SU(5) Grand Unified Theory

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Plan

The Standard Model

- Particles.
- Lagrangian.
- Spontaneous symmetry breaking.

Towards Unification

SU(5) Grand Unified Theory

-SU(5) motivation.
-SU(5) Generators.
-SU(5) Fermions Content
-Gauge bosons.
-Interactions.
-Spontaneous symmetry breaking Of SU(5)

The Standard Model

The Way where we are

The S.M. is a theory based on the symmetry group $SU_c(3) \times SU_L(2) \times U_Y(1)$

Fermions Content

	$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	e_R	: $\begin{pmatrix} d \\ u \end{pmatrix}_L^{\alpha}$	u^{α}_R	d^{α}_R
$SU_c(3)$:	1_c	1_c	3_c	3_c	3_c
$SU_L(2)$:	2_L	1_L	2_L	1_L	1_L
$SU_Y(1)$:	-1	-2	1/3	4/3	-2/3

Where $\alpha = 1,2,3 = N_c$.
The electromagnetic charge Q_{em}

$$Q_{em} = T_{3L} + Y/2$$

Considering only the first fermions family

Lagrangian

Consider only $SU_L(2) \times U_Y(1)$ electroweak model & Ignore $Su_c(3)$ For the time being

$$L = L_{\text{G. bosons}} + L_{fermions} + L_{\Phi} + L_Y$$

Where:

$$L_{G\ bosons} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} F^i_{\mu\nu} F_i^{\mu\nu}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\,\epsilon^{ijk} A^j_\mu A^k_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

The symmetry invariance holds under the following transformations :

$$\frac{\vec{\tau}\cdot\vec{A}_\mu}{2} \longrightarrow U(\theta)\frac{\vec{\tau}\cdot\vec{A}_\mu}{2}U(\theta)^{-1} - \frac{i}{g}\left[\partial_\mu U(\theta)\right]U(\theta)^{-1}$$

Transformation law for the- SU(2) gauge field

$$U(\theta) = e^{-i\,\frac{\vec{\tau}}{2}.\vec{\theta}(x)} \qquad SU(2)$$

Gives for $\theta(x) \ll 1$

$$A_\mu^{i\prime} = A_\mu^i - \frac{1}{g}\partial_\mu\theta^i + \underbrace{\epsilon_{ijk}\theta^j A_\mu^k}$$

The second term is the transformation law for a triplet under SU(2), so that A^i_μ s (i=1,2,3) carry charges.

$$\frac{\vec{\tau}}{2}\cdot\vec{F}'_{\mu\nu} = U(\theta)\frac{\vec{\tau}}{2}\cdot\vec{F}_{\mu\nu}U(\theta)^{-1}$$

For small $\theta(x)$,

$$F_{\mu\nu}^{i}{}' = F_{\mu\nu}^i + \underbrace{g\epsilon_{ijk}\theta^j F_{\mu\nu}^k}$$

Transformation law for SU(2) tensor

Similarly the transformation laws For U(1) gauge field (photon) and U(1) tensor:

$$B'_\mu = B_\mu + \frac{1}{g'}\partial_\mu\theta(x)$$

$$B'_{\mu\nu} = B_{\mu\nu}$$

Fermions part :

$$\mathcal{L}_f = \sum_f i\, \bar{f}\, \gamma_\mu\, D^\mu\, f$$

$$D_\mu = \partial_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{A}_\mu - ig'\frac{Y}{2}B_\mu$$

Invariant under

$U_Y(1)$:

$$f \rightarrow e^{-i\,g\,Y/2}\, f$$

$SU_L(2)$:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow e^{-i\,\frac{\vec{\tau}}{2}.\vec{\theta}(x)} \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$u_R \rightarrow u_R$$

$$d_R \rightarrow d_R$$

To be read as:

$$D_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L = \partial_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L - \left(ig\frac{\vec{\tau}}{2}\cdot\vec{A}_\mu + ig'\frac{1}{6}B_\mu \right) \begin{pmatrix} u \\ d \end{pmatrix}_L$$

$$D_\mu u_R = \partial_\mu u_R - ig'\frac{2}{3}B_\mu u_R$$

Spontaneous symmetry breaking

?

Symmetry { 4 massless gauge fields ! / all fermions massless ! → SSB

The electroweak symmetry is spontaneously broken by Higgs scalars

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$$

$\Phi : (1_c\, , 2_L\, , 1_Y)$

$$L_\Phi = \frac{1}{2}(D_\mu \Phi)^+ (D_\mu \Phi) - V(\Phi)$$

$$D_\mu \Phi = (\partial_\mu - \frac{i}{2}\, g' B_\mu - \frac{i}{2}\, g \vec{\tau} \cdot \vec{A_\mu})\, \Phi$$

$$V(\Phi) = -\frac{\mu^2}{2}\, \Phi^+ \Phi + \frac{\lambda}{4}\, (\Phi^+ \Phi)^2$$

$\mu^2 > 0$

The vacuum state, chosen to correspond to the vacuum expectation value (VEV.)

$$<\Phi>_0 = |<0|\Phi|0>| = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad with \qquad v = \left(\frac{\mu^2}{\lambda}\right)^{1/2}$$

Verifying That $SU_L(2) \times SU_Y(1) \xrightarrow[<\Phi>_0]{} SU_{em}(1)$

$$T_3 <\Phi>_0 = \frac{\tau_3}{2} <\Phi>_0 \neq 0$$

$$Y <\Phi>_0 = Y_\Phi <\Phi>_0 \neq 0$$

But

$$Q_{em} <\Phi>_0 = 0$$

Idea: the vacuum state
$<\Phi>_0$ is invariant under
a symmetry operation exp ($i\alpha G$)
corresponding to the generator G
if exp ($i\alpha G$) $<\Phi>_0 = <\Phi>_0$
exp ($i\alpha G$) $<\Phi>_0 = 0$.

The covariant derivative in L_Φ :

$$\frac{1}{2}(D_\mu \Phi)^+ (D_\mu \Phi)$$

Will yields the gauge bosons masses :

For Ex.: $$W^\pm = \frac{1}{\sqrt{2}}(A^1_\mu \mp i A^2_\mu) \qquad M^2_W = g^2 v^2/4.$$

While The Yukawa coupling between scalars and fermions :

$$L_Y = f^{(e)}(\bar{\nu} \quad \bar{e})_L \Phi\, e_R + f^{(u)}(\bar{u} \quad \bar{d})_L \tilde{\Phi}\, u_R + f^{(d)}(\bar{u} \quad \bar{d})_L \Phi\, d_R + h.c.$$

Will produce fermions masses : $$\tilde{\Phi} = i\tau_2\Phi^*.$$

$$m_e = f^{(e)} v/\sqrt{2},$$

$$m_u = f^{(u)} v/\sqrt{2}, \qquad m_d = f^{(d)} v/\sqrt{2}.$$

Towards Unification

* **The aim** is constructing a grand unified theory of strong, weak, and electromagnetic interactions.

* So that the different gauge couplings g_s , g and g′ will be unified in one coupling: g_G

What is required for the new grand unified group ?

In principle, it should be large enough to contain the SU(3) x SU(2) x U(1) group of S.M. as a subgroup;

thus it must be at least of rank 4.

SU(5) motivation

SU(5) motivation

∗ Listing the Lie groups having rank 4 :

$[SU(3)]^2$, $[SU(2)]^4$, O(9), O(8), Sp(8), SU(5), F_4 {Exponential group}.

That a group of rank l:

SU(l + 1), O(2l+1), Sp(2l), O(2l).

∗ All these possibilities are excluded, except $[SU(3)]^2$ & SU(5) groups.

Since they do not have complex representations. That we must have complex representations for fermions, because in the S.M. the fermions are not equivalent to their complex conjugates. ($e^+_L \neq e_R$)

∗ The remaining $[SU(3)]^2$ is also excluded.

Since leptons will be described by quantum numbers of color and flavor, that they really don't have.

∗ The SU(5), being of rank-4, is the smallest group that can contain SU(3) x SU(2) x U(I) without introducing any new fermions. It has complex representations and has the right quantum numbers to fit leptons and quarks.

SU(5) is the unique theory for the simplest grand unification scheme.

SU(5) Grand Unified Theory

SU(5) Generators

The generators $T^a = \lambda^a / 2$ of SU(5) for the fundamental 5 X5 are represented by the generalization of the Gell-Mann matrices λ^a (a = 1,..., 24), which are hermitian and traceless and with normalization $\mathrm{tr}(\lambda^a \lambda^b) = 2\delta^{ab}$.

To obtain the SU(3) X SU(2) subgroup structure of Su(5). We keep SU(3) color group acts on the first 3 rows & columns and SU(2) acts on the 4 th & 5 th ones.

So :

$$T^a = \frac{\lambda^a}{2} = \frac{1}{2} \left(\begin{array}{ccccc} \multicolumn{3}{c}{\overbrace{\quad\quad\quad}^{SU(3)}} & 0 & 0 \\ & \zeta^a & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Ex: $$\lambda_3 = \begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$$

(a=1,...,8) are the SU(3) generators

call ζ^a the usual 3X3 Gell- Mann matrices

$$T^{21,22,23} = \frac{\lambda^{21,22,23}}{2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \multicolumn{2}{c}{\sigma^{1,2,3}} \\ 0 & 0 & 0 & & \end{pmatrix}$$

SU(2)

SU(2) generators

σ^i are the usual 2x2 pauli matrices

Ex: $T^{23} \equiv T_3$ the weak isospin = $\begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & +\frac{1}{2} & \\ & & & & -\frac{1}{2} \end{pmatrix}$

The factor $(5/3)^{1/2}$ is to guarantee λ_{24} is normalized

Did we miss some thing ?

It remains to identify the electroweak hypercharge generator , that is;

$$\frac{Y}{2} = Q - T_3$$

Q is known consistently with the charges of the quarks and leptons . So that Y/2 could be constructed.

$$\frac{Y}{2} = \sqrt{\frac{5}{3}}\,\frac{\lambda_{24}}{2} \quad with \ \lambda_{24} = \frac{1}{\sqrt{15}}\,Diagonal(-2,-2,-2,3,3)$$

Notice that Y is on the diagonal that doesn't belong to Su(2) nor SU(3)

And with Q_{em} generator:

$$Q(5) = \begin{pmatrix} -1/3 & & & & \\ & -1/3 & & & \\ & & -1/3 & & \\ & & & 1 & \\ & & & & 0 \end{pmatrix}$$

Finally, the rest of the 24 matrices (12 generators Which will correspond to new gauge bosons) act in the off- diagonal space, and obtained by inserting pauli matrices appropriately;

Ex:

$$\lambda_{13} = \begin{pmatrix} & \mathcal{O} & & 1 & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ 1 & 0 & 0 & & \\ 0 & 0 & 0 & \multicolumn{2}{c}{\mathcal{O}} \end{pmatrix}, \quad \lambda_{14} = \begin{pmatrix} & \mathcal{O} & & -i & 0 \\ & & & 0 & 0 \\ & & & 0 & 0 \\ i & 0 & 0 & & \\ 0 & 0 & 0 & \multicolumn{2}{c}{\mathcal{O}} \end{pmatrix}$$

Notice the number of diagonal generators

SU(5) Gauge Bosons

The gauge fields A_μ belong to the 24-dimensional adjoint rep. of SU(5), written as a 5 x 5 matrix

$$A^\mu \equiv A^\mu_a T_a = A^\mu_a \frac{\lambda_a}{2}$$

$$a = 1 \ldots\ldots 24$$

The 8 SU(3) G.B. (gluons)

New G.B. (lepto- quarks)

$$2A_\mu = \left(\begin{array}{ccc|cc}
A^3_\mu + \frac{1}{\sqrt{3}}A^8_\mu - \frac{2}{\sqrt{15}}B_\mu & A^1_\mu - iA^2_\mu & A^4_\mu - iA^5_\mu & \overline{X^1_\mu} & \overline{Y^1_\mu} \\
A^1_\mu + iA^2_\mu & -A^3_\mu + A^8_\mu - \frac{2}{\sqrt{15}}B_\mu & A^6_\mu - iA^7_\mu & \overline{X^2_\mu} & \overline{Y^2_\mu} \\
A^4_\mu + iA^5_\mu & A^6_\mu + iA^7_\mu & -\frac{2}{\sqrt{3}}A^8_\mu - \frac{2}{\sqrt{15}}B_\mu & \overline{X^3_\mu} & \overline{Y^3_\mu} \\
\hline
X^1_\mu & X^2_\mu & X^3_\mu & W^3_\mu + \frac{3}{\sqrt{15}}B_\mu & W^+_\mu \\
Y^1_\mu & Y^2_\mu & Y^3_\mu & W^-_\mu & -W^{23}_\mu + \frac{3}{\sqrt{15}}B_\mu
\end{array}\right)$$

New G.B. (lepto- quarks)

SU(2)G.B.

The electroweak gauge bosons are defined by:

$W^3_\mu = A^{23}_\mu$

$B_\mu = A^{24}_\mu$

$W^\pm = \frac{1}{\sqrt{2}}\left(A^{21}_\mu \mp A^{22}_\mu\right)$

The new gauge fields, X^μ_i and X^μ_i ,i= 1,2,3 defined by:

$$X^\mu_1 = \frac{1}{\sqrt{2}}(A^\mu_9 + iA^\mu_{10})$$

$$Y^\mu_1 = \frac{1}{\sqrt{2}}(A^\mu_{16} + iA^\mu_{17})$$

etc.

So :

$$\sqrt{2}\,A^\mu = \left(\begin{array}{ccc|cc}
\frac{1}{\sqrt{2}}A^\mu_3 + \frac{1}{\sqrt{6}}A^\mu_8 - \frac{2B^\mu}{\sqrt{30}} & \frac{1}{\sqrt{2}}A^\mu_{1-i2} & \frac{1}{\sqrt{2}}A^\mu_{4-i5} & \bar{X}^\mu_1 & \bar{Y}^\mu_1 \\
\frac{1}{\sqrt{2}}A^\mu_{1+i2} & -\frac{A^\mu_3}{\sqrt{2}} + \frac{A^\mu_8}{\sqrt{6}} - \frac{2B^\mu}{\sqrt{30}} & \frac{A^\mu_{6-i7}}{\sqrt{2}} & \bar{X}^\mu_2 & \bar{Y}^\mu_2 \\
\frac{1}{\sqrt{2}}A^\mu_{4+i5} & \frac{1}{\sqrt{2}}A^\mu_{6+i7} & -\sqrt{\frac{2}{3}}A^\mu_8 - \frac{2B^\mu}{\sqrt{30}} & \bar{X}^\mu_3 & \bar{Y}^\mu_3 \\
\hline
X^\mu_1 & X^\mu_2 & X^\mu_3 & \frac{W^\mu_3}{\sqrt{2}} + \frac{3B^\mu}{\sqrt{30}} & W^\mu_+ \\
Y^\mu_1 & Y^\mu_2 & Y^\mu_3 & W^\mu_- & -\frac{W^\mu_3}{\sqrt{2}} + \frac{3B^\mu}{\sqrt{30}}
\end{array}\right)$$

Notice that B_μ still on the diagonal

SU(5) Fermions content

The standard model $SU_c(3) \times SU_L(2) \times U_Y(1)$ contains 15 left handed fields as follows:

2	$(\nu_e, e^-)_L$: (1,2)	$\Longrightarrow$ colour singlet, isospin doublet.
6	$(u^\alpha, d^\alpha)_L$: (3,1)	$\Longrightarrow$ colour triplet, isospin singlet.
1	e_L^+:(1,1)	$\Longrightarrow$ Singlet.
3	$u_L^{\alpha c}$:($\bar{3}$,1)	$\Longrightarrow$ colour triplet (antiparticles), isospin singlet.
3	$d_L^{\alpha c}$:($\bar{3}$,1)	$\Longrightarrow$ colour triplet(antiparticles), isospin singlet.

The SU(5) theory unify both quarks and leptons (leading to baryon number and lepton number violation) in $\bar{5}$ and 10 dim. irreducible representation.

$$\bar{5} \equiv \psi^i_L = \left(\begin{array}{c} d^c_{r(1)} \\ d^c_{g(2)} \\ d^c_{b(3)} \\ e^- \\ -\nu_e \end{array}\right)_L \begin{array}{l} \left.\vphantom{\begin{array}{c}a\\a\\a\end{array}}\right\}\text{SU(3)c} \\ \left.\vphantom{\begin{array}{c}a\\a\end{array}}\right\}\text{SU}_L(2) \end{array} ;10 \equiv (\chi_{ij})_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^c_3 & -u^c_2 & -u^1 & -d^1 \\ -u^c_3 & 0 & u^c_1 & -u^2 & -d^2 \\ u^c_2 & -u^c_1 & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & u^3 & e^+ & 0 \end{pmatrix}_L$$

The assignment of quarks and leptons to multiplets of the grand unified group SU(5) explaining **(-1/3) quark charges** (not explained by QCD nor E.W.), that (for 5 rep.):

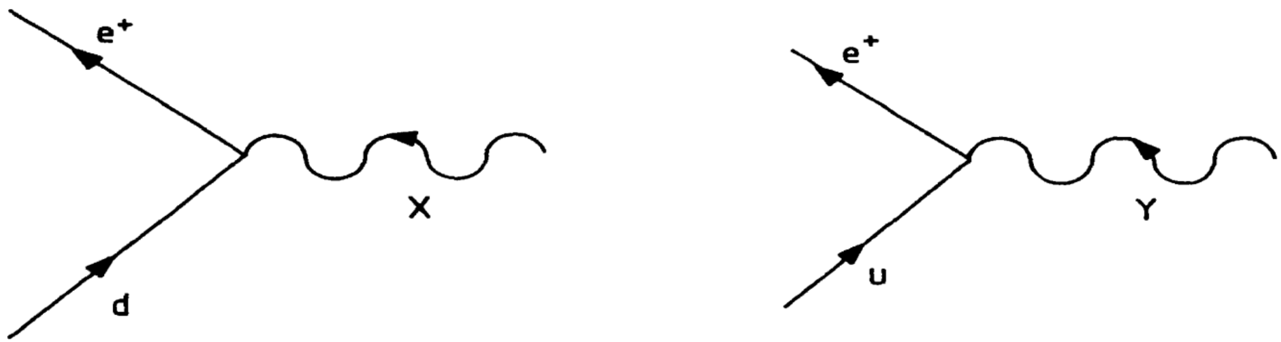
$$-3Q_d + Q_e = 0 \quad \rightarrow \quad Q_d = -Q_e/3.$$

Charge quantization

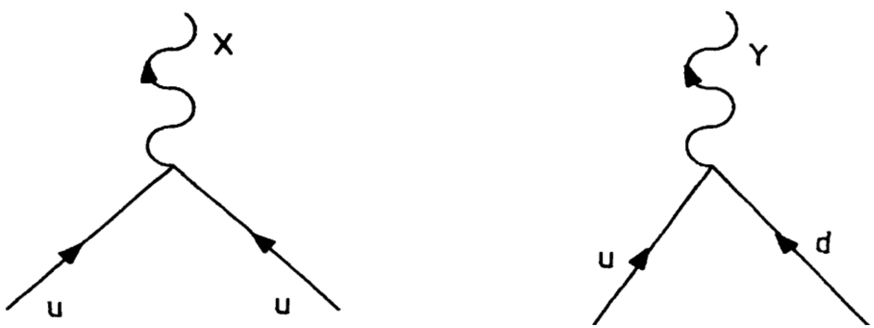
Taking the charge conjugation of $\bar{5}$

$$;5 \equiv \psi_R = \begin{pmatrix} d_{1(r)} \\ d_{2(g)} \\ d_{3(b)} \\ e^+ \\ -\nu^c_e \end{pmatrix}_R$$

Notice that we will have the following vertices:



Vertices changing a quark into a lepton.



Baryon number violation!

Vertices with two quarks annihilating.

Interactions

$$L = L_{GB} + L_f + L_\Phi + L_Y$$

Considering the fermion part of the Lagrangian :

$$L_f = i\overline{\psi}^p \gamma^\mu D_\mu \psi_p + i\overline{\chi}^{pq} \gamma^\mu D_\mu \chi_{pq}$$

$$D_\mu = \partial_\mu - i g_G \sum_{a=1}^{24} \frac{\lambda_a}{2} A_\mu^a \equiv \partial_\mu - i g_G \, A_\mu$$

$$D_\mu \, \psi_p = [\partial_\mu \delta_p^q - i g_G \, (A_\mu)_{pq}] \psi_q = \partial_\mu \psi_p - i g_G \, (A_\mu)_{pq} \psi_q$$

$$D_\mu \chi_{pq} = \partial_\mu \chi_{pq} - i g_G \left(A_\mu\right)^{pr} \chi_{rq} - i g_G \left(A_\mu\right)^{qs} \chi_{ps} = \partial_\mu \chi_{pq} - 2 i g_G \left(A_\mu\right)^{pr} \chi_{rq}$$

So, The fermions Lagrangian becomes;

$$L_f = i\overline{\psi}^p \gamma^\mu \partial_\mu \psi_p + g_G\, \overline{\psi}^p \gamma^\mu (A_\mu)_{pq} \psi_q + i\overline{\chi}^{pq} \gamma^\mu \partial_\mu \chi_{pq} - 2 g_G\, \overline{\chi}_{pq} (A_\mu)^{pr} \chi_{rq}$$

K.T.

Interaction T.

$$g_G\, \overline{\psi}^p \gamma^\mu (A_\mu)_{pq} \psi_q \; - \; 2 g_G\, \overline{\chi}_{pq} (A_\mu)^{pr} \chi_{rq}$$

2

1

Proton decay

From (1) we can get terms like:

$$L_I = \frac{g_G}{\sqrt{2}} \left(\overline{e_L^-} \; \gamma^\mu \; X_\mu^\alpha \; d_\alpha^c \; + h.c. \right)$$

And From (2) we can get terms like:

$$L_I = \frac{g_G}{\sqrt{2}} \left(\varepsilon_{\alpha\beta\lambda} \, \bar{u}_L^\beta \; \gamma^\mu \, X_\mu^\alpha \; u^{c\lambda} \; + h.c. \right)$$

Leading to a proton decay:

The amplitude of the process $\propto$ g^2_G / M^2_X .
Squaring the amplitude to get the decay rate.
Then the life time of the proton $\tau_p \propto M^4_X / g^4_G$.
As predict from the G.U.T. scale : m_x is of order 10^{14} Gev, giving τ_p of order 10^{30} years !

While exp. show that proton is stable and having live time $\tau_p \geq 6 \times 10^{32}$ years !

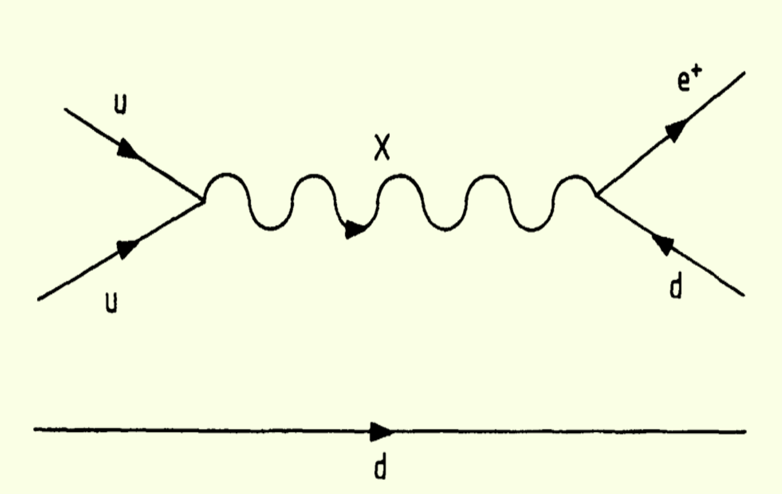


Diagram for $p \rightarrow \pi^0 e^+$.

SU(5) SSB.

$$\mathbf{SU(5) \to SU_C(3) \times SU_L(2) \times U(1) \to SU_C(3) \times U_Q(1).}$$

M_X & M_Y of order 10^{14}Gev

G.U.T. scale

M_W & M_Z of order 100 Gev

2 different masses scales

Suitable choice

SU(5) → $SU_c(3) \times SU_L(2) \times U_Y(1)$

$$\Phi \equiv \sum_{a=1}^{24} \phi_a T_a$$

Corresponding to the adjoint rep.

$SU_c(3) \times SU_L(2) \times U_Y(1) \to SU_c(3) \times U_Q(1)$

$$H = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ \varphi^+ \\ \varphi^0 \end{pmatrix}$$

Corresponding to the fundamental rep.

So we need 2 multiplets of Higgs scalars with very diff. vacuum expectation values .

$$\mathbf{SU(5)} \xrightarrow[\mathbf{24}]{} \mathbf{SU_C(3) \times SU_L(2) \times U(1)}$$

The Lagrangian for the Higgs scalars Φ :

$$L_{\Phi} = Tr(D_\mu \Phi)^2 - m_1^2 Tr\Phi^2 + \lambda_1 Tr(\Phi^2)^2 + \lambda_2 Tr\Phi^4$$

With the effective potential:

$$V_{\Phi} = m_1^2 Tr\Phi^2 + \lambda_1 Tr(\Phi^2)^2 + \lambda_2 Tr\Phi^4$$

Taking Φ in the T^{24} direction

$$\Phi = \phi^{24} T^{24} = \frac{1}{\sqrt{15}} \phi_{24}\, Diagonal(-2, -2, -2, 3, 3)$$

This choice will fulfill our target, which is :

* Breaking SU(5) generators . (so that: X, Y gauge bosons . will acquire masses)
* Keeping S.M. generators unbroken. (S.M. gauge bosons still massless)

But

The condition of symmetry invariance :

$[\Phi , T] = 0$ symmetry invariant

Where T is a generator of a symmetry transformation $U = \exp(-i\, \alpha.\mathbf{T})$.

With a transformation property of Φ under U:

$$\Phi \longrightarrow U \Phi U^{+}$$

Easily you can check that $\langle\Phi\rangle$ commutes with $SU_c(3) \times SU_L(2) \times U_Y(1)$ generators , leaving S.M. symmetry unbroken. Will it doesn't commute with the remaining 12 G.B. of SU(5) (SU(5) symmetry has broken)

Taking the minimum of the potential gives the form of expectation value corresponding to this symmetry breaking:

$$< \Phi >= \frac{v_\Phi}{\sqrt{15}} \quad Diagonal(-2,-2,-2,3,3)$$

At the minimum:

$$v_\Phi^2 = \frac{m^2}{4\lambda_1 + \frac{14}{15}\lambda_2}$$

With the condition:

$$\lambda_2 > 0, \qquad \lambda_1 > -\frac{7}{30}\lambda_2$$

Gauge Bosons masses

The covariant derivative term in L_Φ :

$$D_\mu \Phi = \partial_\mu \Phi + i g_G [A_\mu, \Phi]$$

Produces gauge fields masses terms:

$$L_{mass} = -g_G^2 Tr([A_\mu, \Phi]^2)$$

By calculating :

$$Tr[A_\mu, \Phi]^2 = \frac{-4v_\Phi^2}{15} \sum_i^3 25(\overline{X}_i^\mu X_\mu^i + \overline{Y}_i^\mu Y_\mu^i)$$

We can see that only X and Y Lepto-quarks acquire masses

$$L_{mass} = \frac{20}{3} g_G^2 v_\Phi^2 \sum_i^3 (\overline{X}_i^\mu X_\mu^i + \overline{Y}_i^\mu Y_\mu^i)$$

That all the 3 colors have the same mass

$$m_X^2 = m_Y^2 = \frac{20}{3} g_G^2 v_\Phi^2$$

So that the vev for the adjoint of Higgses is of order 10^{15} Gev

$$\mathbf{SU_C(3) \times SU_L(2) \times U(1) \underset{5}{\rightarrow} SU_C(3) \times U_Q(1).}$$

The general Lagrangian for the 5 of Higgs scalars H

$$L_H = \frac{m_2^2}{2} H_5^+ H_5 + \frac{\lambda_3}{4} (H_5^+ H_5)^2$$

Manage as in the S.M.

To break $SU_c(3) \times SU_L(2) \times U_Y(1)$ to $SU_c(3) \times U_Q(1)$ we must take the vev in the neutral, $SU_L(2)$ doublet, colour singlet component of H.

$$< H >= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_H \end{pmatrix}$$

Remember:

$T_{23} <H> \neq 0$	Broken
$T_{24} \propto Y \ <H> \neq 0$	Broken
$Q<H> = 0$	Unbroken

With the effective potential:

$$V(H) = -\frac{m_2^2}{2} H^+ H - \frac{\lambda_3}{4}(H^+ H)^2$$

and at the minimum

$$H_5^2 = v_H^2 = -m_2^2/\lambda_3$$

This vev give masses to $W^{\pm}$ and Z^0 bosons as in S.M. :

As we write V(Φ) & V (H). Now considering the coupling between two sectors and write down the general SU(5)-invariant fourth-order potential:

$$\begin{aligned} V(\Phi, H) &= m_1^2 Tr\Phi^2 + \lambda_1 Tr(\Phi^2)^2 + \lambda_2 Tr\Phi^4 + \frac{m_2^2}{2} H_5^+ H_5 + \frac{\lambda_3}{4}(H_5^+ H_5)^2 \\ &\quad + \underbrace{\alpha H^+ H Tr\Phi^2 + \beta H^+ \Phi^2 H} \end{aligned}$$

The underlined coupling terms will yields coupling terms between h_3 & h_2 (h_3 are the triplet higgs scalars and h_2 the two higgs duplets)

So the total Higgs masses:

$$(2\,\alpha + \frac{4}{15}\,\beta)\,v_\Phi^2\;H_3^+ H_3 + \;(2\,\alpha + \frac{9}{15}\,\beta)\,v_\Phi^2 H_2^+ H_2$$

As it should be (as we 'll see later) $m_{h2} \approx$ G.U.T. scale

Duplet –triplet splitting problem

Not right because m_{h2} of order 100 Gev

Making the second term equal to zero, requires fine tuning of parameters , that:

$$\alpha = -\frac{9}{30}\beta.$$

$\beta < 0$

Fermions masses & Yukawa Interactions in SU(5)

In the minimal SU(5) theory the fermion masses may originate only through the coupling to the 5-dim. Higgs rep. H_5 .
While Φ_{24} decouples from the fermions, for it 'd give them masses of order Gut scale.

Writing the Yukawa couplings of fermions with the H_5 Higgs :

$$\mathcal{L}_Y = f_d \bar{\psi}_R \chi \Phi^\dagger + f_u \frac{1}{2} \chi^T C \chi \Phi + h.c.$$

When gets its vacuum expectation value $< H >^T = (0\ 0\ 0\ 0\ \upsilon_w)$, we get for the fermionic masses:

$$\mathcal{L}_m = -[f_d v_W (\bar{d}d + \bar{e}e) - f_u v_W \bar{u}u]$$

Fermions must
not pick mass Before E.W.
symmetry breaking

Notice ?

As in the Standard Model $m_f = f\ \upsilon$. But we predict the electron and down quark masses being equal.

This prediction appears very bad (we know that $m_d \simeq 10$MeV , $m_e \simeq 0.5$MeV), but we must recall that it is valid only at the large scale M_x where the whole SU(5) symmetry becomes operative.

Higgs scalars h_α (α = r, g, b) interactions with fermions:

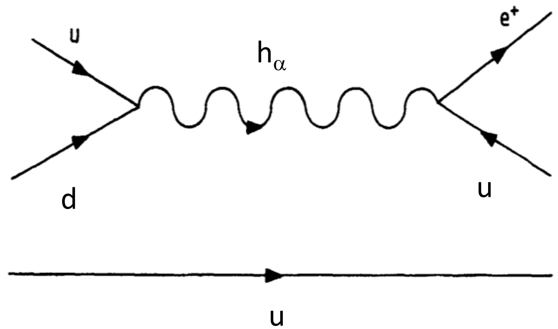
$$\mathcal{L}_h = f_d \bar{\psi}_{R\,i} \chi^{i\,\alpha} h_\alpha^+ + f_u \epsilon_{ijkl\alpha} (\chi^T)^{ij} C \chi^{kl} h^\alpha$$

Yields:

$$\begin{aligned} \mathcal{L}_h \quad = \quad & \{ f_d \left(\epsilon^{\alpha\beta\gamma} \bar{u}^c_{L\,\beta}\, d^\gamma_R + \bar{u}^\alpha_L\, e^+_R + \bar{d}^\alpha_L\, \nu^c_R \right) \\ & + f_u \left(\epsilon^{\alpha\beta\gamma} \bar{u}^c_{R\,\beta}\, d^\gamma_L + \bar{u}^\alpha_R\, e^+_L \right) \} \, h^\alpha \end{aligned}$$

Like the situation before for the X and Y bosons, we have the possible exchanges of h_α which leads to the proton decay.

Of course, the amplitude is proportional to the small Yukawa coupling and the corresponding limit on m_h is somewhat less strict. From (τp)exp ≥ 1032 yr, the following lower limit on $m_h \geq 10^{12}$GeV .



Another channel for p decay mediated by h_α

This is the famous doublet-triplet splitting phenomenon: $m_{h3} \gg m_{h2}$.

Conclusion

SU(5) is a smart theory

But it may be needs many modifications to solve some problems within it

Thanks for
The nice
listening