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Self-correcting inspection procedure under inspection errors

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In this paper we present a novel treatment of the inspection-system design problem when inspection is unreliable and subject to classification errors. Our approach, based on the theory of Error-Correcting Codes (ECC), leads to the development of a Self-Correcting Inspection (SCI) decision rule that does not require complete knowledge of inspection error probabilities. We show that the proposed rule assures correct classification, if the number of inspection errors is less than a certain number. We analyze the performance of the SCI decision rule under different inspection situations, including some situations that are uncommon in the field of error-correcting codes. Then, we show how the underlying mathematical structure can be applied to determine the number of inspections and the level of inspection reliability in order to minimize the sum of inspection-related costs. The practical contribution of this work lies in that it expands the ability of the designer of inspection systems to deal with cases where there is very little or no information regarding the reliability of the inspection operations.

1. Introduction and literature review

Inspections are performed in virtually every production system. Their purpose is to verify that the production operations were carried out properly and that the production output meets the expectations of the customer. Inspection operations can often be seen as procedures used to classify a product unit into two or more classes according to its conformance to a given set of requirements. There is a large body of evidence, e.g., see Raz and Thomas (1990), pointing to the fact that inspection operations can be unreliable, resulting in classification errors. These errors have both cost and quality implications. One common approach to deal with inspection errors is to introduce redundancy into the inspection procedure by carrying out multiple inspections – either identical or unique – on each product unit and to base the classification decision on their combined results.

A significant amount of research has been devoted to the problem of optimal design of multiple inspection systems subject to errors. Some of the earlier work in this area was surveyed by Raz and Thomas (1990). Moskowitz and Tsai (1988) presented a model for a two-stage sequential screening procedure based on correlated variables. Their model can be used to calculate the acceptance ratio and the average outgoing quality for given levels of inspection errors at the two stages. Later on, Moskowitz *et al.* (1991) developed a procedure for selecting the parameters of an inspection plan that optimizes a cost function reflecting inspection risk preferences and inspection misclassification. Moskowitz *et al.* (1993) expanded this work to include a multiple-stage screening model that controls the maximum as well as the average misclassification error. The papers mentioned so far deal with the inspection of multiple *items* drawn from a single lot or process. In certain cases multiple inspections are carried out on each individual item, which brings up the issue of determining the disposition of each item based on the results of all the inspections. Some typical areas of application where this situation occurs include the manufacture of gear cases, as reported by Raz and Bousum (1990), and assembly of circuit boards, as studied by Chevalier and Wein (1997). Another application, that of wafer inspection in a FAB, is discussed in Section 4.1.

Chen and Chung (1994) addressed the issue of determining the optimal specification levels to optimize net profit while considering inspection error. Hong and Elsayed (1999) followed this line of research and developed a model for determining jointly the economically optimal process mean and specification limit under inspection measurement error. Tang and Schneider (1988) developed

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a model for determining the level of inspection error that will minimize the sum of inspection and disposition costs.

The research published so far relating to inspection errors has assumed that the inspection error probabilities are known or can be accurately estimated. Such estimation, however, is not always possible, especially when new types of inspection technologies are applied or when inspection is too expensive to allow experimentation to estimate error probabilities. Moreover, most of the research related to inspection errors deals with lot acceptance, where the acceptance/rejection decision is based on multiple identical inspections carried out on a sample drawn from the lot. Here we consider a different situation, where a number of inspections, not necessarily identical, are applied to each product unit, which is to be classified into two or more classes.

In this paper we present a novel treatment of the inspection-system design problem. Our approach is based on an analogy between inspection operations and message transmission through communications channels. A related mathematical isomorphism is defined in Ben-Gal and Levitin (1998, 1999). Particularly, we establish an analogy between codewords (a vector of symbols which constitutes a message) and vectors of inspection results (elements of the vector represent the result of various inspections carried out on a *single* part) and apply some known results from the theory of Error-Correcting Codes (ECC), which are widely used in the design of information transmission systems. However, and in contrast to the prevailing literature on communication channels, our method deals also with cases where the vector set of inspection results does not have a special mathematical structure (such as forming a linear subspace as is the case with linear codes) and with situations when inspection errors are not independent and identically distributed. Based on ECC theory, we develop a decision rule for classifying items that does not require knowledge of the inspection error probabilities. Moreover, our decision rule assures correct classification, as long as the number of inspection errors does not exceed a given threshold.

The practical contribution of the work presented here lies in that it expands the ability of the designer of inspection systems to deal with cases where there is very little or no information regarding the reliability of the inspection operations. Furthermore, the solution presented here is also applicable to situations when classification decisions are made upon different inspections applied to a single part type. A key practical advantage of our approach is that the decision rule is very simple to implement, in that it involves straightforward calculations.

This paper is organized as follows. In Section 2 we draw the analogy between an unreliable inspection process and the transmission of a message via a noisy communication channel. This section also contains our notation and definitions. In Section 3 we present the required theoretical background and analyze the performance of our decision rule. This is done by deriving probabilistic expressions for the various outcomes of the inspection procedure. In Section 4 we analyze the performance of the decision rule under different inspection conditions. Section 5 shows how the decision rule can be applied to the design of inspection procedures. We conclude with some suggestions for further work in Section 6.

2. Unreliable inspection and noisy communication channels

2.1. *The analogy*

Consider a Product Unit (PU) that belongs to a certain quality class. For instance, it could be "acceptable", "not acceptable", or "second rate". Initially we do not know to which quality class the PU belongs. In order to correctly classify the PU, it is subjected to a set of Inspection Operations (IOs), which may all be identical. A particular IO might be based on detecting the presence or absence of certain attributes, or might include measurement on a continuous or discrete scale. In any case, the outcome of each IO is binary: "0" or "1", denoting, for example, whether the PU passes or fails the IO, or whether the IO measure is above or below a certain threshold. The results of the set of inspections on a single PU can be represented by a vector, where the ith component in the vector represents the outcome of the ith IO. There is a mapping from the set of error-free result vectors (called *valid vectors*) to the set of quality classes. Often, however, the IOs are not perfectly reliable. Each IO may involve two types of errors: reporting a "0" results when a "1" should have been reported and *vice versa*. For this reason, the set of IOs often includes otherwise redundant tests whose purpose is to help detect inspection errors and classify PUs. The issue at hand is how to classify the PUs.

Consider now a message that needs to be transmitted from a source to a receiver by a communications channel. In order to increase the reliability of the transmission, the message is encoded by a finite set of codewords. Each codeword is, in fact, a vector of binary symbols, say "0"s and "1"s. Transmission noise in the communications channel might corrupt (transform) some of the symbols. As a result, the received vector may differ from the initially transmitted codeword. The receiver faces the problem of correctly decoding the received vector. If the received vector is not one of the codewords, then the receiver detects that an error has occurred during the transmission, and can try to correct it, that is, to associate it with one of the codewords.

An analogy between the inspection process and message transmission can be drawn as shown in Fig. 1. The source of the coded message is analogous to the quality class of the PU that is being inspected. The transmission

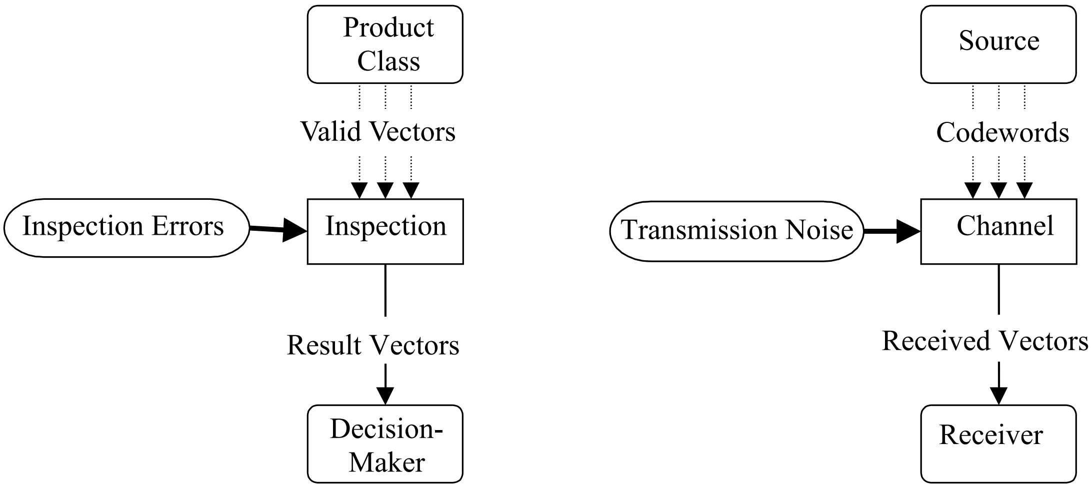

Fig. 1. Analogy between the inspection process and the transmission of coded messages via a communication channel.

of a codeword through a communication channel corresponds to the inspection operations performed on the PU. The receiver of the codeword is regarded as the decision-maker that classifies the PU. The PU belongs to one of a number of predetermined quality classes, e.g., conforming, nonconforming and partially conforming. Any valid vector, which belongs to the set of error-free inspection results, can be mapped to one of these classes. This vector is analogous to the binary codeword transmitted by the channel source. The result vector that contains the actual results of all inspections corresponds to the received vector in the communication channel.

Thus, a perfectly reliable inspection procedure is isomorphic to a perfectly reliable communication channel, where the received vectors (result vectors) are identical to the codewords (valid vectors), guaranteeing a correct decoding of the message (classification of the PU) by the receiver (decision-maker). However, if the transmission (inspection) is subject to noise (errors), then the received vectors (result vectors) may be different from the codewords (valid vectors). In such a case, the received vectors (result vectors) have to be decoded, that is, associated with one of the codewords (valid vectors) and then interpreted (classified) accordingly.

2.2. *Notation, assumptions and definitions*

The following notation will be used throughout the paper.

$n \triangleq$ number of Inspection Operations (IOs) that are carried out on each PU;

$M \triangleq$ number of valid vectors (classes);

$\mathbf{c}_j \triangleq$ a valid vector $\mathbf{c}_j = (c_j^1, c_j^2, \ldots, c_j^n)$, $j = 1, 2, \ldots, M$; valid vectors contain the error-free inspection results that would have been obtained under perfectly reliable inspections;

$\mathbf{C} \triangleq$ the set of all valid vectors, $\mathbf{C} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_M\}$;

$\hat{\mathbf{c}}_k \triangleq$ a binary vector of inspection results carried out on a given product unit in the presence of inspection errors, $\hat{\mathbf{c}}_k = (\hat{c}_k^1, \hat{c}_k^2, \ldots, \hat{c}_k^n)$, $k = 1, 2 \ldots, 2^n$;

$\hat{\mathbf{C}} \triangleq$ the set of all binary vectors of length n (forming a binary vector space of dimension n) where $\hat{\mathbf{C}} = \{\hat{\mathbf{c}}_1, \hat{\mathbf{c}}_2, \ldots, \hat{\mathbf{c}}_{2^n}\}$. Note that $\mathbf{C} \subseteq \hat{\mathbf{C}}$;

$q_j \triangleq$ the *a priori* probability that a given PU's error-free result vector is $\mathbf{c}_j$;

$p_j^i \triangleq$ the unknown probability of an inspection error in the ith IO when the error-free result vector is $\mathbf{c}_j$.

We assume that inspection errors are mutually independent Bernoulli random events. We also assume that $p_j^i < 0.5$ for all $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, M$, which means that, for each and every IO, a correct result is more likely than an erroneous one. An important special case, called the *equierror case*, occurs when errors in each IO are independent of the product class and the inspection error probabilities are identical. That is, $p_j^i = p$ for all $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, M$ (in ECC terms these assumptions are known as the *memory less binary symmetric channel* assumptions). We further assume for the equierror case that $q_j = 1/M$ for all $j = 1, \ldots, M$. Although the equierror case does not always occur in practice, it is a reasonable assumption requiring the least amount of information when probability estimates are not available. For the sake of readability, the subscripts on the vectors, $\mathbf{c}_j$,$\hat{\mathbf{c}}_k$, and their respective elements will be omitted when discussing general properties.

We now define the *Self-Correcting Inspection* (*SCI*) decision rule. The SCI rule either associates (i.e., corrects) the vector $\hat{\mathbf{c}} \in \hat{\mathbf{C}}$ with a vector $\mathbf{c} \in \mathbf{C}$, or determines that the inspection error cannot be corrected. For example, consider the case of six identical IOs, where there are only two valid vectors: $\mathbf{c}_1 = (0, 0, 0, 0, 0, 0)$ corresponding to a conforming PU, and $\mathbf{c}_2 = (1, 1, 1, 1, 1, 1)$ corresponding to a nonconforming PU. The set of all binary vectors $\hat{\mathbf{C}}$ contains result vectors that reflect the presence of inspection errors, i.e., vectors that carry both "1" and "0" values. If the result of the IOs is $(0,0,1,0,0,1)$ and we assume that at most two inspection errors have occurred, then we can classify the unit as conforming, regardless of probability of an inspection error (by the majority vote rule). As we shall see in Section 4.2, the proposed SCI decision rule does, indeed, associate $(0,0,1,0,0,1)$ with $\mathbf{c}_1$.

3. Performance analysis of the SCI decision rule

The following analysis is based on the theory of Error-Correcting Codes (ECC) and on the performance analysis of linear-block codes (for further information see, for example, Peterson and Weldon (1972), Lin and Costello (1983) and Wicker (1995). Certain modifications have been made to apply ECC performance analysis to unreliable inspection procedures, where the valid vectors do not necessarily form a special algebraic structure (which is often the case with codewords). Moreover, and in contrast to the *memoryless binary symmetric* channel assumptions, we do not assume that the probabilities of inspection errors are constant or independent of the class of the PU. We start with some known ECC definitions.

Definition 1. The *Hamming distance*, d_H, between two (binary) vectors of equal length $\mathbf{v}_1$ and $\mathbf{v}_2$, expressed by $d_H(\mathbf{v}_1, \mathbf{v}_2)$, is equal to the number of vector elements (symbols) in which they differ. For example, the Hamming distance between (10011) and (10101) is equal to two since their third and fourth symbols are different.

Definition 2. The *distance*, d, of a set of valid vectors $\mathbf{C}$ is the minimum Hamming distance between any two valid vectors from $\mathbf{C}$. Thus,

$$d = \min_{\mathbf{c}_i, \mathbf{c}_j \in \mathbf{C}} d_H(\mathbf{c}_i, \mathbf{c}_j); \mathbf{c}_i \neq \mathbf{c}_j.$$

Definition 3. *The error-correction capability* of a set of valid vectors $\mathbf{C}$ is given by the parameter $t = \lfloor (d-1)/2 \rfloor$. It has been proven (Wicker, 1995) that given a set $\mathbf{C}$ with distance d, the ECC decoding procedure can correct up to t errors.

Observation 1: Following the analogy presented above, given a set $\mathbf{C}$ with distance d, one can assure correct classification of PUs, if the number of inspection errors does not exceed the error-correction capability t.

Definition 4. An *error vector*, $\mathbf{e} = (e^1, \ldots, e^n)$ is a binary vector where

$$e^i = \begin{cases} 0 & \text{if the } i\text{th IO is error-free,} \\ 1 & \text{if the } i\text{th IO has been modified} \\ & \text{by an inspection error.} \end{cases}$$

Of course, the value of the error vector is unknown to the decision-maker.

Definition 5. The *weight* of a vector $\mathbf{v}$, denoted by $w(\mathbf{v})$, is the number of non-zero elements in the vector (in binary vectors it is simply the number of "1"s symbols). For example, $w(100110) = 3$.

Under the additive noise model presented in Fig. 2, each result vector $\hat{\mathbf{c}}$ can be considered as the sum modulo 2 (denoted by $\oplus$) of a valid result vector $\mathbf{c}$ and an error vector $\mathbf{e}$. Thus, the reliability of the inspection process determines the likelihood that each of the 2^n possible error vectors $\mathbf{e}$ occurs. This model enables us to find the likelihood of an inspection error by calculating the probability that an error vector $\mathbf{e}$ corrupts a valid vector $\mathbf{c}$ changing it to $\hat{\mathbf{c}}$, where $\hat{\mathbf{c}} = \mathbf{c} \oplus \mathbf{e}$.

3.1. *The self-correcting inspection (SCI) decision rule*

In this section we develop the rationale for our decision rule, which we call the Self-Correcting Inspection (SCI) decision rule. The decision process operates as follows. Once the inspections are carried out and the result vector

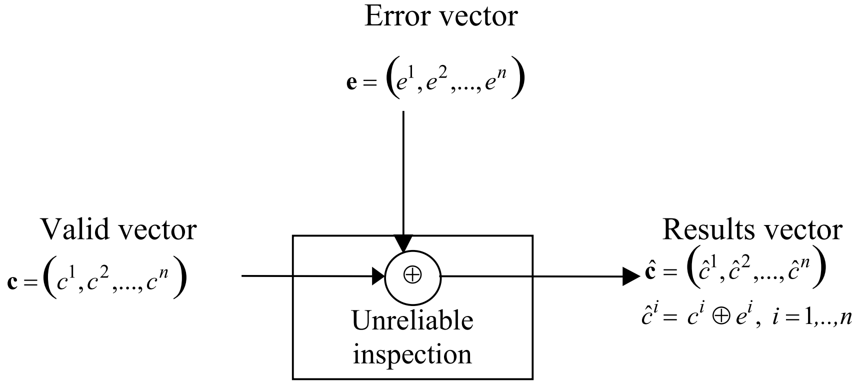

Fig. 2. The additive noise model for an inspection process.

becomes known, we examine the result vector and determine whether or not it is valid. If the result vector is valid, then we classify the PU accordingly. If the result vector is invalid, then we conclude that one or more inspection errors have occurred, i.e., that the error vector $\mathbf{e}$ associated with this particular PU contains at least one "1" symbol. This step of determining the existence of errors in the result vector is called *error detection*. An error vector is *undetectable* if and only if it causes the result vector to look like a valid vector other than the error-free result vector. Hence, given a valid vector $\mathbf{c}_j$, there are $M-1$ valid vectors other than $\mathbf{c}_j$ and thus $M-1$ undetectable error vectors.

Once an error is detected, knowledge about the structure of $\mathbf{C}$ can be applied to determine which of the valid vectors has apparently been corrupted. Specifically, our rule selects the valid vector that is closest to the result vector in terms of its Hamming distance. This step is referred to as *error-correction*. If the minimal distance valid vector is not unique, then we say that the result vector is *unclassifiable*.

We define $D(\hat{\mathbf{c}})$ as the classification function. This function returns the identifier of the valid vector associated with the result vector $\hat{\mathbf{c}}$. The decision rule can be stated as follows where:

$$y = \arg\min_x f(x) \text{ if } f(y) = \min_x f(x)$$

$$D(\hat{\mathbf{c}}) = \begin{cases} \arg\min_{1\le j\le M} d_{\mathrm{H}}\left(\mathbf{c}_j, \hat{\mathbf{c}}\right) & \text{if the minimum is unique,} \\ 0 & \text{otherwise, and we say that } \\ & \hat{\mathbf{c}} \text{ is unclassifiable.} \end{cases}$$

Note that the SCI decision rule does not depend on any estimates of inspection error probabilities, nor does it depend on the probability distribution of the result vectors. However, using the results from maximum *a posteriori* decoder theory it can be shown (Wicker, 1995), that for the equierror case when no result vectors are unclassifiable, the SCI decision rule maximizes, over all non-randomized decision rules, the probability of correctly classifying the PUs. The reason is that the closest valid vector in terms of the Hamming distance is the valid vector $\mathbf{c}_j$ that maximizes the *a posteriori* decoding probability,

$$\Pr\{\mathbf{c} = \mathbf{c}_j | \hat{\mathbf{c}}_k\}.$$

Thus, classifying by minimizing the Hamming distance is equivalent to classifying by maximizing the *a posteriori* probability. It should be noted that if there are unclassifiable vectors, $\hat{\mathbf{c}}$, then the result still holds if we let $D(\hat{\mathbf{c}})$ be identified with one of the multiple vectors that satisfy

$$\arg\min_{1\le j\le M} d_{\mathrm{H}}\left(\hat{\mathbf{c}}_j, \hat{\mathbf{c}}\right).$$

3.2. *Error detection performance*

We now describe the possible events associated with the additive noise model and derive their respective probabilities. These events and their respective notation are presented in Fig. 3 and are characterized by the following relations:

$$\begin{aligned} P_{\mathrm{N}} + P_{\mathrm{U}} + P_{\mathrm{D}} &= 1, \\ P_{\mathrm{D}} &= P_{\mathrm{C}} + P_{\mathrm{NC}}, \\ P_{\mathrm{NC}} &= P_{\mathrm{E}} + P_{\mathrm{F}}. \end{aligned} \tag{1}$$

We now develop expressions for the probabilities of the various events. The probability that an error vector $\mathbf{e}$ will occur when the error-free result vector is $\mathbf{c}_j$ is denoted by $\mathscr{P}_j(\boldsymbol{e})$ and is equal to

$$\mathscr{P}_j(\boldsymbol{e}) = \prod_{i=1}^{n}\left[e^i p_j^i + \left(1-e^i\right)\left(1-p_j^i\right)\right]. \tag{2}$$

Note that for the equierror case $\mathscr{P}(e) = p^{w(e)}(1-p)^{n-w(e)}$. The probability that no inspection errors occurred in any of the n IOs, P_{N}, is given by

$$P_{\mathrm{N}} = \sum_{j=1}^{m} q_j \mathscr{P}_j(\mathbf{0}) = \sum_{j=1}^{m} q_j \prod_{i=1}^{n}\left(1-p_j^i\right), \tag{3}$$

where $\mathbf{0}$ denotes the error-free vector $(0, 0, \ldots, 0)$. For the equierror case $P_{\mathrm{N}} = (1-p)^n$. The probability that one or more inspection errors have occurred is $1 - P_{\mathrm{N}}$. This is also an upper bound on P_{D} – the probability of detecting an inspection error (see Equation (1)), i.e.,

$$P_{\mathrm{D}} \le 1 - P_{\mathrm{N}}. \tag{4}$$

For a set $\mathbf{C}$ with distance d, one can detect all error vectors caused by up to $d-1$ inspection errors (since undetectable error vectors are those that cause valid vectors to look like other valid vectors). Consequently, the probability of an undetectable error, P_{U}, is bounded above by the probability of having d or more inspection errors. Hence, for the equierror case,

$$\begin{aligned} P_{\mathrm{U}} &\le \sum_{j=d}^{n}\binom{n}{j} p^j (1-p)^{n-j} \\ &= 1 - \sum_{j=0}^{d-1}\binom{n}{j} p^j (1-p)^{n-j}. \end{aligned} \tag{5}$$

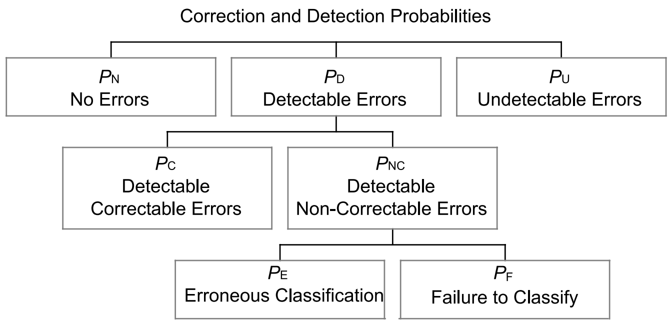

Fig. 3. Possible events for the additive noise model.

The bound on P_D which is given in Equation (4) is usually good if $M \ll 2^n$. However, in this case, the bound on P_U, found in Equation (5), is not tight. The reason is that if the valid vectors are sparsely distributed in the results space, then it is highly unlikely that an arbitrary error vector will yield an inspection result that coincides with another valid vector.

An alternative bound on P_U, for the equierror case, results from the fact that there are exactly $M-1$ error vectors with weight not less than d that result in other valid vectors, thus,

$$P_U \leq (M-1)p^d(1-p)^{n-d}. \tag{6}$$

Given the set of all valid vectors, $\mathbf{C}$, one can obtain exact expressions for the probabilities of detectable and undetectable errors. For any pair of valid vectors $\mathbf{c}_i, \mathbf{c}_j \in \mathbf{C}$, $\mathbf{c}_i \neq \mathbf{c}_j$ let us define $w(i,j)$ as the weight of the error vector $\mathbf{e} = \mathbf{c}_i \oplus \mathbf{c}_j$ that causes $\mathbf{c}_i$ to look like $\mathbf{c}_j$, i.e., $w(i,j) = d_H(\mathbf{c}_i, \mathbf{c}_j) = w(\mathbf{c}_i \oplus \mathbf{c}_j)$. We shall denote by $P_U(\mathbf{c}_j)$ the probability of having an undetectable error when the error-free result vector is $\mathbf{c}_j$. Then we obtain the following

$$P_U(\mathbf{c}_j) = \sum_{i \neq j} \mathscr{P}_j(\mathbf{c}_i \oplus \mathbf{c}_j), \tag{7}$$

where for the equierror case,

$$P_U(\mathbf{c}_j) \sum_{i \neq j} p^{w(i,j)}(1-p)^{n-w(i,j)}. \tag{8}$$

Thus, the probability of an undetectable error is as follows,

$$P_U = \sum_{j=1}^{M} q_j P_U(\mathbf{c}_j). \tag{9}$$

The probability of having a detectable error is now calculated by

$$P_D = 1 - P_N - P_U. \tag{10}$$

3.3. *Error-correction performance*

From Observation 1 we note that the SCI decision rule is guaranteed to correctly classify the PUs if the number of inspection errors is less than $t = \lfloor (d-1)/2 \rfloor$.

Note that the SCI decision rule classifies PUs by selecting the valid vector, which is the closest to the result vector in terms of the Hamming distance, even if this distance is larger than t (in ECC this is known as the *complete decoders*). Accordingly, the probability of having at least one inspection error and a correct classification, P_C, is bounded from below, for the equierror case, by

$$P_C \geq \sum_{j=1}^{t} \binom{n}{j} p^j (1-p)^{n-j}. \tag{11}$$

This bound becomes tighter, as the probability of an inspection error, p, gets smaller, that is when

$$p^j(1-p)^{n-j} \gg p^{j+1}(1-p)^{n-(j+1)}.$$

An exact expression for P_C can be obtained if we are willing to examine all vectors in $\hat{\mathbf{C}}$. In this case, P_C can be written as

$$P_C = \sum_j q_j \sum_{\hat{\mathbf{c}} \in \hat{\mathbf{C}} \setminus \mathbf{C} | D(\hat{\mathbf{c}}) = j} \mathscr{P}_j(\mathbf{c}_j \oplus \hat{\mathbf{c}}), \tag{12}$$

where $\hat{\mathbf{C}} \setminus \mathbf{C}$ is the set of elements in $\hat{\mathbf{C}}$ which are *not* in $\mathbf{C}$ and the notation | should be read "such that". The sum is taken over the set $\hat{\mathbf{C}} \setminus \mathbf{C}$ since the probability of correctly classifying a valid vector is accounted for in P_N. The probability of a detectable non-correctable error, P_{NC}, is defined as the probability that a detectable error has occurred but has not been properly corrected. Thus,

$$P_{NC} = P_D - P_C. \tag{13}$$

The probability of detecting an inspection error and having a classification error, P_E, is bounded from above by the probability of having an error vector of weight greater than t. For the equierror case, this probability is:

$$P_E \leq \sum_{j=t+1}^{n} \binom{n}{j} p^j (1-p)^{n-j} = 1 - \sum_{j=0}^{t} \binom{n}{j} p^j (1-p)^{n-j}. \tag{14}$$

The same bound can be applied to the probability of a classification failure, P_F. A classification failure occurs when the identity of the closest valid vector is not unique. An exact expression for P_F can be found by summing, for each unclassifiable vector, the ways by which it can be obtained,

$$P_F = \sum_{\hat{\mathbf{c}} \in \hat{\mathbf{C}} | D(\hat{\mathbf{c}}) = 0} \sum_j q_j \mathscr{P}_j(\hat{\mathbf{c}} \oplus \mathbf{c}_j). \tag{15}$$

Now, the probability of an erroneous classification, i.e., the probability of a detectable non-correctable error that yields an incorrect classification of the PU, is given by

$$P_E = P_{NC} - P_F. \tag{16}$$

Note that if errors occurred in the IOs such that the result vector is a valid vector, then the classification will be incorrect, but we do not include this case in the probability P_E. Table 1 summarizes the different inspection events and their associated classification outcomes.

Finally, the probability of correct classification of a PU is given by

$$P_{CC} = P_N + P_C, \tag{17}$$

while the probability of not having a correct classification is given by

$$P_{UC} = P_U + P_{NC} = P_U + P_E + P_F. \tag{18}$$

When inspection is relatively cheap (e.g., in the case of automated inspection) it might be reasonable to reinspect all PUs that are unclassifiable. Doing so and

Table 1. Inspection events and their outcomes

	Event	*Notation*	*Equation*	*Classification*
1	No errors	P_N	(3)	correct
2	Undetectable errors	P_U	(9)	incorrect
3	Detectable errors	P_D	(10)	depends
3.1	Correctable errors	P_C	(12)	correct
3.2	Non-correctable errors	P_{NC}	(13)	depends
3.2.1	Failure to classify	P_F	(15)	none
3.2.2	Erroneous classification	P_E	(16)	incorrect

assuming that the system has no memory, the new probabilities of the inspection events are as follows:

$$P'_N = P_N/(1-P_F); P'_U = P_U/(1-P_F);$$
$$P'_D = 1 - P'_N - P'_U; P'_C = P_C/(1-P_F);$$
$$P'_{NC} = P_{NC}/(1-P_F); P_E = P_E/(1-P_F) \text{ and } P'_F = 0. \quad (19)$$

4. Analysis of SCI cases

The performance analysis presented in this section is based on the equierror assumption. This assumption requires the least amount of information about the error characteristics of the IOs and follows directly from the principle of maximum entropy (Cover and Thomas, 1991). As experience with the inspection system accumulates, more information about the error characteristics of the IOs may become available. A performance analysis could then be carried out along the same lines as presented here, but with more accurate inspection error probability estimates. Then, for example, the SCI rule can implement a weighted Hamming distance to maintain the one-to-one correspondence between the distance and the *a posteriori* decoding probability.

We begin with the least restrictive case, where **C** is an arbitrary set of valid vectors. Following this, we consider the case where **C** forms a linear vector space (equivalent to linear-block codes). This situation is less practical since the IOs cannot always be selected such that their error-free result vectors form a linear vector space. However, if such selection is possible, then certain results of linear-block codes can be applied to further analyze the correction and detection ability of the SCI decision rule.

4.1. *The set of valid vectors is an arbitrary set*

To illustrate the application of the SCI decision rule when the valid vectors constitutes an arbitrary set, we present a real-life example taken from the inspection of wafers in a FAB. In the deposition chamber, wafers pass through the lithography module to undergo three operations: (i) deposition; (ii) print; and (iii) development. Occasionally, the process goes out of control and when it does there are three equally probable causes: (i) a general deposition problem; (ii) a contamination (hot spot) problem; or (iii) a stepper problem. Defective wafers are subjected to a series of five inspections to determine the cause of the problem (because of the system's reliable nature and the high intensity of system monitoring, it is assumed that only one problem occurs at a time). The inspections are called: (i) overlay (checking whether the different layers are properly positioned one above the other); (ii) layer thickness (measuring the photo resist material); (iii) critical dimensions; (iv) defect macro; and (v) defect micro. Each of the possible causes of defects has its own 'signature' in terms of error-free inspection results. If there is a general deposition problem, then the wafer should fail inspections (ii) and (iii), but pass the rest (result vector $\mathbf{c}_1 = (01100)$). If there is a contamination problem, then the wafer should fail all but the third test (result vector $\mathbf{c}_2 = (11011)$). Finally, if there is a stepper problem, then the part should fail the first inspection and pass all the others (result vector $\mathbf{c}_3 = (10000)$). Given this scenario, there are three valid vectors ($M = 3$). Since the IOs are binary, generally two IOs ($\lceil \log_2 3 \rceil = 2$) are required in order to classify the PUs (in this example the first and second inspections would suffice). However, if we use only two inspections, then the minimum distance between valid vectors is one, and any inspection error can result in misclassification or failure to classify. Increasing the number of IOs increases the distances between valid vectors and allows the SCI decision rule to correct inspection errors. We thus obtain: $w(1,2) = 4$, $w(1,3) = w(2,3) = 3$, $d = 3$ and $t = \lfloor (3-1)/2 \rfloor = 1$.

The performance of the SCI decision rule can be investigated by generating error vectors systematically, adding them to the error-free valid vectors and checking whether the result vectors are still closer to the original valid vector. The results are shown in Table 2. It appears that only the five error vectors with weight one can be corrected for all the valid vectors. Eight other error vectors, each with weight two, can also be corrected for one of the valid vectors. Other error vectors cannot be corrected by the SCI decision rule, since the result vectors are closer to other valid vectors or are unclassifiable. For example, consider the weight two error vector $\mathbf{e} = (00011)$. Note that $d_H(\mathbf{c}_1 \oplus \mathbf{e}, \mathbf{c}_1) = d_H(\mathbf{c}_1 \oplus \mathbf{e}, \mathbf{c}_2) = 2$, resulting in classification failure. Similarly, note that $d_H(\mathbf{c}_3 \oplus \mathbf{e}, \mathbf{c}_3) = 2$ whereas $d_H(\mathbf{c}_3 \oplus \mathbf{e}, \mathbf{c}_2) = 1$, resulting in a classification error.

It is possible to identify the probabilities of the various events either by using the expressions developed in Section 3 or by turning directly to Table 2. The probability that the result vector is error-free is given by (3), namely

Table 2. Inspection result vectors (for the sake of clarity we have omitted the vector parentheses)

Error vectors	$\mathscr{P}_j(\boldsymbol{e})$	*Error-free result vector is* $\boldsymbol{c}_1$ = *01100*			*Error-free result vector is* $\boldsymbol{c}_2$ = *11011*			*Error-free result vector is* $\boldsymbol{c}_3$ = *10000*		
		$\hat{\boldsymbol{c}}$	$\boldsymbol{c}_{D(\hat{\boldsymbol{c}})}$		$\hat{\boldsymbol{c}}$	$\boldsymbol{c}_{D(\hat{\boldsymbol{c}})}$		$\hat{\boldsymbol{c}}$	$\boldsymbol{c}_{D(\hat{\boldsymbol{c}})}$	
00000	$(1-p)^5$	01100	01100	N	11011	11011	N	10000	10000	N
00001	$p(1-p)^4$	01101	01100	C	11010	11011	C	10001	10000	C
00010	$p(1-p)^4$	01110	01100	C	11001	11011	C	10010	10000	C
00011	$p^2(1-p)^3$	01111	–	F	11000	10000	E	10011	11011	E
00100	$p(1-p)^4$	01000	01100	C	11111	11011	C	10100	10000	C
00101	$p^2(1-p)^3$	01001	–	F	11110	–	F	10101	10000	C
00110	$p^2(1-p)^3$	01010	–	F	11101	–	F	10110	10000	C
00111	$p^3(1-p)^2$	01011	11011	E	11100	01100	E	10111	11011	E
01000	$p(1-p)^4$	00100	01100	C	10011	11011	C	11000	10000	C
01001	$p^2(1-p)^3$	00101	01100	C	10010	10000	E	11001	11011	E
01010	$p^2(1-p)^3$	00110	01100	C	10001	10000	E	11010	11011	E
01011	$p^3(1-p)^2$	00111	–	F	10000	10000	U	11011	11011	U
01100	$p^2(1-p)^3$	00000	10000	E	10111	11011	C	11100	01100	E
01101	$p^3(1-p)^2$	00001	10000	E	10110	10000	E	11101	–	F
01110	$p^3(1-p)^2$	00010	10000	E	10101	10000	E	11110	–	F
01111	$p^4(1-p)$	00011	11011	E	10100	10000	E	11111	11011	E
10000	$p(1-p)^4$	11100	01100	C	01011	11011	C	00000	10000	C
10001	$p^2(1-p)^3$	11101	–	F	01010	–	F	00001	10000	C
10010	$p^2(1-p)^3$	11110	–	F	01001	–	F	00010	10000	C
10011	$p^3(1-p)^2$	11111	11011	E	01000	01100	E	00011	11011	E
10100	$p^2(1-p)^3$	11000	10000	E	01111	–	F	00100	01100	E
10101	$p^3(1-p)^2$	11001	11011	E	01110	01100	E	00101	01100	E
10110	$p^3(1-p)^2$	11010	11011	E	01101	01100	E	00110	01100	E
10111	$p^4(1-p)$	11011	11011	U	01100	01100	U	00111	–	F
11000	$p^2(1-p)^3$	10100	10000	E	00011	11011	C	01000	01100	E
11001	$p^3(1-p)^2$	10101	10000	E	00010	10000	E	01001	–	F
11010	$p^3(1-p)^2$	10110	10000	E	00001	10000	E	01010	–	F
11011	$p^4(1-p)$	10111	11011	E	00000	10000	E	01011	11011	E
11100	$p^3(1-p)^2$	10000	10000	U	00111	–	F	01100	01100	U
11101	$p^4(1-p)$	10001	10000	E	00110	01100	E	01101	01100	E
11110	$p^4(1-p)$	10010	10000	E	00101	01100	E	01110	01100	E
11111	$p^5(1-p)$	10011	11011	E	00100	01100	E	01111	–	F

N – no errors; U – undetectable errors; C – correctable errors; E – erroneous classification; F – failure to classify.

$P_{\mathrm{N}} = (1-p)^5$. The probability that an error vector modifies the result vector to look like another valid vector is given by (8) and (9) as

$$P_{\mathrm{U}} = \sum_{i=1}^{3} \tfrac{1}{3} \sum_{j \neq i} p^{w(i,j)} (1-p)^{n-w(i,j)} = \tfrac{1}{3}\left[2p^4(1-p) + 4p^3(1-p)^2\right]. \quad (20)$$

The probability of detecting an error is given by (10), $P_{\mathrm{D}} = 1 - P_{\mathrm{N}} - P_{\mathrm{U}}$. Using (12) we have

$$P_{\mathrm{C}} = 5p(1-p)^4 + \tfrac{1}{3}[8p^2(1-p)^3]. \quad (21)$$

The probability of a detectable, non-correctable error is given by (13), $P_{\mathrm{NC}} = P_{\mathrm{D}} - P_{\mathrm{C}}$. The probability of classification failure, P_{F}, is given by (15) and is equal to

$$P_{\mathrm{F}} = \tfrac{1}{3}[9p^2(1-p)^3 + 7p^3(1-p)^2 + p^4(1-p) + p^5]. \quad (22)$$

The probability of erroneous classification, is given in (16), and is equal to

$$P_{\mathrm{E}} = \tfrac{1}{3}[18p^2(1-p)^3 + 22p^3(1-p)^2 + 12p^4(1-p) + p^5] \quad (23)$$

The probability of a correct classification, P_{CC}, and the probability of not having a correct classification, P_{UC}, are obtained by (17) and (18), respectively.

4.2. *The set of valid vectors forms a linear vector space*

Additional ECC applications are available for the case where the valid vectors that belong to **C** form a linear vector space. This happens when we are able to select the IOs such that the valid vectors form a linear subspace of $\hat{\mathbf{C}}$. In such a case, one can apply classification methods based on known linear-block codes results (Lin and Costello, 1983; Wicker, 1995).

The main advantage of **C** having such a special structure lies in the fact that there is no need to enumerate all the error vectors, as done in (7), (12) and (15). The undetectable error vectors are the valid vectors themselves, since each and every linear combination of valid vectors results in a valid vector. Likewise, the undetectable error vectors are independent of the result vector (Wicker, 1995). Therefore, it follows that if the weights of the vectors in **C** are known, one can obtain exact and simple expressions for the probability of an undetectable error P_U, given in (9), and the probability of having inspection error(s) and a correct classification, P_C, given in (12). In particular, letting A_j be the number of valid vectors of weight j, then

$$P_U = \sum_{j=d}^{n} A_j p^j (1-p)^{n-j}. \quad (24)$$

Let α_l be the number of error vectors of weight l that can be corrected. Note that due to the fact that the valid vectors form a linear vector space, if an error vector renders a correct classification for one valid vector, then it does so for all valid vectors. Thus we have:

$$P_C = \sum_{j=1}^{t} \binom{n}{j} p^j (1-p)^{n-j} + \sum_{l=t+1}^{n} \alpha_l p^l (1-p)^{n-l}. \quad (25)$$

The simplest case in which **C** forms a linear vector space is the equierror case when each PU goes through n identical inspections. Since the outcome of each inspection is binary, a PU can be in one of two possible classes: either it conforms to all n inspections (a "conforming" unit), or it does not conform to all of them (a "non-conforming" unit). The set of valid vectors, **C**, contains the 'all-zeros' vector and the 'all-ones' vector. In particular, the inspection parameters are $d = n$, $t = \lfloor (n-1)/2 \rfloor$, $M = 2$, and the weight distribution is $A_0 = 1$, $A_n = 1$ and $A_i = 0$, $i = 1, \ldots, n-1$. The probability that a result vector is error-free is given by (3), namely $P_N = (1-p)^n$. There is one instance in which we will not be able to detect the occurrence of inspection errors: when all n inspections provide incorrect results. The probability of such an event is given by (9) or alternatively by (24) as

$$P_U = p^n. \quad (26)$$

The probability of having a detectable error is obtained by (10), i.e.,

$$P_D = 1 - (1-p)^n - p^n. \quad (27)$$

Note that this is the probability that the error vector weight is between one and $n-1$. The probability of a correctable error, given by (12), is in fact the probability that the weight of the error vector is between one and $t = \lfloor (n-1)/2 \rfloor$, i.e.,

$$P_C = \sum_{j=1}^{t} \binom{n}{j} p^j (1-p)^{n-j}. \quad (28)$$

The probability of a detectable non-correctable error, given by (13), is in fact the probability that the weight of the error vector is between $t+1$ to $n-1$, i.e.,

$$P_{NC} = [1 - (1-p)^n - p^n] - \sum_{j=1}^{t} \binom{n}{j} p^j (1-p)^{n-j} = \sum_{j=t+1}^{n-1} \binom{n}{j} p^j (1-p)^{n-j}. \quad (29)$$

The probability of an erroneous classification, P_E, is in fact the probability that the result vector is within a distance of one to t from the other valid vector, i.e.,

$$P_E = \sum_{k=1}^{t} \binom{n}{k} p^{n-k} (1-p)^k. \quad (30)$$

The probability of classification failure, P_F, can be obtained using (1) or by noting that it is in fact the probability that the result vector falls exactly within a Hamming distance of $t+1$ from both valid vectors. Thus,

$$P_F = \sum_{j=t+1}^{n-1} \binom{n}{j} p^j (1-p)^{n-j} - \sum_{k=1}^{t} \binom{n}{k} p^{n-k} (1-p)^k, = \begin{cases} \binom{n}{n/2} p^{n/2} (1-p)^{n/2} & n \text{ even}, \\ 0 & n \text{ odd}. \end{cases} \quad (31)$$

Note that a classification failure can occur only if there is an even number of inspections. When the number of inspections is odd, $P_F = 0$. This leads us to the following (apparently) paradoxical observation.

Observation 2: Using the identical binary inspection procedure, reducing the number of IOs can actually improve the SCI performance. Decreasing the number of inspections by one (and sometimes even by a larger odd number) from an even number of inspections causes the probability of not having a correct classification, P_{UC}, (18), to decrease and hence, the probability of having a correct classification, P_{CC}, to increase.

Observation 2 is demonstrated graphically in Fig. 4, where we plot P_{UC} against the number of inspections for $p = 0.3$. This observation can be explained intuitively by examining what happens when the number of IOs is decreased by one from an even number, n, to an odd number, $n-1$. First, let us look at the disposition decision that would have been obtained from the $n-1$ IOs. If the majority of the $n-1$ IOs indicate acceptance or rejection by a margin greater than one, then the nth IO

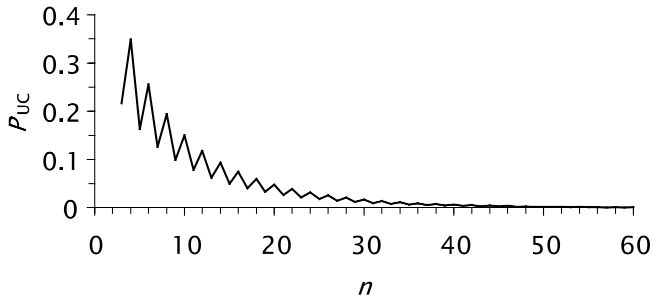

Fig. 4. The probability of not having a correct classification, P_{UC}, plotted as a function of the number of IOs, $p = 0.3$ and $n = 3, 4, 5, \ldots, 60$.

cannot change the decision, and consequently its removal will have no effect. The same is true if the majority of the $n-1$ inspections indicate acceptance or rejection by a margin of exactly one and the result of the nth inspection coincides with this majority. However, if the $n-1$ IOs indicate acceptance (rejection) by a margin of only one and the nth IO indicates rejection (acceptance) – i.e., has a different indication from the majority of the $n-1$ IOs – then the nth inspection changes the decision to failure to classify. If the PU is non-conforming (conforming) then the removal of the nth IO will change the decision from failure to classify to an incorrect classification, which has no affect on the probability of incorrect classification. On the other hand, if the PU is indeed conforming (non-conforming) then the removal of the nth IO will change the decision from failure to classify to a correct classification! In summary, in all cases the removal of the nth IO does not prevent us from classifying the PU correctly and in one situation allows us to correctly classify a PU that we would otherwise deem to be unclassifiable.

The counterintuitive outcome of Observation 2, as shown by the discussion above, is based on the fact that we do not classify a PU when the number of IOs indicating rejection is equal to the number indicating acceptance. There are two obvious alternatives for dealing with a classification failure. The first is to determine the disposition of the part arbitrarily or randomly. With this policy one can show that there is no difference between the probability of a correct classification when using $n-1$ or n inspections (where n is an even number). The second alternative is to repeat the entire inspection set. This indeed solves the counterintuitive outcome above (if n is even, n inspections are better than $n-1$), but a similar, albeit reversed, counterintuitive outcome results. Namely, for any even number n, the probability of a correct classification is always higher with n inspections rather than with $n+1$ inspections. The analysis is similar to that of Observation 2 and is thus omitted.

Another case where **C** forms a linear vector space occurs when all the valid vectors are identical to the codewords of a known linear-block code. In such a case, additional results from ECC theory become applicable to the classification problem. For the sake of compactness, we omit this discussion.

5. Optimal design of inspection systems operating under the SCI decision rule

In this section we demonstrate how the probabilistic expressions that were derived for the various outcomes of the SCI decision rule can be used as building blocks in inspection-related design problems. In particular, we will show how we can determine the number of IOs and the individual inspection reliability level for inspections that are to operate under the SCI decision rule. As stated earlier, the application of the SCI decision rule does not require knowledge of the actual inspection error probabilities. However, in order to proceed with the optimization we need to make some assumptions regarding the inspection error probabilities. In the following we will assume the equierror case as the least restrictive assumption.

We first address the problem of finding the optimal number of IOs in order to minimize costs. We are concerned with two costs: the direct inspection cost, $F^i(n)$, and the misclassification cost, $F^e(n,p)$, and we wish to minimize their sum, denoted by TC. In general, $F^i(n)$ increases with n and $F^e(n,p)$ decreases with n. The problem can thus be stated as follows:

$$TC^* = \min_n TC = \min_n \{F^i(n) + F^e(n,p)\}. \quad (32)$$

We further assume:

$F^i(n) = nC_I$ – a linear cost function in n – where C_I is the unit inspection cost.

$F^e(n,p) = P_{UC} \times C_P$, where C_P is the misclassification cost per unit.

Note that based on Observation 2 we will always choose n to be odd. Accordingly, and using (18), (26) and (29), the objective function becomes

$$\begin{aligned} TC^* &= \min_{n|n \text{ odd}} \{nC_I + P_{UC} \times C_P\} \\ &= \min_{n|n \text{ odd}} \left\{ nC_I + C_P \left[p^n + \sum_{j=(n+1)/2}^{n-1} \binom{n}{j} p^j (1-p)^{n-j} \right] \right\}. \quad (33) \end{aligned}$$

Equation (33) can be optimized numerically for a given inspection error probability p and cost constants C_I and C_P. Figure 5 presents the total cost (TC) as a function of odd numbers of IOs, where $p = 0.3$, and $C_I = 1$. Three different misclassification costs are considered: $C_P = 55$; 80; and 120. The optimal solutions and costs are, respectively, ($n = 7$; $TC^* = 13.93$), ($n = 9$; $TC^* = 16.9$) and ($n = 11$; $TC^* = 20.38$).

The second design issue that we address is determining the desired inspection reliability level, as represented by the probability of inspection error p. The approach is illustrated using the example from Section 4.1. Recall that

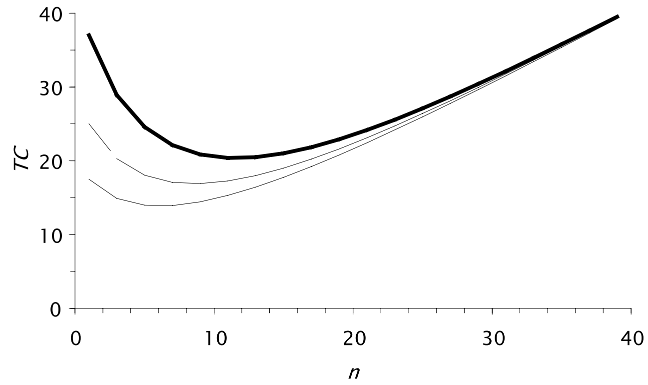

Fig. 5. The total cost (*TC*) as a function of an odd number of IOs, where $p = 0.3$, $C_P = 1$ and $n = 1, 3, \ldots, 39$. Three different misclassification costs are considered: $C_P = 55$ (bottom line); $C_P = 80$ (middle line) and $C_P = 120$ (bold line).

each valid vector points to one of three possible production problems. Here we assume the equierror case; however, we will assume some prior knowledge about the distribution of the production problems. In particular, the $\mathbf{c}_1$ vector, which is associated with a general deposition problem, appears with probability 0.7 and can be repaired for a cost of C_g dollars per unit. The $\mathbf{c}_2$ vector, which is associated with a contamination problem, appears with probability 0.2 and has a rework cost of C_c dollars per unit. The $\mathbf{c}_3$ vector, which is associated with a stepper problem, appears with probability 0.1 and has a rework cost of C_s dollars per unit. Each non-conforming unit that is mistakenly shipped to the customer generates a failure cost (penalty) of C_f dollars. In addition, there is a cost, C_r, associated with increasing the reliability of the inspection equipment that is used by the IOs. For the sake of simplicity, we assume that the reliability cost function is linear and is given by $(1-p)C_r$ dollars per unit. All other costs are assumed to be negligible. Table 3 presents the costs of the different inspection outcomes.

Expected costs are obtained from Table 3 by multiplying the last two columns – the cost column by the classification-probability column. This simple optimization problem can be formulated and solved if the cost constants are known. For example, with the cost constants $C_g = 1$, $C_c = 1$, $C_s = 3$, $C_f = 10$, and $C_r = 7$, the following optimization problem is obtained:

$$\min_p TC = \min_p[\text{classification costs} + \text{reliability cost}]$$
$$= \min_p[8.2 - 7p + 29.7p^2 - 47.6p^3 + 37.8p^4 - 11.2p^5]. \quad (34)$$

Using standard techniques the optimal solution is ($p^* = 0.185$, $TC^* = 7.66$ per unit).

Table 3. Analysis of the example from Section 4.1

Valid vector $\mathbf{c}_j$	*Classification decision* $\mathbf{c}_{D(\hat{\mathbf{c}})}$	*Probability* $Pr\{D(\hat{\mathbf{c}})\|\mathbf{c}_j\}$	*Cost [\$ per unit]*
$\mathbf{c}_1$	$\mathbf{c}_1$ (correct)	$(1-p)^5 + 5p(1-p)^4$	C_g
$q_1 = 0.7$	$\mathbf{c}_2$	$p^3(4 - 7p + 4p^2)$	$C_c + C_f$
	$\mathbf{c}_3$	$p^2(3 - 8p + 9p^2 - 4p^3)$	$C_s + C_f$
$\mathbf{c}_2$	$\mathbf{c}_1$	$p^3(4 - 7p + 4p^2)$	$C_g + C_f$
$q_2 = 0.2$	$\mathbf{c}_2$ (correct)	$(1-p)^5 + 5p(1-p)^4$	C_c
	$\mathbf{c}_3$	$p^2(3 - 8p + 9p^2 - 4p^3)$	$C_s + C_f$
$\mathbf{c}_3$	$\mathbf{c}_1$	$p^2(3 - 8p + 9p^2 - 4p^3)$	$C_g + C_f$
$q_3 = 0.1$	$\mathbf{c}_2$	$p^2(3 - 8p + 9p^2 - 4p^3)$	$C_c + C_f$
	$\mathbf{c}_3$ (correct)	$(1-p)^5 + 5p(1-p)^4$	C_s

6. Concluding remarks

In this paper we have introduced a novel approach for dealing with inspection subject to errors. This approach, based on the theory of Error-Correcting Codes (ECC), leads to the development of a Self-Correcting Inspection (SCI) decision rule that does not require knowledge of individual inspection error probabilities. The main part of the paper was devoted to the analysis of the performance of the SCI decision rule under certain basic assumptions. We believe that this work could be extended in a number of ways. First, it will be worthwhile to develop probability expression for the cases when the inspection error probabilities are not symmetric and vary across IOs. It will also be interesting to develop expressions for nonbinary IOs. We predict that most of these expressions will be mathematically complicated, and that it should be worthwhile to look for results from communications theory in order to facilitate the development. An important issue to be studied is the sensitivity of the classification decision, and of other system parameters such as total cost and overall classification reliability, to the various simplifying assumptions that were made to facilitate the mathematical treatment. A potential research direction is to consider cases where the error probabilities are known to be unequal, possibly using a weighted Hamming distance to maximize the a *posteriori* probability of correct classification. Overall, we believe that the analogy between communications theory and inspection is worthwhile researching and can lead to new and effective solutions to practical problems, as illustrated in our example.

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References

Ben-Gal, I. and Levitin, L. (1998) Bounds on code distance and efficient fractional factorial experiments, in *Proceedings of* *the IEEE International Symposium on Information Theory*, Boston, MA, p. 436.

Ben-Gal, I. and Levitin, L. (1999) Application of information theory and error-correcting codes to fractional factorial experiments. *Journal of Statistical Planning and Inference*, **92**(1–2), 267–282.

Chen, S.-L. and Chung, K.-J. (1994) Inspection error effects on economic selection of target value for a production process. *European Journal of Operational Research*, **79**, 311–324.

Chevalier, P.B. and Wein, L.M. (1997) Inspection for circuit board assembly. *Management Science*, **43**(9), 1198–1213.

Cover, T.M. and Thomas, J.A. (1991) *Elements of Information Theory*, Wiley & Sons, New York, NY.

Hong, S.H. and Elsayed, E.A. (1999) The optimum mean for processes with normally distributed measurement error. *Journal of Quality Technology*, **31**(3), 338–344.

Lin, S. and Costello, D.J.C. (1983) *Error Control Coding Fundamentals and Applications*, Prentice-Hall, Englewood Cliffs, NJ.

Moskowitz, H., Plante, R. and Tsai, H.-T. (1991) Single-sided economic screening models incorporating individual unit misclassification error and risk preference. *European Journal of Operational Research*, **53**, 228–243.

Moskowitz, H., Plante, R. and Tsai, H.-T. (1993) A multistage screening model for evaluation and control of misclassification error in the detection of hypertension. *Management Science*, **39**(3), 307–321.

Moskowitz, H. and Tsai, H.-T. (1988) A one-sided double screening procedure using individual unit misclassification error. *Management Science*, **34**(9), 1139–1153.

Peterson, W. and Weldon, E.J. (1972) *Error-Correcting Codes*, 2nd edn, MIT Press, Cambridge, MA.

Raz, T. and Bousum, K. (1990) A recursive partitioning method for the selection of quality assurance tests. *Quality Engineering*, **2**(3), 319–329.

Raz, T. and Thomas, M.U. (eds.) (1990) *Design of Inspection Systems*, Quality Press, Milwaukee, WI.

Tang, K. and Schneider, H. (1988) Selection of the optimal inspection precision level for a complete inspection plan. *Journal of Quality Technology*, **20**(3), 153–156.

Wicker, S.B. (1995) *Error Control Systems for Digital Communication and Storage*, Prentice-Hall, Englewood Cliffs, NJ.

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