had α=1% of the capacitors to have failed? This would be approximately 0.01*MTBF = 50 kh. To calculate more precisely we may set equation (6) to $F(t_\alpha)=0.01$ and solve for t_α as follows:

$$t_\alpha = -\ln(1-\alpha)/\lambda = 50{,}252 \text{ h} \quad (9)$$

(C) What percentage of the banks will have failed at the time when 1% of the capacitors have failed? Since there are 10 capacitors per bank and 1% of the capacitors have failed, we might expect there to be 10% bank failures. This is indeed approximately true. The best way to calculate the cumulative proportion of bank failures β precisely is to subtract from unity the probability of survival of all N=10 capacitors when we expect proportion α of the capacitors to have failed. Thus we have a cumulative bank failure value of

$$\beta = 1-(1-\alpha)^N = 1 - (1-.01)^{10} = 9.6\% \quad (10)$$

bank failures at t=50 kh.

(D) We have 1,000 banks in the field operating 4,300 hours per year. How many field returns do we expect per year? The FIT rate of the capacitor is 200 and of the bank is 200N=2,000 FIT, so we expect 2 of every thousand banks to fail every thousand hours. For 4,300 hours per year operating, that's about 9 bank failures per year.

Fortunately, most of CDE capacitors are exhibiting far lower FIT rates than 200 in the field.

Calculating attrited lifetime

In practice, it is apparent that the lifetime of a large group of capacitors may be shorter than prescribed by the wearout life rating L calculated from

equation (1). This situation may occur when life is defined in a manner to describe the time at which a small percentage of large capacitor banks will have failed. For example, if only 1% cumulative capacitor bank failures can be tolerated, and the banks contain 24 large high-voltage capacitors, it is quite possible that this will yield an “attrited life” of smaller duration than given by the wearout life calculation, as the example calculations of a bank of ten capacitors in the preceding section suggested.

The level at which attrited life t_α from equation (9) will be less than the wearout life L from equation (1) depends on the maximum tolerable proportion of capacitor failures α and upon the ratio of L/MTBF. It turns out that by implementing a few approximations and performing some algebra we have an approximate relationship that the capacitor’s attrited life will be less than the wearout life L when the tolerable proportion of random prewearout failures α meets the following condition:

$$\alpha < \frac{NVr\sqrt{C}}{25{,}000(Va/Vr)^6} \qquad (11)$$

For example, when there are N=10 capacitors in a bank, each rated 10 mF at 500 Vdc, and the applied DC voltage is 429 Vdc, we will experience a capacitor bank life that is shorter than the predicted wearout life when we limit the proportion of random bank failures to 5%. Another way of looking at this is to say that we predict the proportion of banks that fail before reaching the predicted wearout life to be about 5%.

Optimally reliable capacitor bank

Notice that the failure rate model of equation (3) penalizes not only for high operating temperatures, but also for high stored energy and for a large number of capacitors in the bank. It is apparent that using a small number of large capacitors is beneficial as are implementing voltage derating and maintaining a small heat rise ΔT.

Continuing the above line of reasoning, perhaps there is an optimum number of capacitors to maintain a moderate temperature from ripple current heating while storing only as much energy as is needed. Adding more capacitors would be less reliable due to the bank size and quantity of capacitors, and using fewer capacitors would result in enough self-heating from the ripple current as to render the bank less reliable.

It turns out that, based on our reliability model, there is indeed an optimum number of capacitors to achieve the lowest bank failure rate, and this can be derived as follows. Neglecting mutual heating, ESR variation with temperature, heat transfer coefficient variation with temperature, variations in thermal resistance with capacitor position within the bank, and the possibility that lowering the bank impedance will change the total ripple current to the bank, the core temperature rise ΔT of the ca-

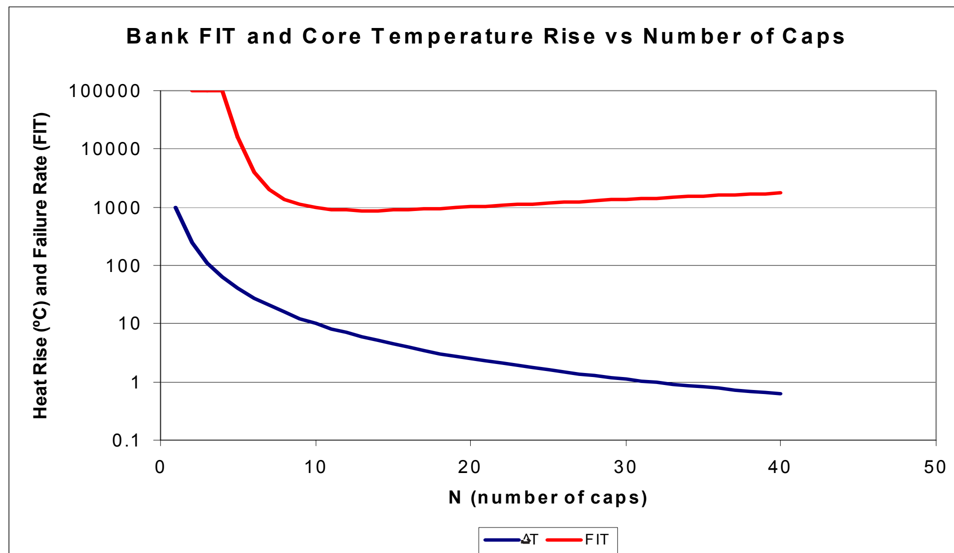

Figure 3: Example of core temperature rise and failure rate versus number of capacitors

pacitors in a bank with a fixed amount of ripple current is inversely proportional to the square of the number of capacitors in the bank. Let us assume that there is a heat rise ΔT_1 for a bank with only one capacitor. In actuality this value may be higher than the melting point of aluminum, but it doesn't invalidate the argument. We have

$$\Delta T = \Delta T_1 / N^2 \qquad (12)$$

and from equation (2)

$$\lambda = \gamma N 2^{\Delta T_1 / N^2} \qquad (13)$$

for constant γ. Setting

$$\partial\lambda/\partial N = 0 \qquad (14)$$

we obtain

$$N = \sqrt{\frac{\Delta T_1 \ln 2}{5}} \qquad (15)$$

so that

$$\Delta T_{OPT} = 5 / \ln 2 = 7.2\ ºC \qquad (16)$$

Thus we have the result that for the optimally reliable capacitor bank for a fixed bank ripple and given capacitor design, *the optimum number of capacitors is that number of capacitors which results in a core temperature rise of 7.2 ºC*. Figure 2 shows the heat rise and FIT rate versus the number of capacitors in the bank for a particular application.

In actuality we find that very few customers design their capacitor banks for such a small heat rise due to cost and size constraints. ΔT = 15 - 20 ºC is actually more typical. But most experienced component engineers would probably admit that in these cases, adding capacitors would not only reduce the operating temperature but also increase the reliability.

Model for wearout life distribution

Obviously not all capacitors fail at precisely the hour they reach the predicted wearout time L. A few will fail before (generally less than 10%) and many after. This brings up the question as to how the wearout failures are distributed in time.

For wearout we assume a Normal Distribution with mean of μ=1.2L and standard deviation of σ=0.1L.

From the mean time μ and the standard deviation σ we compute a normalized coordinate

$$z \equiv \frac{t - \mu}{\sigma} \qquad (17)$$

and from this we calculate the PDF as

$$f_{nor}(t) = \phi_{nor}(z) / \sigma = \frac{e^{-z^2/2}}{\sigma\sqrt{2\pi}} \qquad (18)$$

There is no closed-form formula for the CDF of the Normal Distribution, but most spreadsheets and engineering software have this built in. The cumulative distribution function is of course defined as

$$F_{nor}(t) = \Phi_{nor}(z) = \int_{-\infty}^{z} \phi_{nor}(w)\, dw \qquad (19)$$

and a hazard function

$$h_{nor}(t) = \phi_{nor}(z)/[1- \Phi_{nor}(z)] \qquad (20)$$

When one looks at Figure 1, one may wonder about how the wearout distribution of a population of capacitors will affect the failure rate of large banks of capacitors. After all, if we say that up to 10% of the capacitors may have failed from wearout by time t=L, then we may infer that 1% will have failed at some previous time, and hence the proportion of failures of large capacitor banks will have possibly exceeded 10% before the rated wearout life of the capacitors. Fortunately the Normal Distribution allows us to consider this problem as follows.

To predict the time distribution of wearout failures of banks of N capacitors, we should shift the Normal Distribution curve's mean value μ for a single capacitor slightly to the left by an amount of time $\Delta\mu$ sufficient to equate the value of the cumulative distribution function to be to equal to a value of α/N instead of 0.10. We find

$$\Delta\mu = F_{nor}^{-1}(0.1) - F_{nor}^{-1}(\alpha/N) \quad (21)$$

then adjust the value of μ as

$$\mu' = \mu - \Delta\mu \quad (22)$$

Because the tails of the PDF function diminish so rapidly (as e^{-t^2}), we generally find that this reduces the predicted wearout life between 10-20% and no more.

Melding pre-wearout and wearout distributions

We have discussed the two major components of the capacitor lifetime distribution, and we calculated the crossover point from wearout-limited to attrition-limited lifetime of capacitor banks. Rather than needing to switch back and forth between these two regimes, we asked ourselves if they could be combined into a single, hybrid distribution with well-behaved properties of their probability density, cumulative distribution, and hazard functions. Cornell Dubilier has recently accomplished this task, and the development is presented here.

The approach we took to develop a well-behaved hybrid CDF that we call the Expo-Normal Hybrid (ENH) Distribution was to define

$$F_{enh}(z) \equiv 1 - [1 - \Phi_{nor}(z')]\, e^{-\lambda t} \quad (23)$$

which leads to a well-defined PDF

$$f_{enh}(z) = [\lambda\, (1 - \Phi_{nor}(z')) + \phi_{nor}(z')\,]\, e^{-\lambda t} \quad (24)$$

where we make a distinction between z and z' by recalling that the mean of the normal distribution is adjusted slightly as discussed and presented in equation (22). We may also derive the hazard function

$$h_{enh}(t) = \lambda + \phi_{nor}(z)/[\sigma(1 - \Phi_{nor}(z))] \quad (25)$$

Table 3 summarizes the distributions used and derived in this paper. Figure 4 on the next page shows the successfully melded hybrid ENH distribution.

Summary

We have presented the CDE wearout lifetime model and discussed the difference between wearout lifetime and "attrited lifetime," defined as lifetime that is limited by reaching a certain portion of open-circuit or short-circuit failures. We presented a model for calculating the failure rate λ, also known as the FIT rate, and the MTBF $1/\lambda$. We discussed the life and reliability of single capacitors versus banks of capacitors. We reviewed the circumstances under which a capacitor in an application would have a lifetime limited by reliability versus by wearout. Finally, we developed and presented a combined wearout - reliability model that we call the Expo-Normal Hybrid (ENH) reliability model.

Hopefully these models will help serve as guidelines for successful capacitor applications. Involving CDE in the design and application development process will help ensure that a suitable level of reliability is achieved.

Distribution	PDF f(t)	CDF F(t)	HF h(t)
Exponential	$\lambda e^{-\lambda t}$	$1-e^{-\lambda t}$	λ
Normal	$\phi_{nor}(z)/\sigma$	$\Phi_{nor}(z)$	$\phi_{nor}(z)/[1-\Phi_{nor}(z)]$
Expo-Normal Hybrid	$[\lambda\,(1-\Phi_{nor}(z)) + \phi_{nor}(z)/\sigma\,]\, e^{-\lambda t}$	$1 - [1-\Phi_{nor}(z)]\, e^{-\lambda t}$	$\lambda + \phi_{nor}(z)/[\sigma(1-\Phi_{nor}(z))]$

$$z \equiv \frac{t-\mu}{\sigma} \qquad \phi_{nor}(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}} \qquad \Phi_{nor}(z) = \int_{-\infty}^{z} \phi_{nor}(w)\, dw$$

Table 3: Summary of properties of statistical distributions related to random and wearout failures

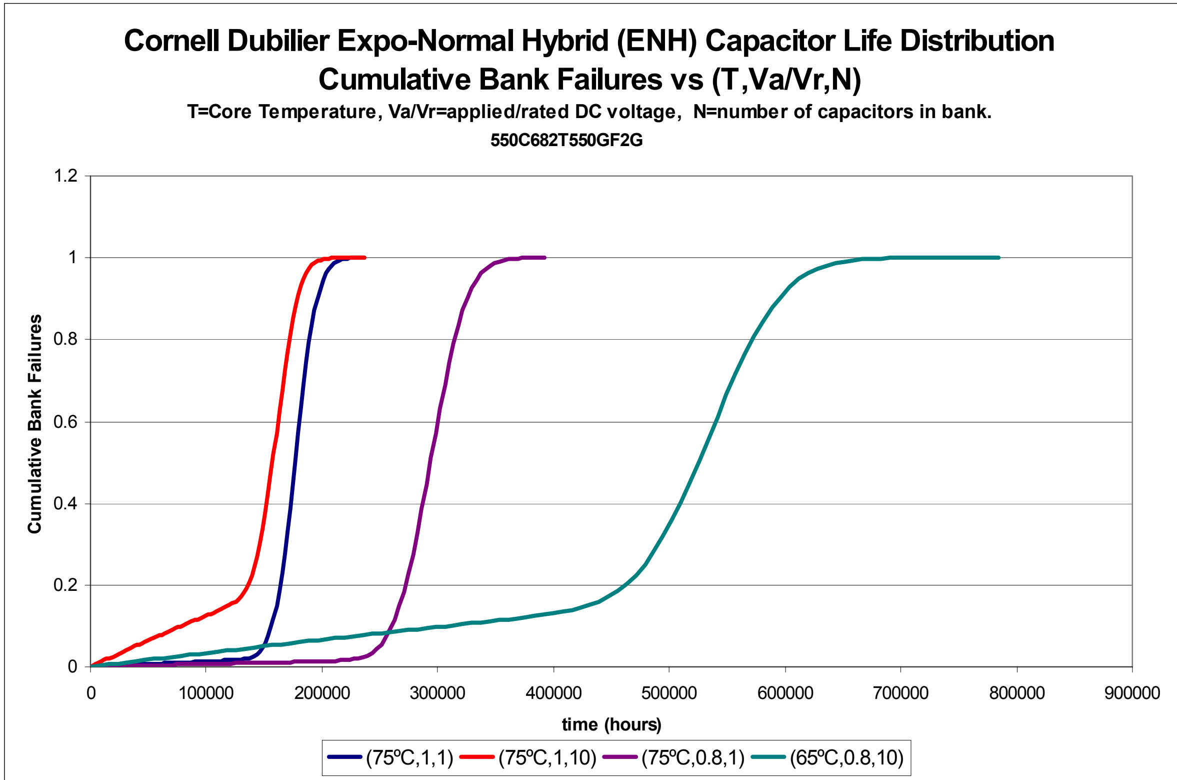

Figure 4: Examples of CDE hybrid life model distribution