Performance-based Pricing Models in Online Advertising: Cost per Click versus Cost per Action

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Abstract

The multibillion-dollar online advertising industry continues to debate whether to use the CPC (cost per click) or CPA (cost per action) model as an industry standard. This article applies the economic framework of incentive contracts to study the trade-offs of these pricing models. In some conditions, the CPA model leads to higher publisher (or advertiser) payoffs than the CPC model. Whether publishers (or advertisers) prefer the CPA model over the CPC model depends on the advertisers' risk aversion, uncertainty in the product market, and the presence of advertisers with low immediate sales ratios. The study findings indicate a conflict of interest between publishers and advertisers in their preferences for these two pricing models. This investigation further considers which pricing model offers greater social welfare.

Key words: Online advertising, cost-per-click through, cost-per-action, pricing model, incentive.

1 Introduction

"CPA, or cost per action, is the Holy Grail for targeted advertising."

- Marissa Mayer, President and CEO, *Yahoo!* (former VP of Search Product and User

Experience, *Google)*

The Internet has emerged as an incredibly important advertising medium. According to a recent report, U.S. advertisers spent $31.7 billion on Internet advertising in 2011, a 22% increase from 2010 (Interactive Advertising Bureau 2012). In the early days of this technology, online advertisers and publishers had simply used a CPM (cost per thousand impressions) model, standard to traditional media advertising, and advertisers paid according to the number of times their advertisement got delivered to consumers. However, the online advertising industry has recently shifted toward performance-based pricing models that tie advertising payments to certain performance metrics. Performance based pricing becomes the most prevalent pricing model since 2006 and approximately 65% of 2011 online advertising revenues were priced on a performance basis (Interactive Advertising Bereau 2012). The first performance-based pricing model to appear used a *cost per click* (CPC) approach, in which advertsiers pay only when viewers click on the advertisment, as invented by Overture (now part of Yahoo!). By 2002, the CPC model had been adopted by both Google and Yahoo! and become the most widely used pricing model in paid search advertising (*The Economist* 2006).

However, the CPC model's dominance currently is being challenged by a new performance-based pricing model that relies on CPA (*cost per action*) and calculates advertising payments according to advertiser-specified "actions," such as email sign-ups, downloads, sales leads, or purchases.[1] In 2006, Google attracted media attention when it started to test a CPA model (e.g., Gonsalves 2006; Helft 2007). As the quote that opened this paper reveals, Google regards CPA as the "Holy Grail" of targeted advertising (Gardiner 2007), and many online advertising companies have adopted it, including not only Google (through its Prouduct Listing Ads) and eBay but also long-time proponents of this model, such as ValueClick and Snap.com. Amazon has also been using the CPA model in its affiliate program (Libai et al. 2003)

The emergence of the CPA model has sparked controversy and debate within the online advertising industry (Cumbrowski 2007; Ezzy 2006; Guanaccia 2006). On one side of the debate,

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[1]Details on how the advertiser defines an "action" and how Google tracks and reports the number of "actions" can be found in Laycock (2007).

advertisers tend to prefer the CPA model, because the CPC model gives publishers little or no incentive to improve the quality of the clicks delivered (they only seek to drive a high volume of clicks to advertisers). Thus, advertisers worry that consumers who click are not actually interested in the products being sold — a problem exacerbated by the potential for fraudulent clicks by third parties that aim to drive up advertisers' costs (Wilbur and Zhu 2009). Because the CPA model ties advertising payments to sales of the advertisers' products, publishers must exert some effort to improve the quality of clicks. The CPA model also helps reduce the risk for advertisers, because if an Internet advertisement fails to produce sales, advertisers do not suffer any further financial loss. Therefore, the CPA model is considered to be a preferred model by advertisers, because it shifts the risk almost entirely to publishers and it allows advertisers to easily manage their campaigns' return on investment.

On the other side of the debate stand web publishers, who often prefer the CPC model. They worry that the CPA model gives advertisers minimal incentives to convert clicks into sales, causing a typical moral hazard problem. If an advertising campaign fails and generates no response, the web publisher receives no payment for displaying the advertisement on their web page. Publishers should be responsible for influencing the consumer, but not closing a deal. Furthermore, some advertisers may take advantage of the CPA arrangement to run a multitude of advertisements that only raise brand awareness, rather than generate immediate sales.[2]

This paper sheds light on this debate over the CPC versus CPA pricing mechanisms. It helps academic researchers and practitioners understand the consequences of adopting one pricing model over the other, as well as the conditions in which each pricing model might perform best. In what circumstances do firms (advertisers or publishers) prefer a particular pricing scheme? Does a CPA model lead to higher purchase rates compared with a CPC model? Does one model produce greater social welfare (defined as the sum of payoffs to all parties) in the online advertising industry?

To the best of our knowledge, this study is the first to investigate the trade-offs between CPC and CPA models. We apply an economic model of incentive contracts to this problem and offer

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[2]One often-mentioned potential problem with the CPA model is the reliability of the technology that tracks actions generated from a CPA campaign. The CPA model may not be successful if publishers can only rely on advertisers' "truthful" reporting of the actions generated from CPA campaigns. However, recent developments in the tracking technology have enabled publishers and advertisers to overcome this issue. For instance, the publisher often requires the advertiser to install certain program which tracks the actual actions at the advertiser's website and reports them to the publisher (Cumbrowski 2007). The privacy issue involved and the incentive for the advertiser to share its information with the publisher are interesting area to further research (see Goldfarb and Tucker 2011a for a recent work in privacy issue), but they are beyond the scope this study.

recommendations regarding when advertisers and publishers should adopt either the CPA or the CPC model. Specifically, we solve a game in which different types of advertisers compete in a second-price auction for the right to display their advertisements on one publisher's Web site. First, we assume that the contract goes to the advertiser with the higher bid in the auction, but later we look at more realistic scenario where the publisher chooses the advertiser based on its expected revenue (not necessarily the highest bidder). Then, the winning advertiser and the publisher decide on the levels of their non-contractible efforts to improve the effectiveness of the advertisement. Finally, both parties realize payoffs. We solve the game with both CPC and CPA pricing models, then compare the results and derive several pertinent propositions.

Our model therefore considers the incentive problem for both publishers and advertisers — an issue largely ignored by existing literature on online advertising. We posit that online advertisers and publishers can exert effort to improve the effectiveness of advertising campaigns. For example, advertisers can invest in user interface, easy of navigation, search, and customized landing pages for different keywords; publishers can also invest in user interface with advertising, recommendation and develop better targeting technologies. However, these costly efforts are not contractible and advertisers and publishers would not invest enough unless they have proper incentives to do so.

Moreover, we incorporate two important and realistic features of online advertising that have rarely modeled in prior research. First, we model the effect of delayed response, which is of central concern for both advertisers and publishers but most extant literaturea has largely overlooked. Delayed response occurs when a consumer who sees the advertiser's offer makes no purchase at that moment but later comes back to the advertiser directly and purchases a product. Such delayed responses can be significant for products that have high value or products that are difficult to be evaluated, such as cars and electronics (Hu 2004). Briggs (2003) reports that an advertiser gets 80 percent of its conversions from these returning consumers. Second, we also allow the possibility of existence of different types of advertisers whose primary goals of advertising campaigns differ (Fulgoni 2009). Some focus on generating a direct and immediate action such as consumer purchase (direct selling advertiser) while others primarly focus on raising awareness about its brand (branding advertiser). By modeling delayed responses and the existence of difference types of advertisers, our analyses reveal that the CPA pricing model increases the possibility that certain types of advertisers win the auction, leading to a potential adverse selection problem.

At first glance, publishers should always prefer the CPC model, and advertisers should al-

ways prefer the CPA model. But our results enhance current industry understanding of these two performance-based pricing models and show that in certain conditions, the CPA model leads to higher publisher payoffs than the CPC model. We posit that the CPA model shifts risk away from advertisers, which may cause them to bid more for advertising space. This effect grows even stronger when advertisers are more risk averse and when uncertainty in the product market is higher. In parallel, we identify conditions in which the CPC model produces higher advertiser payoffs than the CPA model. The CPA model increases the probability that a branding advertiser will win the auction, which again creates an adverse selection problem that reduces advertiser payoffs.

We also compare the conditions in which publishers prefer the CPA model with conditions in which advertisers prefer it and thereby identify scenarios in which both parties' payoffs are higher (or lower) if they use the CPA rather than the CPC model. In other scenarios, publisher payoffs are higher but advertiser payoffs suffer, such that there exists a conflict of interest between publishers and advertisers. Finally, we study which pricing model leads to greater social welfare and thus the conditions in which the CPA model is preferable.

The remainder of this article proceeds as follows. In Section 2, we review related literature, and then in Section 3, we introduce our basic model. We characterize the equilibrium outcomes for the CPC and CPA pricing schemes in Section 4. In Section 5, we identify the conditions for choosing one pricing scheme over the other. In Section 6, we extend our basic model to capture the more realistic situation where the publisher can have a prior information about the advertiser's type and chooses the advertiser based on its expected revenue. Finally, we conclude in Section 6 with a summary of our findings and some broader implications.

2   Literature Review

This research contributes to a growing literature on online advertising. Motivated by the real-world models employed by Google and Yahoo!, several analytical studies in economics and marketing have focused on the design of auction mechanisms and advertisers' bidding strategies. Edelman et al. (2007) study the generalized second-price auction mechanisms used in sponsored search advertising and derive many of its properties; in a separate study, Varian (2007) obtains similar results. They all find the similar equilibrium result that the general auction mechanism employed by Google and Yahoo does not have a dominant bidding strategy, but can be reduced to a simple second-price auction under certain conditions. More recently, Athey and Ellison (2011) examine

advertisers' bidding strategies, consumers' search strategies, and the division of surplus among consumers, search engines, and advertisers. They find that paid search advertising can provide information about sellers' products to consumers and, thus, provide a welfare benefit by making consumer search more efficient. Agarwal et al. (2010) focus on the new CPA pricing mechanism and study how the CPA mechanism can bias the advertisers' bidding strategies.

There are also several papers which focus mainly on the features of sponsored search advertising. Katona and Sarvary (2010) model the bidding behavior of advertisers and paid ad placements and find an interaction between non-sponsored (or "organic") search results and sponsored search advertising; differences in click-through rates across advertisers can also influence advertisers' bidding behaviors. Wilbur and Zhu (2009) investigate how click fraud influences search engines' revenues in a second-price auction. Feng et al. (2011) compare different mechanisms of ranking advertisers and their bids, whereas Weber and Zheng (2007) build a model of search intermediaries in a vertically differentiated product market and derive advertisers' bids and consumer surplus. Liu et al. (2009) compare the different auction mechanisms used by Google and Yahoo!; Feng and Xie (2011) investigate how impression- or performance-based online advertising may signal product quality in search advertising markets. While extant research on online advertising mostly take the pricing mechanism as given, we investigate the choice of pricing scheme and its implications on equilibrium behaviors of advertisers and publishers.

Empirical research on online advertising focuses primarily on banner advertising. For example, Sherman and Deighton (2001) use Web site–level data to suggest optimal placements of advertisements. Chatterjee et al. (2003) examine how click-through rates may be influenced by exposure to banner advertisements, and Manchanda et al. (2006) consider the effect of banner advertising on actual purchasing patterns. These findings suggest that the number of exposures, Web sites, and pages all have positive impacts on consumers' purchasing probabilities. More recent empirical studies investigate keyword searches in the context of paid search advertising (Ghose and Yang 2009; Goldfarb and Tucker 2011b; Rutz and Bucklin 2011; Yao and Mela 2011).

Our model follows traditional principal–agent models that recognize moral hazard (Holmstrom 1979; Holmstrom and Milgrom 1987, 1991). Principal–agent models appear in studies of incentive contracts in various contexts, including retail franchising (e.g., Lafontaine and Slade 1996), executive compensation (for a review, see Murphy 1999), sales force compensation (e.g., Banker et al. 1996), and customer satisfaction incentives (e.g., Hauser et al. 1994). Our study is one of the first to apply

it, together with the economic framework of incentive contracts, to online advertising. We view the CPC or CPA contract between the publisher and the advertiser as a contract that allocates market risks between the parties and that may or may not provide each party with appropriate incentives to make adequate, non-contractible efforts. This new view of the contract between the publisher and the advertiser enables us to develop innovative insights that have important implications for the online advertising industry.

Finally, we note that our paper closely relates to three recent studies of pricing models in online advertising. Hu (2004) is the first paper, which studies online advertising pricing schemes as an optimal contract desing problem, but he only compares traditional CPM and CPC models in a monopolistic advertiser–publisher relationship. We extend his argument to the issue of performance-advertising mechanisms (CPC and CPA) under competition. Zhu and Wilbur (2011) study advertisers' bidding strategies in a hybrid auction, in which advertisers can choose a CPM or CPC bid, and derive the unique properties of the mechanism. They find that publishes should offer multiple bid types to advertisers. Liu and Viswanathan (2010) identify conditions under which publishers prefer the CPM model over performance-based models (CPC or CPA). Unlike these studies, we do not study solely the CPM model. Instead, we focus on the incentive problems in performance-based advertising schemes, and therefore, we analyze the trade-offs between CPA and CPC, with a particular focus on the incentive issues (adverse selection and moral hazard) arising under different pricing schemes.

3 Model

3.1 The online advertisers and online publishers

We model the advertising contract between multiple online advertisers and an online publisher. Each advertiser sells a product to consumers through the online channel. To boost its sales or brand awareness, an advertiser can launch an online advertising campaign in third party's website or blog (which we call a publisher). The advertiser designs an advertisement and contracts with a publisher, tasking the publisher with delivering the advertiser's advertisement to consumers who visit the publisher's website or blog. Every time the advertisement is delivered to a consumer's browser, the consumer may choose to ignore or click on the advertisement. If he or she clicks, the consumer goes to the advertiser's online store, after which this consumer may make a purchase or

leave without purchasing. We define the purchase rate (θ) as the ratio of purchases to clicks.

In the advertising industry, a popular dichotomy differentiates direct response advertising from brand advertising: The former focuses on strategies to drive a particular action, such as purchase, whereas the latter aims to raise awareness and build brand equity (Fulgoni 2009). We assume an advertiser can either be a *direct selling* or a *branding* advertiser. A direct selling advertiser (which we call type D) has a primary goal of generating a direct and immediate action by consumers, such as sale, sign-up, or download, through its advertising campaign. A branding advertiser (type B) instead aims primarily to raise awareness about its brand or build brand equity, which leads to higher future indirect and delayed responses. Of course, the discrete classification of all advertisers into direct selling versus branding advertisers is difficult; most advertising campaigns serve both objectives in practice.[3] Therefore, the classification is based on *relative* terms and the key difference between type D and type B advertisers is whether their advertising goal is relatively to generate a large proportion of direct and immediate sales or a large proportion of delayed responses in the long run.

For simplicity, we consider the problem of two advertisers competing for one advertisement slot on the publisher's Web site using a second-price sealed bid auction, consistent with Agarwal et al. (2010).[4] The advertisers are heterogeneous in the profits they obtain from each sale (m_i) and the ratio of immediate to total sales (ρ_i). We assume that each advertiser's profit margin m_i is randomly drawn from a standard uniform distribution on $[0,1]$. Also, one advertiser is a direct selling, whereas the other is a branding advertiser: $i \in \{B, D\}$. We assume that the direct selling, type D advertiser attains an immediate sales ratio of α (i.e., $\rho_D = \alpha$), but that the branding, type B advertiser experiences an immediate sales ratio of β (i.e., $\rho_B = \beta$), where $0 < \beta \le \alpha < 1$.[5] In the special case in which both advertisers are the same type, we can easily set $\rho_D = \rho_B = \alpha = \beta$.

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[3]To capture our intuition in the simplest manner, we classify them according to their primary goals, such that our simple classification of types D and B approximates what is really a continuum.

[4]We use a stylized model of advertising auction in which we assume that there is only one slot with two advertisers using a second-price auction. This preserves the main incentives of real world CPA and CPC auction while simplifying the analysis significantly (Agarwal et al. 2010; Athey and Levin 2001).

[5]We can easily relax this assumption of two types of advertisers by allowing a more general case, where each advertiser's immediate sales ratio ρ is randomly drawn from a distribution on $F[0,1]$. In this case, we can think of the advertiser whose immediate sales ratio is higher as type D and the other as type B. In this case, α is the expected immediate sales ratio of advertiser D; $\alpha = E[\rho_D] = E[max\{\rho_i, \rho_{-i}\}] = 2\int_0^1 \rho F(\rho)f(\rho)d\rho$, and β is the expected immediate sales ratio of advertiser B; $\beta = E[\rho_B] = E[min\{\rho_i, \rho_{-i}\}] = 2\int_0^1 \rho(1 - F(\rho))f(\rho)d\rho$, where $0 < \beta \le \alpha < 1$. For example, under uniform distribution, $\alpha = 2\int_0^1 \rho F(\rho)f(\rho)d\rho = \frac{2}{3}$, and $\beta = 2\int_0^1 \rho(1 - F(\rho))f(\rho)d\rho = \frac{1}{3}$.

3.2 Incremental efforts

Advertisers can greatly influence the purchase rate once those online prospects land through an online advertising campaign. First, the advertiser can affect the purchase rate by improving its online transaction process or managing its Web server capacity and bandwidth better. A complicated transaction process or a slow Web server increases consumer inconvenience for check-out and thus reduces immediate purchase rate (Mookerjee 2012, Tillett 2001).[6] Second, the advertiser can improve its purchase rate by having a professional and trustworthy website layout, design or using proper wording (Puscher 2009). Third, setting up customized landing pages and closely linking products to keywords can greatly increase purchase rates (Mitchell 2007). Finally, the advertiser can also use recommendation engines, advanced search and navigation tools to improve stickiness and purchase rates (*BusinessWire* 2007). These efforts are costly though, often requiring professional staff or advertising agencies to manage the website. While some of those features can be specified in contract ex ante, most of factors are unobservable to the publisher and are hard to be specified in contract (or at least non-verifiable).[7] We focus on these non-contractible efforts that advertiser i can make and call them e_i.

Similarly, the publisher can work to improve the purchase rate resulting from an advertising campaign. For example, whether the publisher closely associates the advertisement with its surrounding content and chooses appropriate wording in its pitch to consumers affects the eventual purchase rate. More importantly, the publisher can provide the advertisement to consumers who are most likely to be interested in it by using a targeting technology based on superior knowledge of its consumers' demographics, geographical location, expressed interests, and other information (Maislin 2001; Needham 1998; Rutz and Bucklin 2011). These efforts, which are rarely specified in the contract between the advertiser and the publisher, represent our main focus, which we refer to as e_p.

Formally, we assume that the purchase rate θ is a linear function of the advertisers' efforts e_i and the publisher's effort e_p plus random noise ε, which is distributed normally with a mean of 0 and a variance of $\sigma^2 > 0$. The variance of ε also can be interpreted as sales randomness or risk in

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[6] According to the market analysis by TRAC Research, on average \$4,100 of revenues are lost due to website slowdowns as more consumers are becoming increasingly intolerant to slowdowns of web server (Choney 2011).

[7] For example, it is infeasible to contract about the utilization of advertiser's web server ex ante: there can always be unforeseen contingency which prevents the full utilization of the server or causes complete breakdown. It is practically impossible for the publisher to verify in a court that the slowdown of online transaction (which lowers the purchase rate) is due to strategic sabotage of the advertiser.

the product market. Also, we impose the condition that the purchase rate cannot be lower than 0. Thus,

$$\theta = Max\left[0,\ e_i + e_p + \varepsilon\right],\ \text{where } \varepsilon \sim N\left(0, \sigma^2\right). \tag{1}$$

Non-contractible efforts are costly to advertisers and the publisher and become more costly as the total effort level increases. We model the advertisers' cost for incremental efforts with a quadratic cost function, as used widely in research in incentive contracts (e.g., Holmstrom and Milgrom 1987, Hauser et al. 1994, Lafontaine and Slade 1996). Formally, the cost of advertiser i's efforts e_i is $C(e_i) = \frac{e_i^2}{2}$. Similarly, the cost of the publisher's efforts is $C(e_p) = \frac{e_p^2}{2}$.

3.3 Payoffs

We use t to denote the monetary transfer from the winning advertiser to the publisher. The publisher's payoff from each click is simply the monetary transfer minus the cost of its efforts,

$$y_p = t - \frac{e_p^2}{2}. \tag{2}$$

Advertiser i obtains a net profit of 0 if it does not win the auction. If it wins, it earns a net profit from each click equal to its profit minus the monetary transfer minus the cost of its efforts,

$$\pi_i = m_i\left(\rho_i + \gamma(1 - \rho_i)\right)\theta - t - \frac{e_i^2}{2}, \tag{3}$$

where γ is the time discount rate, which we assume $\gamma = 1$ for simplicity. Because of the randomness of sales in the product market (ε), we also incorporate risk aversion in the model. We assume that the advertisers have exponential utility functions with a CARA (constant absolute risk aversion) parameter of r, that is, $u\left(\pi_i\right) = 1 - exp\left(-r\pi_i\right)$. Thus, the advertisers' payoff can be written as the certainty equivalence of their net profit ($CE\left(\pi\right)$), which is,

$$y_i = CE(\pi_i) = E(\pi_i) - r\frac{Var(\pi_i)}{2}. \tag{4}$$

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3.4 Timeline

We consider a game in which two advertisers bid on one slot through a second-price sealed bid auction. First, advertisers submit their bids. Second, the publisher awards the slot (and the contract) to the advertiser with the highest bid, at the price of the second highest bid (i.e., the other advertiser's bid). Later in our model extension, we relax this assumption and look at more realistic scenario where the publisher chooses the advertiser based on its expected revenue (not necessarily the highest bidder). By assuming a second-price auction, we can focus our analysis on the incentive problems of both the publisher and advertisers. In a second-price auction, a weakly dominant strategy is for advertisers to bid their true value (Vickrey 1961), so we refer to this outcome as the standard result in our analysis.[8]

Third, the advertiser that wins the slot decides the level of its incremental efforts e_i, and the publisher decides the level of its incremental effort e_p. Finally, advertisers and the publisher observe the actual purchase rate and realize their separate payoffs. We summarize the timeline of the game in Figure 1.

*** Figure 1 ***

4 Analysis

We characterize the equilibrium outcomes under two performance-based pricing models: the cost per click (CPC) and the cost per action (CPA). We then compare and investigate the trade-offs between the two models and identify the conditions in which firms (publisher or advertisers) prefer one pricing model over the other, as well as the conditions in which one pricing model leads to greater social welfare.

4.1 Cost per click pricing model

In the CPC pricing model, the monetary transfer between the publisher and the winning advertiser is a flat fee of t_c for each click. Each advertiser bids on the amount of a payment t_c per click, and this bid, $b(m_i, \rho_i)$, is a function of advertiser i's profit margin m_i and the immediate sales ratio ρ_i. As long as the payment t_c is greater than the reservation value, $t_c > u_0$, the publisher accepts the bid.

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[8]However, under multiple auction case (i.e., advertising slots), a second-price auction will then diverge from the Vickery-Clarke-Groves mechanism, and true-valuation bidding is generally not an equilibrium outcome (Edelman et al. 2007). We discuss this important limitation in conclusion.

Without loss of generality, we assume that $u_0 = 0$, which implies that the publisher always accepts a non-negative bid, $b(m_i, \rho_i) > 0$, which results in a contract between the two parties. Hereafter, we assume that $r\sigma^2 < 1$, which guarantees non-negative bids from both advertisers in any pricing mechanism.

By substituting Equations (1) and (3) into Equation (4), we can obtain advertiser i's payoff if it wins the auction, which is

$$y_i = m_i(e_i + e_p) - t_c - \frac{e_i^2}{2} - \frac{r\sigma^2 m_i^2}{2}. \tag{5}$$

The publisher's payoff is simply

$$y_p = t_c - \frac{e_p^2}{2}. \tag{6}$$

In Lemma 1, we charcterize (1) advertisers' bidding strategy, and (2) the incentives for the publisher and the winning advertiser to undertake incremental efforts under the CPC model.

Lemma 1. *In the CPC pricing model,*

1. *Advertiser i (with marginal profit m_i and immediate sales ratio ρ_i) bids $b^{CPC}(m_i, \rho_i) = \frac{1}{2}\left(1 - r\sigma^2\right) m_i^2$. Moreover, the probability that an advertiser who has higher immediate sales ratio ρ_i (i.e., type B) wins the auction is $E\left[Pr(w = B)\right]^{CPC} = \frac{1}{2}$.*

2. *If advertiser i wins the auction, it exerts the following effort: $e_i^{CPC} = m_i$. The publisher also exerts the following effort: $e_p^{CPC} = 0$.*

We provide proofs of all the lemmas and propositions in the Appendix. Lemma 1 suggests that the marginal profit earned by the advertiser has a positive effect on its submitted bid. However, advertisers' bids are independent of their immediate sales ratio; both advertisers have the same bidding strategy. As a result, the winning advertiser is simply the one with greater marginal profit. Therefore, we can conclude that the CPC model provides a level playing field for both advertisers, and both of them have an equal probability of winning the auction.

The advertiser's bid is negatively influenced by its risk aversion parameter (r) and the level of market risk (σ^2), in line with the following intuition. The advertiser assumes all the market risk under the CPC model: the winning advertiser must pay the publisher for each and every click, even when those clicks fail to lead to any purchases of the winning advertiser's product. Thus, an

advertiser with a higher risk aversion parameter and more market risk will obtain a lower payoff when it wins the auction. Naturally, this advertiser submits a lower bid.

Lemma 1 further shows that a greater marginal profit per purchase (m_i) induces the advertiser to undertake more incremental efforts. The optimal level of the advertiser's effort is independent of the per click payment (t_c), because when a consumer clicks and enters the advertiser's Web site, the cost of that click becomes a sunk cost.

On the other hand, the publisher has no incentive to make incremental efforts under the CPC model, because the publisher's payoff is not tied to purchases. The lack of publisher incentives to improve the purchase rate represents a typical moral hazard problem in contract theory. We discuss how this moral hazard problem affects the expected payoff of both advertisers and the publisher subsequently.

Finally, we can explicitly calculate the payoffs to the advertisers and the publisher, respectively in the CPC pricing model (please see the Appendix for the detailed derivation).

$$E(y_i) = \frac{1}{6}\left(1 - r\sigma^2\right), \tag{7}$$
$$E(y_p) = \frac{1}{12}\left(1 - r\sigma^2\right).$$

In summary, with the CPC model, the advertiser assumes all the risk in the product market. A higher level of market risk or larger risk aversion parameter then directly lowers the advertiser's payoff; it also indirectly lowers the publisher's payoff because the auction bids made by advertisers are lower (Lemma 1-(1)). Although an advertiser bears all the risk, this risk affects only its bidding behavior, not its effort level (Lemma 1-(2)).

4.2   Cost per action pricing model

In the CPA pricing model, the monetary transfer between the publisher and the advertiser i that wins the auction is $\rho_i \cdot \theta \cdot t_a$, where ρ_i is the winning advertiser's immediate purchase ratio, θ is the purchase rate, and t_a is the per-action (i.e., per purchase) payment. By substituting Equations (1) and (3) into Equation (4), we can obtain advertiser i's payoff if it wins the auction;

$$y_i^{CPA} = (m_i - \rho_i \cdot t_a)(e_i + e_p) - \frac{e_i^2}{2} - \frac{r\sigma^2}{2}(m_i - \rho_i \cdot t_a)^2. \tag{8}$$

The publisher's payoff is simply

$$y_p^{CPA} = \rho_i \cdot t_a \left(e_i + e_p \right) - \frac{e_p^2}{2}. \tag{9}$$

Lemma 2. *In the CPA pricing model,*

1. *Advertiser i (with marginal profit m_i and immediate sales ratio ρ_i) bids $b^{CPA}\left(m_i, \rho_i\right) = \frac{m_i}{\rho_i}$. Moreover, the probability that an advertiser who has higher immediate sales ratio ρ_i (i.e., type B) wins the auction is $E\left[Pr(w = B)\right]^{CPA} = 1 - \frac{\beta}{2\alpha}$.*

2. *If advertiser i wins the auction, it exerts the following effort: $e_i^{CPA} = m_i - \rho_i \cdot t_a$. The publisher also exerts the following effort: $e_p^{CPA} = E(\rho_i) \cdot t_a$.*

Lemma 2 suggests that the bid submitted by advertiser i in the CPA model is equal to its marginal profit (m_i) divided by its immediate purchase ratio (ρ_i). Thus, in contrast with the CPC case, the advertiser with lower immediate ratio (i.e., advertiser B) tends to submit a higher bid, with a higher probability of winning the auction, than advertiser D in the CPA model.

In stark contrast with the CPC model, the advertiser's optimal bidding behavior with a CPA contract does not depend on the risk aversion parameter (r) or the level of market risk (σ^2). Because the advertiser pays only if the consumer purchases a product, payment occurs after the uncertainty in the purchase market is realized, which means all risk arising from purchase uncertainty becomes fully insured. In this sense, the advertiser secures against the unnecessary advertising costs associated with unexpectedly low product sales by transferring the risk to the publisher, which gets paid only when the product sells.

Also, Lemma 2 shows that the winning advertiser's incentive to exert incremental efforts depends on its marginal profit per purchase and the payment to the publisher. A higher marginal profit (m_i), lower immediate purchase ratio (ρ_i), and lower per purchase payment (t_a) all induce the advertiser to undertake greater incremental efforts, because the winning advertiser obtains a profit from each purchase, whether that purchase is immediate or delayed, but it pays the publisher only for each immediate purchase. Therefore, in the CPA model, the branding advertiser (B) with a lower immediate purchase ratio experiences a greater incentive to exert incremental efforts than does advertiser D, who has a higher immediate purchase ratio.

Also unlike the CPC case, the publisher's incentives to exert incremental efforts in the CPA model depend on the per purchase payment (t_a) and the publisher's belief about the winning

advertiser's immediate purchase ratio ($E(\rho_i)$).[9] The publisher does not directly observe the wining advertiser's immediate purchase ratio and therefore must form a belief on the basis of the bids submitted by advertisers. The publisher then chooses its efforts according to this belief.

Finally, we can calculate the payoffs to advertisers and the publisher, respectively (see the Appendix for the derivation).

$$
\begin{aligned}
E(y_i) &= \frac{1}{2}\left(1 - r\sigma^2\right) \cdot \left(\frac{\beta}{3\alpha} - \frac{\beta^2}{3\alpha^2} + \frac{\beta^3}{6\alpha^3}\right) + \frac{(\alpha + \beta)^2 \beta}{48\alpha^3}, \\
E(y_p) &= \frac{15\beta}{48\alpha} - \frac{\beta^2}{8\alpha^2} - \frac{\beta^3}{48\alpha^3}.
\end{aligned}
\tag{10}
$$

Because the CPA models shifts the market risk from the advertiser to the publisher, the advertiser's bidding behavior is not affected by market risk or risk aversion (Lemma 2-(1)). In turn, the publisher's expected payoff, which is determined by the advertiser's bid, is independent of these factors as well. The publisher's payoff depends only on the advertiser's immediate sales ratio, because the total payment is tied solely to immediate sales.

5 Comparing the CPC and CPA pricing models

We can derive several interesting results by studying the trade-offs between the two advertising pricing schemes. We also consider how various factors influence preferences for one pricing scheme over the other and then investigate the social welfare that results from each advertising pricing scheme to determine when the publisher's preference aligns with social welfare.

5.1 Adverse selection problem in the CPA pricing model

First, we investigate the issue which types of advertisers would benefit from different pricing schemes, and what the implications are for the publisher's profit.

Proposition 1. *The expected probability that the advertiser with a lower immediate purchase rate (type B) wins the auction is greater in the CPA than the CPC pricing model. Furthermore, the expected marginal profit of the winning advertiser is lower in the CPA than the CPC pricing model:* $E(m_i)^{CPA} \leq E(m_i)^{CPC}$.

---

[9]The exact expression for e_p^{CPA} is $e_p^{CPA} = \frac{2\beta}{3\alpha} - \frac{\beta^2}{3\alpha^2}$ (see the proof of Proposition 2 in the Appendix for the detail derivation).

The intuition behind this proposition is as follows: In the CPC model, both types of advertisers have an equal probability of winning the auction, and the winner is the advertiser with a higher marginal profit (m_i). In contrast, the CPA pricing model gives the advertiser B a competitive advantage, because it has a smaller immediate purchase ratio ($\beta \leq \alpha$), as is reflected in its bidding function $b(m_i, \rho_i) = \frac{m_i}{\rho_i}$. Therefore, in the first part of Proposition 1, the CPA pricing model increases the probability that the branding advertiser wins the auction. Furthermore, because the CPA model gives the advertiser B a competitive advantage, it potentially wins the auction even with a significantly smaller profit margin than the advertiser D (i.e., $m_B < m_D$). This situation leads to a decline in the expected marginal profit of the winning advertiser in the CPA model. Hence, Proposition 1 reveals the CPA pricing model's adverse selection problem. The branding, advertiser B is more likely to win the auction and can design advertisements that raise awareness but do not necessarily generate immediate sales which increase its total advertising costs.

This proposition therefore has important practical implications. Some publishers have adopted the CPA model, in the hope that adopting this model can help them attract more direct selling advertisers that measure campaign effectiveness by purchases rather than clicks. However, adopting the CPA model can lead to some unintended results for these publishers, in that it attracts branding rather than direct selling advertisers. Specifically, the winning advertiser in the CPA model is more likely to be an advertiser with relatively low immediate purchase rate. To make things worse, this adverse selection problem increases the possibility that an advertiser with a smaller profit margin wins the auction. As we show subsequently, the winning advertiser's profit margin has a positive effect on social welfare (i.e., total expected payoffs to all parties) in the online advertising industry. Therefore, the adverse selection problem of the CPA model limits its potential to improve social welfare in this industry.

5.2 Incremental efforts and purchase rate

Next, we explicitly compare the advertisers' and publisher's incentives to exert incremental efforts when they use the two different advertising pricing schemes and how those different incentives affect the final purchase rate.

Proposition 2.

1. *The advertiser i (with profit margin m_i and immediate sales ratio ρ_i) that wins the auction*

exerts less incremental effort in the CPA pricing model than the CPC pricing model: $e_i^{CPC} > e_i^{CPA}$.

2. *The expected level of incremental effort by the winning advertiser is lower in the CPA than the CPC pricing model: $E\left[e_i^{CPC}\right] > E\left[e_i^{CPA}\right]$.*

3. *The level of incremental effort made by the publisher is higher in the CPA than the CPC pricing model: $e_p^{CPA} = \frac{2\beta}{3\alpha} - \frac{\beta^2}{3\alpha^2} > 0 = e_p^{CPC}$.*

First two parts highlight two related but different points. The first part suggests that under the two advertising pricing schemes, the same advertiser behaves differently when it wins the auction, because it receives different incentives. The second part demonstrates that the expected effort by the winning advertiser (different advertisers would win in different pricing schemes) is lower for CPA than CPC. The intuition behind the first part indicates that with a CPA contract, the winning advertiser must share its sales gains with the publisher. This reduces its incentives to make costly incremental efforts, compared with those related to the CPC pricing model. This classic underinvestment problem arises because the advertiser cannot extract all the surplus it creates from its costly effort. The second part in turn reflects two different effects. The winning advertiser experiences reduced incentive to exert incremental efforts with the CPA model. As Lemma 2 shows, the optimal effort level depends on the marginal profit in a CPA contract ($e_i^{CPA} = m_i - \rho_i \cdot t_a$), and the expected marginal profit of the winning advertiser is lower for the CPA model (Proposition 1), which further reduces the expected level of incremental efforts made by this winning advertiser.

Finally, the intuition for the result of the publisher and its incentives in the two pricing schemes is straightforward. In the CPC model, the publisher's payoff is not tied to purchases, so the publisher has no incentive to exert incremental efforts. In contrast, the CPA pricing model ties the publisher's payoff to purchases, so the publisher has strong incentives to undertake incremental efforts.

Next, we also investigate how the expected purchase rate changes with the two different pricing schemes, in line with the varying incentives provided to advertisers and publisher.

Proposition 3. *The expected purchase rate is higher in the CPC than the CPA pricing model: $E\left[\theta^{CPC}\right] \geq E\left[\theta^{CPA}\right]$.*

This result is both interesting and counterintuitive. One might expect that the CPA pricing model leads to a higher expected purchase rate than the CPC model, given the fact that lower

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purchase rate due to the lack of proper incentive for the publisher to improve the quality of the clickes delivered is the main concern of the CPC pricing model. However, there are several forces that we need to take into consideration to fully understand the effects of different pricing mechanisms on the expected purchase rate.

On the one hand, the CPA model lets the winning advertiser and publisher share the potential payoffs and losses, leading to reduced incentives for the winning advertiser and increased incentives for the publisher to exert efforts, as we noted in Proposition 2. These two effects work in the opposite direction for the purchase rate and can cancel out each other. Hence, it is a prior unclear whether the CPA increases or decreases the purchase rate. In addition, the CPA model creates another effect, that is, the adverse selection problem from Proposition 1. This adverse selection problem means that the winning advertiser is more likely to be an advertiser with a smaller marginal profit (m_i), which further reduces the winning advertiser's incentives to undertake efforts; this in turn lowers the expected purchase rate.

On the other hand, in the CPC model, the winning advertiser has very strong incentives to undertake incremental efforts to improve the expected purchase rate (Proposition 2), because it obtains all the potential payoffs when the purchase rate is high but suffers all the potential losses when the purchase rate is low. Adding these three effects together, we find that the CPA model leads to an expected purchase rate which is lower than that for the CPC model.

5.3 Social welfare

Finally, we consider how the choice of pricing models might affect expected social welfare, that is, the total sum of the advertisers' and publisher's expected payoffs.

Proposition 4. *Denote the ratio of β to α as k (i.e., $k = \frac{\beta}{\alpha}$). There exists a threshold value of k_1 such that, when $k \geq k_1$, expected social welfare is greater for the CPA pricing model than the CPC pricing model: $E\left[y_i^{CPC} + y_p^{CPC}\right] \leq E\left[y_i^{CPA} + y_p^{CPA}\right]$. Otherwise ($k < k_1$), $E\left[y_i^{CPC} + y_p^{CPC}\right] > E\left[y_i^{CPA} + y_p^{CPA}\right]$.*

Proposition 4 relies on the existence of two opposing effects. On the one hand, the CPA model's adverse selection problem leads to a lower expected purchase rate and lower expected marginal profits for the winning advertiser. These declines contribute to decreases in the expected social welfare. On the other hand, the CPA model increases expected social welfare by enabling the winning advertiser to share a portion of the market risk with the publisher. Social welfare generally

is higher when risk gets shared among different parties, rather than shouldered by one party. This is a standard optimal risk-sharing result in principal agent models (Holmstrom 1979; Holmstrom and Milgrom 1987). In our particular setting where the publisher is risk-neutral and advertiser is risk-averse, to shift risk from a risk-averse agent to a risk-neutral party can generally increase social welfare.

Furthermore, Proposition 4 recognizes that the total effect depends on the relative size of these two competing effects. When parameter k (ratio of β to α) is above a certain threshold (k_1), the difference between the two advertisers is small, so the CPA model's adverse selection problem is not severe, and the total effect is positive; that is, increased expected social welfare. In contrast, when the parameter k is below a threshold k_1, the CPA model's adverse selection problem becomes severe, and the total effect is negative.

We highlight the important managerial implications of this proposition for the online advertising industry. Specifically, if participation in the CPA pricing model is limited to advertisers with sufficiently high immediate purchase rates, the difference between β and α will be small, and the parameter k (the ratio of β to α) should stay above a certain threshold (k_1). Therefore, the industry would likely benefit from moving to CPA contracts; it would achieve greater overall social welfare compared with that resulting from the CPC model. The online advertising industry (particularly, networks that strive to maximize total payoffs to all parties, because they serve both advertises and publishers) shares this view and is attempting to develop screens for advertisers that wish to use the CPA pricing model. For example, *Affiliate Fuel*, a CPA advertising network, requires all new advertisers to run a test campaign that demonstrates their likelihood of creating direct purchases before they can enter into a larger-scale contract. *Affiliate Fuel*'s prescreening process also examines advertisers' ads and landing pages to ensure they are designed to convert browsers into buyers (*Affiliate Fuel* 2010).[10]

The choice of pricing models clearly affects advertisers' and the publisher's expected payoff.

Corollary. *Denote the ratio of β to α as k (i.e., $k = \frac{\beta}{\alpha}$). There exists a threshold value of k_2 and k_3 such that:*

1. *When $k \geq k_2$, the publisher's expected payoff is higher in the CPA than in the CPC pricing model:, $E\left[y_p^{CPC}\right] \leq E\left[y_p^{CPA}\right]$. Otherwise ($k < k_2$), $E\left[y_p^{CPC}\right] > E\left[y_p^{CPA}\right]$.*

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[10] A similar screening process is used by *Commission Junction*, a leading CPA advertising network (Commission Junction 2010).

2. When $k \geq k_3$, the advertisers' expected payoff is higher in the CPA than in the CPC pricing model: $E\left[y_i^{CPC}\right] \leq E\left[y_i^{CPA}\right]$. Otherwise ($k < k_3$), $E\left[y_i^{CPC}\right] > E\left[y_i^{CPA}\right]$.

3. Moreover, it is always the case that $k_2 \leq k_3$.

The total effect is the sum of a negative effect caused by the CPA model's adverse selection problem and a positive effect caused by risk sharing under the CPA model. Thus, whether the CPA or CPC model leads to greater expected payoffs for the publisher (or advertisers) depends on the relative size of these two competing effects.

The publisher often appears to be the party resisting CPA adoption, as advertisers seemingly clamor for its adoption. This corollary suggests this scenario is not always the case. Rather, the publisher can benefit from adopting the CPA model in certain scenarios ($k > k_2$), as advertisers can suffer from adopting the CPA model in other scenarios ($k < k_3$).

Also, the corollary shows that it is always the case that $k_2 < k_3$; when the publisher prefers CPC (i.e., $k < k_2$), both the publisher and the advertiser are better off ($k < k_3$). Similarly, when the advertiser prefers CPA (i.e., $k > k_3$), the advertiser is better off, and the publisher's payoff increases ($k > k_2$) if it adopts the CPA model.[11]

However, the corollary also indicates a region of parameter k (ratio of β to α) in which the incentives of the publisher and the advertiser are misaligned (i.e., $k_2 < k < k_3$), such that the publisher prefers CPA but the expected payoff for the advertiser is greater for the CPC model. This region highlights the conflict of interest between parties, which results from the adverse selection and risk sharing that exists in the CPA model.

To highlight the results pertaining to the choice of performance-based pricing schemes and the conflict of interest between the publisher and advertisers, we illustrate the differences in the publisher's and advertisers' expected payoffs for the CPC and CPA settings in Figure 2, for which we set $r = 0.5$ and $\sigma^2 = 1$. The publisher prefers the CPC model if $k < k_2 \simeq 0.142$, and the advertisers' expected payoffs are greater in the CPC model when $k < k_3 \simeq 0.765$. Thus, when the publisher chooses the CPC model (i.e., $k < k_2 \simeq 0.142$), the advertisers' expected payoffs are

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[11]If we compare all the thresholds levels, k_1, k_2, k_3, from Propositions 4 and Corollary, we can easily confirm that $k_2 < k_1 < k_3$ (it is obvious given that k_1 is the cutoff for social welfare which is the sum of the publisher's and advertisers' expected payoff). That is, when the publisher chooses CPC (i.e., $k < k_2$), the publisher is better off and advertiser's payoff ($k < k_3$) as well as social welfare also increases ($k < k_1$). The opposite reasoning applies to the advertiser's choice of CPA (i.e., $k > k_3$): the advertiser is better off and the publihser's payoff ($k > k_2$) as well as social welfare also increases ($k > k_1$).

greater. In contrast, when $0.142 < k < 0.765$, the advertisers' expected payoffs increase in the CPC model, but the publisher prefers the CPA model.

*** Figure 2 ***

5.4 Uncertainty and risk aversion

Next, we study how various factors influence preferences for one pricing scheme over another. Define $\Delta E\left(y_p\right) = E\left(y_p^{CPA}\right) - E\left(y_p^{CPC}\right)$ and $\Delta E\left(y_i\right) = E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right)$. From Lemma 1 and 2, we can easily observe that the advertisers' risk aversion parameter (r) and market risk (σ^2) can have negative impacts on the payoffs to both the publisher and advertisers. A unique feature of the CPA pricing model is its ability to enable the winning advertiser to share a portion of the market risk with the publisher. This risk-sharing arrangement can mitigate the negative impact of both the advertisers' risk aversion parameter (r) and market risk (σ^2).

In the following proposition, we outline how these factors affect the publisher's and advertisers' preferences for each performance-based pricing scheme.

Proposition 5. *As uncertainty in the product market (σ^2) increases or advertisers become more risk averse (r), the difference in the publisher's and advertisers' expected profits in the CPA versus CPC pricing model, $\Delta E\left(y_p\right)$ and $\Delta E\left(y_i\right)$, monotonically increases:* $\frac{\partial(\Delta E(y_p))}{\partial r} \geq 0$, $\frac{\partial(\Delta E(y_p))}{\partial \sigma^2} \geq 0$, $\frac{\partial(\Delta E(y_i))}{\partial r} \geq 0$, *and* $\frac{\partial(\Delta E(y_i))}{\partial \sigma^2} \geq 0$.

Proposition 5 shows that as uncertainty in the product market increases or advertisers become more risk averse, the publisher can benefit from adopting the CPA model instead of the CPC model. Its CPC-related payoffs suffer from product uncertainty (σ^2) and the risk aversion parameter r, but payoffs are not affected by these factors in the CPA model. If an advertiser is subject to uncertainty, as represented by σ^2, it is less willing to pay and, therefore, this risk-averse advertiser bids a lower price per click in the CPC model. The publisher benefits from CPA pricing, because the advertiser's payment does not depend on σ^2. With a CPC contract, advertisers shoulder most of the risk of unexpected sales, which lowers the bidding price.

Similarly, as advertisers become more risk averse, they offer a lower payment per click to compensate for their own risk, which arises from uncertainty. However, with the CPA model, the advertising payments are tied to purchases, so the burden of bearing the uncertainty risk shifts

from the advertiser to the publisher, and the payment from the advertiser to the publisher is independent of the risk aversion factor. When r is relatively small, advertisers shoulder more risks, and the CPC model appears more favorable to both the publisher and advertisers. Otherwise, the CPA model is more appealing. These findings suggest that the CPA model can balance risk sharing between the publisher and advertisers.

Dicussion

Proposition 5 sheds some light on online advertising industry by revealing which types of advertisers and products represent good candidates for contracts that tie advertising payments to purchases. The CPA model is particularly suitable for advertisers that are risk averse and products that have high levels of market uncertainty. In these conditions, the CPA model can help improve social welfare and the payoffs to both the publisher and advertisers. Advertisers that are more risk averse and sell products with high levels of market uncertainty likely make low bids in the CPC pricing model, because they would have been forced to shoulder all the market risk. However, with the CPA model, which ties advertising payments to purchases, the risk burden due to product uncertainty shifts from the advertisers to the publisher, so advertisers are more willing to participate and more likely to offer high bids. Such a risk-sharing arrangement directly increases the adertisers' payoff and indirectly increases the publisher's payoff through the advertisers' bids.

Conventional wisdom suggests smaller firms are more risk averse because of their inability to suffer through large market risks. Hence, the CPA pricing model is particularly beneficial to small advertisers that otherwise would have not participated in online advertising, for fear of the market risks involved in CPC deals. In addition, advertisers that sell products with strong seasonality and unpredictable demand are good candidates for CPA deals. These findings are consistent with trends in the online advertising industry. For example, the previously mentioned Affiliate Fuel network has indicated its great interest in hosting products that are time sensitive and seasonal (Affliliate Fuel 2010).

6 Extension

One could argue that the publisher can learn about advertisers' types through multiple noisy signals, such as (1) the repeated past interaction with advertisers, or (2) the estimate from a test campaign

(*Commission Junction* 2010). Using this information, the publisher can calculate the expected revenue from each advertiser under CPA. Hence, the publisher's knowledge of advertisers' types could weaken the adverse selection problem identified in the current model. In this section, we extend our main model to explore this issue.

In practice, the publisher's knowledge of advertisers' types is far from perfect. For example, even if the publisher knows an advertiser's past performance (such as immediate conversion rate) through repeated interaction, it is possible that this advertiser adopts different strategies (or goals) in different product campaigns: it might have used a direct selling strategy (or goal) in one product campaign and a branding strategy (or goal) in another. To capture the reality that the publisher can have noisy information about the advertiser's type in a parsimonious way, we assume that the publisher obtains an *ex ante* signal regarding each advertiser's type. We model the uncertainty of such a noisy signal as ϕ such that the signal is correct with probability ϕ, and incorrect with probability $1 - \phi$. In other words, with probability ϕ, type D (B) advertiser is correctly identified as direct seller (branding advertiser), and with probability $1 - \phi$, type D (B) advertiser is incorrectly identified as a branding advertiser (direct seller).

Moreover, we now allow that the publisher uses its knowledge of each advertiser's type to choose the advertiser based on its expected revenue (not the highest bidder). Such a process is similar to how Google uses historical data to adjust each advertiser's bid and determine the winner of the auction. Since the publisher's objective is to maximize its expected payoff, which directly relates to each advertiser's immediate sales ratio, the publisher will adjust each advertiser's bid based on its knowledge of each advertiser's immediate sales ratio. More specifically, if the publisher receives a signal which indicates it is a type D advertiser, it will adjust the advertiser's bid by multiplying α, which is type D advertiser's immediate sales ratio; if the publisher receives a signal which indicates it is a type B advertiser, it will adjust the advertiser's bid by multiplying type B advertiser's immediate sales ratio, β.[12]

Because there are two advertisers – one with higher immediate sales ration and thus more focused on direct selling (type D advertiser) and the other with lower immediate sales ratio and thus more focused on building brand equity (type B advertiser), along with a signal, there are two cases to consider: *i*) with probability ϕ, the publisher correctly identify both type D and type B advertisers;

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[12]Although we do not know for certain what publishers (for example, Googles or ValueClick) do when choosing the winning bidder, it is well known that publishers maximizes its expected revenue based on advertisers' type. Our specification captures this spirit in a simplest possible way.

ii) with probability $1 - \phi$, the publisher has incorrectly identify type D and type B advertisers. We then investigate whether allowing the publisher to use a prior knowledge of advertisers' types changes the adverse selection problem discussed in Proposition 1.

Proposition 6. *Suppose that the publisher receives signals about the advertisers' type and use them to adjust advertisers' biddings. The expected probability that the branding (type B) advertiser wins the auction in the CPA model is $E\left[Pr(w = B)\right]^{CPA} = 1 - \frac{1}{2}\left[\phi + (1 - \phi)\left(\frac{\beta}{\alpha}\right)^2\right]$. This expected probability that the branding (type B) advertiser wins the auction is still greater in the CPA than the CPC pricing model: $E\left[Pr(w = B)\right]^{CPC} \leq E\left[Pr(w = B)\right]^{CPA}$.*

The intuition behind this proposition is as follows. In case (i) mentioned above, the publisher has correct signals and can correctly adjust each advertiser's bid. Thus, the expected probablity that the type B (branding) advertiser wins the auction in the CPA model becomes the same as that in the CPC model, which is $\frac{1}{2}$. In the CPA model without such signals, the expected probability that the branding advertiser wins the auction is $1 - \frac{\beta}{2\alpha}$. In case ($ii$), the publisher's incorrect signals regarding both advertisers prevent the publisher from correctly adjusting each advertiser's bid. The expected probability that the branding advertiser wins the auction is $1 - \frac{\beta^2}{2\alpha^2}$, even higher than that in the CPA model without such signals.

Overall, we conclude that our main results are robust even if the publisher can learn about advertisers' types as long as the publisher's knowledge is far from perfect, which is almost the case in reality. Allowing the publisher to have knowledge of each advertiser's type and choose the advertiser based on its expected revenue would only weaken the adverse selection problem of the CPA model. However, this problem would still persist and sometimes would be more severe when the quality of the publisher's knowledge is low (i.e., ϕ is small).

7   Conclusions

Since 2002, the online advertising industry increasingly has adopted the CPC pricing model, which ties advertising payments to clicks. More recently, several large companies have started to pursue CPA pricing, which calculates advertising payments on the basis of purchases. Which model leads to better outcomes for advertisers, publishers, and the industry as a whole? Is CPA really the future of online advertising? This study offers a first step in understanding this crucial debate. We apply a formal economic framework to analyses of the trade-offs between CPA and CPC, with a

particular focus on the non-contractible efforts that publishers and advertisers will exert to improve the effectiveness of advertising campaigns. Unlike existing literature, we view pricing models as contracts that give publishers and advertisers incentives to exert non-contractible efforts, as well as allocate the market risk between advertisers and publishes. This unique angle on these two popular online advertising pricing models leads to several interesting and new insights.

Our results also have important implications for all parties involved in online advertising: advertisers, publishers, and advertising networks. We outline the conditions in which one pricing model is more desirable than the other in terms of increasing the payoffs to each party. We also note which parameters influence the trade-offs between the CPC and CPA models and how the use of different pricing models affects social welfare in this industry. Such insights can help advertising networks design efficient marketplaces for their clients (i.e., advertisers and publishers), as well as help resolve the strident debate about the future of pricing models in online advertising, with billions of advertising dollars in the balance.

There are a number of limitations to the current work and our results could be extended in further research. First, we assume that the publisher has a single slot in spirit of trying to capture the reality that most firms have limited capacity of advertising space in their website. This also allowed us to keep our analysis tractable by guaranteeing the advertisers' bid represent their true valuations in a second-price auction. Nevertheless, most publishers often have more than one slot in which they can place ads in their website. A second-price auction will then diverge from the Vickery-Clarke-Groves mechanism, and true-valuation bidding is generally not an equilibrium strategy (Edelman et al. 2007). Broader analysis encompassing multiple-slots, even if technically challenging, would be insightful for generalization of our finidings.

Second, the utility functional form and risk-aversion parameter that we use are the most common versions in extant economic research pertaining to incentive contracts. We conjecture that including different utility functions and nonlinear mappings from effort to purchases, would not qualitatively impact our analysis. Some extensions could be analyzed in the context of a linear contract between the advertiser and the publisher; others would require nonlinear contracts.

Third, we offer several propositions regarding the influence of various factors on the use of CPC and CPA models. These factors, including the advertisers' risk aversion, uncertainty in the product market, and the proportion of delayed responses, differ for various advertisers and publishers. It therefore would be interesting to test these propositions using empirical data. Finally, we predict

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how the adoption of a CPA model (rather than a CPC model) influences purchase (conversion) rates, firm profits, and social welfare. Additional research should test these predictions empirically as well.

The focus on the current work is not to come up with the optimal auction mechanism of online advertising, but try to highlight the costs and benefits of two different performance-based pricing, which are widely used in practice. By doing so, we are hoping that we shed some insight for the managerial decision about which way the industry should move on.

Appendix

Proof of Lemma 1.

We solve the second part first. In the CPC pricing model, the publisher's payoff is $y_p^{CPC} = t_c - \frac{e_p^2}{2}$. The optimal effort level by the publisher is $e_p^{CPC} = \text{argmax } y_p^{CPC} = 0$. Advertiser i's payoff if it wins the auction is $y_i^{CPC} = m_i(e_i + e_p) - t_c - \frac{e_i^2}{2} - \frac{r\sigma^2 m_i^2}{2}$. The optimal effort level by advertiser i is $e_i^{CPC} = \text{argmax}_{e_i} \left(y_i^{CPC} \right) = m_i$.

Using the result from above that $e_p^{CPC} = 0$ and $e_i^{CPC} = m_i$, advertiser i's payoff if it wins the auction is $y_i^{CPC} = \frac{1}{2} \left(1 - r\sigma^2 \right) \cdot m_i^2 - t_c$. Because the advertisers bid their true values in a second-price Vickery auction, $b(m_i, \rho_i) = \frac{1}{2} \left(1 - r\sigma^2 \right) \cdot m_i^2$. The advertiser with a higher m_i wins the auction. Because $m_i(i = D, B)$ is randomly drawn from a standard uniform distribution on $[0, 1]$, the probability that the branding advertiser wins the auction is $E\left[\Pr(w = B) \right]^{CPC} = E\left[\Pr(m_B \geq m_D) \right]^{CPC} = \int_0^1 \int_{m_D}^1 dm_B \, dm_D = \frac{1}{2}$. **Q.E.D.**

Derivation of the payoffs to the advertisers and the publisher in the CPC model.

In the CPC pricing model, advertiser i bids $b(m_i, \rho_i) = \frac{1}{2} \left(1 - r\sigma^2 \right) \cdot m_i^2$. When $r\sigma^2 < 1$, both bids are nonnegative, and a contract results between the two parties. The advertiser with a higher realized m_i wins the auction and pays the lower bid. Let m_D, m_B each be random draws from a standard uniform distribution on $[0, 1]$. Thus,

$$t_c^* = \frac{1}{2} \left(1 - r\sigma^2 \right) \cdot \min \left(m_D^2, m_B^2 \right). \tag{11}$$

The winning advertiser's profit is:

$$y_i^{CPC} = \frac{1}{2} \left(1 - r\sigma^2 \right) \cdot \left(\max \left(m_D^2, m_B^2 \right) - \min \left(m_D^2, m_B^2 \right) \right). \tag{12}$$

The publisher's expected profit is:

$$y_p^{CPC} = t_c^* = \frac{1}{2} \left(1 - r\sigma^2 \right) \cdot \min \left(m_D^2, m_B^2 \right). \tag{13}$$

Integrating Equation (12) over the two uniform distributions, we get:

$$E(y_i^{CPC}) = \frac{1}{2}\left(1 - r\sigma^2\right) \cdot \left\{ \left(\int_0^1 \int_0^{m_D} m_D^2 \, dm_B \, dm_D + \int_0^1 \int_{m_D}^1 m_B^2 \, dm_B \, dm_D \right) \right. \tag{14}$$

$$\left. - \left(\int_0^1 \int_0^{m_D} m_B^2 \, dm_B \, dm_D + \int_0^1 \int_{m_D}^1 m_D^2 \, dm_B \, dm_D \right) \right\}$$

$$= \frac{1}{6}\left(1 - r\sigma^2\right).$$

Similarly, by integrating Equation (13) over the two uniform distributions, we get: $E(y_p^{CPC}) = \frac{1}{12}\left(1 - r\sigma^2\right)$. **Q.E.D.**

Proof of Lemma 2.

We solve the second part of the Lemma first. In the CPA pricing model, the winning advertiser forms an expectation regarding the publisher's effort e_p, given the winning bid t_a. Advertiser $i's$ payoff, if it wins the auction, is $y_i^{CPA} = (m_i - \rho_i \cdot t_a)(e_i + e_p) - \frac{e_i^2}{2} - \frac{r\sigma^2}{2}(m_i - \rho_i \cdot t_a)^2$. The advertiser does not know the publisher's e_p for sure but forms an expectation about it. Thus, the advertiser's payoff becomes $y_i^{CPA} = (m_i - \rho_i \cdot t_a)(e_i + E(e_p)) - \frac{e_i^2}{2} - \frac{r\sigma^2}{2}(m_i - \rho_i \cdot t_a)^2$. The optimal effort level by the advertiser i in turn is the solution to $e_i^{CPA} = argmax_{e_i} y_i^{CPA} = m_i - \rho_i \cdot t_a$.

The publisher's payoff is $y_p^{CPA} = \rho_i \cdot t_a(e_i + e_p) - \frac{e_p^2}{2}$. The publisher does not know the winning advertiser's ρ_i and e_i and therefor must form expectations about these values. Thus, the publisher's payoff becomes $y_p^{CPA} = E(\rho_i) \cdot t_a(E(e_i) + e_p) - \frac{e_p^2}{2}$, and the optimal effort level bythe publisher is the solution to

$$e_p^{CPA} = argmax_{e_p} E(y_p^{CPA}) = E(\rho_i) \cdot t_a. \tag{15}$$

Using the result from the above that $e_i^{CPA} = m_i - \rho_i \cdot t_a$ and $e_p^{CPA} = E(\rho_i) \cdot t_a$, in the CPA pricing model, advertiser $i's$ payoff, if it wins the auction, is

$$y_i^{CPA} = \frac{1}{2}\left(1 - r\sigma^2\right) \cdot (m_i - \rho_i \cdot t_a)^2 + (m_i - \rho_i \cdot t_a) \cdot E(e_p).$$

Because advertisers bid their true values in the second-price Vickery auction, $b(m_i, \rho_i) = \frac{m_i}{\rho_i}$. Let $\frac{m_D}{\alpha} = n_D$ and $\frac{m_B}{\beta} = n_B$ equal the bids from the direct selling and branding advertisers, respectively. Then $n_D \sim U\left[0, \frac{1}{\alpha}\right]$ and $n_B \sim U\left[0, \frac{1}{\beta}\right]$. The probability that the branding advertiser

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Figure 1: Advertiser's bidding behavior and equilibrium outcome
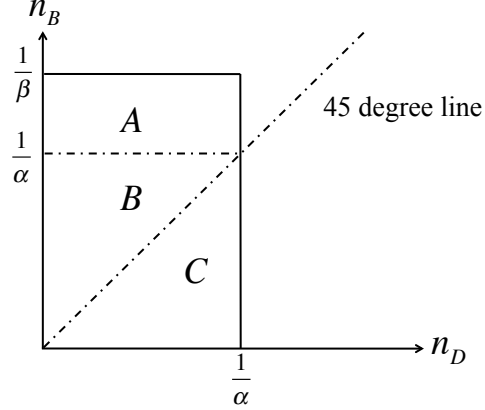
wins the auction $(E\left[\Pr(w=B)\right]^{CPA})$ is

$$\Pr[n_B \geq n_D] = \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} f(n_B)f(n_D)\,dn_B\,dn_D + \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\alpha}} f(n_B)f(n_D)\,dn_B\,dn_D \qquad (16)$$

$$= \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \alpha\beta\,dn_B\,dn_D + \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\alpha}} \alpha\beta\,dn_B\,dn_D$$

$$= 1 - \frac{\beta}{2\alpha}.$$

Q.E.D.

Derivation of the payoffs to the advertisers and the publisher in the CPA model.

Let $\frac{m_D}{\alpha} = n_D$ and $\frac{m_B}{\beta} = n_B$ be the bids from the direct selling and branding advertiser, respectively. Then, $n_D \sim U\left[0, \frac{1}{\alpha}\right]$ and $n_B \sim U\left[0, \frac{1}{\beta}\right]$. Three different regions arise in equilibrium and lead to different analysis (see Figure 3).

1. When $\frac{1}{\alpha} < n_B$ (region A in Figure 3): The type B advertiser (branding type) wins the auction, because $n_D < n_B$ always holds. Thus, $t_a = n_D$. The publisher can determine that the winning advertiser is a type B advertiser, because its bid is $n_B > \frac{1}{\alpha}$, and a type D advertiser would never bid greater than $\frac{1}{\alpha}$. Thus, the publisher's expectation of the winning advertiser's type is $E(\rho_i) = \beta$, and the publisher undertakes effort $E(e_p) = \beta \cdot n_D$. The expected payoff for

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the winning advertiser is,

$$E(y_i^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \left\{ \frac{1}{2} \left(1 - r\sigma^2\right) \cdot (\beta\, n_B - \beta\, n_D)^2 + (\beta\, n_B - \beta\, n_D)\, \beta\, n_D \right\} \alpha\beta\, dn_B\, dn_D$$

$$\text{(17)}$$

$$= \frac{1}{2} \left(1 - r\sigma^2\right) \cdot \left(\frac{1}{3} - \frac{\beta}{2\alpha} + \frac{\beta^2}{3\alpha^2} - \frac{\beta^3}{6\alpha^3} \right) + \left(\frac{\beta}{4\alpha} - \frac{\beta^2}{3\alpha^2} + \frac{\beta^3}{12\alpha^3} \right).$$

Whereas the expected payoff for the publisher is,

$$E(y_p^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \left\{ \beta\, n_D \left(\beta\, n_D + \beta\, n_B - \beta\, n_D \right) - \frac{1}{2} \left(\beta\, n_D \right)^2 \right\} \alpha\beta\, dn_2\, dn_1 \qquad \text{(18)}$$

$$= \frac{\beta}{4\alpha} - \frac{\beta^2}{6\alpha^2} - \frac{\beta^3}{12\alpha^3}.$$

2. When $n_D < n_B \leq \frac{1}{\alpha}$ (region B in Figure 3): The type B advertiser wins the auction. Thus, $t_a = n_D$. The winning advertiser's bid is $n_B \leq \frac{1}{\alpha}$, which could be made by either type of advertiser. Thus, the publisher cannot anticipate the winning advertiser's type (i.e., its expectation of the winning advertiser's type is $E(\rho_i) = \frac{\alpha+\beta}{2}$), and the publisher does not exert effort, $E(e_p) = \frac{\alpha+\beta}{2} n_D$. The expected payoff of the winning advertiser then is,

$$E(y_i^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\beta}} \left\{ \frac{1}{2} \left(1 - r\sigma^2\right) \cdot (\beta\, n_B - \beta\, n_D)^2 + (\beta\, n_B - \beta\, n_D) \frac{\alpha+\beta}{2} \cdot n_D \right\} \alpha\beta\, dn_B\, dn_D$$

$$\text{(19)}$$

$$= \frac{1}{2} \left(1 - r\sigma^2\right) \cdot \left(\frac{\beta^3}{12\alpha^3} \right) + \frac{(\alpha+\beta)\beta^2}{48\alpha^3}.$$

Whereas the expected payoff for the publisher is,

$$E(y_p^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\beta}} \left\{ \beta\, n_D \left(\frac{\alpha+\beta}{2} \cdot n_D + \beta\, n_B - \beta\, n_D \right) - \frac{1}{2} \left(\frac{\alpha+\beta}{2} \cdot n_D \right)^2 \right\} \alpha\beta\, dn_B\, dn_D$$

$$\text{(20)}$$

$$= -\frac{\beta}{96\alpha} + \frac{\beta^2}{48\alpha^2} + \frac{7\beta^3}{96\alpha^3}.$$

3. When $n_B \leq n_D$ (region C in Figure 3): The type D advertiser wins the auction. Thus, $t_a = n_B$. The winning advertiser's bid, $n_D \leq \frac{1}{\alpha}$, could be made by either type of advertiser,

and again, the publisher cannot anticipate the winning advertiser's type. The publisher's expectation about the winning advertiser's type is $E(\rho_i) = \frac{\alpha+\beta}{2}$, and the publisher undertakes efforts $E(e_p) = \frac{\alpha+\beta}{2} n_B$. The expected payoff of the winning advertiser is,

$$E(y_i^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_0^{n_D} \left\{ \frac{1}{2} \left(1 - r\sigma^2 \right) \cdot (\alpha\, n_D - \alpha\, n_B)^2 + (\alpha\, n_D - \alpha\, n_B) \frac{\alpha+\beta}{2} \cdot n_B \right\} \alpha\beta\, dn_B\, dn_D \tag{21}$$

$$= \frac{1}{2} \left(1 - r\sigma^2 \right) \cdot \left(\frac{\beta}{12\alpha} \right) + \frac{(\alpha+\beta)\beta}{48\alpha^2}.$$

Whereas the expected payoff for the publisher is,

$$E(y_p^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_0^{n_D} \left\{ \alpha\, n_B \left(\frac{\alpha+\beta}{2} \cdot n_B + \alpha\, n_D - \alpha\, n_B \right) - \frac{1}{2} \left(\frac{\alpha+\beta}{2} \cdot n_B \right)^2 \right\} \alpha\beta\, dn_B\, dn_D \tag{22}$$

$$= \frac{7\beta}{96\alpha} + \frac{\beta^2}{48\alpha^2} - \frac{\beta^3}{96\alpha^3}.$$

Therefore, in the CPA pricing model, the winning advertiser's expected payoff is the sum of the advertiser's payoffs in regions A, B, and C in Figure 3. Thus,

$$E(y_i^{CPA}) = \frac{1}{2} \left(1 - r\sigma^2 \right) \cdot \left(\frac{\beta}{3\alpha} - \frac{\beta^2}{3\alpha^2} + \frac{\beta^3}{6\alpha^3} \right) + \frac{(\alpha+\beta)^2\beta}{48\alpha^3}. \tag{23}$$

Similarly, the publisher's expected payoff is the sum of the advertiser's payoffs in regions A, B, and C in Figure 3, and

$$E(y_p^{CPA}) = \frac{15\beta}{48\alpha} - \frac{\beta^2}{8\alpha^2} - \frac{\beta^3}{48\alpha^3}. \tag{24}$$

Q.E.D.

Proof of Proposition 1.

1. From Lemma 1 and Lemma 2, we know that $E(\Pr(w = B)]^{CPC} = \frac{1}{2}$ and $E(\Pr(w = B)]^{CPA} = 1 - \frac{\beta}{2\alpha}$. Because $0 < \beta \leq \alpha < 1$, we also know $E(\Pr(w = B)]^{CPC} = \frac{1}{2} < E(\Pr(w = B)]^{CPA} = 1 - \frac{\beta}{2\alpha}$.

2. In the CPC pricing model, the expected marginal profit of the winning advertiser is

$$E(m_i)^{CPC} = \int_0^1 \int_0^{m_D} m_D \, dm_B \, dm_D + \int_0^1 \int_0^{m_B} m_B \, dm_B \, dm_D = \frac{2}{3}.$$

In the CPA pricing model, let $\frac{m_D}{\alpha} = n_D$ and $\frac{m_B}{\beta} = n_B$ be the bids from the direct selling and branding advertisers, respectively. Then, $n_D \sim U\left[0, \frac{1}{\alpha}\right]$ and $n_B \sim U\left[0, \frac{1}{\beta}\right]$. The expected marginal profit of the winning advertiser $E(m_i)^{CPA}$ is the sum of the expected marginal profit for the three regions.

(a) When $\frac{1}{\alpha} < n_B : E(m_i) = \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \alpha\beta \cdot \beta \, n_B \, dn_B \, dn_D = \frac{1}{2}\left(1 - \frac{\beta^2}{\alpha^2}\right).$

(b) When $n_D < n_B \leq \frac{1}{\alpha} : E(m_i) = \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\alpha}} \alpha\beta \cdot \beta \, n_B \, dn_B \, dn_D = \frac{\beta^2}{3\alpha^2}.$

(c) When $n_B \leq n_D : E(m_i) = \int_0^{\frac{1}{\alpha}} \int_0^{n_D} \alpha\beta \cdot \alpha n_D \, dn_B \, dn_D = \frac{\beta}{3\alpha}.$

Thus, the expected marginal profit of the winning advertiser is

$$E(m_i)^{CPA} = \frac{1}{2} + \frac{\beta}{3\alpha} - \frac{\beta^2}{6\alpha^2}.$$

The function $f(x) = \frac{1}{2} + \frac{1}{3}x - \frac{1}{6}x^2$ is a increasing function on [0,1] and reaches its maximum of $f(1) = \frac{2}{3}$ in this region. Therefore, we have proven

$$E(m_i)^{CPA} = \frac{1}{2} + \frac{\beta}{3\alpha} - \frac{\beta^2}{6\alpha^2} \leq \frac{2}{3} = E(m_i)^{CPC}.$$

Q.E.D.

Proof of Proposition 2.

1. Directly from the comparison of Lemma 1 and Lemma 2, $e_i^{CPC} = m_i > e_i^{CPA} = m_i - \rho_i t_a.$

2. First, in the CPC pricing model, $E(e_i^{CPC}) = \frac{2}{3}$, whereas in the CPA pricing model, the expected level of effort made by the winning advertiser $E(e_i^{CPA})$ is the sum of the expected level of incremental effort made by the winning advertiser in the three regions.

(a) When $\frac{1}{\alpha} < n_B : E(e_i^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \alpha\beta \cdot (\beta \, n_B - \beta \, n_D) \, dn_B \, dn_D = \frac{1}{2}\left(1 - \frac{\beta}{\alpha}\right).$

(b) When $n_D < n_B \leq \frac{1}{\alpha} : E(e_i^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\alpha}} \alpha\beta \cdot (\beta \, n_B - \beta \, n_D) \, dn_B \, dn_D = \frac{\beta^2}{6\alpha^2}.$

(c) When $n_B \leq n_D : E(e_i^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_0^{n_D} \alpha\beta \cdot (\alpha \, n_D - \alpha \, n_B) \, dn_B \, dn_D = \frac{\beta}{6\alpha}.$

Thus, the expected level of incremental effort by the advertiser is

$$E(e_i^{CPA}) = \frac{1}{2} - \frac{\beta}{3\alpha} + \frac{\beta^2}{6\alpha^2}.$$

The function $f(x) = \frac{1}{2} - \frac{1}{3}x + \frac{1}{6}x^2$ is a decreasing function on [0,1] and reaches its maximum of $f(0) = \frac{1}{2}$ in this region. Thus, we have proven

$$E(e_i^{CPA}) = \frac{1}{2} - \frac{\beta}{3\alpha} + \frac{\beta^2}{6\alpha^2} < \frac{2}{3} = E(e_i^{CPC}).$$

3. Directly from the comparison of Lemma 1 and Lemma 2, $e_p^{CPC} = 0 < e_p^{CPA} = E(\rho_i)t_a$. Becuase the level of effort by the publisher in the CPA model is a function of $E(\rho_i)$ and the outcome of advertisers' bidding behavior is t_a, we consider three different regions:

(a) When $\frac{1}{\alpha} < n_B$: $E(e_p^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{\frac{1}{\alpha}}^{\frac{1}{\beta}} \alpha\beta\beta n_D \, dn_B \, dn_D = \frac{1}{2}\left(\frac{\beta}{\alpha} - \frac{\beta^2}{\alpha^2}\right).$

(b) When $n_D < n_B \leq \frac{1}{\alpha}$: : $E(e_p^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{1}{\alpha}} \alpha\beta \cdot \frac{\alpha+\beta}{2} n_D \, dn_B \, dn_D = \frac{1}{12}\left(\frac{\beta}{\alpha} + \frac{\beta^2}{\alpha^2}\right).$

(c) When $n_B \leq n_D$: $E(e_p^{CPA}) = \int_0^{\frac{1}{\alpha}} \int_0^{n_D} \alpha\beta \cdot \frac{\alpha+\beta}{2} n_B \, dn_B \, dn_D = \frac{1}{12}\left(\frac{\beta}{\alpha} + \frac{\beta^2}{\alpha^2}\right).$

The level of incremental effort exerted by the publisher is the sum of these three cases:

$$e_p^{CPA} = \frac{2\beta}{3\alpha} - \frac{\beta^2}{3\alpha^2}.$$

The function $f(x) = \frac{2}{3}x + \frac{1}{3}x^2$ is an increasing function on [0,1] that reaches its minimum of $f(0) = 0$ in this region. Thus, we have proven

$$e_p^{CPA} = \frac{2\beta}{3\alpha} - \frac{\beta^2}{3\alpha^2} > 0 = e_p^{CPC}.$$

Q.E.D.

Proof of Proposition 3.

Note that $E(\theta) = E(e_i) + E(e_p)$. From the results in the proofs of Propositions 2, we know $E(\theta^{CPC}) = E(e_i^{CPC}) + E(e_p^{CPC}) = \frac{2}{3}$ and $E(\theta^{CPA}) = E(e_i^{CPA}) + E(e_p^{CPA}) = \left(\frac{1}{2} - \frac{\beta}{3\alpha} + \frac{\beta^2}{6\alpha^2}\right) + \left(\frac{2\beta}{3\alpha} - \frac{\beta^2}{3\alpha^2}\right) = \frac{1}{2} + \frac{\beta}{3\alpha} - \frac{\beta^2}{6\alpha^2}$. The function $f(x) = \frac{1}{2} + \frac{1}{3}x - \frac{1}{6}x^2$ is a increasing function on [0,1] that reaches its maximum of $f(1) = \frac{2}{3}$ in this region. Thus, we have proven

$$E\left(\theta^{CPA}\right) = \frac{1}{2} + \frac{\beta}{3\alpha} - \frac{\beta^2}{6\alpha^2} \leq \frac{2}{3} = E\left(\theta^{CPC}\right).$$

Q.E.D.

Proof of Proposition 4.

Let $k = \frac{\beta}{\alpha}$. From Lemma 1 and Lemma 2, $E\left(y_i^{CPC} + y_p^{CPC}\right) = \frac{1}{4}(1 - r\sigma^2)$ and $E\left(y_i^{CPA} + y_p^{CPA}\right) = \frac{1}{2}\left(1 - r\sigma^2\right) \cdot \left(\frac{2k - 2k^2 + k^3}{6}\right) + \left(\frac{4k - k^2}{12}\right)$. Thus, $E\left(y_i^{CPA} + y_p^{CPA}\right) - E\left(y_i^{CPC} + y_p^{CPC}\right) = \frac{1}{2}\left(1 - r\sigma^2\right) \cdot \left(\frac{2k - 2k^2 + k^3}{6} - \frac{1}{2}\right) + \left(\frac{4k - k^2}{12}\right)$. When $k = 0$, $E\left(y_i^{CPA} + y_p^{CPA}\right) - E\left(y_i^{CPC} + y_p^{CPC}\right) < 0$; when $k = 1$, $E\left(y_i^{CPA} + y_p^{CPA}\right) - E\left(y_i^{CPC} + y_p^{CPC}\right) > 0$. Moreover,

$$\frac{\partial\left(E\left(y_i^{CPA} + y_p^{CPA}\right) - E\left(y_i^{CPC} + y_p^{CPC}\right)\right)}{\partial k} = \frac{1}{2}\left(1 - r\sigma^2\right) \cdot \left(\frac{(3k - 2)^2 + 2}{18}\right) + \frac{2 - k}{6} > 0, \text{ for } \forall k \in [0, 1].$$

Therefore, there exists a threshold value of $k_1 \in (0, 1)$, such that when $k \geq k_1$, $E\left(y_i^{CPC} + y_p^{CPC}\right) \leq E\left(y_i^{CPA} + y_p^{CPA}\right)$ but when $k < k_1$, $E\left(y_i^{CPC} + y_p^{CPC}\right) > E\left(y_i^{CPA} + y_p^{CPA}\right)$. **Q.E.D.**

Proof of Collorary.

1. Let $k = \frac{\beta}{\alpha}$. From Lemma 1 and Lemma 2, $E\left(y_p^{CPC}\right) = \frac{1}{12}\left(1 - r\sigma^2\right)$ and $E\left(y_p^{CPA}\right) = \frac{15k - 6k^2 - k^3}{48}$. When $k = 0$, $E\left(y_p^{CPA}\right) - E\left(y_p^{CPC}\right) < 0$; when $k = 1$, $E\left(y_p^{CPA}\right) - E\left(y_p^{CPC}\right) > 0$. Moreover, $\frac{\partial\left(E\left(y_p^{CPA}\right) - E\left(y_p^{CPC}\right)\right)}{\partial k} = \frac{15 - 12k - 3k^2}{48} > 0$, for $\forall k \in [0, 1]$. Therefore, there exists a threshold value of $k_2 \in (0, 1)$, such that when $k \geq k_2$, $E\left(y_p^{CPC}\right) \leq E\left(y_p^{CPA}\right)$ but when $k < k_2$, $E\left(y_p^{CPC}\right) > E\left(y_p^{CPA}\right)$.

2. From Lemma 1 and Lemma 2, $E\left(y_i^{CPC}\right) = \frac{1}{6}\left(1 - r\sigma^2\right)$ and $E\left(y_i^{CPA}\right) = \frac{1}{2}\left(1 - r\sigma^2\right) \cdot \left(\frac{2k - 2k^2 + k^3}{6}\right) + \left(\frac{k + 2k^2 + k^3}{48}\right)$. When $k = 0$, $E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right) < 0$; when $k = 1$, $E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right) > 0$. Moreover, $\frac{\partial\left(E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right)\right)}{\partial k} = \frac{1}{2}\left(1 - r\sigma^2\right) \cdot \left(\frac{2 - 4k + 3k^2}{6}\right) + \frac{1 + 4k + 3k^2}{48} > 0$, for $\forall k \in [0, 1]$. Therefore, there exists a threshold value of $k_3 \in (0, 1)$, such that when $k \geq k_3$, $E\left(y_i^{CPC}\right) \leq E\left(y_i^{CPA}\right)$, and when $k < k_3$, $E\left(y_i^{CPC}\right) > E\left(y_i^{CPA}\right)$.

3. We define $\Delta E\left(y_p\right) = E\left(y_p^{CPA}\right) - E\left(y_p^{CPC}\right)$ and $\Delta E\left(y_i\right) = E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right)$. Then, $\Delta E\left(y_p\right) - \Delta E\left(y_i\right) = \frac{1}{12}\left(1 - r\sigma^2\right)\left(1 - 2k + 2k^2 - k^3\right) + \frac{1}{48}\left(14k - 8k^2 - 2k^3\right)$. In turn, it is easy to see that for $\forall k \in [0, 1]$, $1 - 2k + 2k^2 - k^3 \geq 0$ and $14k - 8k^2 - 2k^3 > 0$. Therefore, we have proven that $\Delta E\left(y_p\right) - \Delta E\left(y_i\right) \geq 0$, for $\forall k \in [0, 1]$. We have already proven that $\Delta E\left(y_p\right)$ and $\Delta E\left(y_i\right)$ are both increasing functions for $\forall k \in [0, 1]$. Therefore, $k_2 < k_3$.

Q.E.D.

Proof of Proposition 5.

1. From Lemma 1 and Lemma 2, $E\left(y_p^{CPC}\right) = \frac{1}{12}\left(1 - r\sigma^2\right)$ and $E\left(y_p^{CPA}\right) = \frac{15k - 6k^2 - k^3}{48}$. In addition, $\frac{\partial\left(E\left(y_p^{CPA}\right) - E\left(y_p^{CPC}\right)\right)}{\partial r} = \frac{1}{12}\sigma^2 \geq 0$ and $\frac{\partial\left(E\left(y_p^{CPA}\right) - E\left(y_p^{CPC}\right)\right)}{\partial \sigma^2} = \frac{1}{12}r \geq 0$.

2. From Lemma 1 and Lemma 2, $E\left(y_i^{CPC}\right) = \frac{1}{6}\left(1 - r\sigma^2\right)$ and $E\left(y_i^{CPA}\right) = \frac{1}{2}\left(1 - r\sigma^2\right) \cdot \left(\frac{2k - 2k^2 + k^3}{6}\right) + \left(\frac{k + 2k^2 + k^3}{48}\right)$. In addition, $\frac{\partial\left(E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right)\right)}{\partial r} = -\frac{1}{2}\sigma^2\left(\frac{2k - 2k^2 + k^3}{6}\right) + \frac{1}{6}\sigma^2$, and $\frac{\partial\left(E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right)\right)}{\partial \sigma^2} = -\frac{1}{2}r\left(\frac{2k - 2k^2 + k^3}{6}\right) + \frac{1}{6}r$. It is easy to prove that $\frac{2k - 2k^2 + k^3}{6} < \frac{1}{3}$, for $\forall k \in [0, 1]$. That is, we have proven that $\frac{\partial\left(E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right)\right)}{\partial r} \geq 0$, and $\frac{\partial\left(E\left(y_i^{CPA}\right) - E\left(y_i^{CPC}\right)\right)}{\partial \sigma^2} \geq 0$.

Q.E.D.

Proof of Proposition 6.

First note that advertiser $i's$ payoff, if it wins the auction is unchanged. Using the result from Lemma 2, advertisers bid $b(m_i, \rho_i) = \frac{m_i}{\rho_i}$. Let $\frac{m_D}{\alpha} = n_D$ and $\frac{m_B}{\beta} = n_B$ equal the bids from the direct selling and branding advertisers, respectively.

Case (i): The publishers has correct signals of both advertisers' types. It uses this to adjust each advertiser's bid to calculate the expected revenue by multiplying α to type D advertiser's bid n_D and multiplying β to type B advertiser's bid n_B. The probability that type B advertiser wins the auction $(E\left[\Pr(w = B)\right]^{CPA})$ is

$$\Pr[\beta n_B \geq \alpha n_D] = \int_0^{\frac{1}{\alpha}} \int_{\frac{\alpha}{\beta}n_D}^{\frac{1}{\beta}} \alpha\beta \, dn_B \, dn_D = \frac{1}{2}.$$

Case (ii): The publishers has incorrect signals of both advertisers' types. It uses this to adjust each advertiser's bid to calculate the expected revenue by multiplying β to the type D advertiser's bid n_D and multiplying α to the type B advertiser's bid n_B. The probability that type B advertiser wins the auction $(E\left[\Pr(w = B)\right]^{CPA})$ is

$$\Pr[\alpha n_B \geq \beta n_D] = \int_0^{\frac{1}{\alpha}} \int_{\frac{\beta}{\alpha^2}}^{\frac{1}{\beta}} \alpha\beta \, dn_B \, dn_D + \int_0^{\frac{1}{\alpha}} \int_{n_D}^{\frac{\beta}{\alpha^2}} \alpha\beta \, dn_B \, dn_D = 1 - \frac{\beta^2}{2\alpha^2}.$$

Thus, the expected probability that type B advertiser wins the acution in the CPA model is

$$E\left[\Pr(w = B)\right]^{CPA} = \frac{1}{2}\phi + (1 - \frac{\beta^2}{2\alpha^2})(1 - \phi) = 1 - \frac{1}{2}\left[\phi + (1 - \phi)\left(\frac{\beta}{\alpha}\right)^2\right].$$

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Because $\frac{\beta}{\alpha} \leq 1$, we have $\phi + (1 - \phi)\left(\frac{\beta}{\alpha}\right)^2 \leq 1$. Therefore, $E\left[\Pr(w = B)\right]^{CPC} = \frac{1}{2} \leq 1 - \frac{1}{2}\left[\phi + (1 - \phi)\left(\frac{\beta}{\alpha}\right)^2\right] = E\left[\Pr(w = B)\right]^{CPA}$. **Q.E.D.**
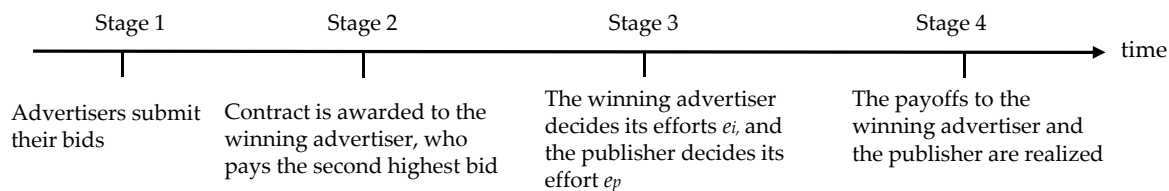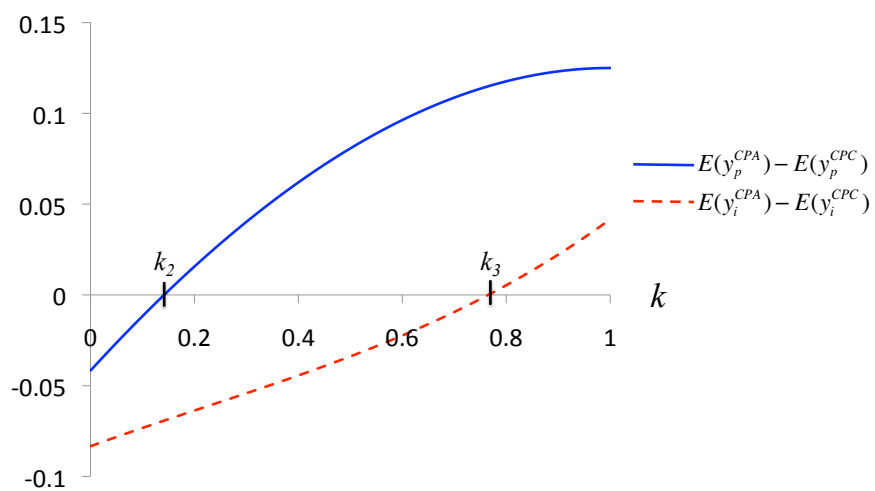
Figure 2: Timeline of the game

Stage 1	Stage 2	Stage 3	Stage 4	time
Advertisers submit their bids	Contract is awarded to the winning advertiser, who pays the second highest bid	The winning advertiser decides its efforts e_i, and the publisher decides its effort e_p	The payoffs to the winning advertiser and the publisher are realized	

Figure 3: The publisher's and advertisers' payoffs

$E(y_p^{CPA}) - E(y_p^{CPC})$

$E(y_i^{CPA}) - E(y_i^{CPC})$

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