Utrecht University
Faculty of Science
Department of Mathematics
Seminar Mathematics in Islamic Arts 2010

Muqarnas

Mathematics in Islamic Arts

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Contents

Preface

As stated on the front, this reader is about muqarnas, which is a topic in mathematics in Islamic art. In here you will find the theory discussed in the workshop, background information and some exercises.

At the end of the workshop a whole muqarnas is to be constructed by the group. In order to achieve this goal, first will be explained what a muqarnas is. In addition to this, some historical information will be given. Next we will consider how the different elements look from which a muqarnas is constructed.

In the second part of the workshop we consider how a two-dimensional muqarnas design can be regarded as a three-dimensional muqarnas structure. Finally, the group will build a muqarnas using the different elements made out of cardboard paper.

CHAPTER 1

Introduction

Muqarnas is the Arabic word for stalactite vault. It is an originally Islamic type of wall or ceiling decoration, which is used to make a smooth transition from the rectangular basis of the building to the vaulted ceiling. On the front page of this reader you can see an example of a muqarnas. However, muqarnas are not only used as a ceiling decoration. You can also find muqarnas on minarets or the eaves of a building, for example.

A muqarnas consists of tiers (layers), which themselves consist of elements. There is a great variety of these elements, yet most of them can be more or less deduced from a small set of basic elements. Among these basic elements we can distinguish cells and intermediate elements. The cells look like small pieces of a vault. They are the most impor-

FIGURE 1.1. Muqarnas niche in the Friday Mosque (Iran).

tant in building a muqarnas, since they provide the 'body' of the muqarnas. The intermediate elements can be used to combine cells together, although they are not essentially needed and can be omitted. We will discuss more about the elements of a muqarnas later.
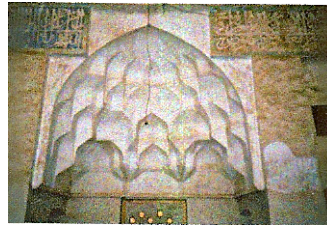
One thing that is very interesting about muqarnas is the way they are designed. Muqarnas designs are two-dimensional. We don't know how these two-dimensional designs were transferred to three-dimensional structures. This is the main problem we will consider in this workshop.
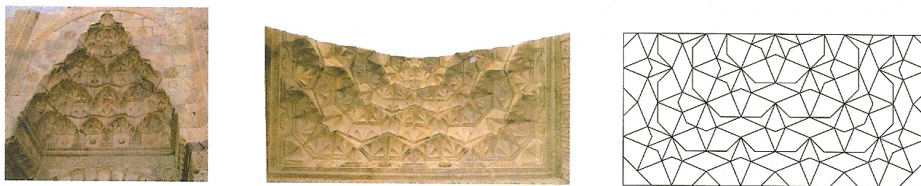

FIGURE 1.2. A muqarnas with its design.

CHAPTER 2

History of muqarnas

In the middle of the tenth century muqarnas began to develop in North Eastern Iran and central North Africa. Although the developments occurred simultaneously, it is not known whether they were related. Muqarnas spread throughout the Islamic world from the eleventh century on.

1. al-Kāshī

The earliest mathematical approach to muqarnas found is given by al-Kāshī. Jamshīd Ghiyath al-Din al-Kāshī was a great mathematician and astronomer born around 1380 in Kashan, Iran. He is mostly known for his work *The Key of Arithmetic*, which he wrote in 1427, two years before he died. In this book al-Kāshī approximates the surface of a muqarnas and gives the earliest definition of muqarnas (see chapter 3).

al-Kāshī distinguishes four types of muqarnas: the simple muqarnas, the clay-plastered muqarnas, the curved muqarnas and the Shirazi muqarnas. The simple and clay-plastered muqarnas consist only of plane surfaces. The clay-plastered muqarnas is similar to the simple muqarnas, except that its tiers are not all of the same height. In the curved and Shirazi muqarnas also curved surfaces are used. The Shirazi muqarnas is like a curved muqarnas but has a greater variety of elements. Where the top views of the simple, clay-plastered and curved muqarnas all consist of triangles and quadrilaterals, a Shirazi muqarnas plan also contains other polygons such as pentagons, hexagons, octagons and multipointed stars.

2. Shiro Takahashi

Another distinction of muqarnas types is given by Shiro Takahashi (1943-). He classifies muqarnas into three different types: the square lattice muqarnas, the pole table muqarnas and 'other style' muqarnas, which do not belong to the first two types.

The most important characteristic of the square lattice muqarnas is that the top view of the muqarnas is filled with squares and 45 degree rhombuses. They were developed in the eleventh century and over the next thousand years they spread throughout the entire Islamic world. Also typical for the square lattice muqarnas is that they have a fourfold rotational symmetry. The stalactite decoration of the Alhambra Palace in
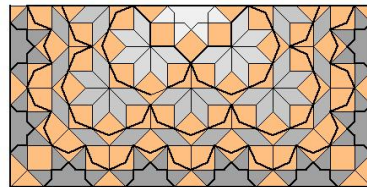

FIGURE 2.1. **Square lattice:** Design of a muqarnas in Samarqand.

Granad from the fourteenth century is a beautiful example of a square lattice muqarnas at its peak.

FIGURE 2.2. **Square lattice:** One of the Two Sisters of the Alhambra Palace.

After the invasion and fall of the Mongols in the fifteenth century the pole table pattern appeared and became widespread in western Asia (Iran included). The pole table muqarnas has no direct link with the architectural structure. The different elements of the muqarnas are first produced on the ground, before being attached to the architectural structure using ribs. The *center* (the top of a muqarnas) of the pole table expanded from 4, 5 and 6 segments to 7 and 11. The star shaped elements also include the 7 and 9 point stars. In the seventeenth century the pole table pattern reached its peak with the Shah Mosque at Isfahan.
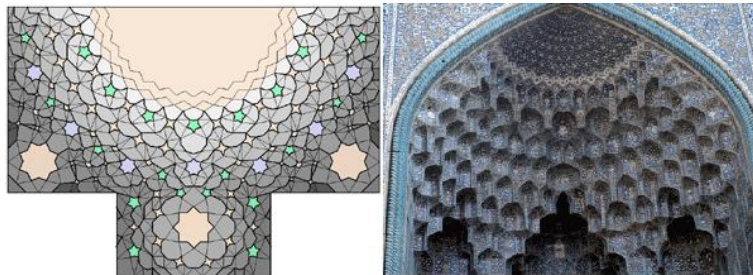

FIGURE 2.3. **Pole table:** The design and a picture of the Shah Mosque.

In Turkey, Syria and Egypt other materials were used than those who were custom in Iran. Instead of sundried bricks, also stone and wood were used to build a muqarnas. This led to the creation and evolution of original muqarnas styles. Therefore there are many different styles of muqarnas which cannot be properly categorized. Unfortunately, after the end of the Safavid dynasty in 1735 the traditional muqarnas culture stagnated because of the modernization.
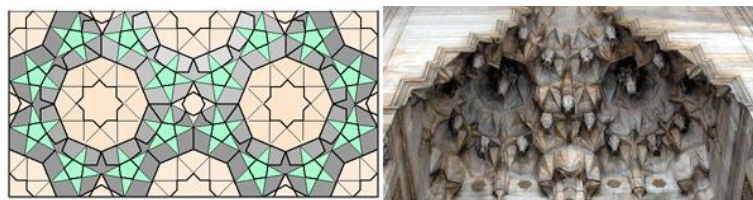

FIGURE 2.4. **Other style:** Muqarnas designed by Sinan in 1557 in the Süleymaniye Mosque, Istanbul, Turkey.

In this workshop we will mainly consider Il-Khanid muqarnas. These muqarnas were constructed during the Il-Khanid dynasty, which took place in the thirteenth century. In this period Iran, along with parts of Iraq, Afghanistan, Turkmenistan, Uzbekistan and Azerbaijan, was under the reign of a Mongolian ruler (a grandson of Ghengis Khan). Il-Khanid muqarnas are a type of curved muqarnas (according to al-Kāshī) and square lattice muqarnas (according to Shiro Takahashi). This means that Il-Khanid muqarnas consist of curved surfaces, with a top view that is filled with squares and 45 degree rhombuses.

3. Muqarnas designs

The three-dimensional muqarnas can be projected to the plane because the elements do not overlap. This results in a design of the muqarnas. The earliest known example of a muqarnas design is found in 1968 by German archaeologists amid the ruins of the Takht-i-Sulaymān palace in Iran. On this thirteen century stucco plate the design of one quarter of a muqarnas vault is engraved. Among the ruins there are also remains of actual muqarnas remains discovered. The elements of these muqarnas seem to be prefabricated and fit well in the definition given by al-Kāshī; the height of the elements is twice their width. Where the original muqarnas could have been located can no longer be determined.

A whole collection of muqarnas designs was discovered in 1986 at Istanbuls Topkapı Palace Museum Library. This is a collection of scrolls painted at the beginning of the sixteenth century, containing 114 drawings. It is the earliest manuscript of its kind to have been found intact. It is now being kept at the Topkapı Palace Museum in Istanbul and therefore it is called the Topkapı Scroll.

FIGURE 2.5. One of the muqarnas designs from the Topkapı Scrolls.

CHAPTER 3

The elements of a muqarnas

1. Definitions of the elements according to al-Kāshī

As said in the introduction, a muqarnas consists of different elements. These elements can be distinguished between cells and intermediate elements. We use the word cell as a translation for the Arabic word *bayt*, which also can be translated to 'house'. al-Kāshī gives the following definition for a cell:

DEFINITION 1. *The muqarnas is a roofed (musaqqaf) [vault] like a staircase (madraj) with facets (ḍil') and a flat roof (saṭh). Every facet intersects the adjacent one at either a right angle, or half a right angle, or their sum, or another combination of these two. The two facets can be thought of as standing on a plane parallel to the horizon. Above them is built either a flat surface not parallel to the horizon, or two surfaces, either flat or curved, that constitute their roof. Both facets together with their roof are called one cell (bayt).*

In this definition al-Kāshī states that a muqarnas consists of cells, and that cells can be divided into facets and a roof. The facets are the straight planes perpendicular to the horizon and the roof is the upper part of the cell. al-Kāshī made this division to make the calculation for finding the surface of the muqarnas easier.
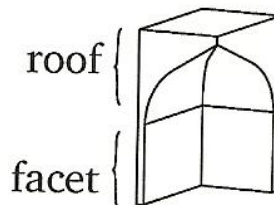

FIGURE 3.1. A muqarnas cell.

al-Kāshī also gives a vague description of intermediate elements which have the form of a triangle and are situated between two adjacent cells:

DEFINITION 2. *Between the roofs of two adjacent cells a curved surface can be located in the form of either a triangle or two triangles.*
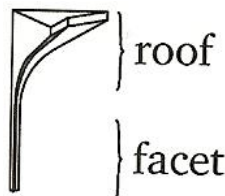

FIGURE 3.2. An intermediate element.

These intermediate elements can also be divided into facets and a roof. The facets of the intermediate elements can however be omitted; then the width of the facets is assumed to be zero. Between two adjacent cells either one intermediate element (one triangle) can be

located or two intermediate elements (two triangles). One intermediate element can thus connect the roofs of two cells or of one cell and another intermediate element. It is also possible that two cells lack the connection of an intermediate element (see figure 3.3).
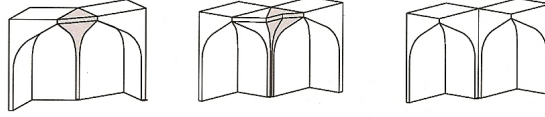

FIGURE 3.3. Left: an intermediate element connects two cells. Center: an intermediate element connects a cell with another intermediate element. Right: two cells without the connection of an intermediate element.

All the different elements of a muqarnas are placed in *tiers*, for which al-Kāshī gives the following definition:

DEFINITION 3. *Adjacent cells, which have their bases on one and the same surface parallel to the horizon, are called one tier (ṭabaqa).*
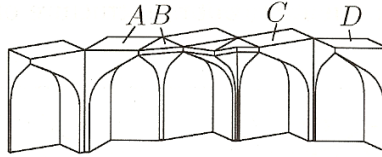

FIGURE 3.4. Part of a tier of the muqarnas niche in the Friday Mosque.

To fit all the different elements together, they are constructed with the same unit of measure. al-Kāshī calls this unit the *module* of the muqarnas and defines it as follows:

DEFINITION 4. *The measure of the base of the largest facet is called the module (miqyās) of the muqarnas.*

2. The curved side

Every element of a muqarnas has two curved sides which all have the same measurements and shape. al-Kāshī developed a construction for this curved side which shows remarkable similarities with the remains of elements found at Takht-i-Sulaymān. This construction can be reproduced as follows:

CONSTRUCTION 1.   (1) *Draw a horizontal line AB.*
(2) *Construct AC perpendicular to AB with length twice the length of AB.*
(3) *Find point D on AC such that $\angle ABD = 30°$.*
(4) *Divide line BD into five equal parts.*
(5) *Find point E on BD such that $|BE| = \frac{3}{5}|BD|$*
(6) *Draw the circle c_1 with middle point D and radius $|DE|$ intersecting DC in point F.*

(7) *Draw two circles with radius $|EF|$. Circle c_2 with middle point E and circle c_3 with middle point F.*

(8) *Define point G as the intersection of c_2 and c_3 below line AB.*

(9) *Draw the arch of the curved side, this is part of the circle with middle point G through the points E and F.*

We then add edges to the curved side, so that it gets thicker. The short vertical line of the curved side, the front side of an element, is called the *apex*. The long vertical line is simply called the *backside*. In a cell the two curved sides join at the apex. In an intermediate element they join at the backside.
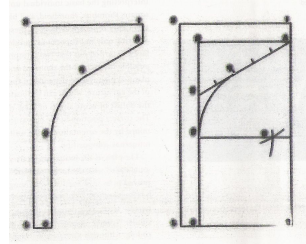

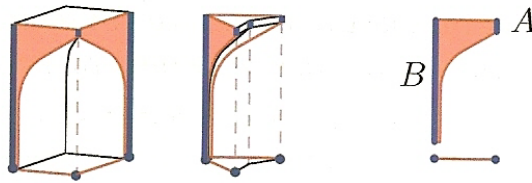FIGURE 3.5. The construction of the curved side.

FIGURE 3.6. The curved sides of a cell and an intermediate element. Side A is the apex; side B is the backside.

3. Basis of the Elements

In principle there is an infinite number of possible muqarnas elements. However, Il-Khanid muqarnas consist of only a small set of elements. The top views of these elements are based on the square and the rhombus. For a unit, the *module (miqyās)*, we choose the length of the side of the square element.

First we will discuss the elements that have the square for a basis.

The top view of the *square* element has by definition ribs of length one and four right angles. This element can appear as a cell and as an intermediate element.

The *jug* element has two ribs of length one. These sides correspond with the curved sides of the element. The short diagonal also has length one. In the Il-Khanid muqarnas the jug only occurs as a cell.

The top view of the *large biped* element is what remains after the projection of a jug element is taken from the projection of a square element. It thus has two sides of length one and a diagonal of length $\sqrt{2} - 1$. This element only occurs as an intermediate element, often in combination with a jug element.

The last element that is based on the square is the *half square* element. It is, as the name suggests, a half-element: the square split over its diagonal. These elements mostly occur as a cell on the edge of a muqarnas. However, it can also
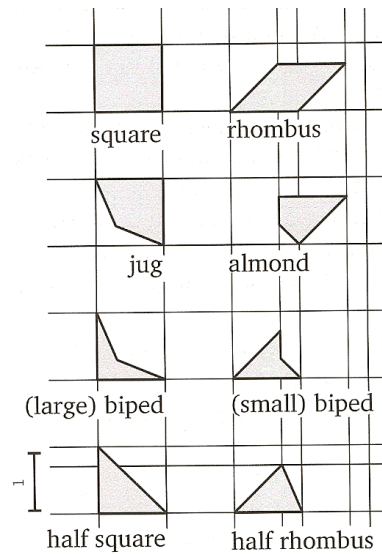
FIGURE 3.7. The top views of the basic elements of Il-Khanid muqarnas.

occur as an intermediate element.

All elements that are based on the rhombus have at least one angle of 45 degrees. The *rhombus* element has four sides of length one, two opposite angles of 45 degrees and two opposite angles of 135 degrees. This element can occur as a cell and as an intermediate element. However, different orientations are possible for the rhombus as intermediate element, see figure 3.8.
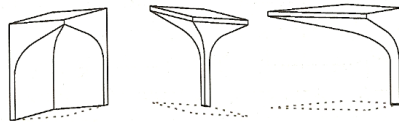

FIGURE 3.8. The rhombus element with different orientations as intermediate element (middle and right). On the left is the rhombus element as a cell.

In this figure we see the rhombus element as a cell and we see two possible orientations for the intermediate element. In the first intermediate element (middle) the curved sides join at the back in an angle of 135 degrees. Here the short diagonal is used for orientation. In the second intermediate element (right) they join in a 45 degree angle. Here the long diagonal is used to orientate the element. In the Il-Khanid muqarnas only the first two orientations occur.

The *almond* and *small biped* elements form together a way to split the *rhombus* similar to how the jug and the large biped split the square. De *almond* has two curved sides of length one that meet in a 45 degree angle. The opposite angle is 135 degrees and the other two angles are right angles of 90 degrees. The almond only occurs as a cell.

The *small biped* is what is left of the rhombus after removing the almond. Also in this element the curved sides meet in an angle of 45 degrees. The opposite angle

is 225 degrees and the other two angles are 45 degrees again. The small biped only occurs as an intermediate element.

The next element is a half-element again. The *half rhombus* element is the rhombus split over the short diagonal. This element can occur as a cell and as an intermediate element. Often, in a muqarnas we find two half rhombus elements (as intermediate elements) in combination with a jug and a square. The top view of this construction forms a hexagon, see figure 3.9.
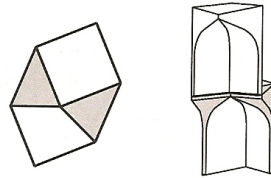

FIGURE 3.9. Left: the hexagon top view of two half rhombus inter-mediate elements in combination with a jug and a square. Right: the hexagon viewed from the front.

Finally, there is one more element that occurs in Il-Khanid muqarnas. The *barley kernel* is a quadrilateral that looks like a rhombus with one half more stretched than the other. It has two sides of length one that meet in a 45 degree angle. The two other sides are longer and they are both of the same length. The barley kernel does not usually appear, except in the upper tier, where it can be used to fill the last and upper part of the vault. Hence, it only occurs as a cell, with orientation as shown in figure 3.8 on the right.
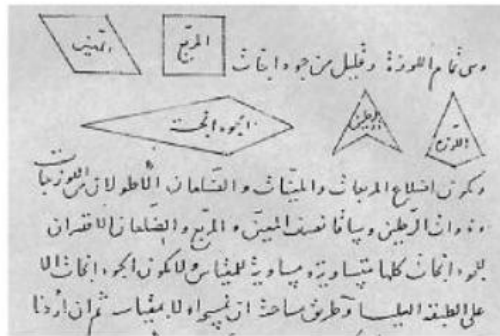

FIGURE 3.10. A sketch of al-Kāshī of different elements, including a barley kernel.

4. Building the square-element

Now we have seen a lot of definitions and descriptions given of muqarnas elements. To gain more insight in the three-dimensional form of an element we developed building boards of the elements based on the construction of the curved side by al-Kāshī. Later, these paper elements can be used to build a real miniature muqarnas.

In the sketch of al-Kāshī the curved side is given thickness. The real building blocks for a muqarnas dont have this thickness and we too will omit this thickness. Creating a building board for the facets of a muqarnas element is very simple. The same applies to the curved sides and the two 'backsides'. The difficult part is creating a building board for the curved front sides of a muqarnas element. The exact calculations of these sides are still under development. Figure 4 shows the building board of the square element as a cell.
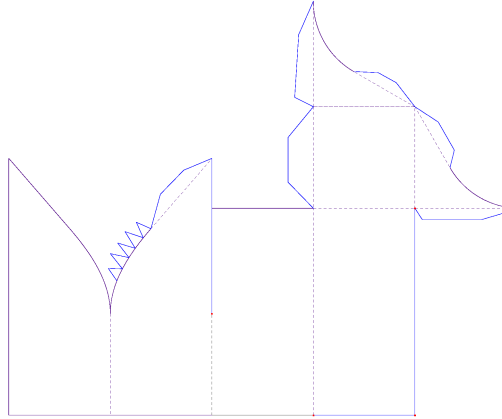

FIGURE 3.11. The building board of a square element with thanks to Aad Goddijn.

EXERCISE 1. *Build the square element from the building board in the appendix.*

5. Combining Muqarnas Elements

al-Kāshī describes a muqarnas as a flight of stairs. We can see a muqarnas as a structure built from elements as if they were in the steps of a stair. We call such a step a *tier* as a translation of the Arabic word *ṯabaqa*. We recall al-Kāshī's definition.

DEFINITION 5. *Adjacent cells, which have their bases on one and the same surface parallel to the horizon, are called one tier (ṭabaqa).*
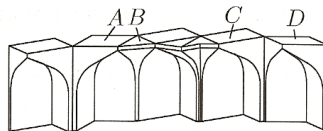

FIGURE 3.12. Part of a tier of a muqarnas in a niche in the Friday Mosque in Isfahan.

In figure 3.12 we can see a part of a tier. Notice that the elements in one tier only join at their curved sides. These sides can join directly, as elements A and B, or just at the back, as elements C and D. The last two lack an intermediate element. Cells can only join other elements at the sides which are the curved sides.

The intermediate elements mostly join other elements at their curved sides too, but they can also meet other intermediate elements at other sides, as in figure 3.13.
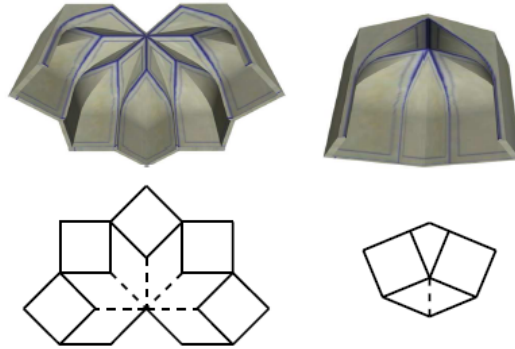

FIGURE 3.13. Here intermediate elements meet at the apex.

Different tiers can be placed on top of each other such that their two-dimensional projections do not overlap. The apex of a curved side in a tier always ends up at the bottom of a curved side of an element in the tier above it. There are different ways to conjoin muqarnas elements:

- Curve to curve: a curved side of an element meets a curved side of a different element in the same tier (*A* en *B* in figure 3.12).
- Front to front: the front of an intermediate element can touch the front of another intermediate element in the same tier (see figure 3.13).
- Back to curved side: The backside of a cell can stand on the curved side of an element in a lower tier. The apex of a curved side of the element in the lower tier then joins the curved side of the cell in the higher tier (see figure 3.14 (a)).
- Back to front: The backside of a cell can stand on the front side of an intermediate element in a lower tier (see figure 3.14 (b) and (c)). An intermediate element can stand on the front of a cell in a lower tier (see figure 3.14 (d)).

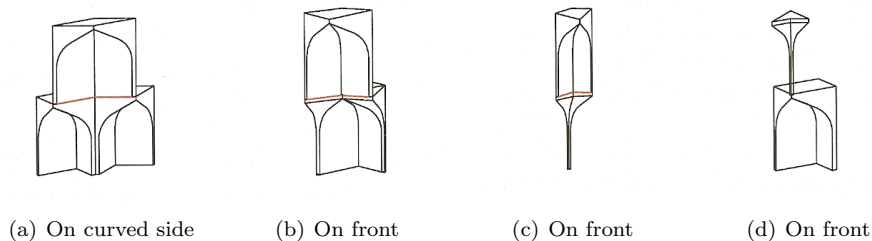(a) On curved side   (b) On front   (c) On front   (d) On front

FIGURE 3.14. Conjoining elements in different tiers.

The bottom tier, which is the tier under which no other tiers stand, is called the *base* of a muqarnas. The highest tier, which is the tier on which no other tiers stand, is called the *center* of the muqarnas.

CHAPTER 4

From design to muqarnas

1. Directing a muqarnas graph

In this chapter we consider the muqarnas designs as graphs which can be given a direction. From this direction we can eventually trace out a three-dimensional muqarnas. In the last chapter we treated the different elements of the Il-Khanid muqarnas and their top views. In a muqarnas design we can recognize the different elements by their top views. When we consider the direction of the top view of an element, we can determine whether it is a cell or an intermediate cell and even in which tier it is located.

Just randomly giving direction to a muqarnas graph does not always lead to a proper corresponding muqarnas. There are certain rules to be followed. The idea and rules of directing a muqarnas graph is thought up by Dr. Silvia Harmsen. Every directed line (from now on called arrow) represents a curved side and points to the *apex* of the associated element.

FIGURE 4.1. The arrows represent a curved side.

Whenever two elements join at their curved sides, their arrows also coincide. This means that an arrow often (not always!) gives the position where two elements join at the curved side.

We remember that the curved sides of a cell join at the apex, therefore the arrows representing a cell point towards the same point. In contrast the arrows of an intermediate element, with their curved sides joining at the backside, point away from the same point. In figure 4.2 you can see all the possible ways to direct the elements that appear in Il-Khanid muqarnas designs (except for the barley kernel, that very seldom appears).

Because we know that for example the almond element only appears as a cell, there is only one way to give direction to the top view of this element. This simplifies the process of directing a muqarnas graph. The first step is to recognize the different elements in the muqarnas graph. Then we can immediately give directions to the edges representing the curved sides of an element that only appears in one way (either a cell or an intermediate element).

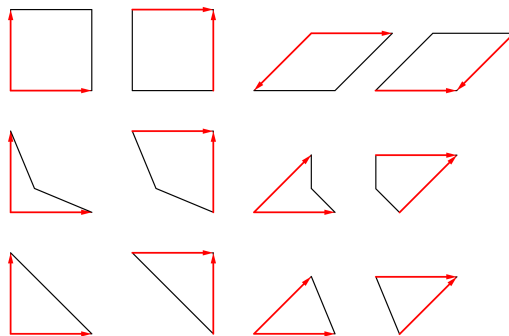To direct the remaining edges we need rules:

FIGURE 4.2. The possible directions of the top views of the elements in Il-Khanid muqarnas designs.

RULE 1. *Only an edge representing a curved side receives a direction.*

This leads to the fact that not all edges receive a direction. Edges without a direction represent the plane backsides of a cell or the front sides of an intermediate element (see figure 4.1). This also states that only edges with length equal to the module receives a direction (but conversely, not all edges of length equal to the module receive a direction!).

RULE 2. *An edge can only have one direction, i.e. there is only one arrow between two points.*

This rule is pretty clear. When an edge represents a curved side it only has one way of going up; it is impossible to go up in both directions. When two elements are combined at their curved sides, the backsides and apexes of the elements have to coincide.

RULE 3. *Parallel edges belonging to the same element get the same direction.*

This rule appears to be in conflict with rule 1, however, one of the parallel arrows represents a curved side of an element above, below or beside it, not the backside of the element. This rule comes in very handy when all elements with only one possible direction have been directed; it gives us the direction of edges of elements that are next to the directed elements in the two-dimensional design.

RULE 4. *Cycles are not allowed, i.e. when starting in a point and following the arrows, you can never return to that same point.*

This rule can be made intuitively clear using a picture of M.C. Escher (see figure 4.4). When a cycle is allowed this would represent a construction similar to this drawing of Escher, which of course is not possible.

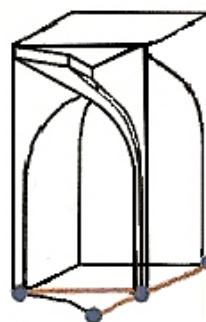RULE 5. *Every arrow points eventually to the top/center of the muqarnas.*

FIGURE 4.3. Illustration where two elements are wrongly connected at their curved sides.
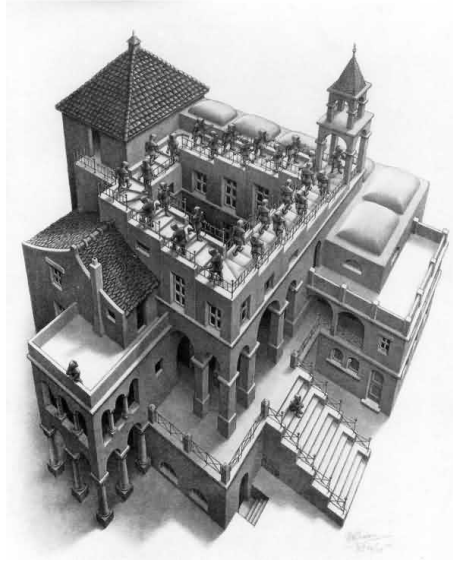
FIGURE 4.4. A drawing of M.C. Escher.

A muqarnas is designed to eventually rise to a top (or center in case of a dome). Therefore, a muqarnas graph cannot arbitrarily point inward and outward. When we have directed almost every line in a muqarnas graph, this rule allows us to direct the remaining edges according to the directions given by the surrounding arrows.

2. Reading a muqarnas graph

When we look at the plate found at Takht-i-Sulaymān we see that it consists almost entirely of squares and rhomboids. Ulrich Harb published the details of the plate and did further research to find the three dimensional design corresponding to the plate. He found that a muqarnas design cannot exist of only squares and rhomboids: it isn't possible to build the corresponding muqarnas. This means that some elements have to be split: a square can be interpreted as a combination of a large biped and a jug, a rhombus can be split into a small biped and an almond.
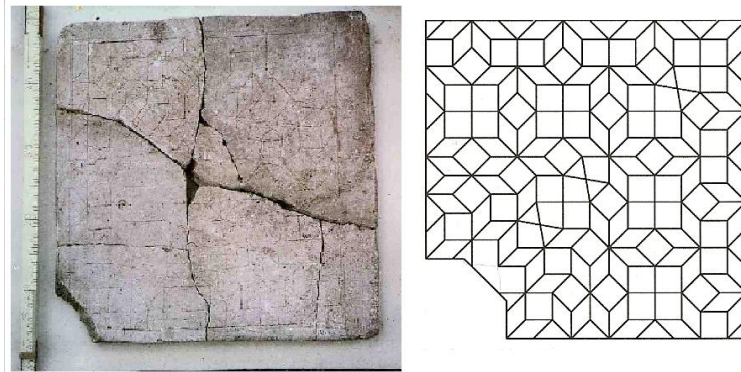

FIGURE 4.5. The lines on the plate are all squares and rhomboids.

This is also the case in some of the drawings of the Topkapı scrolls. Here some elements are drawn as one, but have to be split into two different elements in order to eventually be able to construct a muqarnas.
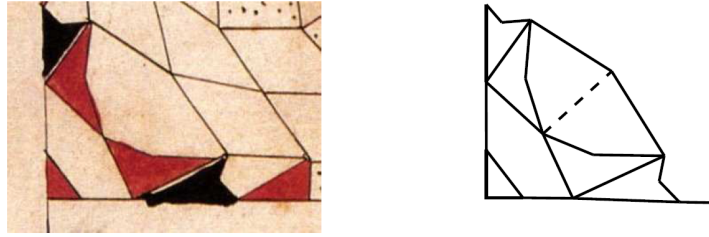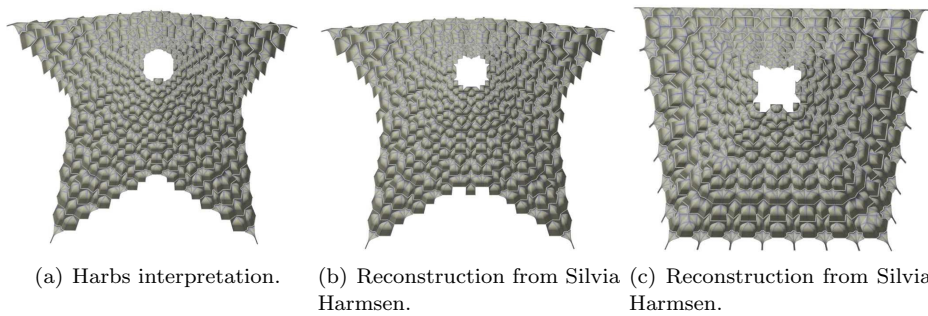
FIGURE 4.6. Part of a drawing from the Topkapı scrolls. Here two jugs have to be split.

This problem of not knowing how to split the elements results in different possible interpretations of the same muqarnas design. Much research has been done on the plate, all resulting in different interpretations. Whether the corresponding muqarnas was ever build is not known.

(a) Harbs interpretation.

(b) Reconstruction from Silvia Harmsen.

(c) Reconstruction from Silvia Harmsen.

FIGURE 4.7. Different interpretations for the plate found at Takht-i-Sulaymān.

3. Directing a subgraph

To give an example of how two direct a muqarnas graph, we will begin with a small subgraph:
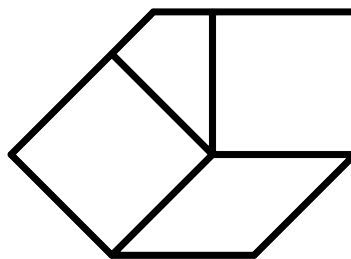

FIGURE 4.8. A subgraph.

First we identify the different elements: an almond, two squares and a rhombus. To direct this subgraph we first use rule 1 and indicate that the almond only has

one direction, that of a cell. This rule tells us that the short edges of the almond receive no direction and gives us the direction of the following two edges:
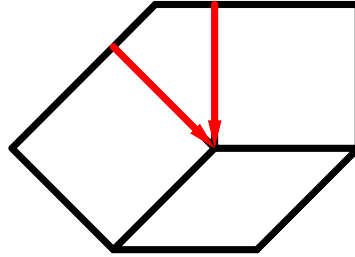

FIGURE 4.9. The first two edges directed.

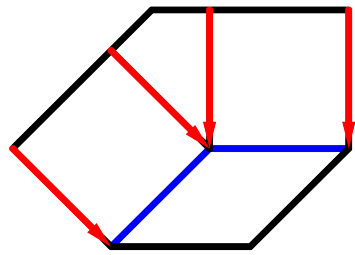Now using rule 3 we can direct the parallel edges belonging to the square:

FIGURE 4.10. Directing the parallel edges.

Now the blue colored edges have to receive a direction, because arrows have to meet in the apex or backside and the red arrows do not meet at all. Following rule 5, these arrows have to point to the top going along with the edges already directed:
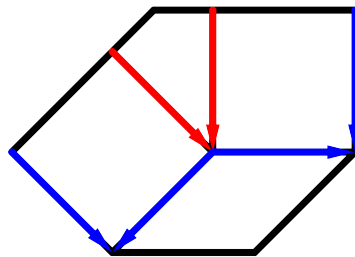

FIGURE 4.11. Directing to the top.

Now the blue arrows represent the squares as cells; the remaining black lines of the squares do not recieve a direction. The last step is again applying rule 3 to find that the rhombus also appears as a cell. Next to the directed subgraph, a three-dimensional figure gives the position of the squares and the rhombus. The almond should be positioned beneath this construction.
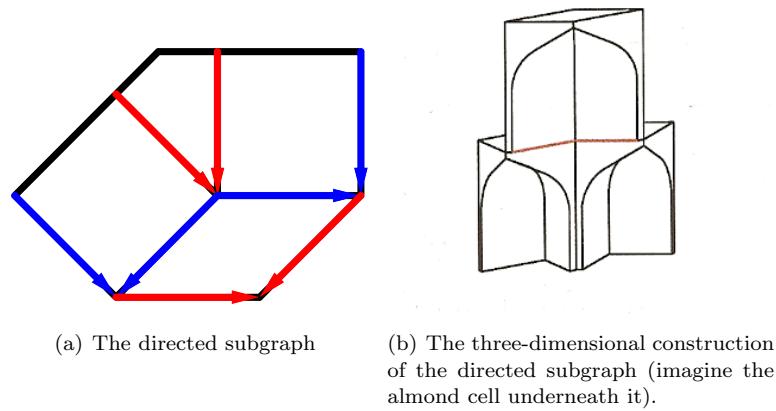
(a) The directed subgraph

(b) The three-dimensional construction of the directed subgraph (imagine the almond cell underneath it).

FIGURE 4.12. The result of directing the subgraph.

EXERCISE 2. *Now you may try to direct a subgraph using the rules. In the appendix you can find two subgraphs: the first one easy, the second one more difficult.*
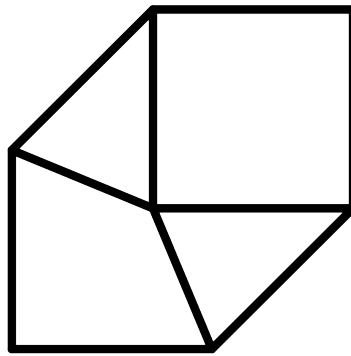

FIGURE 4.13. A subgraph to be directed as exercise.

4. Determining the height of a muqarnas

Now that we have directed a muqarnas graph, we can determine the height of the corresponding muqarnas. For this we use an algorithm from Dr. Silvia Harmsen, although we present a simplified version.

ALGORITHM 1. *From base to center.*
  (1) *All vertices from which arrows are only leaving receive height 0.*
  (2) *From every vertex with known height h, follow all arrows to another vertex. These vertices (the end points of the arrows) will receive height $h + 1$.*
  (3) *Stop when all vertices have received some height. The maximum height is the height of the muqarnas. Call this height H.*

This method seems to be going well, but there is a problem. Some arrows connect vertices with heights that differ with more than one step, as bottom right in figure 4.14. Here the height goes from 1 to 3 in one step. This cannot happen, since the arrows represent sides of an element in the muqarnas, and all elements have the same height.
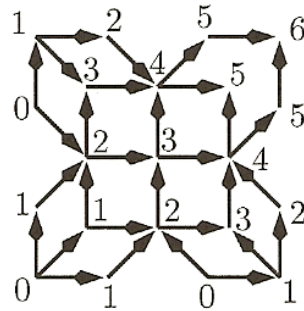
FIGURE 4.14. Following algorithm 1.

To be sure that this problem will not occur we have to make an adjustment. This can be done by demanding that, beginning in the center, every arrow backward has to connect a vertex with height h with a vertex of height $h - 1$.

ALGORITHM 2. *From center to base.*
  (1) *All points in which arrows only come together receive height H.*
  (2) *From every vertex with known height h, follow all arrows backwards to another vertex. These vertices (the starting points of the arrows) will receive height $h - 1$.*

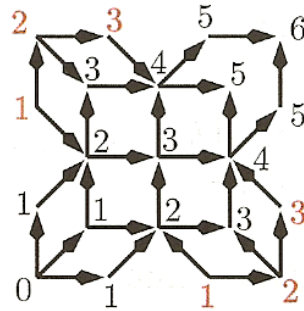This second algorithm will correct any mistakes made with algorithm 1 (see figure 4.15).

FIGURE 4.15. Following algorithm 2. The red numbers are the corrections.

EXERCISE 3. *Try to determine the height of the muqarnas from the appendix that you just directed. Figure C.1 is an easy muqarnas graph, figure C.2 is more challenging.*

5. Building a muqarnas

This section is one great exercise. From a directed muqarnas graph you can now read off the positions of cells and intermediate elements and build a three-dimensional muqarnas. You will be handed out the building boards of all the Il-Khanid muqarnas elements that you need to complete the muqarnas.
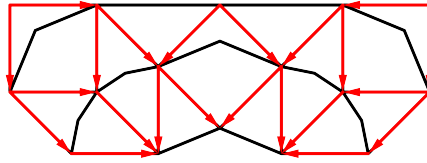
FIGURE 4.16. Muqarnas graph for exercise 3.

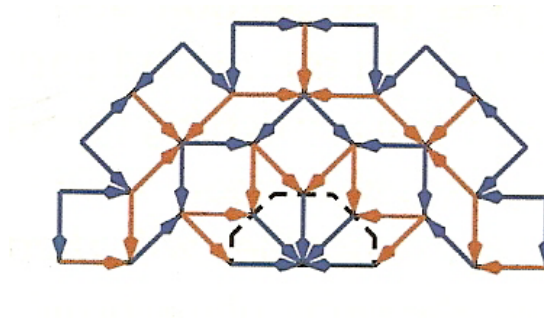EXERCISE 4. *Now try and build a muqarnas yourself!*

FIGURE 4.17. Directed muqarnas design which can be easily build using the building boards.

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[6] Post, B., Jeene, K., van der Veen, M., 2009. *Muqarnas - Wiskunde in de Islamitische kunst*. Reader for course Concrete Meetkunde spring 2009, University Utrecht. Also online: http://www.xs4all.nl/∼ nirmala/muqarnas.pdf

List of words

1. Math concepts

Adjacent: Two elements that are placed next to each other are called adjacent.

Diagonal: In a quadrilateral, the line connecting two opposite angles is called a diagonal.

Perpendicular: Two lines or surfaces are called perpendicular when they meet in a 90° angle.

Quadrilateral: A quadrilateral is a plane figure with four sides and four angles. Examples of quadrilaterals are the square and the rhombus.

Rhombus: A rhombus is a plane figure with four equal sides. It looks like this: ◊.

Rotational symmetry: When a figure can be rotated in such a way that the rotated figure looks the same as the original figure, we say that it has a rotational symmetry. If a figure can be rotated four times to get back to the original figure, we say it has a fourfold rotational symmetry. An example of a figure with a fourfold rotational symmetry is the square.

2. Muqarnas concepts

Apex: The short vertical line of the curved side is called the apex (see figure 3.6).

Backside: The long vertical line of the curved side is called the backside (see figure 3.6).

Center: The upper tier of a muqarnas, above which no other tiers stand, is called the center of the muqarnas.
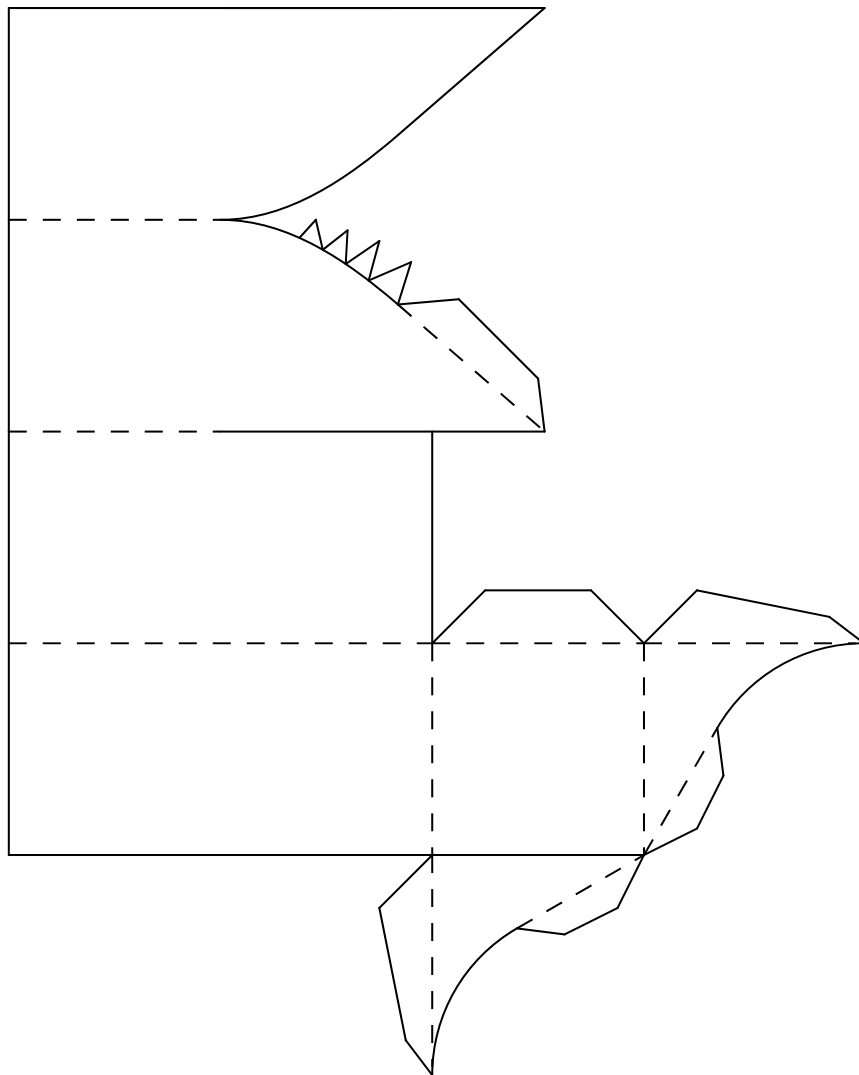
Curved side: The orange sides in figure 3.6 are the curved sides of an element. The curved side has the same shape and measurements in all elements.

Tier: One layer of a muqarnas, in which elements are only placed next to each other, is called a tier.

Building Board

1. Square cell

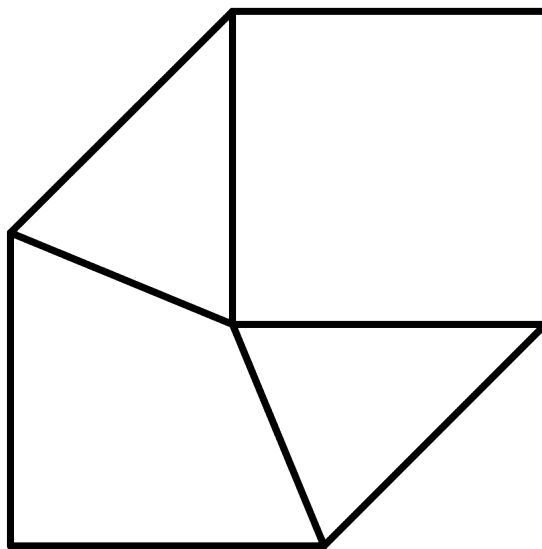APPENDIX C

Directing a subgraph

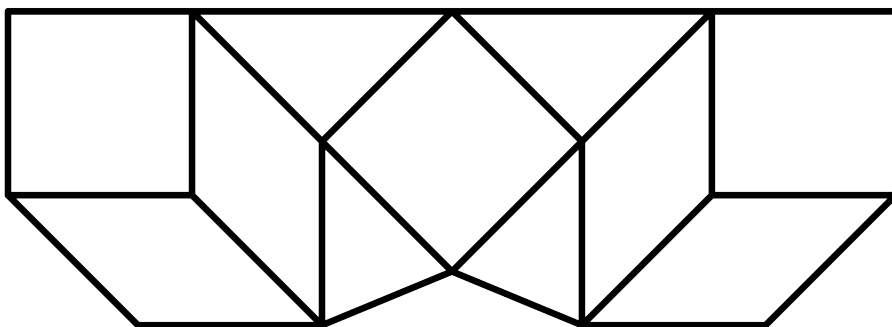FIGURE C.1. **Easy:** A subgraph to be directed as exercise.

FIGURE C.2. **Advanced:** A subgraph to be directed as exercise.