GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Lecture 11

The Quark model

WS2010/11: ‚Introduction to Nuclear and Particle Physics'

Quark model

The quark model is a classification scheme for hadrons in terms of their valence quarks — the quarks and antiquarks which give rise to the quantum numbers of the hadrons.

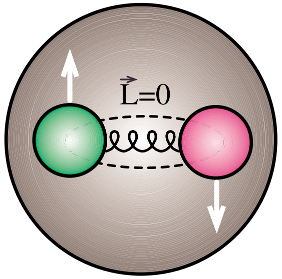
The quark model in its modern form was developed by Murray Gell-Mann - american physicist who received the 1969 Nobel Prize in physics for his work on the theory of elementary particles. He is currently the Presidential Professor of Physics and Medicine at the University of Southern California.

1929 (age 81)

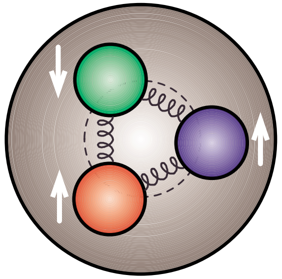
- Hadrons are not ‚fundamental', but they are built from ‚valence quarks', i.e. quarks and antiquarks, which give the quantum numbers of the hadrons

$|\text{Baryon}\rangle = |qqq\rangle \quad |\text{Meson}\rangle = |q\bar{q}\rangle$

q= quarks, $\bar{q}$ – antiquarks



Meson ($q\bar{q}$)



Baryon (qqq)

Quark quantum numbers

The quark quantum numbers:

- **flavor (6): u (up-), d (down-), s (strange-), c (charm-), t (top-), b(bottom-) quarks**

  anti-flavor for anti-quarks $\bar{q}$: $\bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{t}, \bar{b}$

- **charge:** $Q = -1/3, +2/3$
- **baryon number: B=1/3 - as baryons are made out of three quarks**
- **spin: s=1/2 - quarks are the fermions!**
- **strangeness:** $S_s = -1, \quad S_{\bar{s}} = 1, \quad S_q = 0 \quad \text{for} \quad q = u,d,c,t,b \ (\text{and } \bar{q})$
- **charm:** $C_c = 1, \quad C_{\bar{c}} = -1, \quad C_q = 0 \quad \text{for } q = u,d,s,t,b \ (\text{and } \bar{q})$
- **bottomness:** $B_b = -1, \quad B_{\bar{b}} = 1, \quad B_q = 0 \quad \text{for} \quad q = u,d,s,c,t \ (\text{and } \bar{q})$
- **topness:** $T_t = 1, \quad T_{\bar{t}} = -1, \quad T_q = 0 \quad \text{for } q = u,d,s,c,b \ (\text{and } \bar{q})$

Quark quantum numbers

The quark quantum numbers:

hypercharge: $Y = B + S + C + \mathcal{B} + T$ (1)

(= baryon charge + strangeness + charm + bottomness +topness)

- I_3 (or I_z or T_3) - 3'd component of isospin

charge (Gell-Mann–Nishijima formula):

$$Q = I_3 + Y/2 \quad (2)$$

(= 3'd component of isospin + hypercharge/2)

Quark quantum numbers

Property \ Quark	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	$+1$	0	0
B – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	$+1$

Quark quantum numbers

The quark model is the follow-up to the Eightfold Way classification scheme (proposed by Murray Gell-Mann and Yuval Ne'eman)

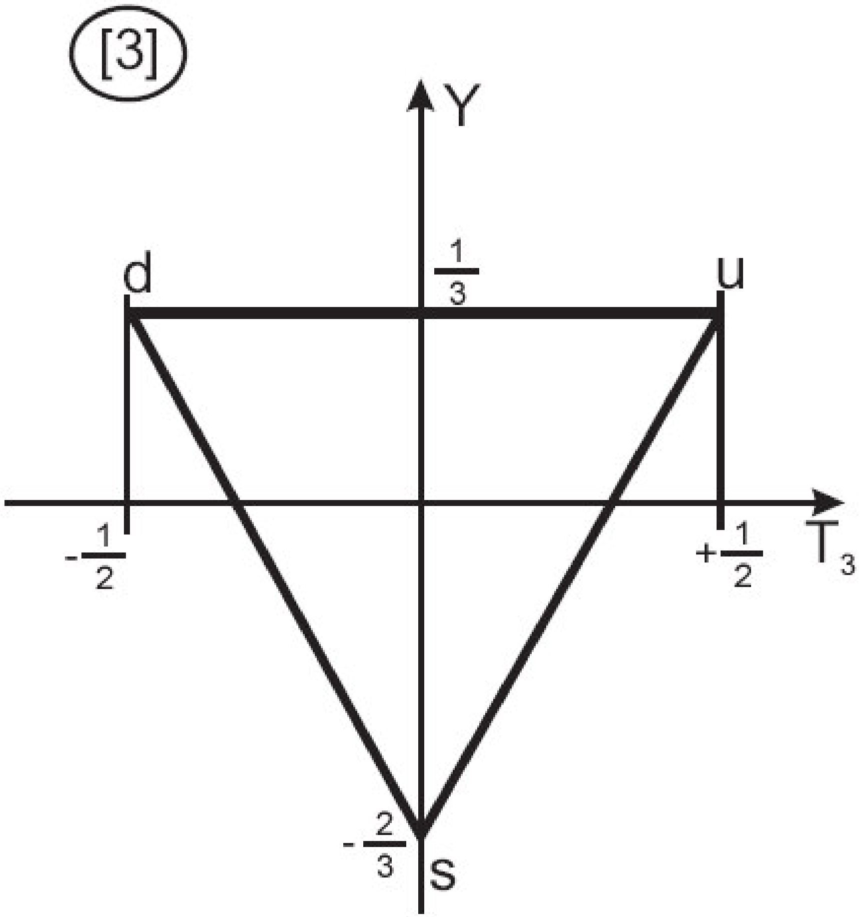
The Eightfold Way may be understood as a consequence of flavor symmetries between various kinds of quarks.
Since the strong nuclear force affects quarks the same way regardless of their flavor, replacing one flavor of a quark with another in a hadron should not alter its mass very much.
Mathematically, this replacement may be described by elements of the SU(3) group.

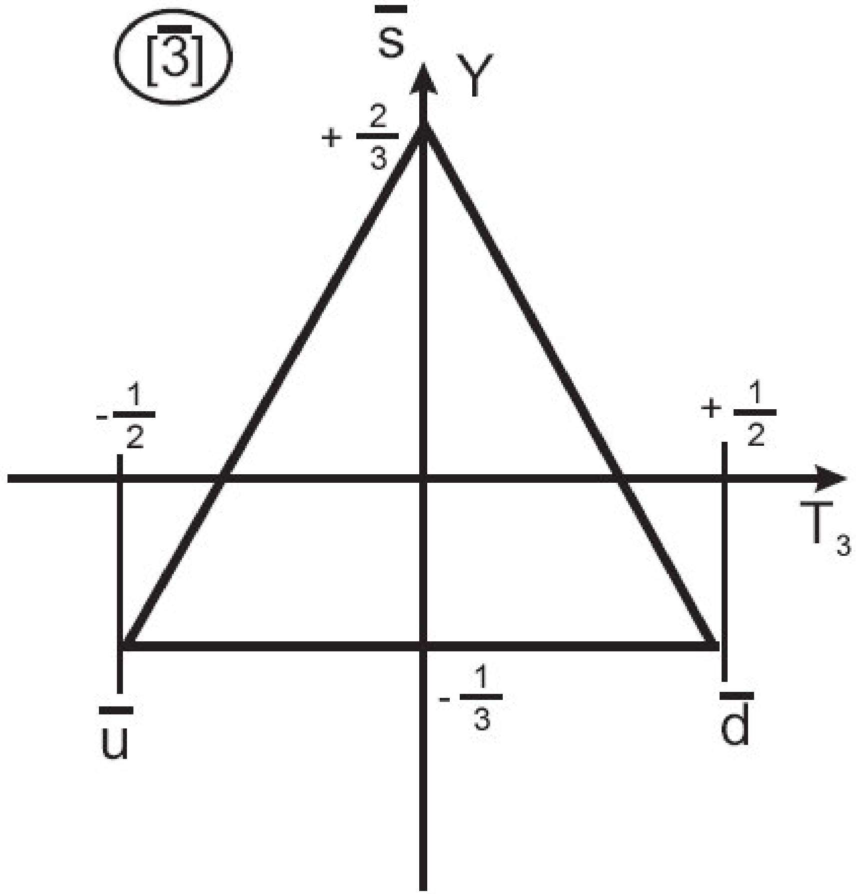
Consider u, d, s quarks :
➔ then the quarks lie in the fundamental representation, 3
(called the triplet) of the flavour group SU(3) : [3]
The antiquarks lie in the complex conjugate representation 3 : $[\bar{3}]$

Quark quantum numbers

triplet in $SU(3)_{flavor}$ group: [3]

anti-triplet in $SU(3)_{flavor}$ group: $[\bar{3}]$



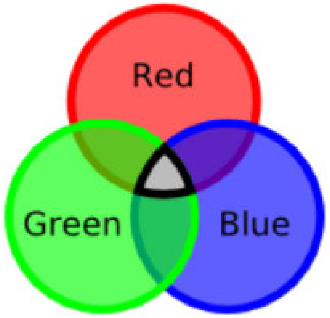


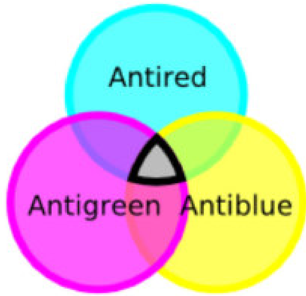
Quark quantum numbers

The quark quantum numbers:

- **Collor 3: red, green and blue ➔ triplet in SU(3)$_{collor}$ group: [3]**

Anticollor 3: antired, antigreen and antiblue ➔ anti-triplet in SU(3)$_{collor}$ group [$\bar{3}$]





- **The quark colors (red, green, blue) combine to be colorless**
- **The quark anticolors (antired, antigreen, antiblue) also combine to be colorless**

All hadrons ➔ color neutral = color singlet in the SU(3)$_{collor}$ group

History: The quantum number ‚color' has been introduced (idea from Greenberg, 1964) to describe the state Δ^{++}(uuu) (Q=+2, J=3/2) , discovered by Fermi in 1951 as π^+p resonance: $\Delta^{++}(uuu) \rightarrow p(uud) + \pi^+(\bar{d}u)$
The state $\Delta^{++}(u\uparrow u\uparrow u\uparrow)$ **with all parallel spins (to achieve J=3/2) is forbidden according to the Fermi statistics (without color) !**

Quark quantum numbers

The current quark masses:

- masses of the quarks

$m_u = 1.7 - 3.3$ MeV/c^2

$m_d = 4.1 – 5.8$ MeV/c^2

$m_s = 70 – 130$ MeV/c^2

$m_c = 1.1 –1.4$ GeV/c^2

$m_b = 4.1 - 4.4$ GeV/c^2

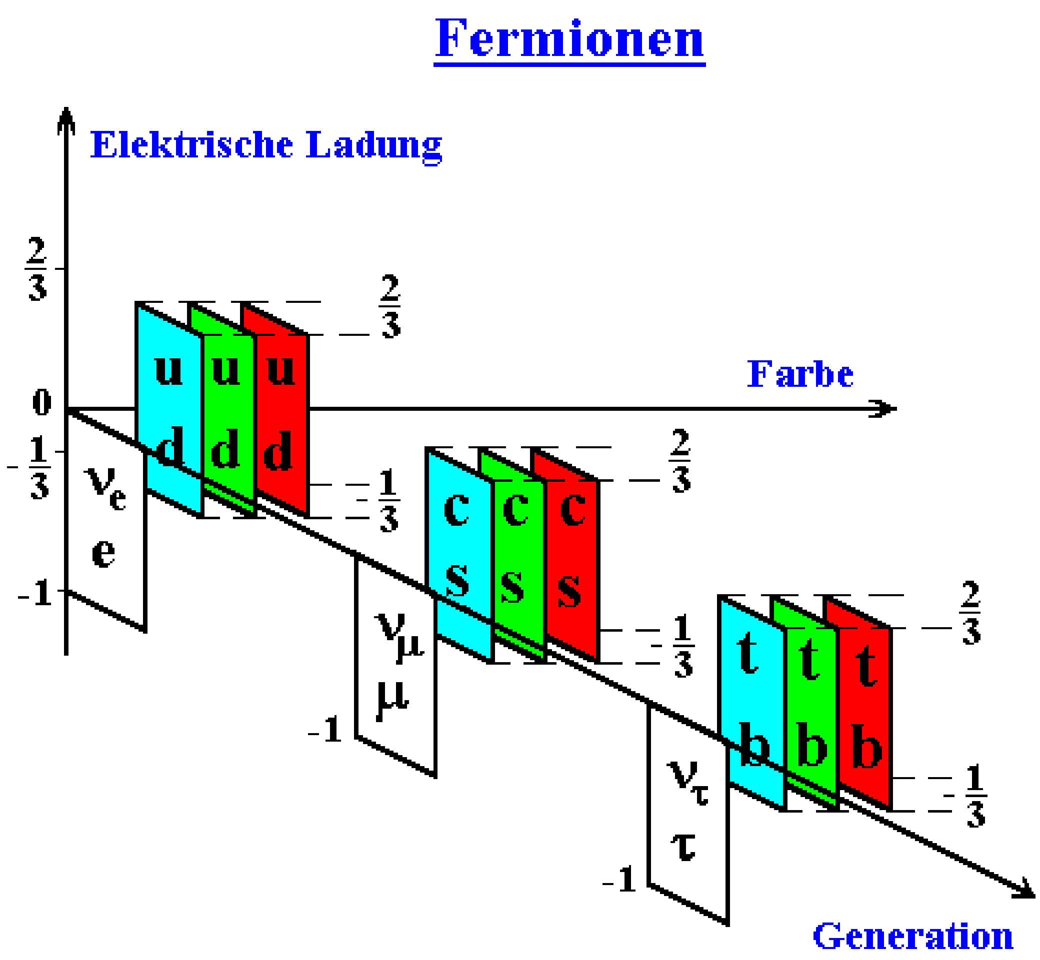
$m_b \sim 180$ GeV/c^2

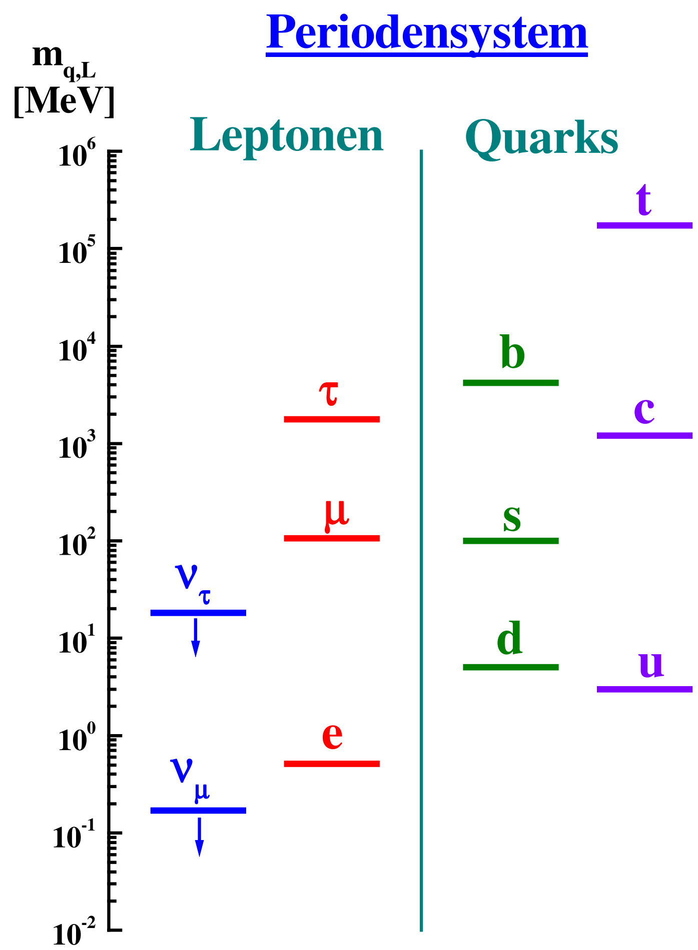
The current quark mass is also called the mass of the 'naked‘ (‚bare‘) quark.

Note: the constituent quark mass is the mass of a 'dressed' current quark, i.e. for quarks surrounded by a cloud of virtual quarks and gluons:

$M_{u(d)}^* \sim 350$ MeV/c^2

Building Blocks of Matter





Hadrons in the Quark model

Gell-Mann (1964): Hadrons are not ‚fundamental', but they are built from ‚valence quarks',

$$|\,\mathrm{Baryon}\,\rangle = |\,qqq\,\rangle \qquad |\,\mathrm{Meson}\,\rangle = |\,q\bar{q}\,\rangle \tag{3}$$

Baryon charge: $B_B = 1$ $\qquad$ $B_m = 0$

Constraints to build hadrons from quarks:

- **strong color interaction (red, green, blue)**
- **confinement**
- **quarks must form color-neutral hadrons**

- **State function for baryons – antisymmetric under interchange of two quarks**

$$\Psi_A \equiv |\,qqq\,\rangle_A = [\,|\,\mathrm{color}\rangle \otimes |\,\mathrm{space}\rangle \otimes |\,\mathrm{spin}\rangle \otimes |\,\mathrm{flavor}\,\rangle]_A \tag{4}$$

Since all hadrons are color neutral, the color part of Ψ_A must be antisymmetric, i.e. a $SU(3)_{color}$ singlet

$$\Psi_A \equiv |\,qqq\,\rangle_A = \underline{|\,\mathrm{color}\rangle}_A \otimes \underbrace{[\,|\,\mathrm{space}\rangle \otimes |\,\mathrm{spin}\rangle \otimes |\,\mathrm{flavor}\,\rangle]_S}_{\text{symmetric}} \tag{5}$$

Hadrons in Quark model

➔ Possible states Ψ_A:

$$\Psi_A = |\text{color}\rangle_A \otimes [|\text{space}\rangle_S \otimes |\text{spin}\rangle_A \otimes |\text{flavor}\rangle_A]_S \quad (6)$$

$$[|\text{space}\rangle_S \otimes |\text{spin}\rangle_S \otimes |\text{flavor}\rangle_S]_S \quad (7)$$

or a linear conbination of (6) and (7):

$$\begin{aligned}\Psi_A &= \alpha\ |\text{color}\rangle_A \otimes [|\text{space}\rangle_S \otimes |\text{spin}\rangle_A \otimes |\text{flavor}\rangle_A]_S \\ &+ \beta\ |\text{color}\rangle_A \otimes [|\text{space}\rangle_S \otimes |\text{spin}\rangle_S \otimes |\text{flavor}\rangle_S]_S\end{aligned} \quad (8)$$

where $\alpha^2 + \beta^2 = 1$

- Consider flavor space (u,d,s quarks) ➔ $SU(3)_{flavor}$ group

Possible states: |flavor> : (6) – antisymmetric
for baryons (7) – symmetric
(8) – mixed symmetry

Mesons in the Quark model

$$|\text{Meson}\rangle = |q\bar{q}\rangle$$

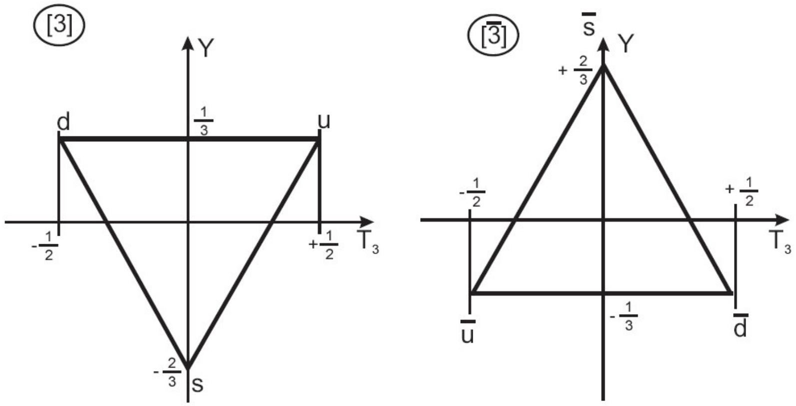
Quark
triplet in $SU(3)_{flavor}$ group: [3]

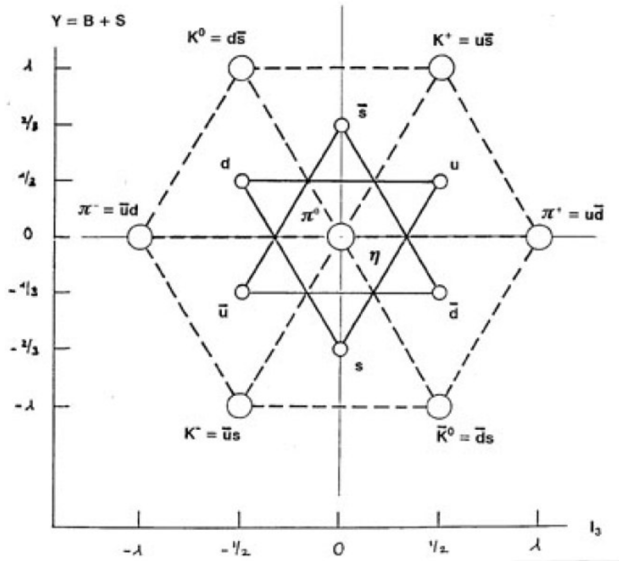
Anti-quark
anti-triplet in $SU(3)_{flavor}$ group: $[\bar{3}]$

From group theory: the nine states (nonet) made out of a pair can be decomposed into the trivial representation, 1 (called the singlet), and the adjoint representation, 8 (called the octet).

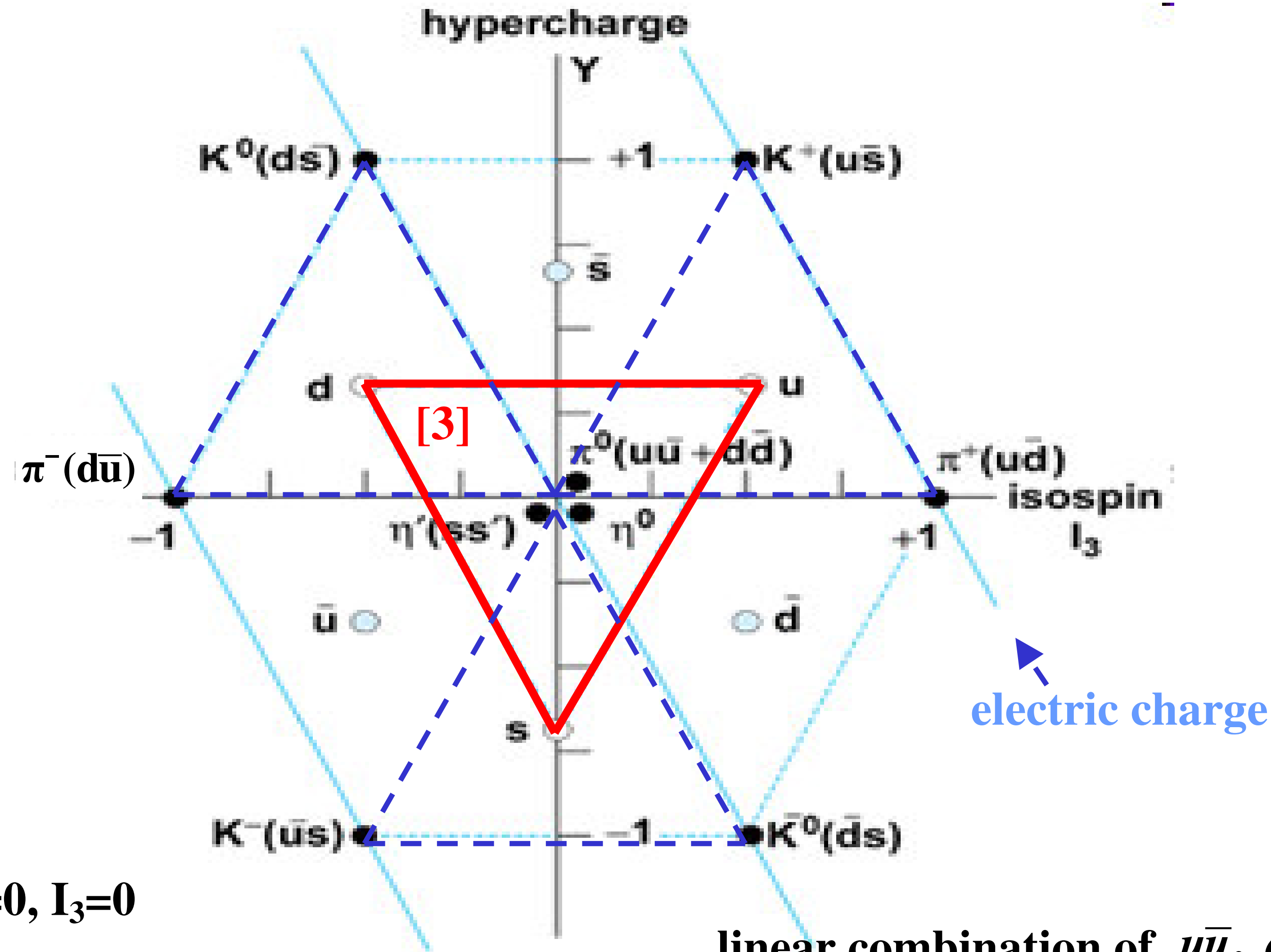
$$[3] \otimes [\bar{3}] = [8] \oplus [1]$$

octet + singlet

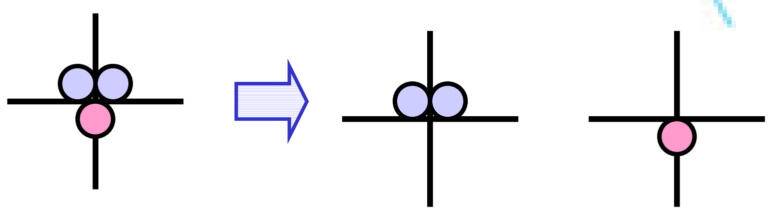




Mesons in the Quark model



3 states: $Y=0$, $I_3=0$



A,B,C: in octet: A,B singlet state C

linear combination of $u\bar{u},\ d\bar{d},\ s\bar{s}$

$$C=\sqrt{\frac{1}{3}}(u\bar{u}+d\bar{d}+s\bar{s})$$

$$A=\sqrt{\frac{1}{2}}(u\bar{u}-d\bar{d}),\quad B=\sqrt{\frac{1}{6}}(u\bar{u}+d\bar{d}-2s\bar{s})$$

Mesons in the Quark model

Classification of mesons:

- **Quantum numbers:**
  - spin *S*
  - orbital angular momentum *L*
  - total angular momentum *J*=*L*+*S*

- **Properties with respect to Poincare transformation:**

1) continuos transformation ➔ Lorentz boost (3 parameters: β)

$U_B \sim e^{i\vec{\beta}\vec{\alpha}}$ Casimir operator (invariant under transformation): $M^2 = p_\mu p^\mu$

2) rotations (3 parameters: Euler angle φ) :

$U_R \sim e^{i\vec{\varphi}\vec{J}}$ Casimir operator: J^2

3) space-time shifts (4 parameters: a_μ)

$U_{st} \sim e^{i\alpha_\mu x^\mu}$ $x'_\mu \to x_\mu + a_\mu$

10 parameters
of Poincare group

Mesons in the Quark model

Classification of mesons:

❖ **Discrete operators:**

4) **parity transformation**: flip in sign of the spacial coordinate $\vec{r} = -\vec{r}$

eigenvalue $P = \pm 1$

$$P = (-1)^{L+1}$$

5) **time reversal**: $t \rightarrow -t$

eigenvalue $T = \pm 1$

6) **charge conjugation**: $C = -C$

C - parity: eigenvalue $C = \pm 1$

$$C = (-1)^{L+S}$$

⇨ **General PCT –theorem:**

$$P \cdot C \cdot T = 1$$

due to the fact that discrete transformations correspond to the U(1) group they are multiplicative .

Properties of the distinguishable (not continuum!) particles are defined by

$$M^2 (or\, M), \quad J^2 (or\, J), \quad P, \; C$$

Mesons in the Quark model

Classification of mesons:

If the quark–antiquark pair is in an orbital angular momentum L state, and has spin S, then

- $|L - S| \leq J \leq L + S$, where $S = 0$ or 1,

- $P = (-1)^{L+1}$, where the ‚+1' arises from the intrinsic parity of the quark–antiquark pair.

- $C = (-1)^{L+S}$ for mesons which have no flavor.

- For isospin $I = 1$ and 0 states, one can define a new multiplicative quantum number called the G-parity such that $G = (-1)^{I+L+S}$.

If $P = (-1)^J$, then it follows that $S = 1$, thus $PC = 1$.
States with these quantum numbers are called *natural parity states* while all other quantum numbers are called *exotic* (for example the state $J^{PC} = 0^{--}$).

Mesons in the Quark model

Classification of mesons:

the mesons are classified in J^{PC} multiplets !

1) **L=0 states**: J=0 or 1, i.e. J=S

$$P = (-1)^{L+1} = -1 \qquad C = (-1)^{L+S} = (-1)^{S} = \begin{cases} +1 \text{ for } S=0 \\ -1 \text{ for } S=1 \end{cases}$$

$$J^{PC} = \begin{cases} 0^{-+} & \text{- pseudoscalar states} \\ 1^{--} & \text{- vector states} \end{cases}$$

2) **L=1 states** - orbital exitations; $P = (-1)^{L+1} = +1$

$J=L+S$: $S=-1$ $J=0$ $J^{PC} = 0^{++}$ - scalar states

$S=0$ $J=1$ 1^{++} - axial vectors

1^{+-} - axial vectors

$S=1$ $J=2$ 2^{++} - tensor

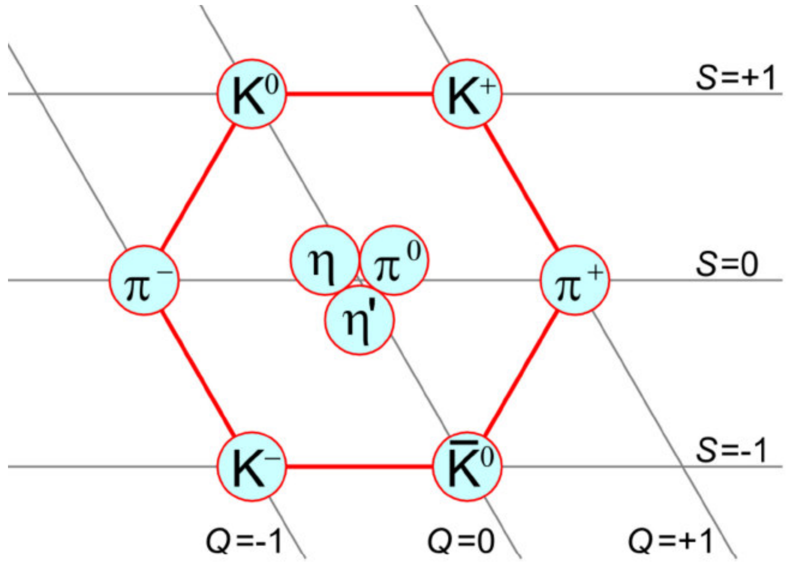
Mesons in the Quark model

			isospin			
L	S	J^{PC}	$I=1$	$I=1/2$	$I=0$	$m\ [MeV]$
$L=0$	$S=0$	0^{-+}	π	K	η,η'	$140\ (m_\pi)-500$
	$S=1$	1^{--}	ρ	K^*	ω,φ	~ 800
$L=1$	$S=0$	1^{+-}	B	Q_2	H	1250
	$S=1$	2^{++}	A_2	K'^*	f,f'	1400
		1^{++}	A_1	Q_1	D	1300
		0^{++}	δ	κ	ε, S^*	1150

Mesons in the Quark model

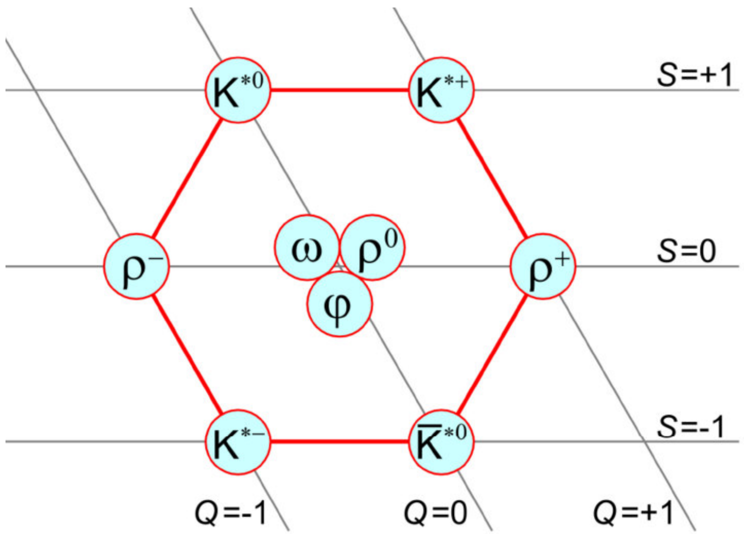
$J^{PC} = 0^{-+}$ - pseudoscalar nonet

particle	symbol	$Mass(MeV)$	$Isospin, I$	I_z
π^+	$u\bar{d}$	140	1	1
π^-	$d\bar{u}$		1	-1
π^0	$(u\bar{u}-d\bar{d})/\sqrt{2}$	135	1	0
K^+	$u\bar{s}$	494	1/2	1/2
K^-	$s\bar{u}$	494	1/2	$-1/2$
K^0	$d\bar{s}$	"	1/2	$-1/2$
$\overline{K^0}$	$s\bar{d}$	"	1/2	$1/2$
η	$(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$	548	0	0
η'	$(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$	958	0	0



$J^{PC} = 1^{--}$ - vector nonet

particle	symbol	$Mass(MeV)$	$Isospin, I$	I_z
ρ^+	$u\bar{d}$	770	1	1
ρ^-	$d\bar{u}$	"	1	-1
ρ^0	$(u\bar{u}-d\bar{d})/\sqrt{2}$	"	1	0
K^{*+}	$u\bar{s}$	892	1/2	1/2
K^{*-}	$s\bar{u}$	"	1/2	$-1/2$
K^{*0}	$d\bar{s}$	"	1/2	$-1/2$
$\overline{K^{*0}}$	$s\bar{d}$	"	1/2	$1/2$
ω	$(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}$	782	0	0
ϕ	$(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$	1020	0	0



Baryons in the Quark model

$$|\text{Baryon}\rangle = |qqq\rangle$$

Quark
triplet in SU(3)$_{flavor}$ group: [3]

Eqs. (4-8): state function for baryons – antisymmetric under interchange of two quarks

From group theory: with three flavours, the decomposition in flavour is

$$[3] \otimes [3] \otimes [3] = ([6]_S \oplus [\bar{3}]_A) \otimes [3] =$$

$$= ([6]_S \otimes [3]) \oplus ([\bar{3}] \otimes [3]) =$$

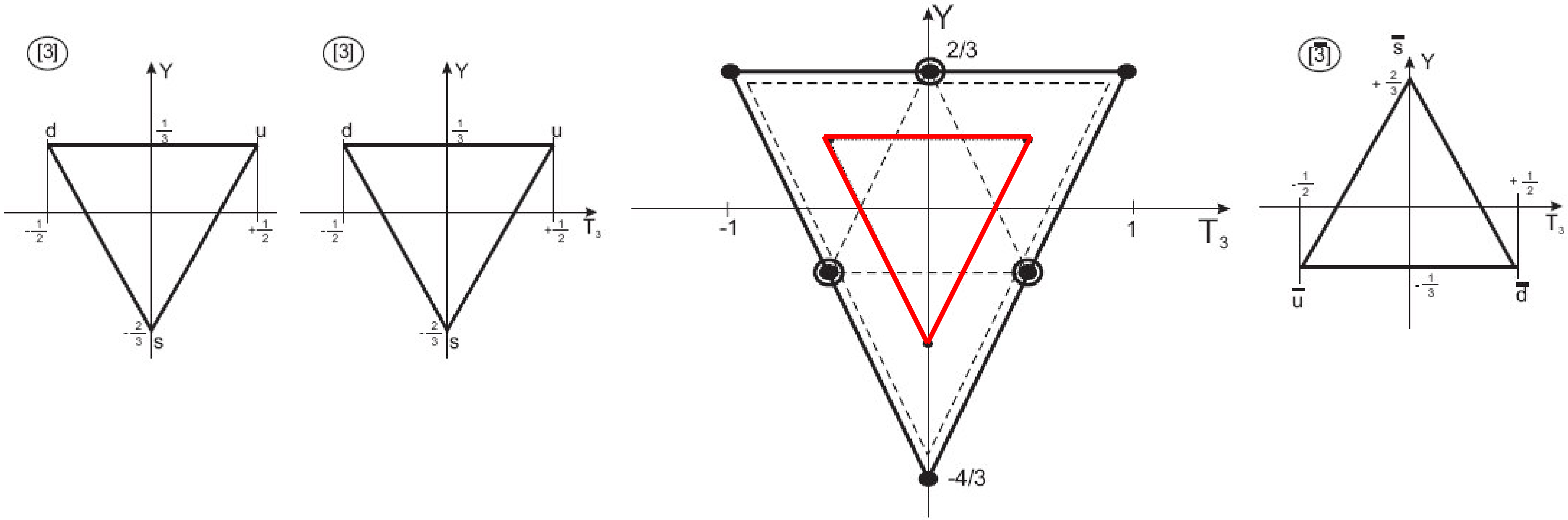
$$= [10]_S \oplus [8]_M \oplus [8]_M \oplus [1]_A$$

The decuplet is symmetric in flavour, the singlet antisymmetric and the two octets have mixed symmetry (they are connected by a unitary transformation and thus describe the same states).
The space and spin parts of the states are then fixed once the orbital angular momentum is given.

Baryons in the Quark model

1) Combine first 2 quark triplets:

$$[3] \otimes [3] = [6]_S \oplus [\bar{3}]_A$$



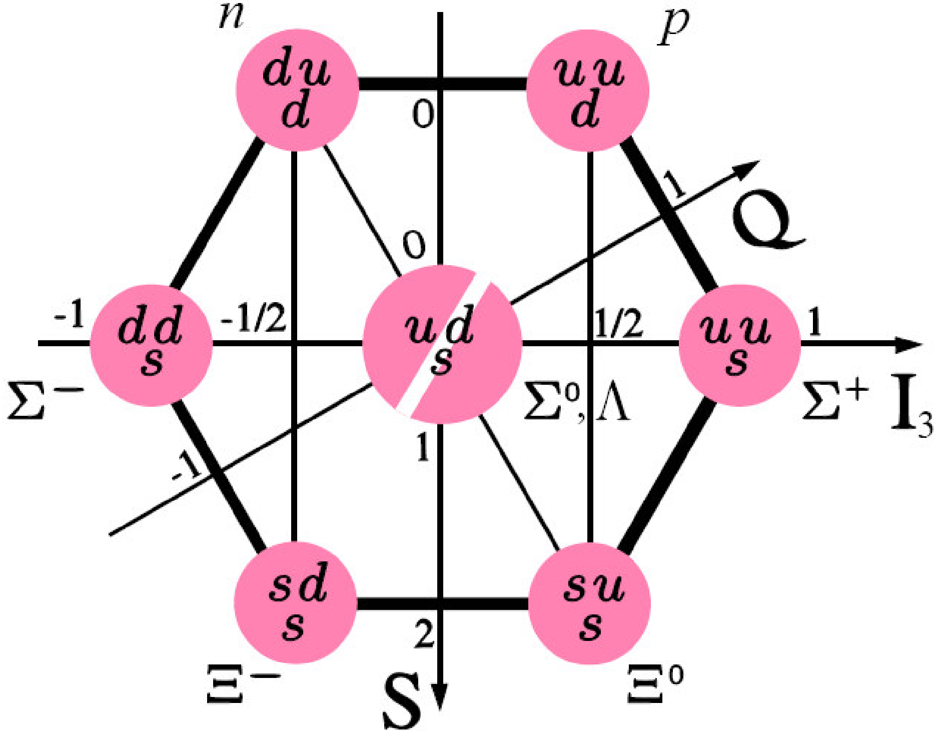
2) Add a 3'd quark:

$$[3] \otimes [3] \otimes [3] = ([6]_S \oplus [\bar{3}]_A) \otimes [3] =$$

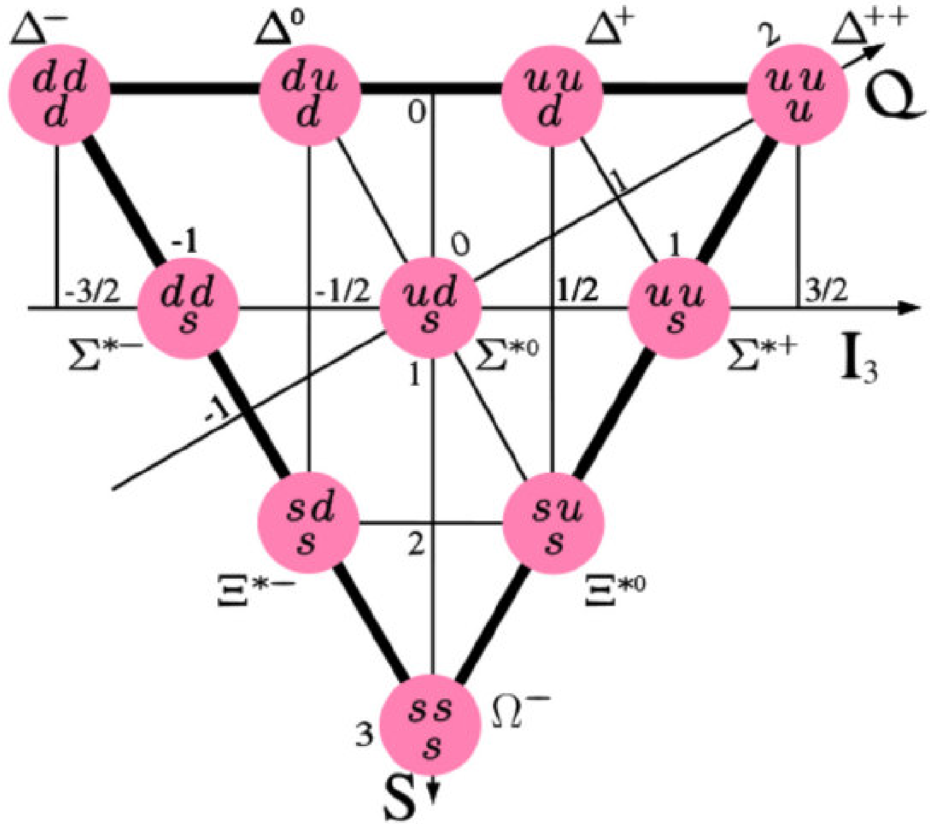
$$= [10]_S \oplus \boxed{[8]_M \oplus [8]_M} \oplus [1]_A$$

Baryons in the Quark model

Octet [8]



Decuplet [10]



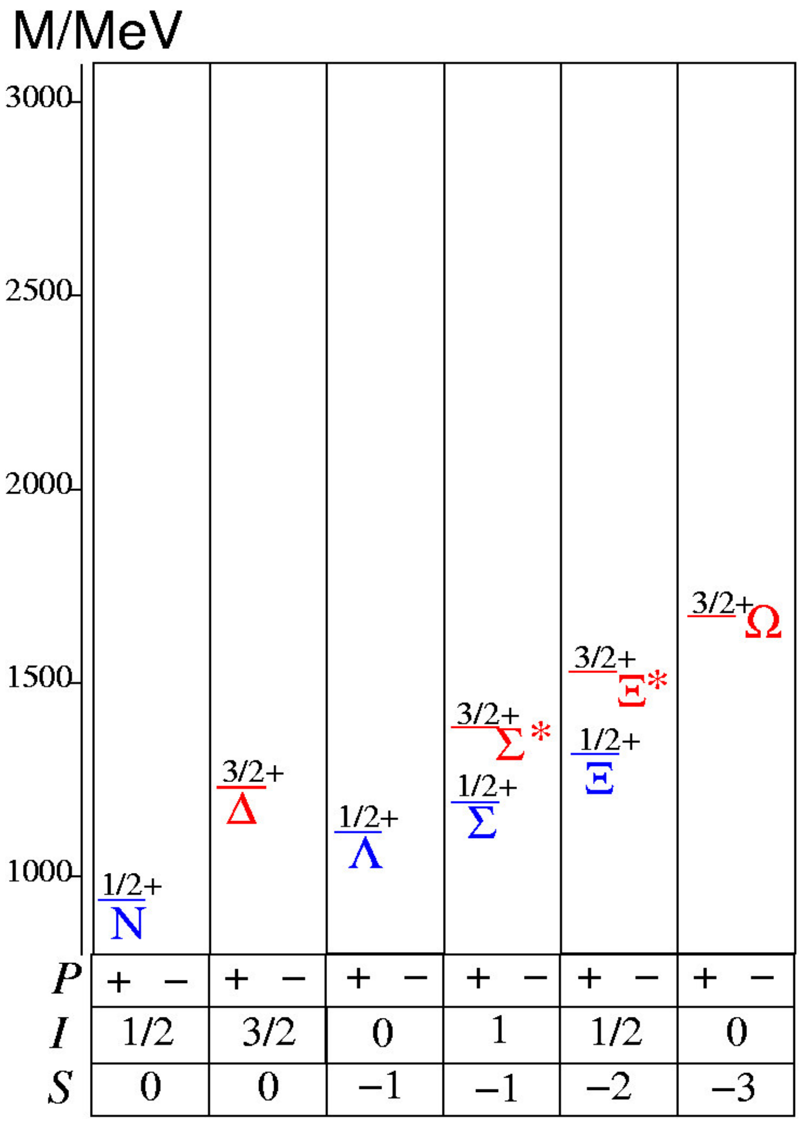
Spin:
J=S

$$J^P = \frac{1}{2}^+$$

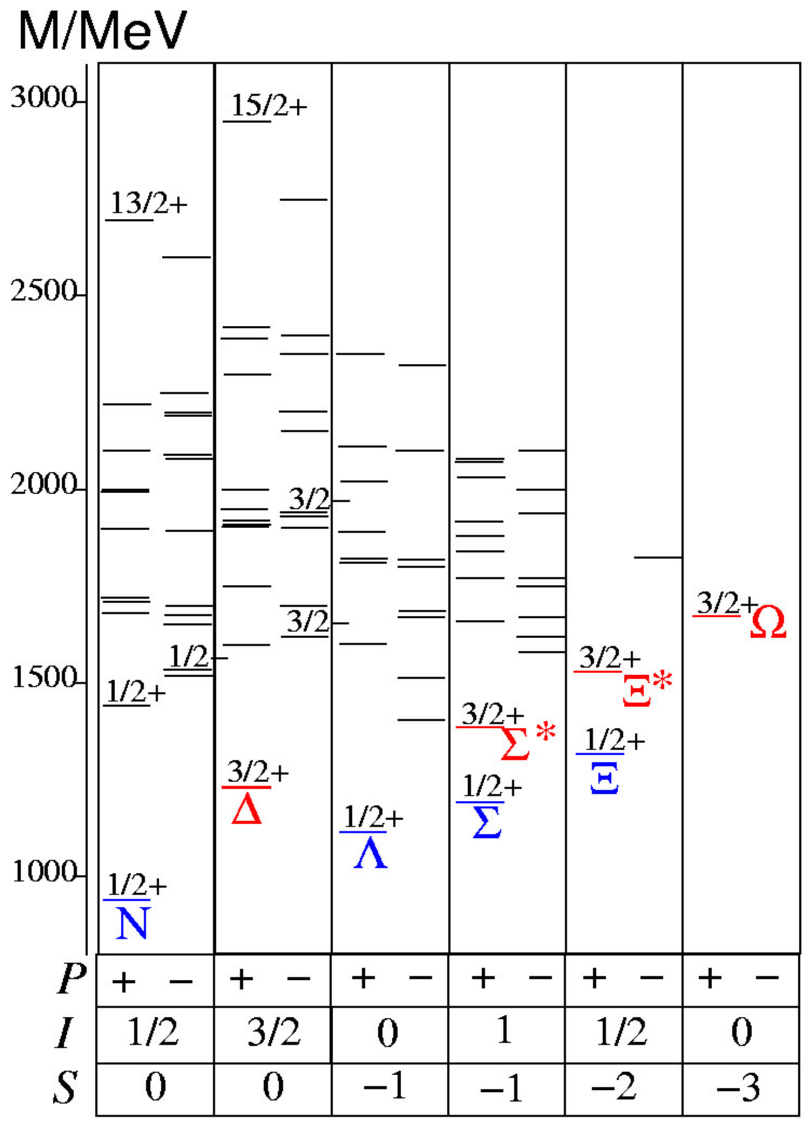
$$J^P = \frac{3}{2}^+$$

Structure of known baryons

Ground states of Baryons



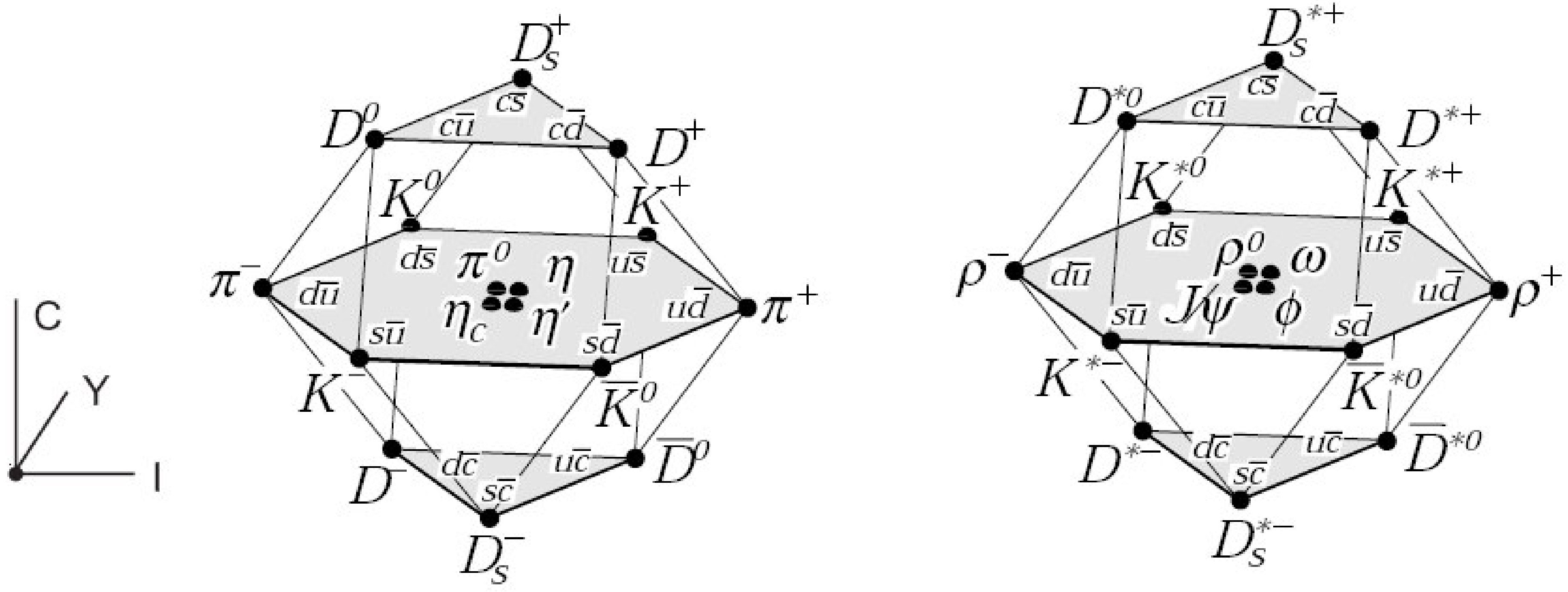
+ exitation spectra



Mesons in the SU(4) flavor Quark model

Now consider the basis states of meons in 4 flavour $SU(4)_{flavor}$: u, d, s, c quarks

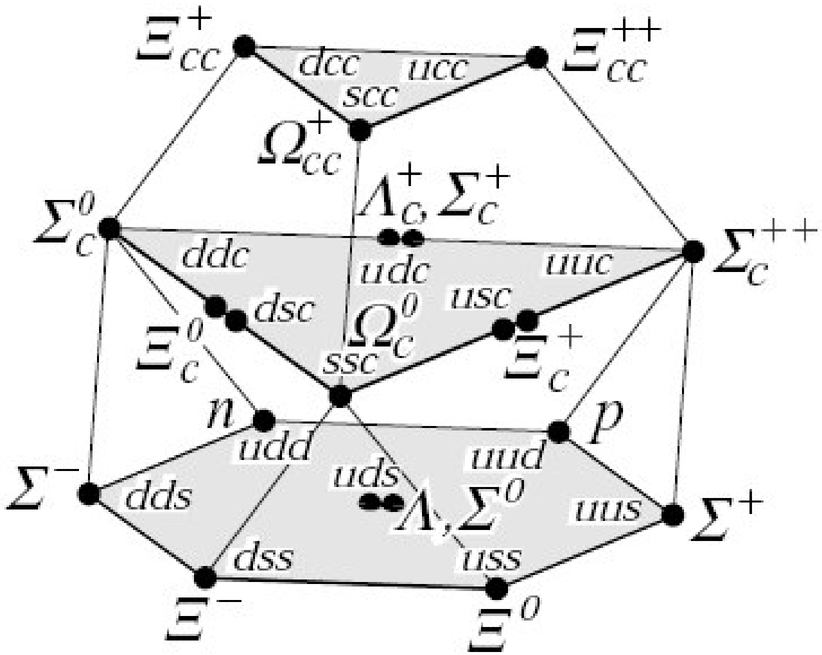
$$[4] \otimes [\bar{4}] = [15] \oplus [1]$$

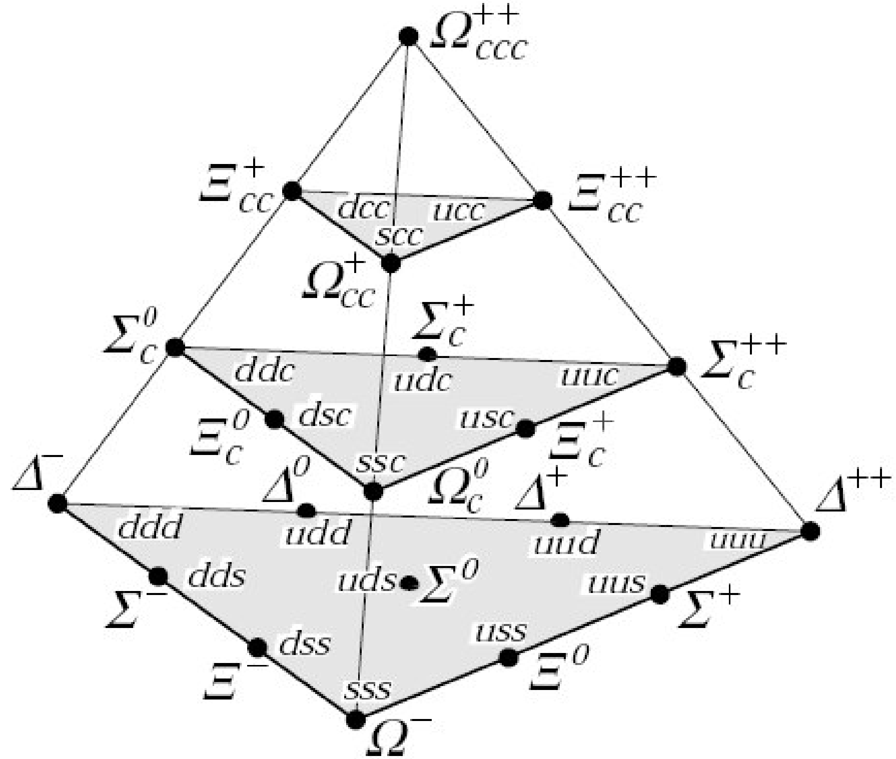


SU(4) weight diagram showing the 16-plets for the pseudoscalar and vector mesons as a function of isospin I, charm C and hypercharge Y. The nonets of light mesons occupy the central planes to which the cc states have been added.

Baryons in the SU(4) flavor Quark model

Now consider the basis states of baryons in 4 flavour $SU(4)_{flavor}$: u, d, s, c quarks





SU(4) multiplets of baryons made of u, d, s, and c quarks:

the 20-plet with an SU(3) octet and the 20-plet with an SU(3) decuplet.

Exotic states

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A SCHEMATIC MODEL OF BARYONS AND MESONS *

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If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

Even if we consider the scattering amplitudes of strongly interacting particles on the mass shell only and treat the matrix elements of the weak, electromagnetic, and gravitational interactions by means

ber $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q\,q\,q)$, $(q\,q\,q\,q\,\bar{q})$, etc., while mesons are made out of $(q\,\bar{q})$, $(q\,q\,\bar{q}\,\bar{q})$, etc. It is assuming that the lowest baryon configuration $(q\,q\,q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q\,\bar{q})$ similarly gives just **1** and **8**.

Exotic states

$$|\text{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}g\rangle + \ldots$$

$$|\text{Baryon}\rangle = |qqq\rangle + |qqqq\bar{q}\rangle + |qqqg\rangle + \ldots$$

$$|\text{Hybrid}\rangle = |q\bar{q}g\rangle + \ldots$$

$$|\text{Baryonium}\rangle = |q\bar{q}q\bar{q}\rangle + \ldots$$

$$|\text{Glueball}\rangle = |gg\rangle + \ldots$$

Experimental evidence:

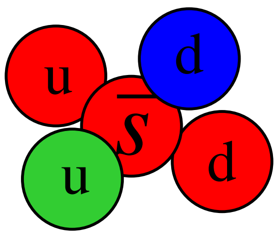
π(1400)

σ(600)

fo(1500)

||

very broad width (200-300 MeV) => short lifetime < 1 fm/c



$$|\text{Pentaquark}\rangle = |qqqq\bar{q}\rangle + \ldots$$