Understanding Linearity and Monotonicity

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Linearity and monotonicity are two terms found in the specifications for many devices, such as DACs and ADCs -- as well as DMMs and sensors -- that seem to cause a great deal of confusion. Monotonicity is a fairly simple concept. However, linearity may be defined as either differential, or integral. And to complicate matters even further there are three, or four, different forms of integral linearity. A basic understanding of these terms is necessary if a designer is to match a component, or an instrument to a specific application.

Monotonicity simply defines the direction that a device's output moves with respect to the direction that the input moves. This is an especially important specification for devices that are used in control system applications, where a non-monotonic device could cause disastrous consequences. That is, for a device to be monotonic, as the input to the device increases in value, the output must also increase in value, ignoring noise effects. Likewise, as the input decreases, the output must also decrease. A DAC is a good example. As the input codes increase in value, the analog output must also increase, if the device is to be considered monotonic (Fig. 1). The important characteristic of monotonicity is that the output direction must follow the input direction; both must increase, or both must decrease. Therefore, a device is either monotonic, or it is not. There are no degrees of monotonicity. Note that nothing is said in this definition about the amount that the output changes with each input change. This is because monotonicity is only concerned with the direction of the change, not the magnitude.

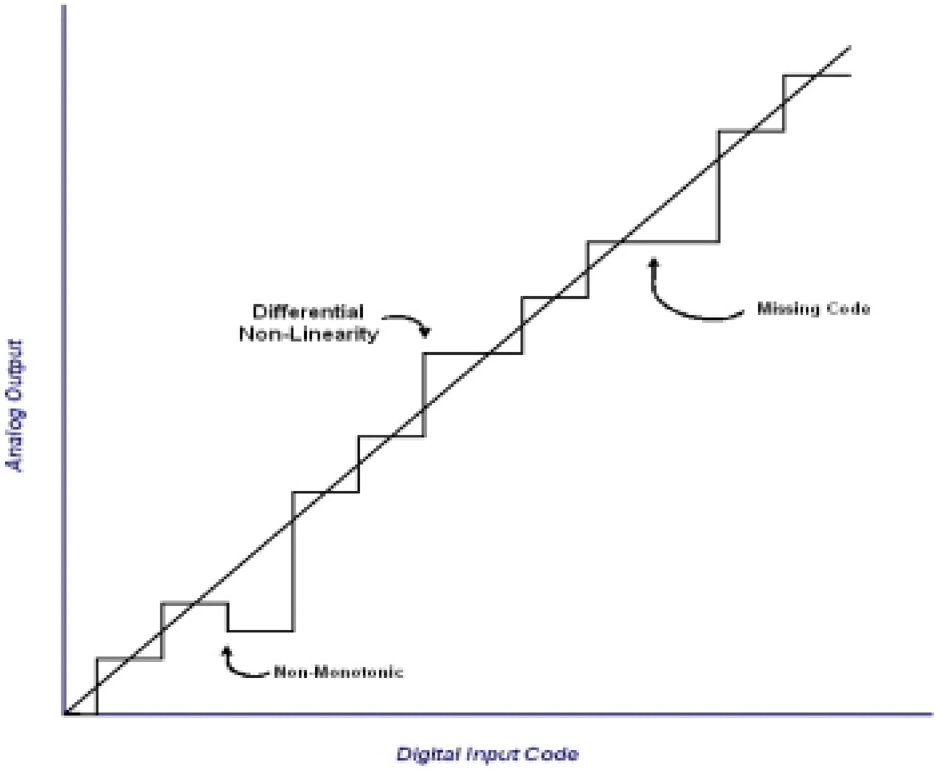

Fig. 1: DAC Example, Non-Monotonic With Differential Non-linearity and Missing Codes

The difference between the changes in a device's actual analog output relative to an ideal single step change (1 LSB), defines its differential non-linearity (DNL) (fig. 1, again).

An ideal device will have a DNL of zero, while a DNL of -1 LSB implies that there is a missing code. Mathematically, DNL is computed as:

DNL = (analog voltage change in LSBs - 1 LSB)

Linearity defines how closely the device's actual output follows an ideal straight line over the entire operating range. However, there are several different ways that linearity may be defined, depending on how the straight line is positioned.

There are three basic definitions for integral linearity in common use: independent linearity, zero-based linearity, and terminal, or end-point, linearity. In each case, linearity defines how well the device's actual performance across a specified operating range approximates a straight line. Linearity is usually measured in terms of a deviation, or non-linearity, from an ideal straight line and it is typically expressed in terms of percent of full scale, or in ppm (parts per million) of full scale. Typically, the straight line is obtained by performing a least-squares fit of the data. The three definitions vary in the manner in which the straight line is positioned relative to the actual device's performance. Also, all three of these definitions ignore any gain, or offset errors that may be present in the actual device's performance characteristics.

Many times a device's specifications will simply refer to linearity, with no other explanation as to which type of linearity is intended. In cases where a specification is expressed simply as linearity, it is assumed to imply independent linearity.

Independent linearity (Fig. 2) is probably the most commonly-used linearity definition and is often found in the specifications for DMMs and ADCs, as well as devices like potentiometers. Independent linearity is defined as the maximum deviation of actual performance relative to a straight line, located such that it minimizes the maximum deviation. In that case there are no constraints placed upon the positioning of the straight line and it may be wherever necessary to minimize the deviations between it and the device's actual performance characteristic.

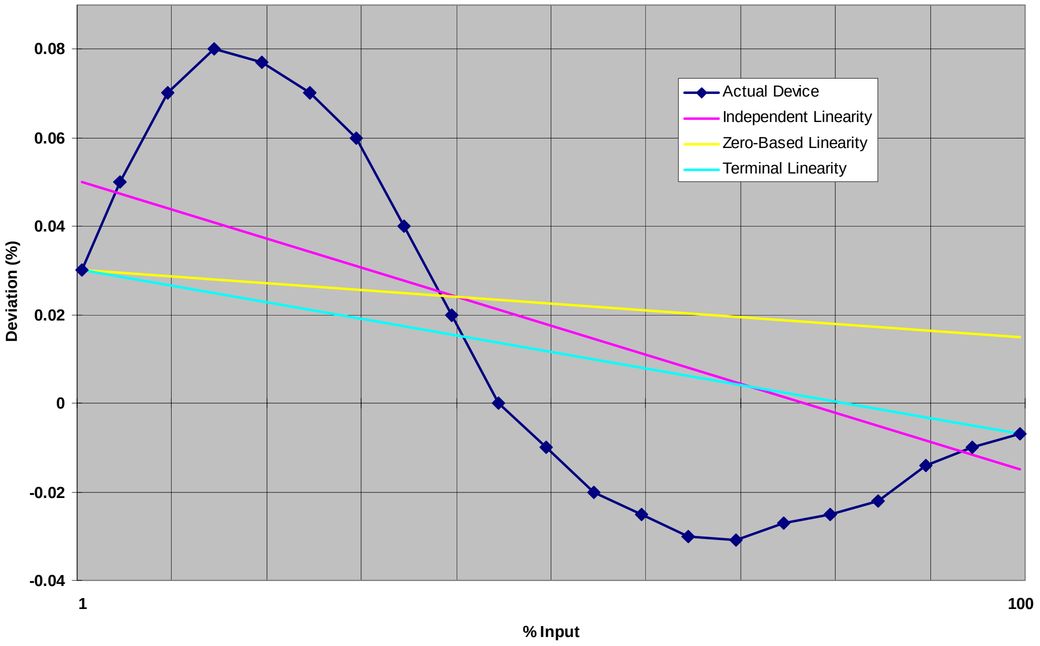

Fig. 2: Linearity Deviations

Zero-based linearity (Fig. 2, again) forces the lower range value of the straight line to be equal to the actual lower range value of the device's characteristic, but it does allow the line to be rotated to minimize the maximum deviation. In this case, since the positioning of the straight line is constrained by the requirement that the lower range values of the line and the device's characteristic be coincident, the non-linearity based on this definition will generally be larger than for independent linearity.

For terminal linearity (also Fig. 2), there is no flexibility allowed in the placement of the straight line in order to minimize the deviations. The straight line must be located such that each of its end-points coincides with the device's actual upper and lower range values. This means that the non-linearity measured by this definition will typically be larger than that measured by the independent, or the zero-based linearity definitions. This definition of linearity is often associated with ADCs, DACs and various sensors.

A fourth linearity definition, absolute linearity, is sometimes also encountered. Absolute linearity is a variation of terminal linearity, in that it allows no flexibility in the placement of the straight line, however in this case the gain and offset errors of the actual device are included in the linearity measurement, making this the most difficult measure of a device's performance. For absolute linearity the end points of the straight line are defined by the ideal upper and lower range values for the device, rather than the actual values. The linearity error in this instance is the maximum deviation of the actual device's performance from ideal.

About The Author

Bertram S Kolts joined Agilent Technologies (Hewlett-Packard) in 1968 and earned a BSEE at Colorado State University in 1975. He has been a test engineer for 28 years, with a background in IC and instrument testing, including the design and construction of test systems (hardware & software) and product testing. Primary areas of expertise include precision automated testing and high-speed digitizer testing. Bert is currently involved with testing source and voltmeter products. His after work hobbies include ham radio, hiking, furniture making, and writing.