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COINCIDENCE CROSS SECTIONS FOR THE DISSOCIATION OF LIGHT IONS IN HIGH-ENERGY COLLISIONS

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Abstract: We make an analysis of the influence of the nuclear and of the Coulomb interaction in the dissociation of weakly-bound and cluster-like nuclei in coincidence experiments at high-energy collisions. We use the theory of diffraction dissociation to account for the nuclear effects and compare it to the Coulomb dissociation in the description of the angular distribution of the fragments, as well as in the total cross sections. The nuclear and the electromagnetic dissociation have very different characteristics and we show how this may help to disentangle their contributions in the study of experimental data.

1. Introduction

Besides the interest on the scattering process itself, the coincidence experiments for break-up of weakly-bound nuclei in high energy collisions can also give precious information about the structure of these nuclei and about the related photonuclear reactions. For example, a study of nuclear reactions of astrophysical interest like, e.g., ^{3}He(^{4}He, γ)^{7}Be has been recently proposed [1,2,3]), by experimentally measuring the electromagnetic transition between a bound state of two nuclear particles and continuum states at small relative energies in the collisions of light nuclei with heavy ones which provide a strong Coulomb field. The proposal suggests to use the nuclear Coulomb field as a source of the photodisintegration processes. By means of the detailed balance theorem these processes can be related to radiative capture reactions of astrophysical interest [2]). The experimental investigation of the photodisintegration processes in the nuclear Coulomb field have to be separated from the nuclear interaction contribution to the projectile fragmentation on hitting the target. Therefore, a theoretical analysis of the angular distribution of the fragments arising from the break-up of the projectile by the nuclear and by the Coulomb interaction with the target is very useful and necessary for the experimental investigations.

Another interesting possibility is the study of the nuclear matter distribution of extremely neutron-rich nuclei, like e.g. ^{11}Li. Some high-energy experiments [4,5,6]) for the break-up of such nuclei are beginning to be available, and seem to be a very promising field of study of such nuclei. Nevertheless, such experiments up to now have been inclusive ones, i.e., only one fragment is observed. Therefore, a summation has to be done over all unobserved channels, leading to a partial loss of information

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about the process. More useful would be the exclusive experiments where the dissociation process of the projectile is separated from the background of other reactions by means of the coincidence detection of the two outgoing fragments together with a simultaneous measurement of their energies. Although these experiments are much harder to perform in high-energy collisions, they are realizable and there are some experimental proposals in this direction (see e.g. refs. [7,8]). Even at the highest energy of 14 GeV/nucleon such coincidence studies are planned [8]). Perhaps, one could determine the momentum transfer in these reactions by a measurement of the recoil energy of the target nucleus.

Due to the scarcity of experimental information at present it seems not appropriate to carry out very detailed calculations of break-up reactions in high-energy collisions. We prefer to make a more qualitative study of these coincidence cross sections and of the role of the nuclear and of the Coulomb interaction. We use some simple assumptions about the structure of the weakly-bound nuclei, disregarding some more specific details for sake of simplicity. We use the diffraction dissociation theory to account for the nuclear interaction. This theoretical approach has been introduced by Akhiezer and Sitenko [9]), Glauber [10]), and Feinberg [11]), to describe the dissociation of highly-energetic deuterons. Also important in this context is the so-called stripping reactions in which one of the clusters of the projectile suffers a strong inelastic collision with the target while the other is diffracted inelastically [10]). The cross sections for the stripping reactions depend much more on the exact knowledge of the nuclear structure and can be only approximately calculated [12,13]).

The decomposition of the contribution of each electromagnetic multipolarity in relativistic Coulomb excitation has been recently performed (see e.g. refs. [14,15])). We use those results to infer the relative importance of these multipolarities (mainly E1 and E2) in the dissociation process. An extension to include other multipolarities, for instance M1, can be easily done. We obtain some useful analytical formulas and, since we envisage the applications in coincidence experiments, we do not consider the case of stripping reactions. Our study is complementary to many previous works on the fragmentation of relativistic particles. We refer for example, to the works of Hüfner and Nemes [16]), Fäldt [12]), and of Evlanov and Sokolov [17]).

In sect. 2 we give the formulas for the amplitudes of Coulomb and of diffraction dissociation of loosely-bound nuclei hitting a heavy nuclear target. In sect. 3 we make a qualitative study of the angular distribution of the fragments, and in sect. 4 we study the dependence of the total cross sections for dissociation as a function of the projectile energy. Our conclusions are given in sect. 5.

2. Dissociation of weakly bound nuclei

The amplitude of the dissociation of the incident projectile on a target nucleus, assumed to stay in its ground state, in the eikonal approximation is

$$f_{\rm d}(\boldsymbol{Q}, \boldsymbol{q}) = \frac{ik}{2\pi} \int {\rm d}^2 b \; {\rm e}^{i\boldsymbol{Q}\cdot\boldsymbol{b}} \Gamma_{\rm d}(\boldsymbol{b}) \,, \qquad (2.1)$$

where $\boldsymbol{k}$ is the c.m. momentum of the projectile, $\boldsymbol{Q}$ is the momentum change in the scattering ($Q = 2k \sin\frac{1}{2}\theta \cong k\theta$, where θ is the scattering angle of the center of mass), $\boldsymbol{q}$ is the relative motion momentum of the outgoing fragments, and $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are the momenta of the corresponding clusters with masses m_1 and m_2, respectively. In non-relativistic collisions $\boldsymbol{q} = (m_2\boldsymbol{k}_1 - m_1\boldsymbol{k}_2)/(m_1+m_2)$, while for high-energy collisions $\boldsymbol{q}$ can be determined by the invariant mass of the two fragments. $\Gamma_d(\boldsymbol{b})$ is the profile function for the dissociation. In the approximations we shall use, it contains contributions from diffraction dissociation on the target surface, and from Coulomb dissociation for impact parameters b larger than the sum of the nuclear interaction radii. Assuming a sharp boundary target, it can be written as $\Gamma_d(\boldsymbol{b}) = \Gamma_N(\boldsymbol{b}) + \Gamma_C(\boldsymbol{b})$, where $\Gamma_N(\boldsymbol{b})$ vanishes for $b \geq R$ and $\Gamma_C(\boldsymbol{b})$ vanishes for $b \leq R$. Therefore we obtain

$$f_d(\boldsymbol{Q}, \boldsymbol{q}) = f_N(\boldsymbol{Q}, \boldsymbol{q}) + f_C(\boldsymbol{Q}, \boldsymbol{q}) . \tag{2.2}$$

The total dissociation cross section is given by

$$d\sigma = |f_d(\boldsymbol{Q}, \boldsymbol{q})|^2 \, d\Omega \frac{d^3q}{(2\pi)^3} , \tag{2.3a}$$

where

$$d\Omega \cong \frac{2\pi}{k^2} Q \, dQ \tag{2.3b}$$

for high-energy collisions.

The relative motion of the clusters within the projectile is described by the wave function

$$\varphi_i(r) = \sqrt{\frac{\eta}{2\pi}} \frac{e^{-\eta r}}{r} , \tag{2.4a}$$

where $\eta = (2\mu\varepsilon/\hbar^2)^{1/2}$ is determined by the separation energy ε of the cluster and μ is the reduced mass of the system (1+2). The relative motion of the clusters released after the disintegration of the projectile is described by the wave function

$$\varphi_f(\boldsymbol{r}) = e^{i\boldsymbol{q}\cdot\boldsymbol{r}} + \frac{1}{iq-\eta} \frac{e^{-iqr}}{r} . \tag{2.4b}$$

These wave functions correspond to the assumption of zero-range nuclear forces between the clusters in the projectile. They are very useful because most of the following calculations can be performed analytically. An extension to the use of more realistic wave functions is straightforward. They form a complete set of orthonormal functions satisfying the relation

$$\varphi_i(\boldsymbol{r})\varphi_i^*(\boldsymbol{r}') + \frac{1}{(2\pi)^3} \int \varphi_f(\boldsymbol{r})\varphi_f^*(\boldsymbol{r}') \, d^3q = \delta(\boldsymbol{r} - \boldsymbol{r}') . \tag{2.4c}$$

The use of such wave functions presupposes a simple model, where no Coulomb repulsion between the clusters are taken into account (as would be important in systems like $\alpha+{}^3$He, d+p, . . .). The Coulomb repulsion between the clusters must lose its importance for high relative motion after their dissociation.

By using the energy and momentum conservation laws we can also express (2.3a) in terms of coincidence cross sections which are of interest in exclusive experiments. One finds

$$\frac{d^3\sigma}{d\Omega_1\, d\Omega_2\, dE_2}=\frac{\mu}{(2\pi)^3\hbar^2}\frac{k_1k_2}{k}|f_d(\boldsymbol{Q},\boldsymbol{q})|^2\,, \tag{2.5}$$

where Ω_1 and Ω_2 are the solid angles of emission of the two fragments and E_2 is energy of one of them. But, since the theoretical analysis is more transparent by using the variables $\boldsymbol{Q}$ and $\boldsymbol{q}$, we shall keep them, and use eqs. (2.3) in what follows.

2.1. AMPLITUDE FOR DIFFRACTION DISSOCIATION

The amplitudes for diffraction dissociation of deuterons by a "black nucleus" were calculated by Akhiezer and Sitenko. The extension to the dissociation of other weakly-bound nuclei gives [17])

$$f_N(\boldsymbol{Q},\boldsymbol{q})=ikR\left\{\frac{J_1(QR)}{Q}[F(-\beta_2\boldsymbol{Q},\boldsymbol{q})+F(\beta_1\boldsymbol{Q},\boldsymbol{q})]\right.$$
$$\left.-\frac{ikR^2}{2\pi}\int d^2Q'\frac{J_1(Q'R)}{Q'}\frac{J_1(|\boldsymbol{Q}-\boldsymbol{Q}'|R)}{|\boldsymbol{Q}-\boldsymbol{Q}'|}F(\beta_1\boldsymbol{Q}-\boldsymbol{Q}',\boldsymbol{q})\right\}, \tag{2.6}$$

where $\beta_1=m_2/(m_1+m_2)$, $\beta_2=m_1/(m_1+m_2)$, $R=1.2\,A_T^{1/3}$ fm is the radius of the target nucleus, and

$$F(\boldsymbol{Q},\boldsymbol{q})=\int d^3r\,\varphi_f^*(\boldsymbol{r})\,e^{i\boldsymbol{Q}\cdot\boldsymbol{r}}\varphi_i(\boldsymbol{r})$$
$$=\sqrt{8\pi n}\left\{\frac{1}{\eta^2+(\boldsymbol{Q}-\boldsymbol{q})^2}+\frac{1}{2Q(i\eta-q)}\ln\left[\frac{q+Q+i\eta}{q-Q+i\eta}\right]\right\}. \tag{2.7}$$

The first term in eq. (2.6) corresponds to the impulse approximation, i.e., the independent scattering of separate clusters by the target. The second term corresponds to the simultaneous scattering of the clusters, also called by eclipse term. It tends to interfere destructively with the first term.

In describing the differential cross sections we shall keep the impulse approximation, which gives reasonable results for small scattering angles. But, in order to obtain the total diffraction dissociation cross sections we have to include the second term since it decreases more slowly with increasing Q, and becomes the dominant contribution to the scattering amplitude (2.6).

2.2. AMPLITUDE FOR COULOMB DISSOCIATION

The amplitude for Coulomb excitation of a high-energetic projectile was calculated in refs. [14,15]), for all multipolarities of the excitation. In ref. [14]) a semiclassical approach was used, while in ref. [15]) the eikonal approximation in first order (which is formally equivalent to the first Born approximation) was used. We use the results of ref. [15]) for the Coulomb excitation amplitude, multiplied by some normalization factors in order to have the same notation for f_C and f_N. We shall restrict ourselves to the electric dipole and to the electric quadrupole dissociation modes, which are the most important ones. We obtain [15])

$$f_C(\boldsymbol{Q}, \boldsymbol{q}) = i\frac{Z_T\alpha}{\gamma}\left(\frac{c}{v}\right)kR^2 \sum_{lm} i^m\sqrt{2l+1}\left(\frac{\omega}{c}\right)^l G_{Elm}\left(\frac{c}{v}\right)\Phi_m(Q)\mathcal{M}(Elm)\,, \qquad (2.8)$$

where the sum runs over $l = 1, 2$ and from $m = -l$ to $m = l$. In this expression Z_T is the nuclear charge of the target, v is the velocity of the projectile, $\gamma = (1 - v^2/c^2)^{-1/2}$ is the relativistic Lorentz factor, α is the fine-structure constant, and

$$\hbar\omega = \varepsilon + E_q = \frac{\hbar^2}{2\mu}(\eta^2 + q^2) \qquad (2.9)$$

is the sum of the absolute value of the binding energy and the kinetic energy of relative motion of the separated clusters.

The adimensional function Φ_m is given by

$$\Phi_m(Q) = \int_1^\infty J_m(QRx)K_m\left(\frac{\omega R}{\gamma v}x\right)x\,dx$$
$$= \frac{1}{(Q^2 + \omega^2/\gamma^2v^2)R}\left\{\frac{\omega}{\gamma v}J_m(QR)K_{m+1}\left(\frac{\omega R}{\gamma v}\right) - QJ_{m+1}(QR)K_m\left(\frac{\omega R}{\gamma v}\right)\right\}. \qquad (2.10)$$

The relativistic Winther-Alder functions $G_{\pi lm}$ are tabulated in the appendix B of ref. [14]), and for the E1 and E2 multipolarities are

$$G_{E11} = \tfrac{1}{3}\sqrt{8\pi}\left(\frac{c}{v}\right), \qquad G_{E10} = -i\tfrac{4}{3}\sqrt{\pi}\frac{c}{\gamma v},$$
$$G_{E22} = -\frac{2}{5}\sqrt{\frac{\pi}{6}}\frac{c^2}{\gamma v^2}, \qquad G_{E21} - i\frac{2}{5}\sqrt{\frac{\pi}{6}}\left(2\frac{c^2}{v^2} - 1\right),$$
$$G_{E20} = \tfrac{2}{5}\sqrt{\pi}\frac{c^2}{\gamma v^2}. \qquad (2.11)$$

For $m < 0$ one can use $G_{El-m} = (-1)^m G_{Elm}$.

The function $\mathcal{M}(Elm)$ is the matrix element for electromagnetic transitions, and for the dissociation of the cluster (1+2) it is given by

$$\mathcal{M}(Elm) = \sum_{k=1,2} Z_k e\int \varphi_f^*(\boldsymbol{r})r_k^l Y_{lm}(\hat{\boldsymbol{n}}_K)\varphi_i(\boldsymbol{r})\,d^3r\,, \qquad (2.12)$$

where $\boldsymbol{r}_1 = \beta_1 \boldsymbol{r}$, $\boldsymbol{r}_2 = -\beta_2 \boldsymbol{r}$ and $\hat{\boldsymbol{n}}_2 = -\hat{\boldsymbol{n}}_1$ are the position and direction of orientation of the clusters 1 and 2 in the center of mass system of the projectile, and Z_k are their respective charges. Inserting the wave functions (2.4) in (2.12), expanding it in multipoles, and using the integral

$$\int_0^\infty e^{-\eta r} r^{l+1} j_l(qr)\, dr = \frac{l!(2q)^l}{(\eta^2+q^2)^{l+1}}, \tag{2.13}$$

we obtain

$$\mathcal{M}(\mathrm{E}lm) = e\sqrt{2\pi\eta}(-i)^l l! 2^{l+1}[Z_1\beta_1^l + (-1)^l Z_2 \beta_2^l] \frac{q^l}{(\eta^2+q^2)^{l+1}} Y_{lm}(\hat{\boldsymbol{q}}) . \tag{2.14}$$

The Coulomb dissociation amplitude is obtained by inserting eqs. (2.9)-(2.11) and (2.14) on (2.8). We observe that for $\beta_1 Z_1 = \beta_2 Z_2$ there will be no electric dipole contribution to the Coulomb dissociation. This is a well-known result and can be readily understood: in this case the electric dipole field pushes the two clusters with the same acceleration in the same direction and does not lead to their separation. In such situations the E2 multipolarity will be the most effective one for dissociating the projectile. This result is a direct consequence of the assumption of a cluster-like structure for the nuclei. For more complicated nuclear wave functions a deviation from this result is to be expected. For example, in the reaction $\gamma + {}^{16}\mathrm{O} \to \alpha + {}^{12}\mathrm{C}$ one indeed finds experimentally an appreciable suppression of the E1 multipolarity, but not completely. In fact it is found that both multipolarities play important roles in such reactions (see, e.g., ref. [1])).

3. Differential cross sections

As an application of the formulas developed in the last section we show in fig. 1 the differential cross section $d^4\sigma/d^3q\, dQ$ for the dissociation of the simplest cluster-like nucleus, i.e. the deuteron, incident on ^{208}Pb with energy $E_d = 200$ MeV. We take $q = \eta$, $Q = 1/R$ and $\theta_q = 90°$, corresponding to the emission of the fragments perpendicular to the beam. ϕ_q is the angle between $\boldsymbol{Q}$ and the component of $\boldsymbol{q}$ perpendicular to the incident beam. We observe that the Coulomb contribution C, as calculated from eqs. (2.3), (2.8)-(2.11), and (2.14), is approximately proportional to $\cos^2 \phi_q$. The nuclear contribution N, as calculated from eqs. (2.3), (2.6) and (2.7), and the interference CN between them, are also shown. The interference tends to be destructive, oscillating around zero with approximately the same amplitude. This is a common trend, valid for all values of $\boldsymbol{q}$ and Q, as can be easily checked.

Next we integrate (2.3) over the angular distribution Ω_q of the relative motion between the fragments. We obtain the differential cross section $d^2\sigma/dq\, dQ$ which can be related to $d^2\sigma/dE_q\, d\Omega$, where E_q is the energy of relative motion of the final

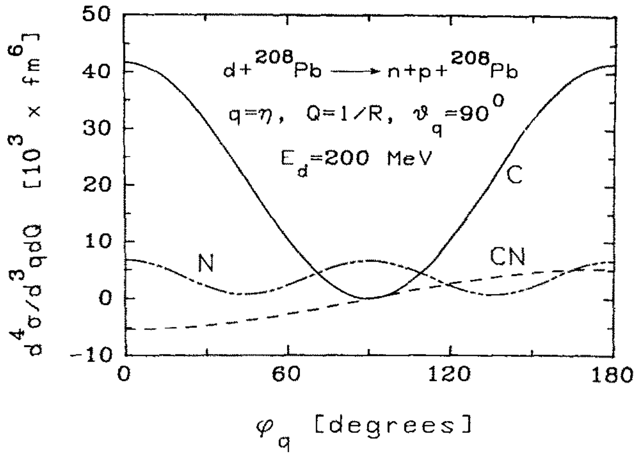

Fig. 1. The differential cross section $d^4\sigma/d^3q\,dQ$ for the dissociation reaction $d+{}^{208}Pb\to n+p+{}^{208}Pb$ for deuteron energies of $E_d=200$ MeV. We used $q=\eta$, $Q=1/R$ and $\theta_q=90°$. ϕ_q is the angle between $\boldsymbol{Q}$ and the component of $\boldsymbol{q}$ perpendicular to the beam.

fragments and Ω is the solid angle of scattering of their center-of-mass. By using the impulse approximation, and eq. (2.7), we find for the nuclear contribution, after performing some tedious, but simple integrals,

$$\frac{d^2\sigma_N}{dq\,dQ}=4q^2R^2\frac{\eta}{Q}J_1^2(QR)$$

$$\times\left\{\frac{2}{[\eta^2+(q+\beta_1Q)^2][\eta^2+(q-\beta_1Q)^2]}+\frac{2}{[\eta^2+(q+\beta_2Q)^2][\eta^2+(q-\beta_2Q)^2]}\right.$$

$$+\frac{M}{qQ(\eta^2+q^2+\beta_1\beta_2Q^2)}+\frac{M}{2qQ^2(q^2+\eta^2)}(\eta N-\tfrac{1}{2}qM)$$

$$\left.+\frac{1}{2Q^2(q^2+\eta^2)}(\tfrac{1}{4}M^2+N^2)\right\}, \tag{3.1a}$$

where

$$M=\frac{1}{\beta_1}\ln\left[\frac{\eta^2+(q+\beta_1Q)^2}{\eta^2+(q-\beta_1Q)^2}\right]+\frac{1}{\beta_2}\ln\left[\frac{\eta^2+(q+\beta_2Q)^2}{\eta^2+(q-\beta_2Q)^2}\right], \tag{3.1b}$$

$$N=\frac{1}{\beta_1}\arctan\left(\frac{2\beta_1\eta Q}{\beta_1^2Q^2-q^2-\eta^2}\right)+\frac{1}{\beta_2}\arctan\left(\frac{2\beta_2\eta Q}{\beta_2^2Q^2-q^2-\eta^2}\right). \tag{3.1c}$$

The Coulomb contribution is easily obtained from the orthonormality of the spherical harmonics and one finds, after inserting (2.8)-(2.11), and (2.14) in (2.3), and summing over m,

$$\frac{d^2\sigma_C}{dq\,dQ}=\frac{d^2\sigma_{E1}}{dq\,dQ}+\frac{d^2\sigma_{E2}}{dq\,dQ}, \tag{3.2a}$$

where

$$\frac{d^2\sigma_{E1}}{dq\,dQ}=128\frac{Z_T^2\alpha^2}{\gamma^2}\left(\frac{c}{v}\right)^2(\beta_1 Z_1-\beta_2 Z_2)^2\eta QR^4\left(\Phi_1^2+\frac{\Phi_0^2}{\gamma^2}\right)\frac{(q^2\omega/c)^2}{(\eta^2+q^2)^4}, \tag{3.2b}$$

$$\frac{d^2\sigma_{E2}}{dq\,dQ}=\frac{512}{15}\,\frac{Z_T^2\alpha^2}{\gamma^2}\left(\frac{c}{v}\right)^6(\beta_1^2 Z_2+\beta_2^2 Z_2)^2\eta QR^4$$

$$\times\left[\frac{\Phi_2^2}{\gamma^2}+\left(2-\frac{v^2}{c^2}\right)^2\Phi_1^2+\frac{3\Phi_0^2}{\gamma^2}\right]\frac{q^2(q\omega/c)^4}{(\eta^2+q^2)^6}. \tag{3.2c}$$

The E1-E2 interference is lost after the integration over ϕ_q. However, in coincidence experiments where $d^4\sigma/d^3q\,dQ$ is measured, the E1-E2 interference is important.

Finally, the interference term between Coulomb and nuclear amplitudes can be found by computing numerically the angular integral in the expression

$$\frac{d^2\sigma_{CN}}{dq\,dQ}=\frac{q^2Q}{(2\pi)^2k^2}\int(f_C^* f_N+f_N^* f_C)\,d\Omega_q. \tag{3.3}$$

In fig. 2 we plot $d^2\sigma/dq\,dQ$ for the reaction $d+{}^{208}Pb\to n+p+{}^{208}Pb$, at deuteron energy $E_d=200$ MeV, for $q=\eta$, and as a function of QR. The Coulomb, C, the

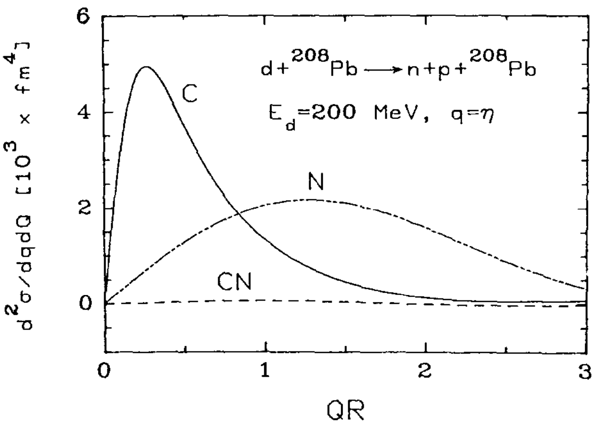

Fig. 2. The double differential cross section $d^2\sigma/dq\,dQ$ for the dissociation reaction $d+{}^{208}Pb\to n+p+{}^{208}Pb$, at deuteron energies $E_d=200$ MeV, for $q=\eta$, and as a function of QR. The curves labelled by C, N and CN correspond to the Coulomb, nuclear and Coulomb-nuclear interference contributions, respectively.

nuclear N, and the interference, CN, contributions are shown separately. One observes that the Coulomb contribution is peaked for low values of Q. Actually, it peaks around $Q_C^{max} \cong \omega/\gamma v$, so that for increasing beam energies the peak moves to lower values of Q, i.e. to more forward angles, and will also increase in height. This is in contrast with the nuclear contribution which, within our approach will always extend to large values of Q, being peaked around $Q_N^{max} \cong 1/R$. This behaviour may help to separate the nuclear and Coulomb dissociation from the measurement of scattering angle of the center-of-mass of the two-cluster system in intermediate energy collisions. Unfortunately, with increasing energy both nuclear and Coulomb dissociation will lead to very forward angular distributions, with $\theta^{max} \cong 1/kR \ll 1$, which makes the experimental measurements very difficult to proceed. For $Z_P Z_T \alpha \gtrsim 1$, the effects of Coulomb repulsion between the projectile and target will considerably change the Q-dependence of the Coulomb dissociation amplitude. A study of these effects based on semiclassical calculations has been performed in ref. [2]). In the present context it implies in the use of Coulomb distorted waves, instead of plane waves, in the calculation leading to the amplitude (2.8). Nevertheless, the relative behaviour between the Coulomb and nuclear angular distribution remains qualitatively the same.

In fig. 3 we plot $d^2\sigma/dq\, dQ$ for the same reaction, as a function of q/η, and for $Q = 1/R$. As a general trend, for fixed Q the Coulomb dissociation is more pronounced for $q \cong \eta$, decreasing very fast for large values of q, while the nuclear dissociation peaks for $q \cong Q$ and decreases slowly with increasing values of q. In both figs. 2 and 3 we see that the Coulomb + nuclear interference is very small, being some orders of magnitude smaller than the nuclear or the Coulomb contribution.

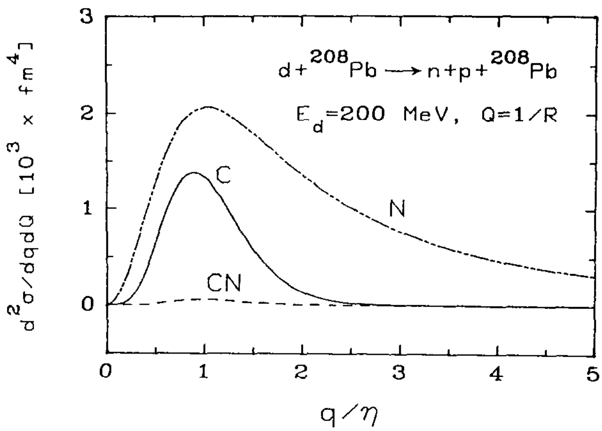

Fig. 3. The double differential cross section $d^2\sigma/dq\, dQ$ for the dissociation reaction $d + {}^{208}Pb \to n + p + {}^{208}Pb$ at deuteron energies $E_d = 200$ MeV, for $Q = 1/R$ and as a function of q/η. The curves labelled by C, N and CN correspond to Coulomb, nuclear and Coulomb-nuclear interference, respectively.

4. Total cross sections

Inserting (2.6) in (2.3a) and using the orthonormality condition (2.4c) the integration over $\boldsymbol{q}$ can be easily performed in the impulse approximation. One gets

$$\frac{d\sigma_N}{dQ}=\frac{2\pi R^2}{Q}J_1^2(QR)\left\{\int d^3r|\varphi_i(\boldsymbol{r})|^2|e^{i\beta_1\boldsymbol{Q}\cdot\boldsymbol{r}}+e^{-i\beta_2\boldsymbol{Q}\cdot\boldsymbol{r}}|^2 - \left|\int d^3r|\varphi_i(\boldsymbol{r})|^2(e^{i\beta_1\boldsymbol{Q}\cdot\boldsymbol{r}}+e^{-\beta_2\boldsymbol{Q}\cdot\boldsymbol{r}})\right|^2\right\}$$
$$=\frac{4\pi R^2}{Q}J_1^2(QR)\left\{1+\frac{2\eta}{Q}\arctan\left(\frac{Q}{2\eta}\right) - \frac{2\eta^2}{Q^2}\left[\frac{1}{\beta_1}\arctan\left(\frac{\beta_1 Q}{2\eta}\right)+\frac{1}{\beta_2}\arctan\left(\frac{\beta_2 Q}{2\eta}\right)\right]\right\}. \tag{4.1}$$

Using (4.1) we find that for $\eta\to\infty$, corresponding to infinite binding energy of the clusters, $d\sigma_N/dQ\to 0$. For $\eta\to 0$, corresponding to very loosely bound nuclei,

$$\frac{d\sigma_N}{dQ}\to\frac{4\pi R^2}{Q}J_1^2(QR),$$

which means that in this case the total nuclear dissociation cross section will be just the sum of the elastic diffraction cross section for each cluster separately. Both limits is what one expects from the simple arguments of the diffraction dissociation theory.

But for larger values of Q the impulse approximation is not more reasonable; the second term of eq. (2.6) will increase in importance for $Q\gg\eta$. Therefore, to obtain the contribution of diffraction dissociation to the total dissociation cross section, one has to integrate (2.3) numerically by using (2.6) and (2.7).

By using the integral

$$R^2\int_0^\infty Q\Phi_m^2(Q)\,dQ=\int_1^\infty xK_m^2\left(\frac{\omega R}{\gamma v}x\right)dx = \tfrac{1}{2}[K_{m+1}(\xi)K_{m-1}(\xi)-K_m^2(\xi)], \tag{4.2}$$

where $\xi=\omega R/\gamma v$, we find for the Coulomb dissociation

$$\frac{d\sigma_C}{dq}=\frac{d\sigma_{E1}}{dq}+\frac{d\sigma_{E2}}{dq} \tag{4.3a}$$

with

$$\frac{d\sigma_{E1}}{dq}=128Z_T^2\alpha^2\left(\frac{c}{v}\right)^2(\beta_1Z_1-\beta_2Z_2)^2 \times\frac{\eta q^4}{(\eta^2+q^2)^4}\left[\xi K_0K_1-\frac{v^2\xi^2}{2c^2}(K_1^2-K_0^2)\right], \tag{4.3b}$$

$$\frac{d\sigma_{E2}}{dq}=\frac{512}{15}Z_T^2\alpha^2\left(\frac{c}{v}\right)^4(\beta_1^2Z_1+\beta_2^2Z_2)^2$$
$$\times\frac{\eta q^6(\omega/c)^2}{(\eta^2+q^2)^6}\left[\frac{2}{\gamma^2}K_1^2+\left(2-\frac{v^2}{c^2}\right)^2\xi K_0K_1-\frac{\xi^2v^4}{2c^4}(K_1^2-K_0^2)\right]. \quad (4.3c)$$

The total Coulomb dissociation cross section $\sigma_C=\sigma_{E1}+\sigma_{E2}$ can be obtained by a numerical integration of eq. (4.3).

In fig. 4 we show the Coulomb and nuclear dissociation cross sections for the reaction $^7Be+^{208}Pb\to\alpha+^3He+^{208}Pb$ as a function of the laboratory energy per nucleon of the ^7Be projectile. In the calculation of the Coulomb dissociation cross sections we use $R+\frac{1}{2}\pi Z_1Z_2e^2/m_{Be}v^2\gamma$, instead of R, as the minimum impact parameter (see ref. [14])). This is a correction due to the Rutherford bending of the projectile trajectory, which is important for E_{lab}/nucleon$\leq$100 MeV. A more detailed discussion of the effects of the Rutherford bending of the trajectories on the excitation process has been given in ref. [2]). We observe that the E1 contribution is by far larger than the E2, and also than the nuclear dissociation. In such a case the study of the experimental data is very simplified, since one can disregard the nuclear dissociation and assume all being due to the Coulomb dissociation, which is more accurately described.

In fig. 5 we plot the values for the dissociation cross section in the reaction $^6Li+^{208}Pb\to\alpha+d+^{208}Pb$. In this case, and within the simple cluster model, the E1 component of Coulomb dissociation vanishes and only the next component, E2, will be effective in order to dissociate the nucleus. This makes the Coulomb cross section smaller than the nuclear one and the separation between these two contributions have to be measured on the basis of the angular distributions, as discussed in the last sections.

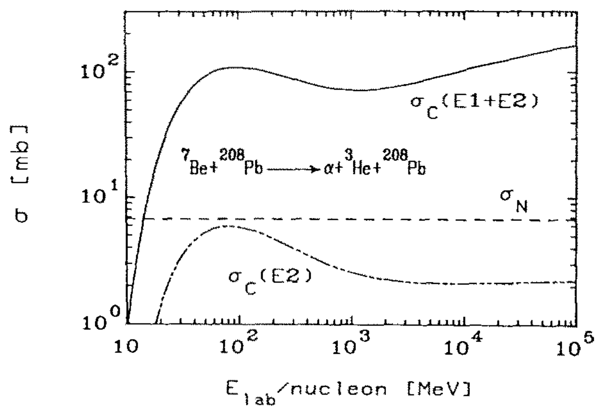

Fig. 4. Dissociation cross sections for the reaction $^7Be+^{208}Pb\to\alpha+^3He+^{208}Pb$ as a function of the laboratory energy per nucleon of the ^7Be projectile. σ_N represents the nuclear diffraction dissociation, σ_C(E1) the contribution of the electric dipole multipolarity to the Coulomb dissociation, and σ_C(E1+E2) is the sum of the contributions of the electric and quadrupole multipolarities.

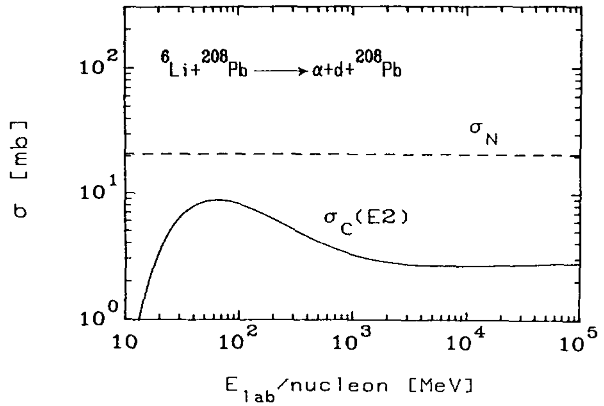

Fig. 5. Same as fig. 4, but for the dissociation reaction $^6Li + ^{208}Pb \to \alpha + d + ^{208}Pb$.

One observes in figs. 4 and 5 that the Coulomb cross sections increase with energy up to a maximum around approximately 100 MeV per nucleon, afterwards it decreases a little and then begins to increase with energy again, approximately proportional to $\ln(E_{lab}/A)$, for very high energies. This behaviour is also found in the Coulomb cross sections for excitation of giant resonances in relativistic heavy ion collisions [18]). It is basically of the same origin. Corrections due to the Rutherford bending of the trajectory, or equivalently the Coulomb distortions on the scattered wave functions, and corrections due to Lorentz contraction, compete in the region of some hundreds of MeV per nucleon. This can, indeed, be easily understood. With increasing energy, the nuclei come closer to each other where the fields are stronger, which increases the probability that they will get Coulomb excited. That is the reason why the cross sections increase with energy for $E_{lab} \lesssim 100$ MeV per nucleon. Above these energies the trajectories are approximately straight lines, and since the collision time decreases with energy the momentum transferred from the electromagnetic field to internal degrees of the nuclei also diminishes. That is the reason for the decreasing of the cross sections for $E_{lab} \gtrsim 100$ MeV per nucleon. But this effect will not continue for too high energies because the electromagnetic field becomes contracted and stronger by a factor equal to the Lorentz parameter γ: $E = \gamma Ze^2/b$. Since the momentum transfer is proportional to the product of the strength of the electromagnetic field and the collision time, which is approximately $\Delta t \cong b/\gamma c$, it will be constant, independent of the beam energy. This simple argument works for impact parameters up to a maximum value given by the adiabatic cutoff $b = \gamma c/\omega$, where $\hbar\omega$ is the excitation energy. That is the reason for the increasing of the cross sections for relativistic energies. A precise analysis (see ref. [18]) shows that $\sigma_C \propto \ln\gamma$ for $\gamma \gg 1$.

One interesting application of the fragmentation of cluster-like nuclei is the possibility of deducing information on the neutron skin of neutron-rich nuclei. For example, the reaction $^{11}Li + x \to {}^9Li$ could give information about the possible stability of the di-neutron system in the presence of a nuclear core [19]). It has been suggested (see also ref. [20])) that the force between two neutrons, itself too weak to form a bound system, under the influence of another nucleus, can lead to a bound state of two particles: a di-neutron system and a nuclear core. The binding energy of the two neutrons in ^{9}Li is about 190 ± 110 keV. Assuming for ^{11}Li the above mentioned cluster-like structure, we find the value $\sigma \cong 2.4$ and 12 b for the Coulomb dissociation cross section for $E_q = 80$ or 300 keV, respectively, in the reaction $^{11}\text{Li} + {}^{208}\text{Pb} \rightarrow 2\text{n} + {}^{9}\text{Li} + {}^{208}\text{Pb}$ at ^{11}Li energies of 0.8 GeV/nucleon. For the diffraction dissociation one finds $\sigma_{\text{N}} \cong 210$–$662$ mb. Recently, the inclusive reaction $^{11}\text{Li} + \text{Pb} \rightarrow {}^{9}\text{Li}$ at this energy has been performed at the LBL Bevalac by Tanihata *et al.* [4–6]). They found the total cross section of about 9.5 b. One important contribution to this cross section is the stripping of the neutrons from the ^{11}Li nucleus. It is about the same order of magnitude as the Coulomb dissociation and depends much more on the assumptions about the neutron excess on the surface of that nucleus. Therefore, the knowledge of the Coulomb dissociation cross section and of the experimental values for inclusive reactions are of great importance for the study of the tail in the nuclear matter distribution. By using several targets and beam energies, one can separate the Coulomb and stripping contributions (diffraction dissociation is of little importance in this case) in these reactions due to their different dependence on the nuclear parameters.

5. Conclusions

Precise coincidence experiments for the dissociation reaction of weakly-bound nuclei at high bombarding energies are only at a beginning. As we discussed above, such exclusive experiments would give valuable information on photodisintegration reactions or, indirectly, of radiative capture reactions of astrophysical interest (see also refs. [1–3])), and also about the distribution of the nuclear density in the nuclear surface.

At high energies both the electromagnetic and the nuclear interaction between projectile and target will be important. Far from being a drawback, this can be of utility to extract complementary information about these different reaction mechanisms in the peripheral collisions. A decomposition of these mechanisms from the analysis of angular distribution of the fragments or from the dependence of the cross sections on the energy, charge and mass parameters, is possible in accurate measurements. In the case of electromagnetic dissociation this decomposition can tell us about the relevance of each multipolarity in the dissociation reaction.

We have made very simple assumptions regarding the structure of the nuclei and pointed out the main theoretical considerations for more detailed calculations. Our results should give a characteristic behavior of the reaction cross sections for loosely-bound nuclei. More specific structure effects, such as e.g. resonances, are expected to appear on a background parameterized by the above equations. The availability of experimental data in the near future will certainly arouse interest on the detailed investigation of such effect.

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