Use of History in Developing ideas of refraction, lenses and rainbow

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Refraction phenomena include many aspects of Optical Phenomena. Usually students consider the refraction of light in different cases of phenomena and they do not apply general principles (A. Singh, Butler P. 1990). It is desirable that the students learn to apply qualitatively Snell's law. Galili and Goldberg (1996) proposed a linear approximation of Snell's law (*Refraction angle = 2/3 of incidence angle* for refraction in water or glass) as a way to improve the understanding of image formation, when it is not desired to burden students with trigonometry. This law is similar to Kepler's approximation of the law of refraction: $D = m \cdot r$ where D is the angle of deviation: $D = i - r$, where i is the incidence angle and r is refraction angle (Galili & Goldberg 1996). As the author have found in in-service teacher training, many teachers appreciate this form of the law, as giving them a grasp on refraction phenomena. This law can be roughly confirmed by doing simple measurements. Students at the primary education department appreciated also this law since they could not in general confirm through their measurements Snell's law. Usually students do not approach the measurements with knowledge of the law and they seem not to be concerned with wrong results. Measurements that are not very good usually frustrate them and they do not try to repeat their measurements.

When students are confronted with a failure to confirm a law, they do not try harder, rather they are discouraged. Students do not try to predict a value of the refraction angle and do not consider errors in measurement.

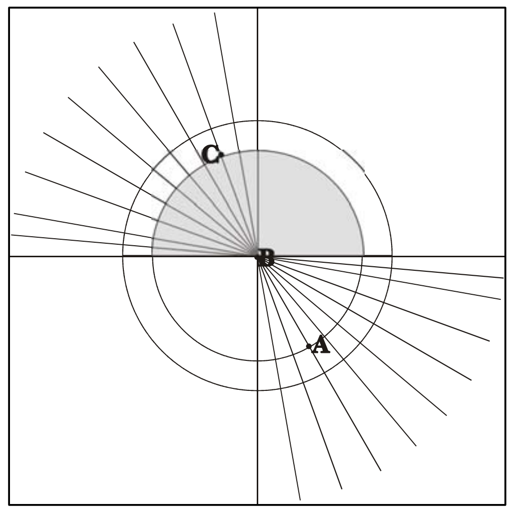

Figure 1 Ptolemy's Method

Ptolemy's Method of measuring the angle of Refraction

In this aspect they can be introduced to historical measurements which show that when people did not know the "correct" law they also produced results which are not very good. This can be seen in the measurements of Ptolemy.

Ptolemy's Refraction experiment results can be compared with students' results. This can be done by asking students to plot their results with Ptolemy's results in Excel and try to find a relation between the angles.

Students were introduced to Ptolemy's method of measuring angles of incidence and refraction. This method can be seen in Figure 1. An observer puts two pins, one in A and one in B. Looking through the glass, from the side of A and B he puts another pin in a point C so the three pins will be seen as one. For the students the method was understood more

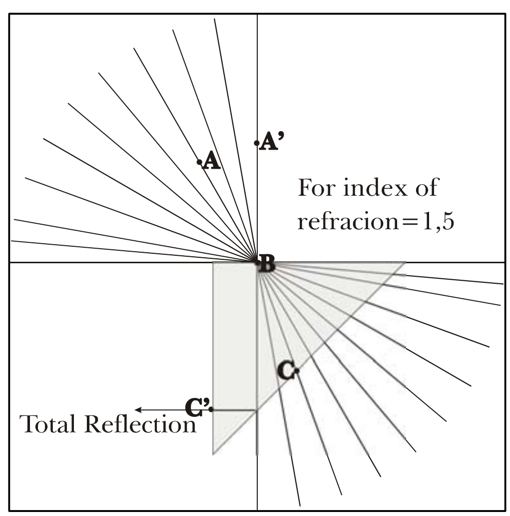

Figure 2 Ptolemy's method for triangular prism

easily when the pin C was totally covered by the glass or water. When the exercise was done with glass objects (semicircle, prisms) some students did not succeed in aligning pins A and B with the imaginary object of C because they confused the bottom part (imaginary) with the top. At first most of the students were unsuccessful in deriving a general method of measuring with the different prisms. They were trying to measure the angles but they did not keep track of what they were doing. Eventually (after one of the students thought of it) they imitated Ptolemy's method of the semicircle in their prisms. This can be seen in figure 2. One unexpected result for the students was the total reflection for the case when the pin A was positioned in A'. By moving the pin C alongside the hypotenuse they could not see it, when pins A' and B where aligned. Finally they found it in C' on the on the side parallel to A'B. This was a pleasant surprise for some of the students. The same procedure can be applied to a prism with parallel edges. The students repeated the exercise and observed the difficulty of measuring the angle of refraction for angles of 80° or more.

Ptolemy's Law of Refraction

Ptolemy presented as a law the analogy of the angle of refraction to the angle of incidence (Galili & Goldberg 1996). Omar (1977) challenges this. He says "the only law of refraction explicitly stated by Ptolemy is: I'/R'>I/R where I' is an angle of incidence greater than I, in the same table, and R', R the corresponding angles of refraction".

If Ptolemy's data are presented in the form that was analyzed by some authors (Nazif 1942, Omar 1977) then it becomes obvious that Ptolemy may had a more complicated relation.

PTOLEMY'S RESULTS FOR REFRACTION FROM AIR TO WATER			
Angle of Incidence	Angle of Refraction	Increase in the Angle of Refraction	Difference of Increase
0°	0°		
		8°00'	
10°	8°		30'
		7°30'	
20°	15°30'		30'
		7°00'	
30°	22°30'		30'
		6°30'	
40°	29°		30'
		6°00'	
50°	35°		30'
		5°30'	
60°	40°30'		30'
		5°00'	
70°	45°30;		30'
		4°30'	
80°	50°		

PTOLEMY'S RESULTS FOR REFRACTION FROM AIR TO GLASS			
Angle of Incidence	Angle of Refraction	Increase in the Angle of Refraction	Difference of Increase
0°	0°		
		7°00'	
10°	7°		30'
		6°30'	
20°	13°30'		30'
		6°00'	
30°	19°30'		30'
		5°30'	
40°	25°		30'
		5°00'	
50°	30°		30'
		4°30'	
60°	34°30'		30'
		4°00'	
70°	38°30;		30'
		3°30'	
80°	42°		

These results were presented to the students to compare with their own results. The students recognized easily the implied relation. The students also questioned the results. The author asked the students to tabulate their own results and to compare with Ptolemy's. They had to examine if the implied relation was holding also for their own results. They did not find this relation to hold in their results. When they plotted their results with Ptolemy's, in a first look they seemed very similar. Ptolemy's results can be explained by a quadratic function. Student's results can be also approximated by a quadratic function. In the following diagram are seen Ptolemy's results for the refraction angle plotted against incidence angle. The curves were the ones given by Excel as a quadratic trend line. Students who succeeded in doing the experiment found similar quadratic trend lines, although their R^2 was not as close to 1 as in the case of Ptolemy.

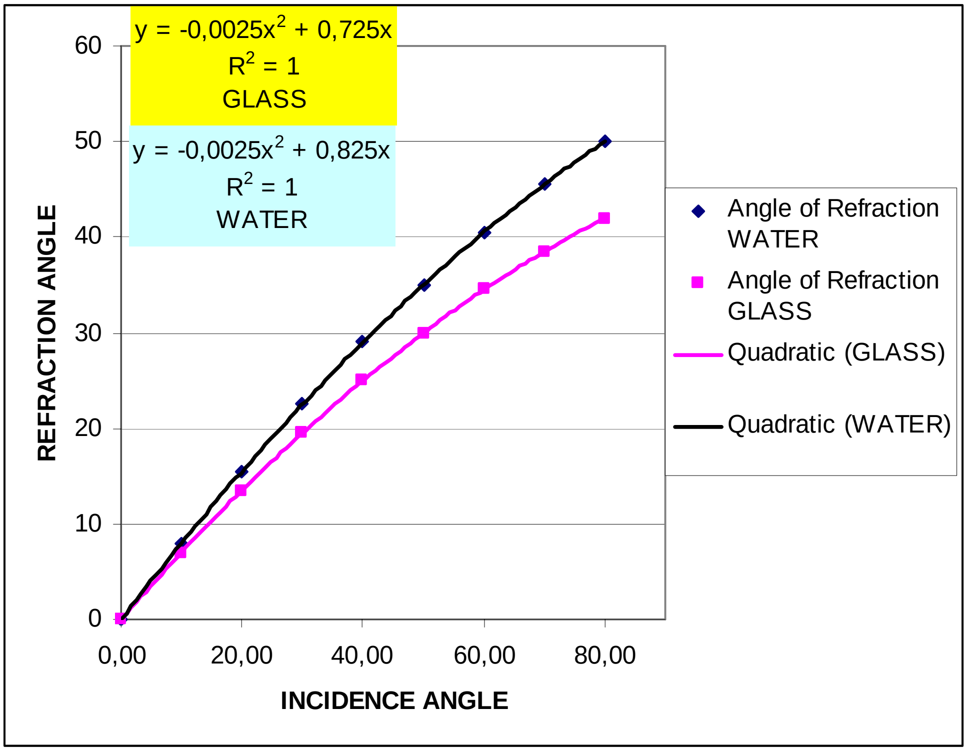

Figure 3 Quadratic fit of Ptolemy's results

So Ptolemy's data conform to a quadratic law which was sought by natural philosophers to correct the linear law (Galili & Goldberg1996). Similar expressions can be found for index of refraction up to n=2,5 with a general equation: *Refraction Angle =-0,0025·Incidence Angle²+b·Incidence Angle* where b an appropriate constant.

A linear trend line for Ptolemy's results conforms to Galili's and Goldberg's approximation of *Refraction angle = 2/3 of incidence angle* for water. For glass the linear trend line would give: *Refraction angle = 4/7 of incidence angle* but the 2/3 law gives better results for small angles although for bigger angles (>40°) the 4/7 law is better.

Al Haytham's laws of refraction

Around 1000 AD Ibn Al Haytham repeated Ptolemy's measurements. He constructed a refraction instrument to measure the angle of deviation of the refracted beam of light (Omar 1977).

The apparatus is depicted in figure 4 for water. For glass he used a segment of a sphere. Although Al Hay-

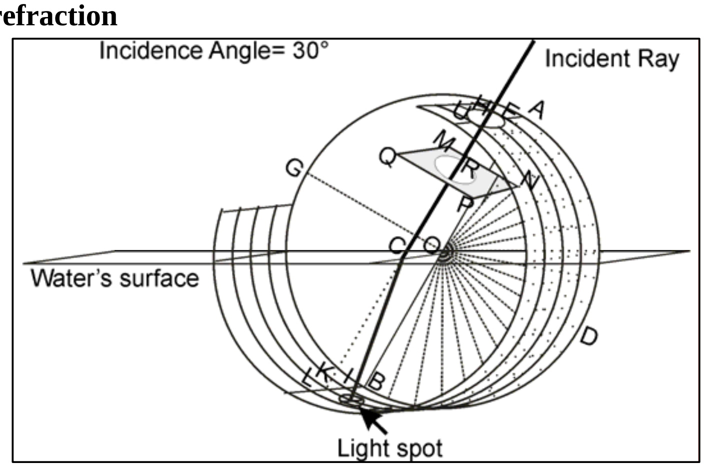

Figure 4 Al Haytham's Instrument

tham was very meticulus in his experimental design his results are for the same angles as Ptolemy's.

A light beam passes through the opening UHF and through another aperture at R. This is to make the beam parallel. He moved a pencil to show that the refraction occurs immediately after the beam reaches the water's surface. It is interesting that he did not measure angles bigger than 80°. Students were asked to consider the difficulties of obtaining strong beams at this time. A student suggested that if he used Sun's Light, it would be difficult to have intense beams at dusk so big angles would have been impossible for sun as a source and only torches and candles for greater angles.

It is interesting that Al Haytham measured the angle of deviation for the same angles of incidence like Ptolemy. According to Wilde (1838) Al Haytham did not present his results in a table, but Witello (Vitello) –who tried to make Al Haytham's work more understandable – presents one. These results are very similar to Ptolemy's. For refraction from air to glass there is only a difference of ½° for angle of incidence of 30°. Al Haytham derived from his results some rules. According to Omar (1977) these rules are:

Let i_1, i_2 represent two angles of incidence, and d_1, d_2, and r_1 and r_2 their respective angles of deviation and refraction, and let $i_1>i_2$. Then:

(1) $d_2>d_1$;

(2) $d_2- d_1< i_2- i_1$;

(3) $d_2/ i_2> d_1/i_1$;

(4) $r_2> r_1$;

(5) In rare to dense refraction, $d<1/2i$;

(6) In dense – to rare refraction, $d < \frac{1}{2}(i+d) \quad \left[d < \frac{1}{2}r \quad \text{when } i < r\right]$

(7) A denser refractive medium deflects the light more toward the normal;

(8) A rarer refractive medium deflects the light more away from the normal.

Nazif(1942) according to Omar, has shown that (2) holds only for rare – to dense re-

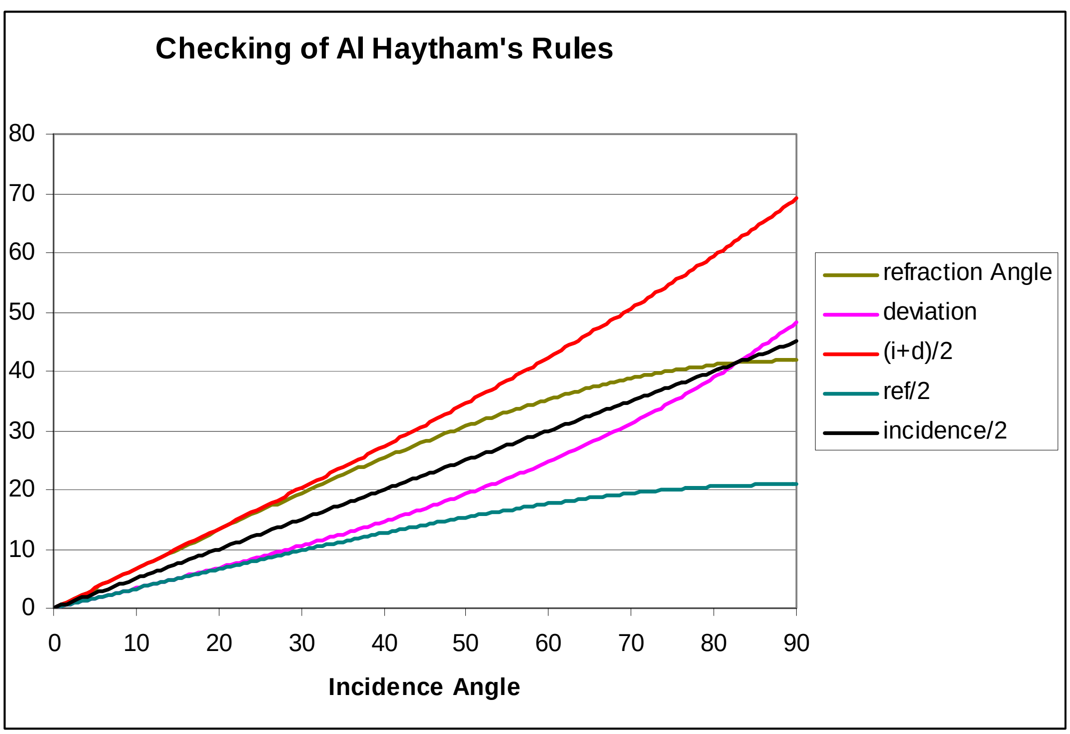

Figure 5 Checking Al Haytham's rule for n=1,5, it can be seen that rule #5 is not true for i>83°

fraction, and (5) and (6) are true only under certain conditions; however, as Nazif also points out, they are correct within the limits of the experiments that were performed by Al Haytham.

Students were asked to check Al Haytham's rules. A few of them succeeded. They were given instructions of how to make an Excel file in which they could change the index of refraction and plot the angles of deviation and refraction, ½(i+d) and ½ r. The students could also check these results from a software "diathlasi" produced by the author. In this way they could "simulate" the results of an experiment, with a graphical representation. In the above case it can be seen that for an angle a little bigger than 80° rule number 5 is not obeyed. Similarly it can be seen that rule number 6 does not hold for all the cases if we have n=2 for refraction from dense to rare.

Interpretation of Refraction phenomena and the introduction of the wave model

The interpretation of Refraction influenced the way scientists made their experiments. Ptolemy considered vision as due to rays emitted by the eye. His apparatus reflected in a way his theory. Al Haytham considered the vision as due to light coming to the eye. Al Haytham's instrument used light beams.

Ptolemy considered the amount of deviation from the original path. He observed that the amount of deviation depended on the degree of density-difference: The greater the difference, the greater the deviation (Sabra 1981). He compared optical refraction with the passage of a projectile from one medium into another more or less resisting medium. For Ibn Al Haytham light moves with very great speed. Al Haytham expressed his ideas of a mechanical model. He argues (Lindberg 1983): Every moved object "must have some its motion altered if it encounters resistance" If this second resistance is strong the motion will be reversed in direction; this is a case of reflection." The idea of the parallelogram of velocities was expressed in earlier times, for the case of reflection, by Hero of Alexandria (Nix and Schmidt 1976) and Ptolemy (Nazif 1942). Al Haytham elaborated these ideas more and expressed the idea that there is a portion (قسط koust) of velocity in a direction parallel to the surface and a portion perpendicular to the surface (Nazif 1942). Al Haytham believed that any transparent body resists the movement of light; the denser they are, the greater the resistance they offer. According to Nazif Ibn Al Haytham assumes that the resistance acts particularly in the direction of the component parallel to the surface. Lindberg (1983, chapter VIII) says that this interpretation is "suspiciously Cartesian". According to him both components continue to exist but with altered magnitudes. To explain his ideas, Al Haytham used some analogies: his famous sword analogy (Nazif 1942). A sword cuts easier when it cuts according to the perpendicular of a rod, so "the motion will be deviated toward a direction in which it is more easily moved than in is original direction". According to Al Haytham, light must be deviated to-

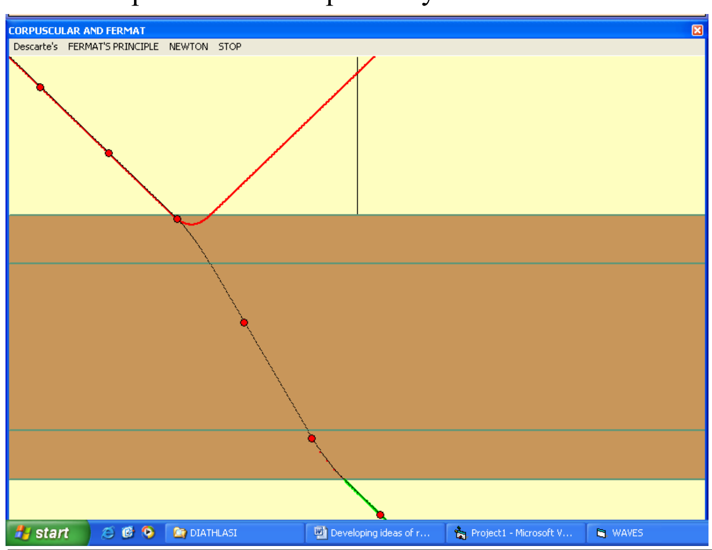

Figure 6 Newton's model of particle nature of light: case of a slab

ward the path of least resistance. A problem existed with this interpretation (Lindberg, 1983,). From the fact that oblique light cannot continue in a straight line when it encounters a denser medium, it does not follow that the light will be deviated towards direction of easier passage through that dense medium. If refraction is a weaker case of reflection, why refraction is not away from the perpendicular?
If the light passes from the denser to the rarer medium, the light moves more swiftly in the rarer medium. The resistance of both media is parallel to the interface.
Nazif (1942) writes that Al Haytham expressed a kind of Fermat's principle. According to Lindberg (1983), all attempts in medieval times to explain the direction of refraction, required refraction in the wrong direction. Descartes explanation with a tennis ball seems very contrived. Newton's explanation with a kind of gravitational field is one way to explain the deviation. Newton's explanation is contrary to the intuition that particles will move faster in the rarer medium.
According to Sabra (1981) Al Haytham's model could have been expressed by the constancy of the velocity perpendicular to the surface and the change of the velocity parallel to the surface. If Al Haytham could express this mathematically he would have derived a "Snell's Law" (Sabra 1981). In Newton's model on the other hand the velocity perpendicular to the surface is the one that changes while the velocity parallel to the surface does not change.

So the same relation $\frac{\sin(\theta_{incidence\,angle})}{\sin(\theta_{refraction\,angle})} = n$, can be interpreted as giving the ratio of velocities either of first to second medium or of the second to the first. The interpretation of the index depends on the theory of the nature of light.

In some books (Prifti *et al* 2003) are presented the cases of refraction of light and sound in water.

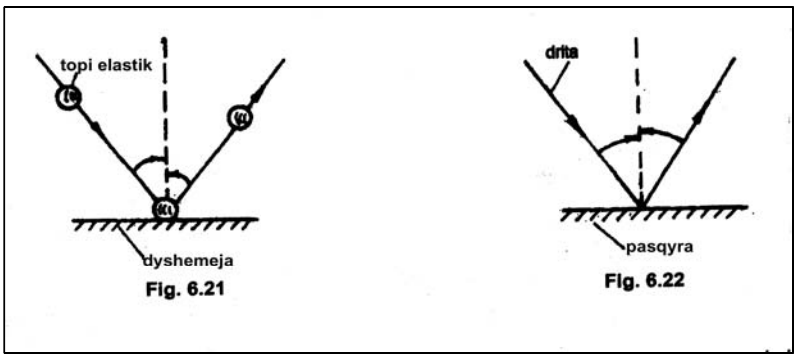

Figure 7 Particle model for reflection in Prifti *et al* (2003): elastic ball on the left impinging on the floor and light on the right impinging on a mirror

At first is emphasized the particle model of light (figure 7) and later the wave model of sound. The refraction of light in water is presented without any mechanical explanation. In the case of sound, although water is a denser medium it is shown that the sound "rays" are

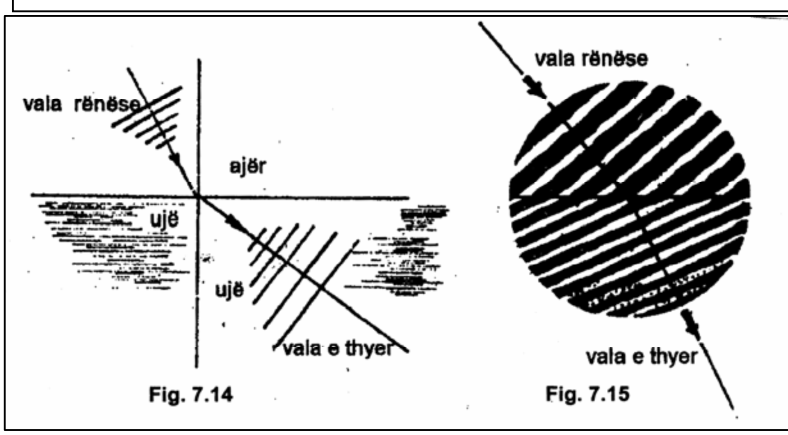

Figure 8 Refraction of sound in water (air in the upper part) (Prifti *et al* 2003). On the right is shown a wave refraction

deviated away from the perpendicular, while for the particles of light, the light rays are deviated towards the perpendicular. The pupils are asked to compare the velocity of wave fronts in the upper part and lower part of figure 7.15 (fig 8). Prifti *et al* (2003) is actually dissociating the word "denser" and "rarer" from the direction of deviation.

In a Greek textbook (Antoniou et al 2000), Fermat's principle is adopted for the refraction phenomena but the nature of light is not stated explicitly.

Students, who study to become Elementary School Teachers, were taught the refraction by the use of a wave model. The students who attend the lessons can be divided in two categories: Students who during their high school years, have been taught much physics (including the refraction of light), and students who have taken more "theoretical" courses in high school. The first category, generally, did not appreciate much the wave model because they felt comfortable to use the ray model, while the "theoretical" students seem to have appreciated more the wave model. This was seen in tests where they had to draw at first the waves and then the rays. Students of the first category generally did much worse in the task of drawing wave fronts than students of the second category (as in figure 9). As one student expressed it "I cannot understand the rays, but the waves I can understand".

As Harrison and Treagust (1993) point out, the wave model of refraction can be taught by the use of suitable analogies.

Ibn Sahl's Law of Refraction and Hyperbolic Lenses

Before Al Haytham's researches on refraction, Ibn Sahl wrote the treatise *On the Burning Instruments.* (Rohsdi 1992). His treatise was composed between 983 and 985. According to Roshdi, Ibn Sahl's treatise makes him the first mathematician known to have studied lenses and shows that in the first half of the tenth century catoptricians were actively working on refraction. The problem Ibn Sahl tackled may be stated as follows: To burn at a given point A, using a distant or near luminous source, by reflection or refraction.

For the first case he studied parabolic and ellipsoidal mirrors. For the second case he studied the hyperbolic lens. To study the hyperbolic lens, Ibn Sahl first considered refraction on a plane surface ZU. According to Roshdi Ibn Sahl wrote:

"Let DC be a light ray in the crystal, which is refracted (figure 9); in the air along GE. The perpendicular to the plane surface ZU at Z intersects line DG at H and the refracted ray at E."

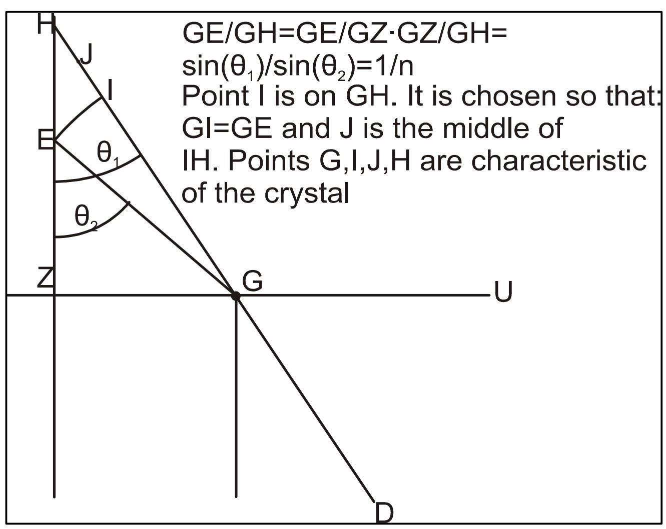

Figure 9 Ibn Sahl's expression of Snell's law

The ratio GE/GH<1, which Ibn Sahl uses throughout his study, is the reciprocal of n.

$$\frac{GE}{GH} = \frac{GE}{GZ} \cdot \frac{GZ}{GH} = \frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{1}{n}$$

The ratio GE/GH characterizes the crystal.

Student's were presented with Ibn Sahl's Law, and were asked to see that it is the same as Snell's law. Snell's law was proposed in 1621 by Snell and was first time published 1637 by Descartes (Sabra 1981 p 101).
The fact that theories that were proposed in ancient or medieval times were ignored or put aside for a long time (for example Aristarchus Heliocentric system) is not something that happened only on the past. In contemporary science history there are accounts of theories that were also put aside or rejected when they first appeared (Ziman 1978). According to Roshdi (1992) Al Haytham knew of Ibn Sahl's treatise and was even sometimes copied in his own hand. Roshdi writes that the main aim of Ibn Sahl's work was on finding a way to construct a burning instrument. When attention is paid to the problems raised by the image of an object observed through the hyperbolic lens, the situation becomes quite different; in this case it is impossible to avoid difficulties such as astigmatism and aberration. Such problems would arise in the work of Ibn Al Haytham. We can only guess why Al Haytham did not adopt Ibn Sahl's law of refraction. One proposed guess was that it was impossible for Al Haytham to check the law with his instrument. His results being so similar with Ptolemy's, show that in a way he accepted the relationship implied by the results and he saw no need for a more complicated law. Another guess was that he could not think of an interpretation of Ibn Sahl's law.

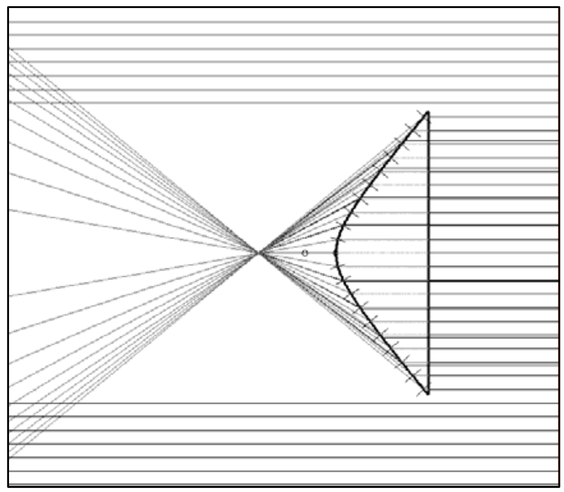

Figure 10 Hyperbolic Lens

One important development by Ibn Sahl was the hyperbolic mirror. With this mirror it is possible to focus perfectly a parallel bundle of light to one point. This is one case where students can be exposed to the difference between a model and the reality. Sometimes books present the laws of thin spherical lenses as "facts". For example the book by Antoniou *et al.*(2000) presents the rules of paraxial rays as facts.
To compare the hyperbolic lens and the spherical lenses a computer program was developed by the author.

Al Farisi's Theory of the formation of the Rainbow

The final two sections correspond to what is called "atmospheric physics".
The first one deals with rainbow phenomena and the last one to the change of appearance of the moon due to atmospheric refraction.
Explanations of the rainbow can be found in Aristotle (Aristotle 1987), Al Haytham. Al Haytham's explanation was based on his work on convex mirrors. He considered the cloud as having a convex spherical form (Nazif 1942).
Usually the explanation of the rainbow is attributed to Descartes (Sabra 1981, Walker 1976).
But actually Al Farisi according to Roshdi(1992) developed the ideas of the Rainbow, based on Al Haytham's work. Al Haytham had derived a rule about the double refraction of light on a sphere:

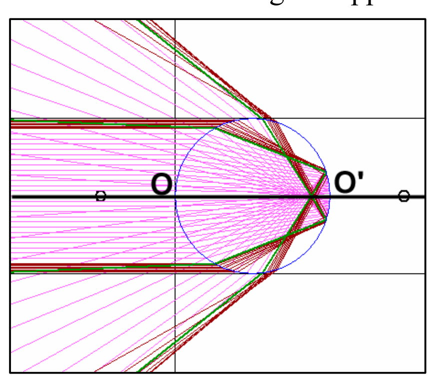

Figure 11 External and Internal cones of Al Farisi

He considered the light rays incident parallel to the axis OO'. The trajectories that the light follows between two refractions are such that as the angle of incidence increases, the distance of the point where the light meets the sphere after its first refraction from the axis increases until the angle of incidence reaches a "critical" value. After this critical value the distance decreases. Al Farisi described the rays as belonging to the "central cone" if their angle of incidence is less than the critical value, and as belonging to the "external cone" if their angle of incidence is bigger than the critical value. Another property of the ray following the critical angle is that the rays of the internal cone are cut by the critical ray outside the sphere, while the rays of the external cone are cut inside the sphere. In the first case the angle of the rays, which is equal to the difference of the deviation angles, is less than half the difference of angles of incidence, while on the second case the angle of rays are more than half the angles of incidence. By taking the limiting value of a ray, for example of the external cone approaching the critical ray, the angle of difference becomes ½ the difference of angles of incidence. This way we have:

$$\lim_{\theta_i \to \theta_c}\left(\arcsin\left(\frac{\sin(\theta_i)}{n}\right) - \arcsin\left(\frac{\sin(\theta_c)}{n}\right)\right) = \frac{1}{2}\lim_{\theta_i \to \theta_c}(\theta_i - \theta_c)$$

After some manipulations we get: $\sin(\theta_c) = \sqrt{\frac{4-n^2}{3}}$, which for water is θ_c=59,58°.

The light after reaching the surface of the sphere either is reflected or refracted. In the first case we have a "zero order" rainbow, which is unobservable for parallel light rays (Look &McCollum 1994). If the light is reflected once and then refracted to the air, we have a "first order" rainbow. This ray corresponds to the "Cartesian" ray (Walker 1976). For more reflections we have higher order rainbows. Software was constructed to explain to the students these ideas. It can be seen that on the formation of the rainbow, the most important rays are that of the external cone. Al Farisi's theory was good for explaining the shape of the rainbow, but Al Farisi's theory of colors was very similar to Aristotle's theory as it is expounded in his Meteorologica (Aristotle 1987).

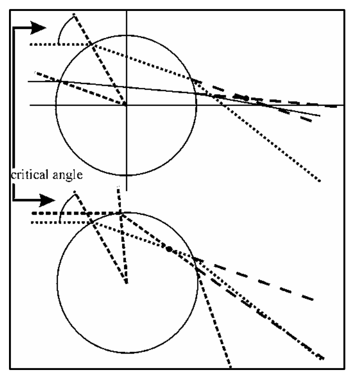

Figure 12 Relation of rays to the special ray of critical angle

Halo phenomena can also be considered.

Al Haytham's Theory about the influence of atmospheric refraction on the observation of astronomical phenomena

The last part deals with Atmospheric Refraction. Atmospheric Refraction is usually presented in some textbooks as a straightforward application of refraction phenomena (Hewitt 1996). In reality it is difficult for students (and teachers) to understand the change of the shape of the moon or the sun as they set down.

Atmospheric refraction is a very complicated phenomenon. Several factors contribute to it: Temperature, presence of vapors etc (Chester 2005).

Refraction was considered by Ptolemy as altering the path of visual rays. Al Haytham tried to explain the influence of the atmosphere to the observation of celestial bodies). Al Haytham explained that the atmospheric refraction produces a change of the height where a star will be seen. Al Haytham tried to explain the fact that atmospheric refraction changes by the influence of the moisture. He gave a theory of the influence of a

layer of vapors over the atmosphere which can change the predicted phenomena (Nazif 1942).
Software was devised to show these results. In this software the student can see that the refraction index of the atmosphere can alter the height where the moon is seen. The student can see that the gradual change of the index from one to a maximum value near the earth produces a larger effect than an abrupt change. He can also compare also the lengthening of the time that the moon is over the horizon in all cases (vapors or no, gradual or abrupt change of n). The student can change the index of refraction of the atmosphere and observe the effect. On figure 13 is shown the case of

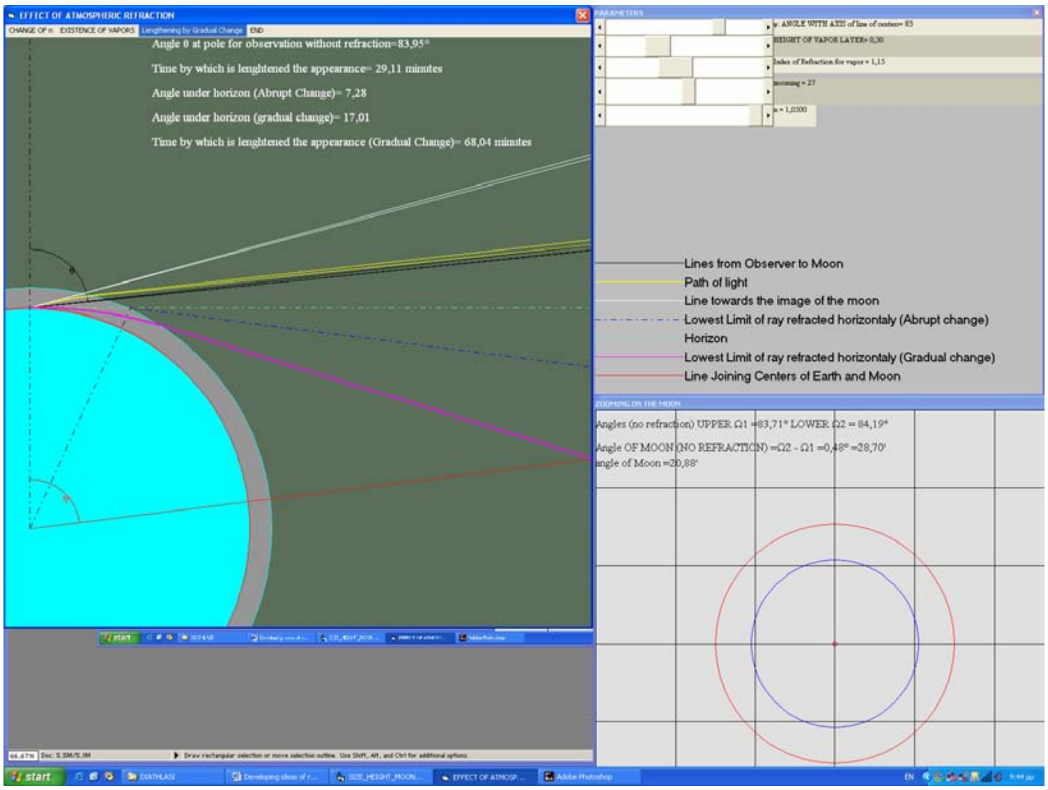

Figure 13 Software depicting the effects of atmospheric refraction

no vapors. On the left form, the lowest curve corresponds to the limiting case where the upper part of the moon is seen. This can be compared to the case of abrupt change of index of refraction (dashed – dotted line). The uppermost white lines correspond to the lines of sight. Their angle corresponds to the size of the moon as perceived by the observed. These lines are tangent to the curves of light. The black lines correspond to the case of no atmospheric refraction.

Final Remarks

Refraction phenomena can serve several purposes of Science Education:

a) One case is the teaching of laboratory techniques. The student can learn to derive "law like relations" (Duschl 1990). Historical data can help the student understand the role of theory on data collection.
b) The student can learn about the interpretation of phenomena. He can compare the corpuscular explanations with wave explanations. Snell's law has a different meaning for Newton's corpuscular theory compared to wave theory. The index of refraction has also a different meaning.

c) The different laws of refraction and the fact that Ibn Sahl's law was actually Snell's law can show that science is not progressing in a straight path but there are many instances of going back to more inadequate laws and theories.
d) The application of refraction laws in lenses is very important. The student can understand more about the "models" of physics by observing how real spherical lenses refract the rays and how this can be compared to the usual model of paraxial rays taught in schools. The students can appreciate that the focusing of rays is actually an approximation and differentiate between models and reality.
e) Al Farisi's explanation of the shape of rainbow is another example that scientists have derived very similar theories in different times. The students can appreciate that even great scientists like Al Haytham can explain in a wrong way a phenomenon. The students can see that in Rainbow both refraction and reflection take place.
f) Atmospheric refraction can be used with suitable software to develop an appreciation of complications in dealing with atmospheric phenomena.

NOTE: Programs can be downloaded from: *http://utopia.duth.gr/~pmichas/aglika*

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