The article published below draws attention to the work of the Kerala school of astronomers, particularly Nilakanta (1500 AD) in modelling planetary motion. In the exchanges between authors and referee, it became clear that this school did not stop with copying their predecessors but attempted to wrestle with the problems of the old (geocentric) system. Whether their work constituted a clean break towards a true heliocentric system, as proposed by Srinivas and colleagues, appears to hinge upon some subtle points of interpretation of the original texts. For example, did the Kerala astronomers maintain the distinction between the mean and the centre of the epicycle of an interior planet, even though both move together in the sky? They could be at different distances, as a referee suggests. In any case, one cannot but note the vitality of this tradition of mathematics and astronomy which even studied infinite series some years later, while the rest of the country was going through an academic dark age.

— Editor

Modification of the earlier Indian planetary theory by the Kerala astronomers (c. 1500 AD) and the implied heliocentric picture of planetary motion

K. Ramasubramanian, M. D. Srinivas and M. S. Sriram

We report on a significant contribution made by the Kerala School of Indian astronomers to planetary theory in the fifteenth century. Nilakantha Somasutvan, the renowned astronomer of the Kerala School, carried out a major revision of the older Indian planetary model for the interior planets, Mercury and Venus, in his treatise Tantrasangraha (1500 AD), and for the first time in the history of astronomy, he arrived at an accurate formulation of the equation of centre for these planets. He also described the implied geometrical picture of planetary motion, where the five planets – Mercury, Venus, Mars, Jupiter and Saturn – move in eccentric orbits around the Sun, which in turn goes around the Earth. The later astronomers of the Kerala School seem to have by and large adopted the planetary model developed by Nilakantha.

It is now widely recognized that the Kerala school of Indian astronomy[1], starting with Madhava of Sangamagrama in the fourteenth century, made important contributions to mathematical analysis much before this subject developed in Europe. The Kerala astronomers obtained the infinite series for π, sine and cosine functions and also developed fast convergent approximations to them[2]. Here we report that the Kerala school also made equally significant discoveries in astronomy, in particular, planetary theory.

We show that Nilakantha Somasutvan of Trkkantiyur (1444–1550 AD) carried out, in his treatise *Tantrasangraha* (1500 AD), a major revision of the earlier Indian planetary model for the interior planets Mercury and Venus. This led Nilakantha to a much better formulation of the equation of centre for these planets than was available either in the earlier Indian works or in the Islamic or European traditions of astronomy till the work of Kepler, which was to come more than a hundred years later.

We also note that Nilakantha in his later works, *Golasara, Siddhanta-darpana* and more importantly the celebrated *Aryabhatiyabhashya*, explains that the computational scheme developed by him implies a geometrical picture of planetary motion, where the five planets Mercury, Venus, Mars, Jupiter and Saturn move in eccentric orbits around the mean Sun, which in turn goes around the Earth. Most of the Kerala astronomers who succeeded Nilakantha, such as Jyesthadeva, Acyuta Pisarati, Putumana Somayaji, etc. seem to have adopted this planetary model.

The conventional planetary model of Indian astronomy

In the Indian astronomical tradition, at least from the time of Aryabhata (499 AD), the procedure for calculating the geocentric longitudes of the five planets, Mercury, Venus, Mars, Jupiter and Saturn involves essentially the following steps[3]. First, the mean longitude (called the *madhyamagraha*) is calculated for the desired day by computing the number of mean civil days elapsed since the epoch (this number is called *ahargana*) and multiplying it by the mean daily motion of the planet. Then two corrections namely *manda samskara* and *sighra samskara* are applied to the mean planet to obtain the true longitude.

The *manda samskara* is equivalent to taking into account the eccentricity

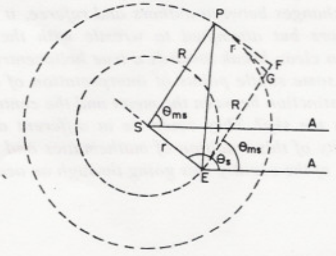

Figure 1. *Sighra samskara* for an exterior planet.

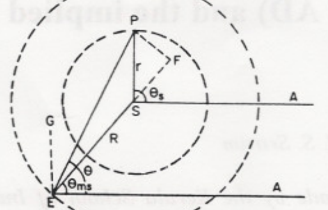

Figure 2. *Sighra samskara* for an interior planet.

of the planet's orbit. Different computational schemes for the *manda samskara* are discussed in Indian astronomical literature. However, the *manda* correction in all these schemes coincides, to first order in eccentricity, with the equation of centre currently calculated in astronomy. The *manda*-corrected mean longitude is called *mandasphutagraha*. As we explain below, for exterior planets, the *mandasphutagraha* is the same as the true heliocentric longitude.

The *sighra samskara* is applied to this *mandasphutagraha* to obtain the true longitude known as *sphutagraha*. The *sighra* correction, as we explain below, is equivalent to converting the heliocentric longitude into the geocentric longitude. The exterior and interior planets are treated differently in applying this correction, and we take them up one after the other.

Exterior planets

For the exterior planets Mars, Jupiter and Saturn, the mean heliocentric sidereal period is identical with the mean geocentric sidereal period. Thus, the mean longitude calculated prior to the *manda samskara* is the same as the mean heliocentric longitude of the planet as we understand today. As the *manda samskara* is applied to this longitude to obtain the *mandasphutagraha*, the latter will be the true heliocentric longitude of the planet.

The *sighra samskara* for the exterior planets can be explained with reference to Figure 1. Longitudes are always measured in Indian astronomy with respect to a fixed point in the Zodiac known as the *Nirayana Meshadi* denoted by A in the figure. E is the Earth and G is the *mandasphutagraha* at a distance R. S is the mean Sun referred to as the *sighrocca* for an exterior planet. Draw $GP = r$ parallel to ES. Then P corresponds to the true planet. We have,

$\angle AEG = \theta_{mS} =$ *Mandasphuta*

$\angle AES = \theta_S =$ Longitude of *sighrocca* (mean Sun)

$\angle AEP = \theta =$ True geocentric longitude of the planet

$\angle GEP = \theta - \theta_{mS} =$ *Sighra* correction.

The difference between the longitudes of the *sighrocca* and the *mandasphuta*, namely,

$$\sigma = \theta_S - \theta_{mS} \tag{1}$$

is called the *sighrakendra* (anomaly of conjunction) in Indian astronomy. Draw PF perpendicular to the extension of the line EG. From the triangle EPF we can easily obtain the result

$$\sin(\theta - \theta_{mS}) = \frac{r\sin\sigma}{[(R + r\cos\sigma)^2 + r^2\sin^2\sigma]^{1/2}} \tag{2}$$

which is the *sighra* correction formula given by Indian astronomers to calculate the geocentric longitude of an exterior planet.

From the figure it is clear that the *sighra samskara* transforms the true heliocentric longitudes into true geocentric longitudes: for, $\angle ASP = \angle AEG$ is the true heliocentric longitude and one has to add $\angle GEP$ to it to get the true geocentric longitude. This is true only if r/R is equal to the ratio of the Earth–Sun and Planet–Sun distances and is indeed very nearly so in the Indian texts. But equation (2) is still an approximation as it is based upon the identification of the mean Sun with the true Sun.

Interior planets

For the interior planets Mercury and Venus, ancient Indian astronomers, at least from the time of Aryabhata, took the mean Sun as the *madhyamagraha* or the mean planet. For these planets, the mean heliocentric period is the period of revolution of the planet around the Sun, while the mean geocentric period is the same as that of the Sun. The ancient astronomers prescribed application of the *manda* correction or the equation of centre characteristic of the planet, to the mean Sun, instead of the mean heliocentric planet as is done in the currently accepted model of the solar system. However, the ancient Indian astronomers introduced a *sighrocca* for these planets whose period is the same as the mean heliocentric period of these planets. Thus the longitude of this *sighrocca* will be the same as the mean heliocentric longitude of the interior planet.

Table 1. Comparison of r/R (variable) in *Aryabhatiya* with modern values (ratio of the mean values of Earth–Sun and Planet–Sun distances for exterior planets and the inverse ratio for interior planets)

Planet	*Aryabhatiya*	Modern value
Mercury	0.361 to 0.387	0.387
Venus	0.712 to 0.737	0.723
Mars	0.637 to 0.662	0.656
Jupiter	0.187 to 0.200	0.192
Saturn	0.114 to 0.162	0.105

The *sighra samskara* for the interior planets can be explained with reference to Figure 2. Here E is the Earth and S is the *mandasphutagraha*. Draw $SP = r$ parallel to EG. Then P corresponds to the true planet. We have,

$\angle AES = \theta_{mS}$ = *Mandasphuta*

$\angle AEG = \theta_S$ = Longitude of *sighrocca*

$\angle AEP = \theta$ = True geocentric longitude of the planet

$\angle SEP = \theta - \theta_{mS}$ = *Sighra* correction.

Again, the *sighrakendra* σ is defined as the difference between the *sighrocca* and the *mandasphutagraha*. Thus,

$$\sigma = \theta_S - \theta_{mS}. \tag{3}$$

Let PF be perpendicular to the line ES. From the triangle EPF we get the same formula

$$\sin(\theta - \theta_{mS}) = \frac{r\sin\sigma}{[(R + r\cos\sigma)^2 + r^2\sin^2\sigma]^{1/2}}. \tag{4}$$

which is the *sighra* correction given in the earlier Indian texts to calculate the geocentric longitude of an interior planet. Both for Mercury and Venus, the value specified for r/R is very nearly equal to the ratio of the Planet–Sun and Earth–Sun distances. In Table 1, we give Aryabhata's values for both the exterior and interior planets along with the modern values based on the mean Earth–Sun and Sun–Planet distances.

Since the *manda* correction or equation of centre for an interior planet was applied to the longitude of the mean Sun instead of the mean heliocentric longitude of the planet, the accuracy of the computed longitudes of the interior planets according to the older Indian planetary models would not have been as good as that achieved for the exterior planets.

Computation of the planetary latitudes

Planetary latitudes (called *vikshepa* in Indian astronomy) play an important role in the prediction of planetary conjunctions, occultation of stars by planets, etc. In Figure 3, P denotes the planet moving in an orbit inclined at angle i to the ecliptic, intersecting the ecliptic at the point N, the node (called *pata* in Indian astronomy). If β is the latitude of the planet, θ_H its heliocentric longitude, and θ_0, the heliocentric longitude of the node, then for small i we have

$$\sin\beta = \sin i \, \sin(\theta_H - \theta_0) \simeq i \sin(\theta_H - \theta_0). \tag{5}$$

This is also essentially the rule for calculating the latitude, as given in Indian texts, at least from the time of Aryabhata. For the exterior planets, it was stipulated that

$$\theta_H = \theta_{mS}. \tag{6}$$

the *mandasphutagraha*, which as we saw earlier, coincides with the heliocentric longitude of the exterior planet. The same rule applied for interior planets would not have worked, because according to the earlier Indian planetary model, the *manda*-corrected mean longitude for the interior planet has nothing to do with its true heliocentric longitude.

However, all the older Indian texts on astronomy stipulated that for interior planets, the latitude is to be calculated from equation (5) with

$$\theta_H = \theta_S + manda \text{ correction}, \tag{7}$$

the *manda*-corrected longitude of the *sighrocca*. Since the longitude of the *sighrocca* for an interior planet, as we

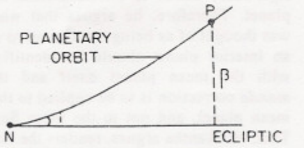

Figure 3. Latitude of a planet.

explained above, is equal to the mean heliocentric longitude of the planet, equation (7) leads to the correct identification, that even for an interior planet, θ_H in equation (5) has to be the true heliocentric longitude.

Thus, we see that the earlier Indian astronomical texts did provide a fairly accurate theory for the planetary latitudes. But they had to live with two entirely different rules for calculating latitudes, one for the exterior planets (equation (6)), where the *mandasphutagraha* appeared and an entirely different one for the interior planets (equation (7)), which involved the *sighrocca* of the planet, with the *manda* correction included.

This peculiarity of the rule for calculating the latitude of an interior planet was repeatedly noticed by various Indian astronomers, at least from the time of Bhaskaracharya I (629 AD), who in his *Aryabhatiyabhashya* drew attention to the fact that the procedure for calculating the latitude of an interior planet is indeed very different from that adopted for the exterior planets[4]. Bhaskaracharya II in his own commentary *Vasanabhashya* on *Siddhantasiromani* (1150 AD) quotes the statement of Chaturveda Prithudakaswamin (860 AD) that this peculiar procedure for the interior planets can be justified only on the ground that this is what has been found to lead to *drigganitaikya*, or predictions which are in conformity with observations[5].

Planetary model of Nilakantha Somasutvan (c 1500 AD)

Nilakantha Somasutvan (1444–1550), the renowned Kerala astronomer, appears to have been led to his important reformulation of the older Indian planetary model, mainly by the fact that there obtained two entirely different rules for the calculation of planetary latitudes. As he explains in his *Aryabhatiyabhashya*[6], the latitude arises from the deflection of the planet (from the ecliptic) and not from that of a *sighrocca*, which is different from the planet. Therefore, he argues that what was thought of as being the *sighrocca* of an interior planet should be identified with the mean planet itself and the *manda* correction is to be applied to this mean planet, and not to the mean Sun. This, Nilakantha argues, renders the rule for calculation of latitudes the same for all planets, exterior or interior.

Nilakantha has presented his improved planetary model for the interior planets in an earlier treatise *Tantrasangraha* which, according to Nilakantha's pupil Sankara Variar, was composed in 1500 AD[7]. We shall describe here, the main features of Nilakantha's model in so far as they differ from the earlier Indian planetary model for the interior planets.

In the first chapter of *Tantrasangraha*, while presenting the mean sidereal periods of planets, Nilakantha gives the usual values of 87.966 days and 224.702 days (which are traditionally ascribed to the *sighroccas* of Mercury and Venus), but asserts that these are '*svaparyayas*', i.e. the mean revolution periods of the planets themselves[8]. As these are the mean heliocentric periods of these planets, the *madhyamagraha* as calculated in Nilakantha's model will be equal to the mean heliocentric longitude of the planet, for the case of interior planets also.

In the second chapter of *Tantrasangraha*, Nilakantha discusses the *manda* correction or the equation of centre and states[9] that this should be applied to the *madhyamagraha* as described above to obtain the *mandasphutagraha*. Thus, in Nilakantha's model, the *mandasphutagraha* will be equal to the true heliocentric longitude for both the interior and exterior planets.

Subsequently, the *sphutagraha* or the geocentric longitude is to be obtained by applying the *sighra* correction. While Nilakantha's formulation of the *sighra* correction is the same as in the earlier planetary theory for the exterior planets, his formulation of the *sighra* correction for the interior planets is different and is explained below.

According to Nilakantha the mean Sun should be taken as the *sighrocca* for interior planets also, just as in the case of exterior planets. In Figure 4, P is the *manda*-corrected planet, E is the Earth and S the *sighrocca* or the mean Sun. We have,

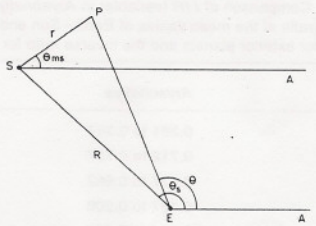

Figure 4. True longitude of an interior planet according to Nilakantha.

$\angle AES = \theta_S = Sighrocca$ (mean Sun),

$\angle ASP = \theta_{mS} = Mandasphuta$,

$\angle AEP = \theta =$ True geocentric longitude of the planet,

$\angle SEP = \theta - \theta_S = Sighra$ correction.

The *sighrakendra* is defined in the usual way by

$$\sigma = \theta_S - \theta_{mS} \quad (8)$$

as the difference between the *sighrocca* and the *mandasphutagraha*. Then from triangle ESP, we get the relation:

$$\sin(\theta - \theta_S) = -\frac{r \sin\sigma}{[(R + r\cos\sigma)^2 + r^2 \sin^2\sigma]^{1/2}} \quad (9)$$

which is the *sighra* correction given by Nilakantha for calculating the geocentric longitude θ of the planet. Comparing equations (8) and (9) with equations (3) and (4), and Figure 4 with Figure 2, we notice that they are the same except for the interchange of the *sighrocca* and the *mandasphutagraha*. The *manda* correction or the equation of centre is now associated with P whereas it was associated with S earlier.

In the seventh chapter of *Tantrasangraha*, Nilakantha gives formula (5) for calculating the latitudes of planets[10], and prescribes that for all planets, both exterior and interior, θ_H in equation (5) should be the *mandasphutagraha*. This is as it should be, for in Nilakantha's model even for an interior planet, the *mandasphutagraha* (the *manda*-corrected mean longitude) coincides with the true heliocentric longitude, just as in the case of the exterior planets. Thus Nilakantha, by his modification of traditional Indian planetary theory, solved the long-standing problem in Indian astronomy, of there being two different rules for calculating the planetary latitudes.

Nilakantha, by 1500 AD, had thus arrived at a consistent formulation of the equation of centre and a reasonable planetary model which is applicable also to the interior planets, perhaps for the first time in the history of astronomy. Just as was the case with the earlier Indian planetary model, the ancient Greek planetary model of Ptolemy and the planetary models developed in the Islamic tradition during the 8th–15th centuries postulated that the equation of centre for an interior planet should be applied to the mean Sun rather than to the mean heliocentric longitude of the planet, as we understand today[11]. In fact, Ptolemy seems to have compounded the confusion by clubbing together Venus along with the exterior planets and singling out Mercury as following a slightly deviant geometrical model of motion[12].

Even the celebrated Copernican revolution brought about no improvement in the planetary theory for the interior planets. As is widely known now[11], the Copernican model was only a reformulation of the Ptolemaic model (with some modifications borrowed from the Maragha School of Astronomy of Nasir ad-Din at-Tusi (1201–74 AD), Ibn ash-Shatir (1304–75) and others) for a heliocentric frame of reference, without altering his computational scheme in any substantial way for the interior planets. The same holds true for the geocentric reformulation of the Copernican system due to Tycho Brahe. Indeed, it appears that the correct rule for applying the equation of centre for an interior planet, to the mean heliocentric planet (as opposed to the mean Sun) was first enunciated in European astronomical tradition only by Kepler in the early 17th century.

Geometrical model of planetary motion

It is well known that the Indian astronomers were mainly interested in the successful computations of the longitudes and latitudes of the Sun, Moon and the planets, and were not much worried about proposing models of the universe. Detailed observations and the following sophistication of their computations of course suggested some geometrical models, and once in a while the Indian astronomers did discuss the geometrical model implied by their computations.

The renowned Kerala astronomer Paramesvara of Vatasseri (1380–1460) has discussed in detail the geometrical model implied in the earlier Indian planetary theory. In the Kerala tradition, Paramesvara has also a great reputation as an observational astronomer. Damodara the son and disciple of Paramesvara was the teacher of Nilakantha. Nilakantha often refers to Paramesvara as *Paramaguru*.

In his commentary on *Aryabhatiya*, Paramesvara briefly discusses in 12 verses[13], the geometrical model of motion as implied by the conventional planetary model of Indian astronomy. In his super-commentary *Siddhantadipika* (on Govindasvamin's commentary on *Mahabhaskariya* of Bhaskaracharya-I (629 AD), Paramesvara gives a more detailed exposition of the geometrical model of planetary motion. He notices that for an interior planet, the final longitude that is calculated ($\angle AEP$ in Figure 2) is the geocentric longitude of what is called the *sighrocca* of the planet (in the conventional planetary model). Paramesvara therefore suggests at the end, that what has been called as the *sighrocca* of an interior planet in conventional planetary model should be identified as the planet itself and the mean Sun should be taken as the *sighrocca* for all the planets, while computing the *sighra* correction. Thus many of the basic ideas which were used

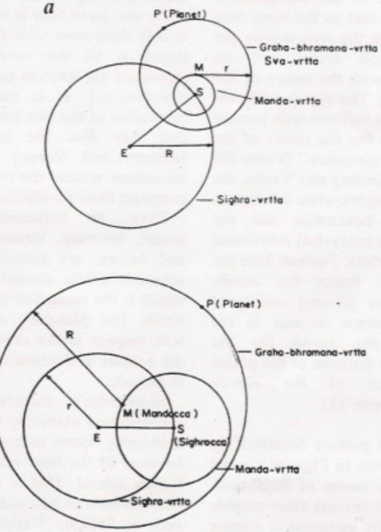

Figure 5. ***a***, Geometrical model of planetary motions according to *Siddhantadarpana* of Nilakantha, illustrated for interior planets. ***b***, Geometrical model of planetary motions according to *Siddhantadarpana* of Nilakantha, illustrated for exterior planets.

by Nilakantha in formulating his new model were already present in the work of Paramesvara.

Nilakantha describes the geometrical picture associated with his model of planetary motion in his works *Golasara*, *Siddhantadarpana* (with his own commentary), and in much greater detail in his *Aryabhatiyabhashya*. There is also a tract of his, on planetary latitudes, *Grahasphutanayane Vikshepavasana*[15], which deals with this topic.

In his *Aryabhatiyabhashya*, Nilakantha explains that the orbits of the planets, i.e. the geometrical model of planetary motion is to be inferred from the computational scheme for calculating the *sphutagraha* (geocentric longitude) and *vikshepa* (latitude of the planets)[16]. The geometrical model valid for both exterior and interior planets as presented by him in verses 19–21 of Chapter I of *Siddhantadarpana*[17] is as follows:

ग्रहभ्रमणवृत्तानि गच्छन्त्युच्चगतीन्यपि ।
मन्दवृत्ते तदर्केन्द्वोर्घनभूमध्यनाभिकम् ॥ १९ ॥
भस्मार्किगुरि चान्त्यो मां तन्मध्यं शीघ्रवृत्तगम् ।
तेषां शीघ्रं भ्रमत्यर्के विक्षिप्तं गोलमध्यगम् ॥ २० ॥
शीघ्रत्वेन तदंशैः स्वं प्रमाणोक्तं ज्ञशुक्रयोः ।
मन्दवृत्तस्य चैवात्र ध्रुवमध्ये स्वकर्णवत् ॥ २१ ॥

'The [eccentric] orbits on which planets move (*graha-bhramanavrtta*) themselves move at the same rate as the apsides (*ucca-gati*) on *manda-vrtta*, [or the *manda* epicycle drawn with its centre coinciding with the centre of the *manda* concentric]. In the case of the Sun and the Moon, the centre of the

Earth is the centre of this *manda-vrtta*.' (Verse 19)

'For the others [namely the planets Mercury, Venus, Mars, Jupiter and Saturn] the centre of the *manda-vrtta* moves at the same rate as the mean Sun (*madhyarkagati*) on the *sighra-vrtta* [or the *sighra* epicycle drawn with its centre coinciding with the centre of the *sighra* concentric]. The *sighra-vrtta* for these planets is not inclined with respect to the ecliptic and has the centre of the celestial sphere as its centre.' (Verse 20)

'In the case of Mercury and Venus, the dimension of the *sighra-vrtta* is taken to be that of the concentric and the dimensions [of the epicycles] mentioned are of their own orbits. Further, here the *manda-vrtta* [and hence the *manda* epicycle of all the planets] undergoes increase and decrease in size in the same way as the *karna* [or the hypotenuse or the distance of the planet from the centre of the *manda* concentric][18].' (Verse 21)

The geometrical picture described by Nilakantha is shown in Figures 5 *a* and *b*. Like the above verses of *Siddhanta-darpana*, there are several other graphic descriptions of this geometrical picture in Nilakantha's works. For the exterior planets, he explains in his tract on planetary latitudes that[19]:

कुजादेर्मान्दाः कक्ष्या भगोलमध्ये दिनकरकक्ष्यास्थ-
मध्यमार्ककेन्द्राः ।

'For Mars and other exterior planets (*Kujadi*), the centre of their *manda-kakshya* [which is also the centre of their *manda* deferent circle], is the mean Sun (*madhyarka*) which lies on the orbit of the Sun on the ecliptic'.

For the case of interior planets, the following is a graphic description of their motion given by Nilakantha in his *Aryabhatiyabhashya*[20]:

तयोः भ्रमणवृत्तेन न भूः परिगृहीता । तयोः बहिरेव
सदा भूः । भगोलैकपार्श्व एव तद्वृत्तस्य परिभ्रमणत्वात् तद्भ्रमणेन
न द्वादशराशिषु चारः स्यात् । तयोरपि वस्तुतः आदित्य-
भ्रमण एव राश्यतिक्रमः । शीघ्रोच्चभ्रमणत्वेन पठिता एव
स्वभ्रमणाः । तस्मात् आदित्यभ्रमणवशादेव द्वादशराशिषु
चारः स्यात् ।

'The earth is not circumscribed by their [i.e. the interior planets, Mercury and Venus] orbits. The Earth is always outside their orbit. Since their orbit is always confined to one side of the geocentric celestial sphere, in completing one revolution they do not go around the twelve signs (*rasis*). For them also really the mean Sun is the *sighrocca*. It is only their own revolutions which are stated to be the revolutions of the *sighrocca* [in ancient texts such as the *Aryabhatiya*]. It is only due to the revolution of the Sun [around the Earth] that they [i.e. the interior planets, Mercury and Venus] complete their movement around the twelve *rasis* [and complete their revolution of the Earth]'.

Thus, in Nilakantha's planetary model, Mercury, Venus, Mars, Jupiter and Saturn, are assumed to move in eccentric orbits around the *sighrocca*, which is the mean Sun going around the Earth. The planetary orbits are tilted with respect to the orbit of the Sun or the ecliptic and hence cause the motion in latitude.

Nilakantha's modification of the conventional planetary model of Indian astronomy seems to have been adopted by most of the later astronomers of the Kerala school. This is not only true of Nilakantha's pupils and contemporaries such as Sankara Variyar (1500–1560), Chitrabhanu (1530), Jyeshtadeva (1500), who is the author of the celebrated *Yuktibhasha*, but also of later astronomers such as Acyuta Pisarati (1550–1621), Putumana Somayaji (1660–1740) and others. They not only adopt Nilakantha's planetary model, but also seem to discuss further improvements. For instance, Acyuta Pisarati in his *Sphutanirnayatantra* and *Rasigola-sphutaniti*[21] discusses in detail the correction to planetary longitudes due to latitudinal effects by the method of reduction to the ecliptic – a point which has been earlier briefly noted by Nilakantha in his *Aryabhatiya-bhashya*[22].

In conclusion it may be noted that there is a vast literature on astronomy (including mathematics) both in Sanskrit and Malayalam, produced by the Kerala school, during the period 14th–19th century. Only a small fraction of it has been published and so far only a few studies of these texts have appeared. What seems to emerge clearly from the source-works already published is that by the later part of the 15th century, if not earlier, Kerala astronomers had arrived at many of the discoveries in mathematical analysis and astronomy which are generally hailed as the signal achievements of the scientific renaissance in Europe during the 16th and 17th centuries. Only more detailed investigations can lead to a correct appreciation and assessment of the work of the Kerala astronomers during the 14–16th centuries and their consequent developments[23].

1. See for instance (a) *Indian Astronomy: A Source Book*, (eds. Subbarayappa, B. V. and Sarma, K. V.), Bombay, 1985; (b) *A Bibliography of Kerala and Kerala-based Astronomy and Astrology*, Sarma, K. V., Hoshiarpur, 1966.
2. See for example (a) Whish, C. M., *Trans. R. Asiatic Soc.*, 1830, **3**, 509; (b) Mukunda Marar, K. and Rajagopal, C. T., *J. Bombay Br. R. Assoc. Soc.* (*NS*), 1944, **20**, 65; (c) Rajagopal, C. T., *Stud. Math.*, 1949, **15**, 201; (d) Rajagopal, C. T. and Rangachari, M. S., *Archiv Hist. Ex. Sc.*, 1976, **18**, 89; (e) Takao Hayashi, *Centauras*, 1990, **33**, 149; (f) Balagangadharan, K., in *Scientific Heritage of India: Mathematics* (ed. Poulose, K. G. Tripunithura) 1991, p. 29; We may note that C. M. Whish's paper which appeared in 1835 is not the first dicussion on the discovery of infinite series by the Kerala astronomers. That there was quite some discussion on the subject in the decades prior to 1835 is indicated by J. Warren in his book *Kala Sankalita* (Madras, 1825, pp. 93, 309–310). Curiously, the whole issue got totally ignored after 1835 till Prof. C. T. Rajagopal and his colleagues resurrected the subject around 1945.
3. For a general review of Indian astronomy, see (a) *A Critical Study of Ancient Hindu Astronomy*, Somayaji, D. A., Karnatak University, Dharwar, 1972; (b) *A History of Indian Astronomy* (eds. Shukla, K. S. and Sen, S. N.) INSA, New Delhi, 1985.
4. Aryabhatiya *With the Commentary of Bhaskara I and Somesvara* (ed. Shukla, K. S.), INSA, New Delhi, 1976, pp. 32, 247.
5. Muralidhara Chaturveda (ed.), *Siddhanta-siromani*, Varanasi, 1981, p. 402.
6. *Aryabhatiyam* with the *Bhashya* of Nilakantha Somasutvan: *Golapada* (ed. Pillai, S. K.), Trivandrum Sanskrit Series, No. 185, 1957, p. 8.
7. *Tantrasangraha* of Nilakantha Somasutvan with the commentary *Laghuvivritti* of Sankara Variar (ed. Pillai, S. K.), Trivandrum Sanskrit Series, No. 188, 1958, p. 2.
8. Ref. 7, p. 8. Nilakantha's modification of traditional planetary model by identifying what were referred to as the *sighroccas* of Mercury and Venus with the planets themselves, seems to have gone unnoticed so far, even in those studies where allusions are made to

Nilakantha's work. For instance, Pingree in his review article on Indian astronomy presents the mean rates of motion of Mercury and Venus given in *Tantrasangraha* as the rates of motion of their *sighroccas* (Pingree, D., History of Mathematical Astronomy in India, in *Dictionary of Scientific Biography*, New York, 1978, vol. XV, p. 622).

9. Ref. 7, pp. 44–46.
10. Ref. 7, p. 139.
11. See for example (a) *A History of Astronomy from Thales to Kepler*, Dreyer, J. L. E., Dover, New York, 1953; (b) *Mathematical Astronomy in Copernicus' De Revolutionibus*, Swerdlow, N. M. and Neugebauer, O., Springer, New York, 1984, 2 vols.
12. See for example *The Almagest* by Ptolemy, Tr. by Taliaferro R. C., in *Great Books of the Western World* (ed. Hutchins, R. M.), Chicago, vol. 16, 1952. For the exterior planets, the ancient Indian planetary model and the model described by Ptolemy are very similar except that, while the Indian astronomers use a variable radius epicycle, Ptolemy introduces the notion of an equant. Ptolemy adopts the same model for Venus also, and presents a slightly different model for Mercury. In both cases the equation of centre is applied to the mean Sun. While the ancient Indian astronomers successfully used the notion of the *sighrocca* to arrive at a satisfactory theory of the latitudes of the interior planets, the Ptolemaic model is totally off the mark when it comes to the question of latitudes of these planets. This difficulty with the computation of latitudes persisted till around the time of Kepler.
13. Kern, B. (ed.), *Aryabhatiyam* with *Vyakhya* of Paramesvara, Leyden, 1885, pp. 68–69.
14. Kuppanna Sastri, T. S. (ed.), *Mahabhaskariyam* with Govindasvamin's *Vyakhya* and *Siddhantadipika* of Paramesvara, Madras Govt. Oriental Series, No. 130, 1957, pp. 233–238.
15. Sarma, K. V. (ed.), *Grahasphutanayane Vikshepavasana* of Nilakantha Somasutvan in *Ganitayuktayah*, Hoshiarpur, 1979, pp. 61–64.
16. Sambasiva Sastri, K. (ed.), *Aryabhatiyam* with the *Bhashya* of Nilakantha Somasutvan: *Kalakriyapada*, Trivandrum Sanskrit Series, No. 110, 1931, p. 53.
17. Sarma, K. V. (editor and translator) *Siddhantadarpana*, Hoshiarpur, 1976, pp. 18–19.
18. This reflects the feature of Indian planetary models, that the *manda* correction or the equation of centre is calculated from a variable radius epicycle model. See for instance *Mahabhaskariya* of Bhaskaracharya, I., ed. and translated by Shukla, K. S., Lucknow, 1960, pp. 136–146.
19. Ref. 15, p. 63.
20. Ref. 6, p. 9.
21. Sarma, K. V. (ed.) *Sphutanirnayatantra* of Acyuta Pisarati, Hoshiarpur, 1974; *Rasigolasphutaniti* of Acyuta Pisarati, ed. and translated by Sarma, K. V., Hoshiarpur, 1977.
22. Ref. 6, p. 6.
23. The well known Orissa astronomer of last century, Chandrasekhara Samanta, who was trained solely in traditional Indian astronomy, seems to have also discussed a model of planetary motion where the five planets go around the Sun, in his work *Siddhantadarpana*. (*Siddhantadarpana* of Mahamahopadhyaya Samanta Sri Chandrasekhara Simha, Calcutta 1897, V. 36. See also the review by W.E.P. in *Nature*, 1532, **59**, 437, 1899). We are grateful to Dr P. Nayak for bringing this important fact to our attention.

ACKNOWLEDGEMENTS. We thank Prof. K. V. Sarma, Dr S. Balachandra Rao, and Dr J. K. Bajaj for many helpful discussions. We also gratefully acknowledge financial support provided by the Department of Science and Technology, New Delhi, under the Nationally Coordinated Project on Foundations and Methodology of Theoretical Sciences in the Indian Tradition.

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