Vol. 37 (1977), No. 3-4, pp. 249—280

Zeitschrift für
Nationalökonomie
Journal of Economics

The Austrian Theory of the Marginal Use and of Ordinal Marginal Utility

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(Received July 22, 1977)

Introduction

The Austrian theory of the marginal use and of ordinal marginal utility has not stood still since its original development in the hands of Menger, Wieser, and Böhm-Bawerk. Over the past hundred years, it has moved far beyond their statement of it, even though this movement sometimes proceeded at a rather leisurely pace.

This paper brings the old theory up to date and extends it. We insist on a new English translation for one of its most important technical terms and call attention to two crucial assumptions which were implicit in the old theory, but which were never stated explicitly until the past decade. A recent mathematical finding implies that the Austrian marginal utility concept is not just ordinal, but in a sense is "intrinsically ordinal".

The restated theory has many important implications for the structure of preferences over commodities, implications which do not follow from the currently orthodox "indifferent" approach. The theory indicates that preferences over commodities are indeed quasi-concave (as Hicks and Allen merely assume), that marginal utility does diminish, even in an ordinalist framework, and that rival and complementary interactions between goods do lead to the Auspitz and Lieben-Edgeworth-Pareto criterion. From this criterion we are then able to deduce that a negative cross substitution elasticity,

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while neither a necessary nor a sufficient condition for net complementarity, is not entirely unrelated to the presence of complementarity. Furthermore, the model provides reason to believe the von Neumann-Morgenstern utility index will in fact exhibit risk-aversion, as modern finance theory merely assumes.

Wants and Utility

What distinguishes the Austrian approach from that of Jevons or Walras is that the Austrians did not accept the utility or subjective value of commodities as given, but rather derived it from the importance of the *wants* that the goods can be used to satisfy[1]. The starting point for inferences about the subjective importance of goods is a subjective rank-ordering of the set of all wants which arranges them in the order of their importance to the individual[2] Given this scale, if we can determine which want is dependent upon the possession of a certain good, we may ascribe the importance of that want to the good. Thus, the utility of a good will essentially be a position on this scale of wants[3].

The Dependent Want and the Implicit Assumptions

The Austrian determination of the dependent want that determines value, as given in the classic expositions of Menger's farmer and Böhm-Bawerk's hunter, runs essentially as follows: Suppose there are three wants, a, b, and c, any one of which can be satisfied by a unit of a certain good, and that the individual prefers a to b and b to c. We represent these preferences by $a \succ b \succ c$, using the symbol $\succ$ rather than $>$, in order to emphasize that this is a preference ordering rather than a numerical inequality. Obviously if the individual has only one unit of the good he will use it to satisfy

[1] See *e. g.* Menger (1950, 116). „*Bedürfnis*" is variously translated as "want" or "need".

[2] See *e. g.* Böhm-Bawerk (1959 II, 137).

[3] Böhm-Bawerk points out that "The expression ['the ranking of wants'] may mean the rank and order of *categories of wants*, or may mean *concrete wants*, that is to say, the individual feelings of want". (1959 II, 137). He goes on to make it explicit that he has in mind a ranking on concrete wants. Thus, we are to enter nothing so general as "the want for food" in the scale of wants, but are to break wants down into specific uses for each portion and type of food.

want a, if he has two units, he will satisfy wants a and b, and if he has three units he will satisfy all three wants. Therefore the value of the first unit is the importance of want a, the value of the second unit is the importance of want b, the value of the third unit is the importance of want c, and any additional units are worthless unless the individual can come up with more wants the good can be used to satisfy.

This conclusion may be obvious, but it is not really warranted, given only the traditional rank-ordering on wants. It is true that if the individual has only one unit he will, by assumption, use it to satisfy want a. However, if he has two units, he may satisfy *any two* wants, that is, he may choose from a and b, a and c, and b and c. In fact, if he feels like it, he may satisfy only one want, a, b, c, or for that matter, he may satisfy no wants at all if he is so inclined. If W is the set of all wants, in this case $W=\{a, b, c\}$, then with two units he may choose from any subset of W with two or fewer elements. In order to infer which subset he will choose, we must be given a preference ordering not just on W, but on W^*, the set of all subsets of W: $W^*=\{P|P\subset W\}$. In our example, $W^*=\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$, where "$\phi$" is the empty set, the subset of W which corresponds to the satisfaction of no wants at all. To eliminate unnecessary clutter, we will omit the braces and commas from the designation of elements of W^*, so that $W^*=\{\phi, a, b, c, ab, ac, bc, abc\}$. If W is finite and has n elements, then W^* has 2^n elements, in this case $2^3=8$. The first implicit assumption that the Austrians made is therefore that the individual's preferences define a linear ordering on W^*, such that an individual with a quantity of some good or goods will use these goods to satisfy the highest rated subset which is feasible, given the supply. This implicit assumption was noted by Georgescu-Roegen (1968, 251), and was also independently discovered by Young (1969) and the present author at about the same time.

And the traditional Austrian formulation makes a second implicit assumption, which somehow implies that if $a \succ b \succ c$ and all are "desirable" so that $a \succ b \succ c \succ \phi$, then it is ab that will be the highest rated subset with two or fewer elements. An article by the later Austrian-school economist Bilimovič (1934, esp. p. 183) provides a clue to what they had in mind. He argues in effect that $b \succ c$ would imply $ab \succ ac$, that $a \succ b$ would imply $ac \succ bc$, and that $c \succ \phi$ would imply $ac \succ a$[4]. These inferences, together with the transitivity

[4] Neurath (1911, 104—105) performs similar operations on his "constellations of pleasures" in his interpretation of Menger and Böhm-

of the ordering on W^*, imply $ab \succ ac \succ bc$ and $ab \succ ac \succ a \succ b \succ c \succ \phi$, so that ab is indeed the highest feasible subset when two units are available. Bilimovič argues as if these inferences were valid deductions from a rank-ordering on W, but that is not the case unless we assume that the wants are unrelated, so that if an additional want or set of wants is added to both sides of a relationship, the elements of the additional set not being contained in either of the sets involved in the original relationship, then the relationship remains undisturbed. We will call this property of the ranking "unrelatedness"[5]. We may loosely refer to the wants as being "unrelated" provided we keep in mind that it is a property of the subjective ordering on W^*, rather than an objective property of the wants themselves. In consumer theory this assumption was first made explicit by Young (1969) and by the present author, working independently at about the same time.

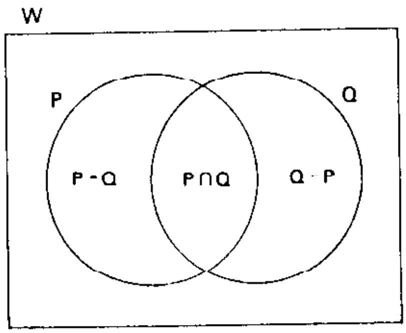

Fig. 1. Unrelatedness in the ranking of subsets of W means that P is preferred to Q if and only if $P-Q$ is preferred to $Q-P$

Unrelatedness is illustrated in the Venn diagram of Fig. 1. The two circles P and Q represent subsets of W, the set of all relevant wants. The set difference $P-Q$ is the set of all wants in P but not in Q, and $Q-P$ is the set of all wants in Q but not in P. The intersection $P \cap Q$ is the set of all wants in *both* P and Q. The unrelatedness assumption states that $P \cap Q$, the wants P and Q have in common, are irrelevant to the relative ordering of P and Q. All that matters is the relative importance of $P-Q$ and $Q-P$, the

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Bawerk. However, his inference that $b_1b_2c_1$ is preferred to a_1b_1 is unwarranted, given only that $a_1 \succ b_1 \succ c_1$ and that c_1 is equal in value to b_2.

[5] Our "unrelatedness" is the same as "additivity" in the nomenclature of Kraft *et al.* (1959, 408).

wants P and Q do *not* have in common. Formally defined, a set of subsets W^* is *unrelatedly ordered* if for any two subsets P and Q of W, we have $P \succ Q$ if and only if $P-Q \succ Q-P$.

The Law of the Marginal Use

Given that wants are unrelated and that $a \succ b \succ c \succ \phi$, it follows that an individual in possession of one unit of our good will use it to satisfy want a, that the use of two units would be the satisfaction of wants a and b, and that the use of three or more units will be the satisfaction of all three wants. Hence, if the individual has only one unit, the use which depends on possession of the last unit will be a, and therefore the value or utility of one unit will be that of a, that is, the place of the set containing only a in the rank-ordering of W^*. If he has two units, the use dependent on either of these units will be the satisfaction of b, the less important of the two uses covered by two units, and hence the utility of one unit will be the position of b on the scale. And if he has three units, the dependent use is the satisfaction of c, the least important of the three uses covered by three units, and hence the utility of the third unit will be the position of c on the scale.

Menger had no name for this use which determines utility, but Wieser proposed one which was subsequently adopted by Böhm-Bawerk:

> I will henceforth refer to that use of a good which is decisive for the value of a single unit of that good as the economically marginal use, or simply as the marginal use, since it stands at the margin of the economically permissible employments... It will be shown that in every instance in which we are concerned with the value of a single unit which is part of a supply of a good, the marginal use determines the magnitude of the value. Economic value is marginal value[6].

If P_n is the set of wants that will be satisfied by n units and P_{n-1} the set that will be satisfied by $n-1$ units, then the set difference $P_n - P_{n-1}$ will be the dependent set of wants or the *marginal use* of the nth unit. If the individual has n units of the good, "the marginal use of one unit" is somewhat ambiguous, since it can refer either

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[6] Wieser (1884, 128). We insist on "marginal use" as the proper translation of "*Grenznutzen*", at least as used by Wieser here and by Böhm-Bawerk. It corresponds to the "*Grenzverwendung*" of Rosenstein-Rodan (1927, 1199, 1202; 1960, 85, 90). For reasons of space, we defer discussion of this point to another paper.

to the marginal use of the last (nth) unit or the marginal use of one additional (the $n+1$st) unit. When necessary, the former may be referred to as the *inner* marginal use and the latter as the *outer* marginal use. The Austrians' Law of the Marginal Use, then, which does not appear in the English literature, is that *the value or utility of a goods-increment is determined by the position of its marginal use on the scale of sets of wants.* A generalized proof of this theorem is given by McCulloch and Smith (1975).

Wieser's *Grenzwerth* or marginal value is the closest term the Austrians had to "marginal utility"[7]. It corresponds exactly to Bernardelli's "conditional utility" (1938). Thus, it makes sense in their framework to speak of the (marginal) value of *two* units of a good, which is determined by the marginal use of two units, in turn the satisfaction of the two least important wants covered by the total supply. Because the Austrians thought in terms of realistic discretely divisible goods instead of hypothetical continuously divisible goods, their "value" corresponds to a non-infinitesimal increment. When only a single unit is at stake, their "value" can be thought of as "marginal utility", provided a distinction is kept in mind between outer marginal utility, corresponding to the outer marginal use, and inner marginal utility, corresponding to the inner marginal use.

Wieser turns the law of the marginal use about to get what might be called a "law of marginal utility", which determines which uses are permissible and which are not:

> Each desire whose importance lies above or is equal to the [marginal] value will be permitted, each whose importance lies below it will be rejected. All economically permissible employments will be included by the marginal value [*Grenzwerth*], and all impermissible ones excluded. (1884, 136, my trans.)

Marginal utility or value thus serves the individual as a mental short-cut to facilitate everyday decisions.

The Law of Diminishing Marginal Utility

An immediate consequence of the Law of the Marginal Use is the fact that if we have a greater quantity of a good, the dependent want will have a lower rank on the scale, and therefore the (marginal) utility of one unit will be lower[8]. If the marginal uses decline

[7] Alt's concept of *Grenzwert* (1936, 163) has no relation to Wieser's. After World War I, "*Grenznutzen*" was confused with marginal utility.

[8] See *e. g.* Menger (1950, 151).

in importance as the available quantity increases, and it is the importance of the marginal use which determines marginal utility, then marginal utility must decrease as the available quantity increases. Note that the Austrian principle of diminishing marginal utility is a theorem, rather than an assumption as with Gossen, Jevons, and Walras[9].

To illustrate this law, suppose that there is one good, "X", and that there are four unrelated wants, *a*, *b*, *c* and *d* that can be satisfied by a unit of X. (An imaginative individual might be able to think up an infinite number of wants he would like to satisfy with a certain commodity. For the sake of brevity, however, we will restrict our examples to finite cases.) A ranking of W^* which could describe an individual's subjective preferences and which satisfies "unrelatedness" is given in Table 1. The sixteen positions

Table 1. Hypothetical Preference Ordering of W^*, With Assigned Ordinal Utility Levels

Set of Wants	Ordinal Utility
abcd	15th
abc	14th
abd	13th
acd	12th
ab	11th
bcd	10th
ac	9th
ad	8th
bc	7th
bd	6th
a	5th
cd	4th
b	3rd
c	2nd
d	1st
∅	0th

Table 2. Total Use and Total Utility of Various Quantities of X

Units of X	Use	Ordinal Utility
0	∅	0th
1	a	5th
2	ab	11th
3	abc	14th
4	abcd	15th
5	abcd	15th

Table 3. Marginal Use and Marginal Utility of 1 Unit of X

Unit of X	Marginal Use of 1X	Ordinal Marginal Utility of 1X
1st	a	5th
2nd	b	3rd
3rd	c	2nd
4th	d	1st
5th	∅	0th

on this scale have been numbered from "0th" to "15th", starting with the lowest position and proceeding up to the highest. These numbers comprise an ordinal utility index, where each number designates a certain utility level. These utility indices are *not* meant to mean that the twelfth utility level is in any sense "twice" as high

[9] Cp. Mises (1966, 243) and Pirou (1945, 64).

as the sixth level, or that the utility of the tenth level equals that of the third level "plus" that of the seventh level. The indices simply give us a convenient method of referring to higher or lower positions on the scale. It seems appropriate to give the empty set ϕ (the set with no elements) the zeroeth position, though it could just as logically be assigned the ninety-seventh, or any other position.

If an individual has n units of X, unrelatedness implies his use of them will be to satisfy the n most important wants, as indicated in Table 2. The utility level of this total use naturally increases with X, as long as we have additional "desirable" wants (that is, ones that are preferred to the empty set).

The marginal use of one unit of X for different quantities is shown in Table 3, along with the utility of this use, which in turn is the marginal utility of a unit of the good. The marginal utility of one additional unit is found to decline from fifth to third to second to first to zeroeth as X increases from 0 to 4.

Notice that the marginal utility is *not* the *arithmetic* difference in the utility level. Rather, the Austrian concept of marginal utility is the utility level of the *set* difference of the respective uses. When von Mises insists, "There are in the sphere of values and valuations no arithmetical operations; there is no such thing as a calculation of values", (1966, 122) he has therefore only gotten at half the truth, for there are, we argue, *set* operations implicit in the Austrian utility analysis. Since the algebra of set manipulation is only a formalization of elementary categories of logic and since it has only recently come into fashion to use set notation, even in mathematics, it is understandable that the Austrians did not make these operations explicit, and in fact, were probably not even consciously aware that they were using them.

The Austrian theorem of diminishing ordinal marginal utility points up the substantial difference between the Austrian tradition and the orthodox theory of utility. Hicks tells us that if we reject cardinal utility and purge our analysis of

> all concepts which are tainted by quantitative utility,. . . the first victim must be marginal utility itself. If total utility is arbitrary so is marginal utility. . . The second victim (a more serious one this time) must be the principle of Diminishing Marginal Utility. If marginal utility has no exact sense, diminishing marginal utility can have no exact sense either. (1946, 19—20).

Yet the Austrians had an ordinal concept of utility in which marginal utility *does* have a meaning, and furthermore, their marginal utility does diminish. For example, in Table 1 we could square each of the ordinal utility index values so that from the top down they read $15^2 = 225$th, $14^2 = 196$th, etc. The marginal utilities in Table 3 would still decline, from 25th to 9th to 4th to 1st to 0th. The Austrian law of diminishing marginal utility is thus invulnerable to monotonic transformations of the utility index.

The Utility of Two Independent Goods

Let us suppose that there are two kinds of goods, X and Y, and that one unit of X will satisfy want *a*, *c* or *e*, and that one unit of Y will satisfy *b* or *d*. We then have $W = \{a, b, c, d, e\}$.

Table 4. Hypothetical Preference Ordering of W^*

Set of Wants	Ordinal Utility
abcde	31st
abcd	30th
abce	29th
abc	28th
abde	27th
acde	26th
abd	25th
abe	24th
acd	23rd
bcde	22nd
ace	21st
ab	20th
bcd	19th
ac	18th
ade	17th
ad	16th
bce	15th
bc	14th
bde	13th
ae	12th
cde	11th
bd	10th
a	9th
be	8th
cd	7th
ce	6th
b	5th
c	4th
de	3rd
d	2nd
e	1st
ø	0th

Table 5. Derived Utility of Combinations of X and Y

Units of		Use	Ordinal Utility
X	Y		
0	0	ø	0th
1	0	a	9th
2	0	ac	18th
3	0	ace	21st
0	1	b	5th
1	1	ab	20th
2	1	abc	28th
3	1	abce	29th
0	2	bd	10th
1	2	abd	25th
2	2	abcd	30th
3	2	abcde	31st

Table 6. Derived Preference Ordering of Combinations of X and Y

Units of		Use	Ordinal Utility
X	Y		
3	2	abcde	31st
2	2	abcd	30th
3	1	abce	29th
2	1	abc	28th
1	2	abd	25th
3	0	ace	21st
1	1	ab	20th
2	0	ac	18th
0	2	bd	10th
1	0	a	9th
0	1	b	5th
0	0	ø	0th

A conceivable preference ordering of W^* is shown in Table 4, along with an ordinal utility index identifying the positions on the scale from zeroeth to thirty-first. If an individual with the preferences of Table 4 has m units of X and n units of Y, unrelatedness of the wants implies that he will use them to satisfy the m most important elements of the set $\{a, c, e\}$, and the n most important elements of the set $\{b, d\}$. Table 5 shows the optimal use which would be made of various combinations of X and Y and the respective utility levels. These utility levels imply a *derived* preference ordering on the commodity bundles. In Table 6 the commodity bundles are arranged in decreasing order of utility. Table 7 lays out the total use and total utility of these bundles in two-dimensional tabular form. In Fig. 2

Table 7. Total Use and Total Utility of Combinations of X and Y

Units of Y \ Units of X	0	1	2	3	4
3	bd 10th	abd 25th	abcd 30th	abcde 31st	abcde 31st
2	bd 10th	abd 25th	abcd 30th	abcde 31st	abcde 31st
1	b 5th	ab 20th	abc 28th	abce 29th	abce 29th
0	ø 0th	a 9th	ac 18th	ace 21st	ace 21st

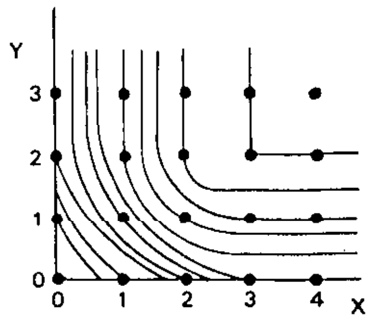

Fig. 2. "Indifference Curves" separating more preferred combinations from less preferred combinations

Table 8. Marginal Use and Marginal Utility of 1 Unit of X

Units of Y \ Unit of X	1st	2nd	3rd	4th
3	a 9th	c 4th	e 1st	ø 0th
2	a 9th	c 4th	e 1st	ø 0th
1	a 9th	c 4th	e 1st	ø 0th
0	a 9th	c 4th	e 1st	ø 0th

Table 9. Marginal Use and Marginal Utility of 1 Unit of Y

Unit of Y \ Units of X	0	1	2	3	4
3rd	ø 0th	ø 0th	ø 0th	ø 0th	ø 0th
2nd	d 2nd	d 2nd	d 2nd	d 2nd	d 2nd
1st	b 5th	b 5th	b 5th	b 5th	b 5th

the horizontal axis represents units of X and the vertical axis units of Y. Lines have been drawn on this graph corresponding to different utility levels. These lines have the property that any point below and to the left of the line has a utility level lower than that corresponding to that of the line, while all points on the line have exactly this utility, and all points above and to the right have at least this utility. Our lines roughly correspond to the "indifference curves" of conventional utility theory. The only difference is that our commodities are "lumpy", rather than infinitely divisible, and therefore the lines usually go through only one point. The reader may, if he objects to indifference curves, think of these lines as "preference curves".

Table 8 shows the marginal use and marginal utility of one unit of X as the total quantities of X and Y vary. Table 9 shows the marginal use and marginal utility of one unit of Y. The marginal utility of X is found to diminish from 9th to 4th to 1st to 0th as X increases from 1 to 4, regardless of the quantity of Y available. Similarly, the marginal utility of Y diminishes from 5th to 2nd to 0th, regardless of the quantity of X available. It could not be otherwise in this case, for the quantity of one good has no bearing on the use that will be made of the other, and therefore no effect on the marginal use. We may therefore state as a general rule that *when X and Y are independent in consumption, i. e.,* when W may be partitioned into two categories of wants such that a unit of X and only a unit of X will satisfy the wants in one category, and a unit of Y and only a unit of Y will satisfy the wants in the other category, the *marginal utility of one good will be independent of the quantity available of the other*[10].

It should be noted that when there is only one good, the concept of marginal utility has no operational significance. So what if a unit of a good has a certain desirability, if there is nothing to compare it to? But when there is more than one good, we have the seemingly trivial but actually important rule, that if an individual is offered a choice between a unit of one good or a unit of another, he will always choose the one with the higher marginal utility, as determined by the marginal use.

The Austrian Resolution of the Paradox of Value

Before the Austrians came on the scene, economists were troubled by the so-called paradox of value. As Adam Smith expressed it,

> The things which have the greatest value in use have frequently little or no value in exchange; and on the contrary, those which

[10] Strotz's concept of a "utility tree" (1957) is undoubtedly related to independence of the goods in question, as is the concept of "additively separable" preferences. The exact connection deserves to be examined in greater detail.

> have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce any thing; scarce any thing can be had in exchange for it. A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had for it. (1776/1937, 28).

The Austrians argued that, if rightly qualified, the value of a good *is* in fact determined by the importance of its usefulness. In Wieser's words, or rather in our translation of Wieser's words,

> For most goods a distinction must be made between the magnitude of their value [*ihres Werthes*] and the magnitude of their use [*ihres Nutzens*]. Only for those goods that are actually employed to bring about the marginal use-performance will the good's own use be the source of its value and will there be agreement between the two judgments. For any other good a different use, which must nevertheless be a use characteristic of that sort of good, will be the basis for the estimate of its value, which accordingly will differ from the estimate of the use-effect it actually brings about; for such a good, the actual use is higher than the dependent use and therefore higher than its value[11].

To illustrate the paradox and its resolution, let us look again at the individual of Tables 4 through 9. Suppose he has 3 X and 2 Y. The total use of X (*ace*, twenty-first position) is more important that the total use of Y (*bd*, tenth position). Furthermore, the *highest* use of X (*a*, ninth position) is more important than the *highest* use of Y (*b*, fifth position). Yet the subjective value, the utility, of a unit of X, even of the very unit that will satisfy want *a*, is lower (first position) than the utility of a unit of Y (second position). The Austrians' answer to this paradox is that the value of a unit of a good is determined, not by the *total* use of goods of that sort, and not necessarily even by its *own* use, but rather by its *marginal* use. Goods do not obtain value from the labor they "contain". Rather, labor derives *its* value from use-value of the goods it is *used* to produce.

Rival Goods

In the example given above, it was assumed that the two goods were used independently of one another. However, the law of the

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[11] Wieser (1884, 128). See also Böhm-Bawerk (1959 I, 135—36 and 1909 A, 234). Note however that Lindgren (1976) puts a completely different interpretation on Smith's meaning.

marginal use is also applicable if the goods must be used together to satisfy some wants or if they can be utilized in place of one another.

Suppose there are two goods, *X* and *Y*, and that a unit of either may be used to satisfy some want or wants, say *c*. Let there be other wants, *a* and *e*, which a unit of *X* can satisfy, and still others, *b* and *d*, which may be satisfied by a unit of *Y*. X and *Y* are then *rivals*, at least with respect to want *c*[12].

Table 10. Hypothetical Preference Ordering

Set of Wants	Ordinal Utility
abcde	31st
abcd	30th
abce	29th
abde	28th
abc	27th
abd	26th
acde	25th
bcde	24th
acd	23rd
abe	22nd
bcd	21st
ab	20th
ace	19th
ade	18th
bce	17th
bde	16th
ac	15th
ad	14th
bc	13th
bd	12th
cde	11th
ae	10th
cd	9th
be	8th
a	7th
b	6th
ce	5th
de	4th
c	3rd
d	2nd
e	1st
∅	0th

Table 11. Use of *X*, Use of *Y*, Total Use and Total Utility

Units of		Use of		Total Use	Total Utility
X	Y	X	Y		
0	0	∅	∅	∅	0th
1	0	a	∅	a	7th
2	0	ac	∅	ac	15th
3	0	ace	∅	ace	19th
0	1	∅	b	b	6th
1	1	a	b	ab	20th
2	1	ac	b	abc	27th
3	1	ace	b	abce	29th
0	2	∅	bc	bc	13th
1	2	a	bc	abc	27th
2	2	ac	bd	abcd	30th
3	2	ace	bd	abcde	31st
0	3	∅	bcd	bcd	21st
1	3	a	bcd	abcd	30th
2	3	ae	bcd	abcde	31th

Table 12. Implied Preference Ordering on Combinations of *X* and *Y*

X	Y	Utility
2	3	31st
3	2	31st
1	3	30th
2	2	30th
3	1	29th
1	2	27th
2	1	27th
0	3	21st
1	1	20th
3	0	19th
2	0	15th
0	2	13th
1	0	7th
0	1	6th
0	0	0th

Let an individual preference-rank the subsets of $W = \{a, b, c, d, e\}$ as shown in Table 10. For various combinations of *X* and *Y*, Table 11

[12] The dictionary definition of "rival" is "one of two or more striving to reach or obtain that which only one can possess" (Webster 1963, 743).

shows the wants X will be used to satisfy, the wants Y will be used to satisfy, the collective use of Y and X, and the corresponding total utility. Want *c* is sometimes satisfied by X and sometimes by Y. Table 12 shows the implied preference ordering on the commodity space. (Note that this ordering is now only semi-linear; it sometimes happens that two different commodity bundles have the same utility.) Tables 13—15 and Fig. 3 are constructed in the same manner as Tables 7—9 and Fig. 2.

Table 13. Total Use and Total Utility of Combinations of X and Y

Units of Y \ Units of X	0	1	2	3	4
4	bcd 21st	abcd 30th	abcde 31st	abcde 31st	abcde 31st
3	bcd 21st	abcd 30th	abcde 31st	abcde 31st	abcde 31st
2	bc 13th	abc 27th	abcd 30th	abcde 31st	abcde 31st
1	b 6th	ab 20th	abc 27th	abce 29th	abce 29th
0	Ø 0th	a 7th	ac 15th	ace 19th	ace 19th

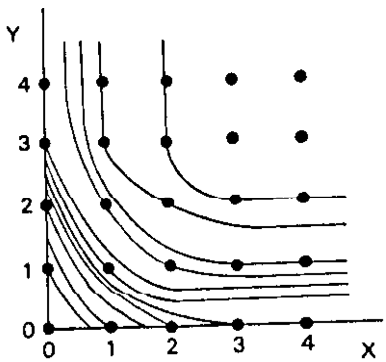

Fig. 3. Indifference curves

Table 14. Marginal Use and Marginal Utility of X

Units of Y \ Unit of X	1st	2nd	3rd	4th
4	a 7th	e 1st	Ø 0th	Ø 0th
3	a 7th	e 1st	Ø 0th	Ø 0th
2	a 7th	d 2nd	e 1st	Ø 0th
1	a 7th	c 3rd	e 1st	Ø 0th
0	a 7th	c 3rd	e 1st	Ø 0th

Table 15. Marginal Use and Marginal Utility of Y

Unit of Y \ Units of X	0	1	2	3	4
4th	Ø 0th	Ø 0th	Ø 0th	Ø 0th	Ø 0th
3rd	d 2nd	d 2nd	e 1st	Ø 0th	Ø 0th
2nd	c 3rd	c 3rd	d 2nd	d 2nd	d 2nd
1st	b 6th	b 6th	b 6th	b 6th	b 6th

As in the case of independent goods, the marginal utility of each good decreases with its own quantity. However, three phenomena in Tables 14 and 15 are different from the case of independent goods and are worthy of note. First, the marginal utility of one good is not independent of the quantity of the other good available. For example, the marginal utility of the second unit of X falls from the third to the second to the first position as Y increases from 1 to 3. Similarly, the marginal utility of the third unit of Y falls from second to first to zeroeth as X increases from 1 to 3. Thus, *when goods are rivals in consumption, the marginal utility of one good tends to fall off as the quantity of the other increases.*

Second, it sometimes happens that the marginal use of one good is the satisfaction of a want which that good cannot itself satisfy. For instance, when the individual has ($1X$, $2Y$), the marginal use of one additional X is d, a want that can only be satisfied by Y. When he has ($2X$, $2Y$), the marginal use of one more Y is e, a want that can only be satisfied by X. Thus Wieser's assertion above (p. 260), that the marginal use must be a use characteristic of the good in question, is not always true.

And third, when goods are rivals, it often happens that their marginal uses coincide. Thus, when ($1X$, $1Y$) is available, the outer marginal use of both X and Y is the satisfaction of want c. When ($2X$, $2Y$) is available, the outer marginal use of both X and Y is the satisfaction of want e.

Complementary Goods

The dictionary definition of "complementary" is "serving to fill out or complete: mutually supplying each other's lack"[13]. Let there be two goods, X and Y, and suppose that one unit of each is required to satisfy some want, b, so that they are complementary with respect to this want. Let both goods have alternative uses in which they are not complements: a and d for X, and c and e for Y. Table 16 shows an "unrelated" ranking of W^* which might reflect an individual's preferences. For various combinations of X and Y, Table 17 shows the best use that can be made of the combination if the satisfaction of b is excluded, the best use if b is included, and the utilities or subjective values of both these uses. The best overall use is the better of these two and is shown with its utility, the derived utility of the combination, in the last two columns.

When we try to derive the marginal uses of X and Y from Table 18, we encounter a new difficulty. For instance when our individual has ($2X$, $1Y$) we find that there is not a simple want

[13] Webster (1973, 169). "*Complémentaire*" has a similar meaning in French. Böhm-Bawerk (1909 A, 276) attributes the Germanization "*komplementär*" to Menger.

dependent on the possession of another unit of X. Rather, an additional unit of X enables him to replace want c with the higher rated want b. We represent this sort of marginal use by the ordered pair (b, c), where the first entry (b) represents the additional want satisfied and the second entry (c) represents the want (if any) whose satisfaction is omitted. Clearly in this case, the unit of X will have higher utility to the individual, the more important b is, and the

Table 16. Hypothetical Preference Ordering

Set of Wants	Ordinal Utility
abcde	31th
abcd	30th
abce	29th
abde	28th
abc	27th
abd	26th
acde	25th
acd	24th
abe	23rd
bcde	22nd
ab	21st
ace	20th
ade	19th
bcd	18th
ac	17th
bce	16th
ad	15th
bde	14th
ae	13th
bc	12th
bd	11th
cde	10th
a	9th
cd	8th
be	7th
b	6th
ce	5th
de	4th
c	3rd
d	2nd
e	1st
ø	0th

Table 17. Best Use of X and Y With and Without Want b, and Best Overall Use

Units of		Best Use if b-- Excluded		Included		Best Use	Utility of Best Use
X	Y	Use	Utility				
0	0	ø	0th	---	---	ø	0th
1	0	a	9th	---	---	a	9th
2	0	ad	15th	---	---	ad	15th
3	0	ad	15th	---	---	ad	15th
0	1	c	3rd	---	---	c	3rd
1	1	ac	17th	b	6th	ac	17th
2	1	acd	24th	ab	21st	acd	24th
3	1	acd	24th	abd	26th	abd	26th
0	2	ce	5th	---	---	ce	5th
1	2	ace	20th	bc	12th	ace	20th
2	2	acde	25th	abc	27th	abc	27th
3	2	acde	25th	abcd	30th	abcd	30th
0	3	ce	5th	---	---	ce	5th
1	3	ace	20th	bce	16th	ace	20th
2	3	acde	25th	abce	29th	abce	29th
3	3	acde	25th	abcde	31st	abcde	31st

less important c is. Menger carelessly describes such a utility as the difference between the utility of b and the utility of c, without telling us what we are to make of this concept (1950, 165).

However, by extending "unrelatedness", we are able to place such "differences" accurately enough for our needs without resorting to cardinality. From Table 16, we have $b \prec cd$. If we "delete" c from both sides of this relation, we obtain $(b, c) \prec d$ (second utility level). Similarly, since $b \succ ce$, we must have $(b, c) \succ e$ (first utility level). Therefore (b, c) is intermediate between the first and second positions. In Table 19 we have indicated this by arbitrarily

Table 18. Total Use and Total Utility of Combinations of X and Y

Units of Y	Units of X: 0	1	2	3	4
4	ce 5th	ace 20th	abce 29th	abcde 31st	abcde 31st
3	ce 5th	ace 20th	abce 29th	abcde 31st	abcde 31st
2	ce 5th	ace 20th	abc 27th	abcd 30th	abcd 30th
1	c 3rd	ac 17th	acd 24th	abd 26th	abd 26th
0	∅ 0th	a 9th	ad 15th	ad 15th	ad 15th

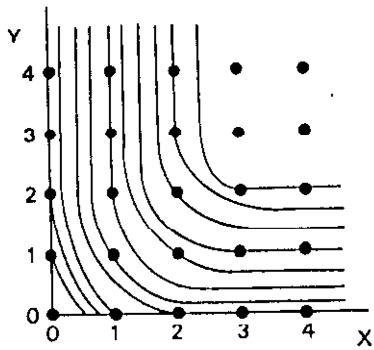

Fig. 4. Indifferences curves

Table 19. Marginal Use and Marginal Utility of 1 Unit of X

Units of Y	Unit of X: 1st	2nd	3rd	4th
4	a 9th	b 6th	d 2nd	∅ 0th
3	a 9th	b 6th	d 2nd	∅ 0th
2	a 9th	(b,e) 3.5th	d 2nd	∅ 0th
1	a 9th	d 2nd	(b,c) 1.5th	∅ 0th
0	a 9th	d 2nd	∅ 0th	∅ 0th

Table 20. Marginal Use and Marginal Utility of 1 Unit of Y

Unit of Y	Units of X: 0	1	2	3	4
4th	∅ 0th	∅ 0th	∅ 0th	∅ 0th	∅ 0th
3rd	∅ 0th	∅ 0th	e 1st	e 1st	e 1st
2nd	e 1st	e 1st	(b,d) 1.7th	c 3rd	c 3rd
1st	c 3rd	c 3rd	c 3rd	b 6th	b 6th

giving it the "1.5th" position. As with a Dewey decimal classification, this is not intended to mean that it is half way between the first and second positions, but merely that it is somewhere in between them.

The marginal use of the second unit of X, given 2Y, is (b, e). This use is not so easy to place on the scale of wants. By comparison to c it can be shown to be higher than the third position. To find an upper bound is more difficult. Because $ae \succ bc$, we have $(a, c) \prec (b, e)$. Furthermore, $cde \succ a$ implies $de \succ (a, c)$. Therefore,

$(b, e) \succ de$ (fourth position). Since (b, e) lies between the third and fourth position, we assign it the "3.5th" position of Table 16.

The marginal utility of the second unit of Y, given 2 X, is the importance of replacing the satisfaction of *d* with that of *b*, or of (b, d) in our notation. This use presumably has the same relation to (b, c) that *bc* has to *bd*. Therefore $(b, d) \succ (b, c)$ (1.5th position). The reader may confirm that $(b, d) \prec (c, e) \prec d$ (second position). We therefore assign (b, d) the "1.7th" utility level[14].

It is still true that the marginal utility of either good always falls as its own quantity increases. When we have 2 units of Y, the marginal utility of a unit of *X* falls from 9th to 3.5th to 2nd, and finally to 0th. When we have 2 *X*, the marginal utility of *Y* falls from 3rd to 1.7th to 1st, and then to 0th.

Furthermore, we note that with complements, we get exactly the opposite of what happens with rival goods: as the quantity of one good increases, the marginal utility of the other *tends to increase*, instead of decrease as was the case with rivals. For instance, when there are 2 *X* available, the marginal utility of another unit of *X* rises from 0th to 1.5th to 2nd as *Y* increases from 0 to 2.

Net Rivals and Complements: The ALEP Criterion

Rivalness and complementarity are not mutually exclusive concepts. Two goods may be rivals with respect to one want and complements with respect to another. Or using them as complements in one proportion may be rival with using them in another proportion, as in the case of production under variable proportions.

Since when rivalness is the only interaction, the marginal utility of one good falls as the quantity of the other increases, and since

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14 The problems that arise when we introduce complementarity indicate that W^* does not contain all of the "uses" of interest. We must consider a more complicated set, say W^{**}, the set of all ordered pairs of disjoint subsets of *W*. Given (P, Q) in W^{**}, *P* is to be interpreted as the additional wants that are to be satisfied, and *Q* the wants whose satisfaction is to be omitted. W^{**} then contains all marginal uses, in the broad sense that we need for complementary (and jointness). It appears that the linear ordering on W^*, together with our extended application of unrelatedness, defines a partial ordering on W^{**} which is sufficient to say which of two goods will be valued more highly in any conceivable situation, to prove diminishing marginal utility, and to establish the ALEP criterion, to be discussed below. See McCulloch and Smith (1975) for a proof of the law of the marginal use involving this extended concept. Cp. Neurath (1911, 96) with respect to "differences in pleasure".

the opposite is true when complementarity is the only interaction, we propose that X and Y be designated *net rivals* in a certain region of the X—Y plane if in that region the marginal utility of the one decreases as the quantity of the other increases holding the quantity of any other goods constant, *net complements* if the opposite is true, and *on net independent* if the marginal utility of one is independent of the quantity available of the other. (It can be shown that these concepts are well defined, that is, that X will have qualitatively the same effect on the marginal utility of Y as Y has on the marginal utility of X, even in the Austrian framework of ordinal marginal utility.) This is actually the definition of rival and complementary goods proposed, though in terms of the cross partial derivatives of a smooth cardinal utility function, by Auspitz and Lieben, Edgeworth, and Pareto[15]. We therefore designate it the "ALEP criterion". Note, however, that while these authors used the ALEP criterion as the *definition* of complements and rivals, the approach of the marginal use theory is to adopt the common English definitions of these concepts in terms of how the goods are used, and then to *demonstrate* a relationship to the ALEP criterion.

Hicks claims that the "Edgeworth-Pareto definition sins against Pareto's own principle of the immeasurability of utility. If utility is not a quantity, but only an index of the consumer's scale of preferences, his definition of complementary and competitive goods will differ according to the arbitrary measure of utility which is adopted", (1946, 43). However, we have shown that in the Austrian concept of ordinal marginal utility, the criterion does indeed have a precise meaning that is invariant with respect to monotonic transformations of the utility index, so Hicks' objection is invalid.

Hicks and Allen instead defined the complementarity of X and Y in terms of Allen's "partial" elasticity of substitution σ_{xy}, which is related to the curvature of the indifference surfaces (Allen 1962/38, 504—505). If it is positive they call the two goods "substitutes" and if it is negative they call them "complements". However, it has never been demonstrated that the sign of the substitution elasticity

[15] Auspitz and Lieben (1889, 482), Edgeworth (1897/1925, 117 n. 1), Pareto (1906/1927, 268—269). It is not actually clear that the functions Auspitz and Lieben and Edgeworth differentiate are really what we would call utility functions. For instance, Edgeworth equates his first derivative to a price. Nevertheless the basic idea is definitely there. While Auspitz and Lieben were Austrians by nationality, they are not considered part of the Austrian school. Their approach was closer to that of Edgeworth. The ALEP criterion has recently been rediscovered by Samuelson (1974, 1264—1264).

has anything to do with whether X and Y are used in combination with one another or in place of one another. It is about time this question be investigated.

We have demonstrated above that the ALEP criterion *is* related to whether the goods are rivals or complements in the *English* sense, if not in the *Hicks-Allen* sense. One implication of the ALEP criterion for the structure of commodity preferences, an implication that was not recognized by Hicks, is that if there is a third good, Z, which is completely independent of the first two goods, then the marginal rate of substitution between X and Z will change in one direction as Y increases holding X and Z constant if X and Y are net complements, and will change in the opposite direction as Y increases if X and Y are net rivals. If goods and wants are finely divisible so that the Allen elasticities exist and are well defined, this implies that

$$\left.\frac{E\,(P_x/P_z)}{E\,Y}\right|_{X,Z} \quad \text{and} \quad \left.\frac{E\,(P_y/P_z)}{E\,X}\right|_{Y,Z}$$

will both be positive, negative or zero, depending on whether X and Y are net complements, rivals, or independents, where E represents the logarithmic differentiation operator:

$$EX = d \log X = dX/X, \text{ etc.,} \tag{1}$$

and P_x, P_y, and P_z represent the prices facing a competitive buyer.

It can be shown that

$$\left.\frac{E\,(P_x/P_z)}{E\,Y}\right|_{X,Z} = \frac{-k_y\,\eta_x\,\sigma_{yy} - (1-k_y\,\eta_y)\,\sigma_{xy}}{k_z\,(\sigma_{xz}\,\sigma_{yy} - \sigma_{xy}^2)} \tag{2}$$

and

$$\left.\frac{E\,(P_y/P_z)}{E\,X}\right|_{Y,Z} = \frac{-k_x\,\eta_y\,\sigma_{yy} - (1-k_x\,\eta_x)\,\sigma_{xy}}{k_y\,(\sigma_{xz}\,\sigma_{yy} - \sigma_{xy}^2)} \tag{3}$$

where the k's and η's are respectively the budget shares and income elasticities of demand for the three goods[16]. Setting (2) and (3) equal to zero as in the case of independent goods and employing the familiar conditions

$$\sum_{j=1}^{3} k_j\,\sigma_{ij} = 0 \tag{4}$$

implies that

$$\eta_x\,\sigma_{yz} = \eta_z\,\sigma_{xy} = \eta_y\,\sigma_{xz}.$$

[16] Expressions (2) and (3) do not necessarily have the same sign unless Z is net independent of X and Y.

Since independent goods will always have positive income elasticities, Eq. (5) implies that all three cross substitution elasticities will have the same sign. Since at most one can be negative, it follows that they must all be positive. Therefore if X and Y are independent (and the third good Z is also independent of both X and Y), σ_{xy} will be positive.

It follows that there will be some small amount of ALEP net complementarity between X and Y for which σ_{xy} remains positive. Therefore σ_{xy} being negative is not a necessary condition for complementarity. Note, however, that as k_y goes to zero, (2) takes on the sign of $-\sigma_{xy}$, and that as k_x goes to zero, (3) does likewise. Therefore if the budget shares of the two goods in question are negligible (and if third goods are independent), a negative σ_{xy} is a necessary and sufficient condition for X and Y to be net complements[17].

Even without the shares going to zero, it may still be a sufficient condition. If σ_{xy} is negative, the numerators of both (2) and (3) will be positive except in unusual cases when some of the income elasticities are negative. Furthermore, it can be shown that the quasi-concavity of preferences implies that the denominators are necessarily positive (McCulloch 1977, 7). Therefore if none of the three goods is inferior (and if the third good is net independent), a negative cross substitution elasticity is a sufficient, if not necessary, condition for net complementarity.

Even though there is some connection between σ_{xy} and the net complementarity or rivalness of X and Y, we cannot say anything for certain unless we know Z's ALEP relation to X and Y. The attempt of Hicks and Allen to infer the complementarity of X and Y from demand parameters alone was therefore futile. But, what is more useful, we *can* make inferences about demand relationships from what we know about how X, Y, and Z are used.

In any event, the Hicks-Allen "definition" of complementarity, which Samuelson (1974, 1528) calls the SHAS definition (after Slutsky, Hicks, Allen and Schultz), should now be rejected once and for all. In its place we propose restoring what might be called the WOLM definition (after Webster, Oxford, Larousse, and Menger), the Websterian version of which we have quoted above. If σ_{xy} is positive, X and Y may be called positive substitutes, and if it is negative, they may be called negative substitutes, pro-

[17] Cp. Hicks (1946, 44). When the shares vanish, income effects can be ignored.

vided the word "substitute" in this sense is not confused with "rival". We have demonstrated above that noninferior negative substitutes are extremely complementary, relative to third goods, but this is a deduction, not a definition[18].

The treatment of complementarity illustrates the substantial methodological difference between the Austrian approach to consumer theory and the current Hicks-Allen orthodoxy. This orthodoxy might appropriately be called the "indifferent" approach, because of its preoccupation with indifference curves and its refusal to look beneath them to examine the relation of goods to underlying wants. The indifferent approach suffers from the positivistic prejudice that science can only take note of "observable" phenomena, and must never attribute human-like motives to its objects of study. The Austrian school, on the other hand, realizes that there is nothing unscientific about attributing human-like motives to human beings. Animism may be impermissible in the natural sciences, but it is indispensable to the social sciences. In any case, the fact that people use water to irrigate their lawns and gasoline to fuel their automobiles, instead of the other way around, is far more observable than the cross substitution elasticities between water, lawns, gasoline and automobiles.

The sterility of the indifferent approach to consumer choice has led many economists working independently of the Austrian school to move in a similar direction. Lancaster (1966, 1971) investigates how goods are used to provide "characteristics", similar to Austrian wants, that are the ultimate objects of consumer preference. Becker and his school (e. g. Michael and Becker 1973) have developed a model in which market goods are combined in a "household production function" to create observable or hypothetical "commodities" which are the ultimate preference objects. However, neither of these approaches insists, as the Austrians do, that preferences on market goods can be broken down in terms of ultimate unrelated preference-objects, nor do they develop the ALEP criterion and its implications, or recognize the ordinal character of marginal utility. Nevertheless, these approaches do belong with the Austrians in the camp of animistic economics.

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[18] Sato and Koizumi (1973) have shown, in the context of a constant-returns-to-scale production function, that negative substitutes imply a positive "elasticity of complementarity", which in turn has the sign of the ALEP-like cross partial derivative of the production function. We approve of this "elasticity of complementarity", but it is unclear what its analog is in terms of utility theory.

Joint Satisfaction

Yet another type of technological interrelationship between goods and wants is that of *jointness*, which arises when one unit of a good can satisfy more than one want simultaneously. This relationship is important when one of the wants can also be satisfied by a second good[19]. The relation between jointness and inferiority (in the sense of having a negative income elasticity) deserves careful analysis. It would appear that inferiority (in some qualified sense applicable to discretely divisible goods) cannot arise in the absence of joint want satisfaction, although we have not been able to demonstrate it[20].

The assumption that wants are unrelated is perfectly natural until it is made explicit. Then it becomes apparent what a restrictive assumption it is. Are we really justified in assuming that preferences have a certain structure? Lancaster allows his underlying "characteristics", which correspond roughly to Austrian wants, to be highly interrelated. However, if we reflect on the types of interrelationships among goods that are likely to occur, we ordinarily find that they can be reduced to purely *technological* interrelationships affecting the satisfaction of *un*related ultimate wants. It would appear that the categories of rivalness, complementarity, and jointness are sufficient to explain any such technological interrelationship.

Convexity of the Indifference Curves

Notice that in Figs. 2, 3, and 4 we were always able to draw indifference curves that were convex to the origin and were never forced to draw a backward-bending portion[21]. We conjecture that it can be proven that convex indifference curves may always be found in the Austrian system of utility, subject only to the reservations given in the next section. This has already been demonstrated in the case of two independent goods by Jeffrey Smith (McCulloch and Smith, 1975). This issue is of great interest for economic theory. Hicks is not satisfied that he has given adequate justifi-

[19] In this case, the first and second goods would correspond to Menger's goods of "superior" (*höher*) and "inferior" (*minderer* or *niederer*) quality (1950, 144—145 and 1934, 118—119).

[20] Grossman (1974, 13—18) demonstrates that jointness can give market goods a different income elasticity than the corresponding "household commodities".

[21] Mathematical economists call this property "quasi-concavity".

cation for his bald assumption of convexity, or what is the same thing, of diminishing marginal rate of substitution:

> Since we know from experience that some points of possible equilibrium do exist on the indifference maps of nearly every one..., it follows that the principle of diminishing marginal rate of substitution must sometimes be true.
> However, for us to make progress in economics, it is not enough for us to know that the principle should be true sometimes; we require a more general validity than that. (1946, 22).

Fortunately, the Austrian utility theory leads to a more satisfying development of this important proposition than does the orthodox "indifferent" approach.

Convex indifference curves were first developed by Edgeworth as an implication of diminishing marginal utility, provided the goods were on net independent or were net complements. Note, however, that they work out to have the usual curvature even in our example of rival goods.

Instances of Increasing Marginal Utility

Suppose that one unit of X will satisfy want a, but that it takes no less than two units to satisfy want b; one unit cannot "half-way" satisfy b. Suppose that b is "much greater" than a. If an individual has one unit of X he will use it to satisfy a. If he has two, he will satisfy b. The marginal utility of the first unit is then the importance of a, while the marginal utility of the second is (b, a), the importance of replacing a with b. If b is sufficiently important and a sufficiently unimportant, the marginal utility of the second unit may actually be higher than that of the first unit[22]. Mises (1966, 125) has recognized that circumstances like these may arise when several units of a good must be used together to provide a given effect, and that they provide valid exceptions to the general principle of diminishing marginal utility.

If more than one unit of a good must be combined to produce a given effect, either by itself or in a complementary package with another good, we would similarly expect to find instances where we are forced to draw concave segments of our indifference curves. Therefore any proof of convexity arising from the Austrian theory

[22] We may say for certain that the second unit has higher marginal utility than the first if there is a third want c, and $b \succ ac \succ c \succ a \succ \phi$. By deleting a from both sides of $b \succ ac$, we get $(b, a) \succ c$, whence $(b, a) \succ a$.

of the marginal use must be qualified to hold only if for each good there is a single quantity in which it enters into the consumption technology.

Nevertheless, we would still expect diminishing marginal utility and convexity to hold for a given individual as a general rule, if not in every instance. Furthermore, when we look at masses of individuals, we might find that any "lumpiness" in the consumption behavior of any individual becomes insignificant in examining the behavior of the group as a whole. Consequently, when describing the reaction of large numbers of individuals to price changes, income transfers, etc., we might expect them to behave, as a general rule, as if for each one decreasing marginal utility and convexity held, even though this may not be exactly true in each individual case.

Is It Really Ordinal?

It may have occurred to the reader that the easiest way to generate an "unrelated" ordering on a set of subsets is to assign a real number, say $m(a_i)$, to each element a_i of W, $i=1, 2, \ldots, n$. For each subset P of W define $m(P) = \sum_{a_i \in P} m(a_i)$. Then for each pair P and Q of subsets of W, let $P \succ Q$ whenever $m(P) > m(Q)$. We will call an ordering generated in this manner "essentially cardinal". Clearly such an ordering obeys unrelatedness, for if $P \succ Q$, then $m(P) = m(P-Q) + m(P \cap Q) > m(Q) = m(Q-P) + m(P \cap Q)$, whence $m(P-Q) > m(Q-P)$, so that $P-Q \succ Q-P$. Similarly, $P-Q \succ Q-P$ implies $P \succ Q$, so that *any essentially cardinal ordering also obeys unrelatedness*.

For example, it is easy to show how the orderings of Tables 1 and 10 can arise from such a cardinal measure. The reader may confirm that measures of 11, 8, 6, and 4 for a, b, c, and d respectively will generate the ordering of Table 1. Similarly, the five wants a, b, c, d, and e can be given the measures 9.5, 8.5, 5.7, 5.2, and 2.2 to generate the ordering of Table 10. Such numbers will in general not be unique if W is finite.

It seems plausible that *all* unrelated orderings, at least on finite sets and sufficiently reasonable infinite sets, must be essentially cardinal. In fact, in 1949 the Italian statistician B. de Finetti conjectured that this is true. If so, it would seem to be mere quibbling to retain an ordinal approach once it is assumed that wants are unrelated, for we could then derive all properties of the ordering from a few numbers which we can manipulate in familiar ways. In any case, it could then be argued that the Austrian utility theory is only superficially ordinal, that their assumptions amount to the same thing as cardinality.

For a decade de Finetti's conjecture remained unsolved. In 1959, Kraft, Pratt and Seidenberg finally proved it false by publishing a counter-example. Take for instance the ordering of Table 4. It contains the four relations $be \succ cd$, $bc \succ ae$, $ce \succ b$, and $ad \succ bce$. If the ordering arose from a measure m ( ), we would have $m(b) + m(e) > m(c) + m(d)$, $m(b) + m(c) > m(a) + m(e)$, $m(c) + m(e) > m(b)$, and $m(a) + m(d) > m(b) + m(c) + m(e)$. Adding these four numerical inequalities together we get that $m(a) + 2m(b) + 2m(c) + m(d) + 2m(e)$ must be strictly greater than itself, a contradiction. Therefore the unrelated ordering of Table 4 cannot be essentially cardinal. Similarly, the ordering of Table 16 contains the four relations $b \succ ce$, $cd \succ be$, $ae \succ bc$, and $bce \succ ad$, which would also imply a contradiction if the ordering were essentially cardinal[23].

Since unrelatedness does not imply measurability, it follows that the Austrian theory of the marginal use is *intrinsically ordinal.* It admits of situations where no cardinal utility function is possible.

The Austrian literature is full of contradictory statements as to whether utility is expressible cardinally. On the ordinal side we may cite Böhm-Bawerk (1959 II, 423, n. 17 to p. 141) and Wieser (1884, 180—181). On the cardinal side, we have Böhm-Bawerk (1959 II, 197—198; 124—136) and Wieser (1884, 196). A much-cited passage in Menger (1950, 183 n.) is often used as evidence that he was an ordinalist, but his meaning is clearly cardinalist if we read it in context. See also Menger (1950, 179, 293, n. 1). Only the later Austrian school economists, such as Mises (1966, 122), Bilimovič (1934), and Rothbard (1956), can be said to take an adamantly ordinal position.

The persistent inconsistency of the older Austrians on the cardinality question is understandable in light of the close relation between measurability and their implicit assumption that wants are unrelated. They can hardly be taken to task for being unclear in the nineteenth century about a distinction which mathematicians did not even state until 1949 and did not resolve until 1959. It is natural to draw on cardinal illustrations to force unrelatedness, even if the

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23 Kraft *et al.* (1959) attribute this conjecture to B. de Finetti (1951, 1—10). The ordering of Table 16 is due to Kraft *et al.* That of Table 4 has, to the best of our knowledge, never been published. See Krantz *et al.* (1971, chapt. 5) for theorems relating to unrelatedness. In McCulloch and Smith (1975) it is demonstrated that if W has 5 elements, there are at least 1920 different intrinsically ordinal unrelated orderings on W^*.

cardinality has no necessary place in the theory. Perhaps Böhm-Bawerk had this in the back of his mind when he added the proviso "or something very much like it" to his statement that utilities may be expressed in multiples of one another.

One situation that does lead to essentially cardinal preferences is the hypothetical one in which goods and wants are perfectly divisible. It can be shown that if W^* is unrelatedly ordered in such a way that W can be partitioned into arbitrarily insignificant subsets, its ordering must be essentially cardinal (Krantz et. al., 1971 I, 206—207)[24]. Thus, if a good such as an automobile could be divided into arbitrarily small pieces satisfying arbitrarily trifling wants which when put together would comprise the important wants satisfied by the whole automobile, utility would be essentially cardinal. Such an assumption is not very realistic, to be sure. We would not want to make it a fundamental postulate of all utility theory. Nevertheless in some applications this convenient simplification might be harmless, provided we recognize it as the simplification it is. When we do indulge in it, the unrelatedness of wants, together with the Austrian logic of choice, will *imply* as a theorem that the derived cardinal marginal utility diminishes.

Probabilistic Cardinalization of Utility

It is a fairly straightforward exercise to adapt the well-known von Neumann-Morgenstern probabilistic axioms[25] to the Aus-

[24] Similarly, Alt (1936) demonstrates that any Bernardelli utility index (1938) that is expressible as a continuous function on commodity space can be monotonically transformed in such a way that Bernardelli's conditional utility is the arithmetic difference of his total utility. If the Bernardelli utility index is not continuous, however, it cannot necessarily be so transformed. As a counterexample, consider the derived commodity preferences that would arise when there are several different indivisible goods, each one of which is capable of satisfying a different basic want, when the ordering on W^* happens to be intrinsically ordinal. (There must be five or more goods for this to happen.) In a published comment on Bernardelli's paper, Samuelson (1939) called attention to crucial flaws in a functional example Bernardelli attempted to work out in his mathematical appendix. Samuelson's comments, however, do not reflect on the text of Bernardelli's paper.

[25] See von Neumann and Morgenstern (1953, Appendix) or any advanced text on microeconomics, and Morgenstern (1976, 809). It is a curious inconsistency in the state of economic doctrine that the leaders of

trian framework and come up with a cardinal utility index for the wants and therefore for commodities. In fact, unrelatedness can be integrated into the traditional von Neumann-Morgenstern axiom system in a way that virtually eliminates one of the traditional axioms. When this is done, the Austrian wants-structure will imply that the resulting cardinal utility index on commodity space will be mathematically concave, and therefore exhibit diminishing marginal utility and indifference curves that are convex toward the origin. What's more, it will imply that consumers really are risk-averse (as is conventionally merely assumed), in terms of their von Neumann-Morgenstern utility index[26].

However, doing this rules out intrinsically ordinal rankings on W^*. Therefore economists cannot have both the von Neumann-Morgenstern axioms and the possibility of intrinsically ordinal preferences. One or the other has to go. Several economists *have* questioned the von Neumann-Morgenstern system. Georgescu-Roegen (1954) argues that perhaps preferences are lexicographic and linear, ruling out the possibility of indifference that is crucial to the von Neumann-Morgenstern approach. Taking a different tack, Quandt (1960) and Meginniss (1976) have questioned whether expected utility maximization is necessary for rationality. These authors argue that there is nothing irrational about consumers who instead maximize expected utility plus a term that depends on the standard error of the utility of the gamble (Quandt), or on the entropy of the gamble (Meginniss). Intrinsically ordinal preferences might not be ruled out for consumers like these.

In summary, the issue of probabilistic cardinalization of utility is still up in the air. We personally find intrinsically ordinal preferences and the von Neumann-Morgenstern axiom system about equally plausible. Until this inconsistency is resolved, however, it should be remembered that the purely Austrian approach does admit intrinsically ordinal marginal utility.

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the profession acknowledged soon after 1944 that the von Neumann-Morgenstern cardinalization of utility was plausible, yet refused for decades to grant that it meant that the 1934 Hicks-Allen objections to the ALEP criterion were no longer valid. Only thirty years later was Samuelson willing to draw this obvious conclusion (1974, 1264—1265). Even so, in the same paper he took pains to deny that he was "back-sliding" from the indifferent tradition (1285, n. 23).

26 Furthermore, the circumstances described above, under which instances of increasing marginal utility can arise, provide a rationale for the Friedman-Savage hypothesis (1948).

Conclusion

The Austrian theory of the marginal use raises almost as many problems as it has solved. We list here a few of these unsolved problems.

Complementarity and rivalness do lead to the ALEP criterion in the examples we worked out above, but we have made no attempt to formalize this rule into a general theorem. Intuitively, the ALEP condition must appear when the complementary or rival relationships are somehow active in the inner or outer marginal uses, but it is not clear exactly what the circumstances are under which this holds.

Although the theory leads to quasi-concavity of commodity preferences over goods in the particular cases we worked out, even when rival or complementary interactions are present, it has only been proven that this must be generally true when there are two goods, and then only in the case when the two goods are independent. Perhaps preferences do not really have to be quasi-concave after all.

And finally, it must be resolved whether the possibility of intrinsically ordinal preferences nullifies the von Neumann-Morgenstern axiom system, or if instead the validity of those axioms rules out intrinsically ordinal preferences.

After over a century, the Austrian theory is still in its youth. Perhaps the day has come for Felix Kaufmann's young *Grenznutzler* to return from the netherworld of economic doctrine:

> There I will quietly lie in wait,
> Amid my neglected writings,
> Until I hear the trumpet call
> of Complementary Goods.
> Then through the sky will gallop Böhm-Bawerk,
> Polemics will thunder and flash!
> Then armed with a quill I'll rise up from the grave,
> To fight for the *Grenznutzen* school![27]

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Printed in Austria