Magnetic Confinement of Plasmas for Fusion

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March 18, 2009

1 Abstract

The research into magnetic confinement of plasmas has been primarily driven by fusion reactor development. One of the most important considerations for a successful fusion reactor is the ability to confine high temperature plasmas in a stable manner. Various configurations have been developed to attempt this feat. In addition to the configuration geometry, other important factors in magnetic plasma confinement are plasma heating methods and energy extraction from plasma undergoing fusion. At present, the technology is still underdeveloped and there is not a clear "best answer" to these problems. This paper will attempt to give a brief understanding of plasma physics, some of the interesting configurations to confine plasmas, and a brief outline of the future of plasma research and fusion technology. Since there are many different configurations, this paper will focus primarily on the most studied configuration, the tokamak, and provide a brief introduction to several other configurations.

2 The Basics of Plasmas

Plasmas are simply ionized states of matter. Thermal excitation provides the energy to ionize gases and the resulting "fourth state of matter" is a plasma. The electrons and positive ions are strongly influenced by Coulomb forces and magnetic fields. The relevant interactions are described by the field of electrodynamics; the basic governing equations are Maxwell's equations. Because a plasma consists of a very large number of charged particles, all interacting and responding to external (or internally created) fields, plasma physics can become quite complicated [3].

A basic understanding of the various plasma configurations can be gleaned by examining the equations of motion of charged particles resulting from the Coulomb's Law, the force on a particle in an electric field, and the Lorentz Force describing the force on charged particles in a magnetic field [4].

$$\mathbf{F_{Coulomb}} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}\mathbf{r} \tag{1}$$

$$\mathbf{F_{elec}} = Q\mathbf{E} \tag{2}$$

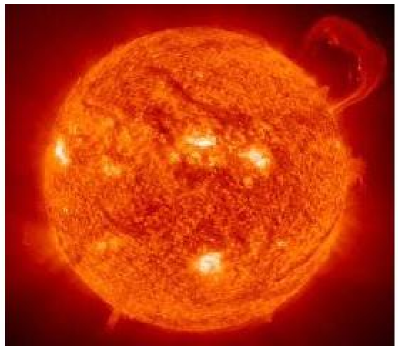

Figure 1: An easily recognizable plasma undergoing fusion

$$\mathbf{F_{mag}} = Q(\mathbf{v} \times \mathbf{B}) \tag{3}$$

Coulomb's Law, equation 1, gives the force on a particle of charge Q due to another particle of charge q, where $\mathbf{r}$ is the relative position vector. In a plasma, even in the absence of any externally applied electric field, the charged particles can create electric fields. Moving charges are influenced by the magnetic force given in Equation 3, which can cause a separation of oppositely charged, moving particles. Equation 2 will apply to the electric field produced by the separation of opposite charges.

All of this theory is basic electromagnetism applied to a large number of particles. Clever manipulations of magnetic fields allow physicists to confine very high temperature and density plasmas without allowing them to come into physical contact with any material surfaces.

2.1 Electrical Resistivity

The resistivity of a plasma is important in applications such as Ohmic Heating. The specific resistivity is defined by $\eta = (E/j)$ where j is the current density. For a frequency of collisions between electrons and ions of ν_{ei}, the resistivity is given by:

$$\eta = \frac{m_e \nu_{ei}}{n_e e^2} \tag{4}$$

For a hydrogen plasma with a thermal energy $k_B T = 1$ keV, the resistivity is about 1.5 times the resistivity of copper at room temperature [5].

3 Electromagnetism and Plasmas

3.1 Cyclotron Motion and Particle Orbits

Combining equations 2 and 3 gives the force acting on a charged particle in an electric and magnetic field. If the particle has mass m and charge q, then [4]:

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{5}$$

If the magnetic field is uniform and the electric field is zero, then for $\mathbf{B} = B_0\tilde{\mathbf{z}}$ the velocity of the particle is described by [5]:

$$
\begin{aligned}
v_x &= -v_\perp sin(\Omega t + \psi) \\
v_y &= v_\perp cos(\Omega t + \psi) \\
v_z &= v_{z0} \\
\Omega &= -\frac{qB}{m} \\
\rho &= \frac{mv_\perp}{|q|B}
\end{aligned} \tag{6}
$$

This is simple Larmor motion, which results in a particle spiraling around a field line at the cyclotron frequency Ω. The Larmor radius ρ is found by balancing the Lorentz Force and the centripetal force of the circular motion. Notice that for electrons, Ω is positive and the motion is right-handed while the opposite is true for the positive ions. The center of this motion, or the axis about which the particle spins, is called the guiding center. In general, when there is a force $\mathbf{F}$ on a particle that has a component perpendicular to the magnetic field, the guiding center drifts with velocity [5]:

$$
v_g = \frac{1}{q}\frac{\mathbf{F} \times \mathbf{B}}{B^2} \tag{7}
$$

This effect is subtle. While the particle orbits the magnetic field line in cyclotron motion, it experiences a force $\mathbf{F}$. When the particle is moving with a component of velocity parallel to $\mathbf{F}$, it is accelerated. Then, when it reaches the point on its orbit where its velocity is perpendicular to $\mathbf{F}$, its velocity is greatest. Similarly, when its velocity is anti-parallel to $\mathbf{F}$, it decelerates, so that its velocity is lowest when it again become perpendicular to $\mathbf{F}$ at π radians from the point where its velocity was maximum. This velocity difference between opposite points on its cyclotron orbit results in a small drift velocity given by Equation 7.

When there is a uniform electric field $\mathbf{E}$, the electric force depends on the charge of the particle and q in Equation 7 drops out. Therefore, the drift velocity of the guiding center for both ions and electrons is given by:

$$
v_g = \frac{\mathbf{E} \times \mathbf{B}}{B^2} \tag{8}
$$

The circulating charge effectively forms a current loop with magnetic moment given by $\mu_m = IS$ for current I and area S. The current is given by $I = q\Omega/2\pi$ and the area is given by $S = \pi\rho^2$. The magnetic moment can be expressed as follows [5]:

$$
\mu_m = \frac{mv_\perp^2}{2B} \tag{9}
$$

The magnetic moment is created by the circular compenent of the velocity, $v_\perp$. If the dot product of (the vector quantity) $\mathbf{v}_\perp$ and Equation 5 is taken, with $\mathbf{v}$ replaced by $\mathbf{v}_\perp$ in Equation 5, then the following relationship holds, since $\mathbf{v}_\perp$ is perpendicular to $\mathbf{v}_\perp \times \mathbf{B}$:

$$
\frac{d}{dt}(\frac{mv_\perp^2}{2}) = q(\mathbf{v}_\perp \cdot \mathbf{E}_\perp) \tag{10}
$$

Integrating Equation 10 over one period of the Larmor orbit gives the change in the kinetic energy over the period, where the last relation is due to Green's Theorem [5]:

$$
\Delta\mathrm{KE} = q\int(\mathbf{v}_\perp \cdot \mathbf{E}_\perp)\,dt = q\oint \mathbf{E}_\perp \cdot ds = q\int(\nabla \times \mathbf{E} \cdot \mathbf{n})\,dS \tag{11}
$$

Faraday's Law gives $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$ which, when inserted into the last (surface) integral in Equation 11, gives:

$$
\Delta\mathrm{KE} = |q|\pi\rho^2\frac{\partial B}{\partial t} \tag{12}
$$

The change in the magnetic field over this period (one Larmor period) is given by:

$$\Delta B = (\frac{\partial B}{\partial t})(\frac{2\pi}{\Omega}) \tag{13}$$

This gives the relationship [5]:

$$\Delta \mathrm{KE} = \frac{mv_\perp^2}{2}\frac{\Delta B}{B} = \mathrm{KE}\,\frac{\Delta B}{B} \tag{14}$$

Applying the quotient rule to KE/B = μ_m gives Δ(KE/B) = $\Delta\mu_m$ = ΔKE/B – ΔBKE/B^2. However, Equation 14 can be rewritten as ΔKE/B = ΔBKE/B^2 simply by multiplying by ΔB/B. Therefore, Δ(KE/B) = $\Delta\mu_m = 0$.

Thus, the magnetic moment μ_m in Equation 9 is conserved when the magnetic field is changed slowly, which implies that the kinetic energy of the particles is increased when the magnetic field is increased. As the field is increased, the particles move with increasing Larmor frequency around a smaller orbit ρ. This is known as adiabatic heating and is a method of increasing the plasma temperature [1].

3.2 Toroidal Fields

Given the toroidal coordinate system shown in Figures 2 and 3, the simple toroidal magnetic field that has magnitude B_0 at $R = R_0$ and falls off in strength with R^{-1} is described by (this could be created by an infinite current-carrying wire):

$$\begin{aligned} B_r &= 0 \\ B_\phi &= \frac{B_0 R_0}{R} \\ B_z &= 0 \end{aligned} \tag{15}$$

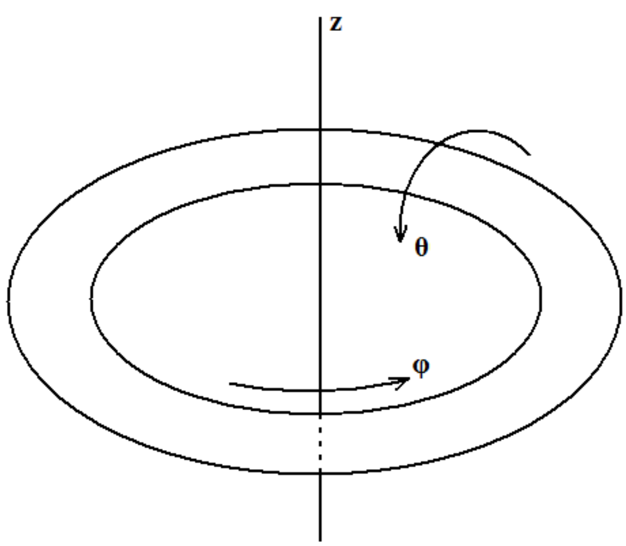

Figure 2: Toroidal coordinates

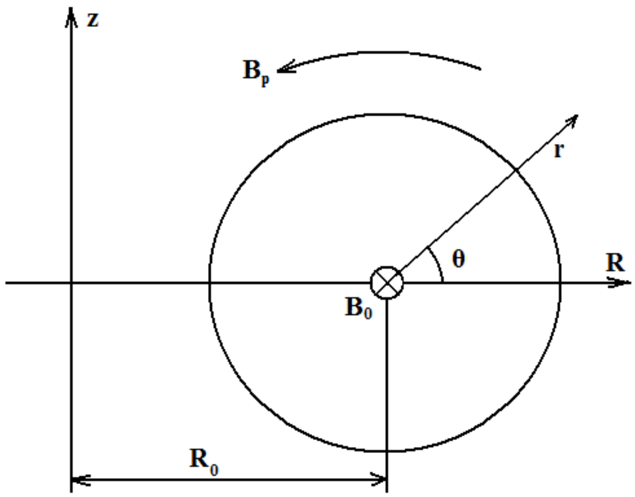

Figure 3: Toroidal coordinates

The guiding center of the Larmor motion moves with velocity given by Equation 16, where $\hat{\phi}$ and $\hat{z}$ are unit vectors [1].

$$\mathbf{v_g} = v_{\|}\hat{\phi} + \frac{m}{qB_{\phi}R}(v_{\|}^2 + \frac{v_{\perp}^2}{2})\hat{z} \tag{16}$$

Electrons and ions circle the torus in Larmor motion, but the second term in Equation 16 implies that ions and electrons slowly drift in opposite directions along the z-axis. This occurs due to the field strength decreasing inversely with R. As a particle moves into a weaker field region, its Larmor radius increases and causes vertical ($\hat{z}$) drift when the particle moves back into stronger fields. This drift creates charge separation, which results in a vertical electric field ($\mathbf{E} = E_0\hat{z}$). This causes ions and electrons to drift radially outwards, with a radial drift velocity given by Equation 8. In order to confine a plasma in the toroidal configuration, one must stop the charge separation that produces the electric field causing the radial drift. Adding an orthogonal component to the magnetic field is one method of doing that.

3.3 Poloidal Fields

If a current is present in the toroidal ($\hat{\phi}$) direction, a magnetic field in the poloidal ($\hat{\theta}$) direction is created. The radial coordinate in the toroidal cross section is given as r in Figure 3. Let the magnitude of the poloidal field be B_p and the magnitude of the toroidal field be B_{ϕ}. Then the rotational transform angle, which describes the amount the magnetic field rotates in the poloidal cross section when ϕ progresses through 2π, is [1]:

$$\epsilon = \frac{2\pi R B_p}{r B_{\phi}} \tag{17}$$

The frequency at which a particle rotates about the minor axis of the torus, every time θ increments by 2π, is:

$$\omega = \frac{\epsilon v_{\|}}{2\pi R_0} \tag{18}$$

This rotation in the poloidal direction has the effect of eliminating the charge segregation that led to radial drift velocities when only toroidal fields are present.

3.4 Banana Orbits

If the magnitude of B_ϕ is much greater than the magnitude of B_p, then the total magnitude of the overall field B is approximated well by $|B_\phi|$ of the simple toroidal field given in Equation 15. Referring to Figure 3 gives [5]:

$$B = \frac{B_0}{1 + (r/R)\cos\theta} \approx B_0(1 - \frac{r}{R_0}\cos\theta) \tag{19}$$

θ can be expressed in terms of the toroidal angle ϕ in Figure 2 and the rotational transform angle ϵ, from Equation 17. If ϕ is also expressed as $\frac{l}{R}$, where l is the length along the field line in the toroidal direction, then:

$$\theta = \frac{\epsilon l}{R} = \frac{B_p l}{r B_0} \tag{20}$$

Equation 19 implies that, in the coordinates of Figure 3, as θ is increased or decreased about zero, at constant r in the weaker magnetic field on the outside of the torus, the magnetic field strengthens. If the component of the particle's velocity along the magnetic field, $v_\parallel$, is much smaller than $v_\perp$, then the particle is confined in a magnetic mirror, described in Section 5.1. In the poloidal plane of Figure 3, the orbits trace out a "banana" pattern on the outer edge of the plasma. The banana orbit is important for steady state current drives in plasmas, discussed in Section 5.3.4 [5].

4 Why Magnetic Confinement is Important

Magnetic confinement is the only method to produce and sustain for long periods of time a plasma capable of fusion. Plasmas must be heated to hundreds of millions of degrees before fusion can take place and sustain the plasma. Very high densities compared to other plasmas are also required. Such high temperatures mean that if a plasma was to contact a material surface, it would immediately cool down and damage the surface. Therefore, there is no solid container that can work; the container must be magnetic.

5 Configurations

Several commonly studied configurations for magnetic confinement are the magnetic mirror, the z-pinch, the tokamak, and the stellarator.

5.1 Magnetic Mirror

The magnetic mirror was one of the first confinement schemes envisioned [5]. Figure 4 shows the magnetic field geometry of a basic magnetic mirror. The mirror ratio is the ratio of the magnitudes of the field at the ends to the middle region:

$$R_M = \frac{B_M}{B_0} \tag{21}$$

The main idea is that the magnetic field is weak at the center and strong at either end. If the electric field is zero, then the kinetic energy of the plasma particles is conserved since the magnetic field does no work. Using this fact and the fact that the magnetic moment μ_m, given in Equation 9, is conserved, the velocity parallel to the field is [5]:

$$v_\parallel = \pm\sqrt{(v^2 - \frac{2}{m}\mu_m B)} \tag{22}$$

Equation 22 means that as the particle moves into a region of larger field, the parallel velocity decreases as the perpindicular velocity accounts for more of the particle's kinetic energy. In the transition region from the low to higher strength field, the magnetic field has a radial component. This radial component of the field interacts

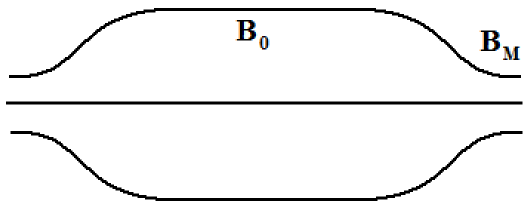

Figure 4: A simple magnetic mirror

with the perpindicular (azimuthal) velocity of the particle and produces a force that directs the particle back into the region of low field strength. Since the field is "pinched" at both ends, the particle will bounce between the regions of increasing field strength, hence the name "magnetic mirror".

This method of confinement has been abandoned as a viable configuration for fusion reactors, due to the fact that particle collisions tend to destroy the velocity distribution required for the mirror to work effectively and particles begin to escape.

5.2 Z-Pinch

Another simple magnetic confinement is the z-pinch, a configuration that relies on the interaction of a current-carrying plasma with the magnetic field it creates. Figure 5 shows a schematic of the ZaP Flow Z-Pinch Experiment at the University of Washington.

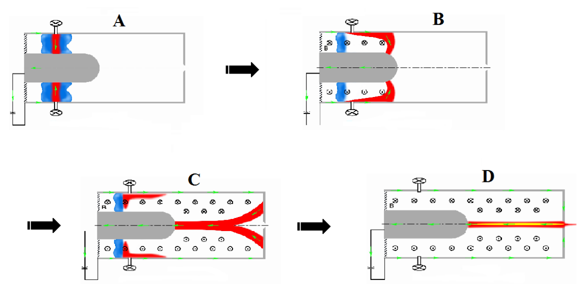

Figure 5: Z-pinch with embedded flow (ZaP Experiment)

In Figure 5, **A** shows a current flow through a plasma from the cathode to the anode (the anode is the grey protrusion on the longitudinal axis and the cathode is the cylindrical vessel). The Lorentz force accelerates plasma down the z-axis and compresses it onto the z-axis in **B** and **C**. Because of the acceleration experienced in creating the z-pinch, the plasma has a large particle velocity in addition to current. This creates a more stable z-pinch in **D** than one without particle flow. Nevertheless, z-pinches have been largely abandoned in favor of more promising confinement methods.

5.3 Tokamak

The Tokamak configuration is the most widely studied magnetic confinement method. For toroidal plasma confinement, both poloidal (B_p) and toroidal (B_ϕ) fields are necessary. The Tokamak is a toroidal configuration with a poloidal field provided by external coils and plasma current, as described in Section 3.3. Figure 6 shows ITER, a modern tokamak currently being built.

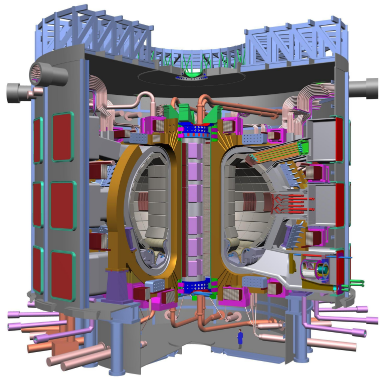

Figure 6: ITER, a large tokamak currently being built as a prototype research reactor

5.3.1 Pressure

The magnetic pressure of a field is $p_M = \frac{B^2}{2\mu_0}$. The pressure of the plasma p is due to the kinetic energy of the particles comprising the plasma. The ratio of the plasma pressure to the magnetic pressure from the externally applied field is called the beta ratio [1]:

$$\beta = \frac{p}{p_M} = \frac{2p\mu_0}{B_0^2} \tag{23}$$

The "strength" of the confinement is reflected in the value of β. If the plasma is confined, $\beta < 1$. β is an important figure of merit for tokamak designs.

5.3.2 External Vertical Fields

The poloidal field due to plasma current is stronger inside of the plasma ring than outside. External vertical fields are added to increase the poloidal field outside of plasma and decrease the field inside, to give a poloidal field geometry similar to the geometry shown in Figure 7. This is necessary for tokamak equilibrium [5].

5.3.3 Inductive and Non-Inductive Current Drives

There are several ways to drive current in the plasma to produce the poloidal fields. The simplest is to use Faraday's Law:

$$\xi = -\frac{d\Phi_B}{dt} \tag{24}$$

Since the plasma is a conducting ring, a change in magnetic flux through it will create a current. However, this is not suitable for steady state operation, since the flux needs to constantly change in order to induce a current.

A non-inductive approach to driving current involves injecting a high energy beam of neutral particles into the plasma across the magnetic fields, with which the beam doesn't interact. Once inside the plasma, the particles are ionized and collisions with the plasma particles drives current.

5.3.4 Bootstrap Current

A subtle effect called the bootstrap current is important in providing steady state current drive to create the required poloidal fields. The banana particles described in Section 3.4 orbit in opposite directions for electrons and ions. These particles are distinct from particles orbiting in mainly the toroidal direction (due to their velocity

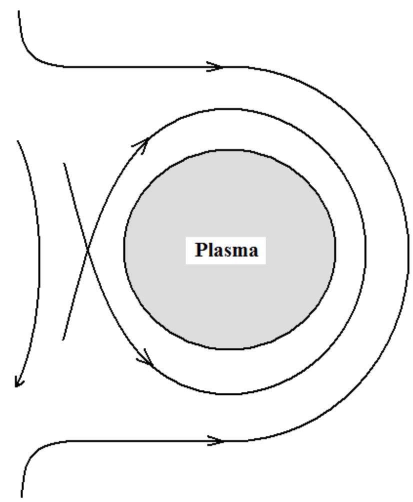

Figure 7: Combined poloidal field resulting from plasma current field and external vertical field

distributions which causes them to be trapped). If a density gradient exists in the radial direction (R) of the plasma, then a banana orbit closer to the center of the torus will have more particles than one farther away. Therefore, at the overlap of consecutive banana orbits (if they are imagined to be somewhat discrete), there is a net movement of particles, electrons in one direction and ions in the other. The particles in banana orbits collide with untrapped particles, causing a net current in the toroidal direction. The bootstrap current density (whose derivation is beyond the scope of this paper) is given by [1]:

$$j_{bootstrap} \approx -\frac{1}{A^{1/2} B_p} \frac{dp}{dr} \tag{25}$$

where $A = R/a$, the ratio of the major to minor axis of the torus. Bootstrap current can provide much of the steady state current drive required to create the poloidal field. Bootstrap current has been shown to provide 70 to 80 percent of the plasma current in some experimental tokamak configurations [5].

Although beyond the scope of this paper, the bootstrap current also stabilizes the ballooning mode of magnetohydrodynamic (MHD) instability.

5.4 Stellarator

A stellarator is another promising configuration. It uses toroidal field magnets that are twisted to create the necessary poloidal field without needing a plasma current. Figure 8 shows an example of the complex magnet coil structure and the twisting of the toroidal field. The rotational transform angle ϵ, given in Equation 17, is therefore provided by the externally applied fields.

6 Fusion Power

A sufficiently heated and compressed plasma, whose constituents support fusion reactions, can undergo fusion. Steady state fusion is the ultimate goal for a fusion reactor, and the main driving force behind the research into magnetic confinement. There are several important issues involved (in addition to confining the plasma in the first place). The first is selecting an appropriate fusion reaction; the second is generating heat in the plasma; and the third is extracting energy from a "burning" plasma.

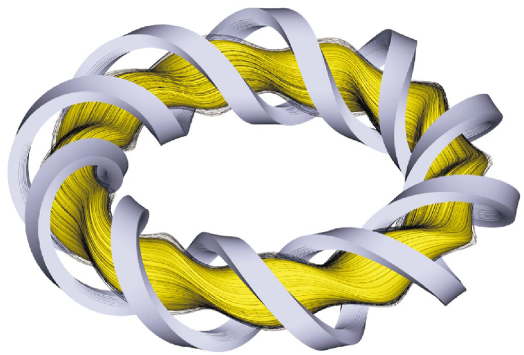

Figure 8: A stellarator configuration, showing the plasma and the magnetic coils

6.1 Fusion Reactions

Several fusion reactions are of interest [5]:

$$\begin{aligned}
\mathrm{D}+\mathrm{D} &\rightarrow \mathrm{T}+p \\
\mathrm{D}+\mathrm{D} &\rightarrow \mathrm{He}^3+n \\
\mathrm{D}+\mathrm{T} &\rightarrow \mathrm{He}^4+n \\
\mathrm{D}+\mathrm{He}^3 &\rightarrow \mathrm{He}^4+p \\
\mathrm{Li}^6+\mathrm{n} &\rightarrow \mathrm{T}+\mathrm{He}^4 \\
\mathrm{Li}^7+\mathrm{n} &\rightarrow \mathrm{T}+\mathrm{He}^4+n
\end{aligned} \tag{26}$$

The easiest fusion reaction to attain, because it requires the lowest energies and the isotopes are relatively easy to extract from seawater, is the D,T fusion reaction. It is the basis for all potential fusion reactor designs, although it does present problems. Unlike the fusion of D and He^3, whose products are charged and thus trapped within the plasma, it releases damaging high energy neutrons. These high energy neutrons can irradiate the reactor and cause material damage as well, presenting additional design challenges.

6.2 Heating the Plasma

Before fusion can occur, the temperature of the plasma must be raised. This is accomplished using several methods. Adiabatic heating occurs when the toroidal field is increased, as discussed in Section 3.1.

Another method is neutral beam injection, which involves injecting a beam of high energy neutral atoms across the magnetic field into the plasma. These atoms are ionized and immediately confined by the magnetic fields, and transfer energy to the plasma through collisions.

Resistive (or ohmic) heating occurs when a current is driven through the plasma in one of the various ways discussed, and heat is generated through resistive losses; the resistivity of a plasma is given in Equation 4.

Finally, it is possible to use RF waves to excite oscillations in the plasma to generate heat [5].

6.3 Extracting Energy

In the D,T fusion reaction, the neutrons carry the majority of the energy. Since they have no charge, they can cross magnetic fields unaffected (the alpha particles will expend most of their energy into further heating of the plasma). The high energy neutrons would be slowed and captured in a lithium blanket. The blanket would protect the structure of the reactor and also breed tritium for the reactor fuel (see the last fusion reaction given in 26). Part of the neutron energy would be transferred into a coolant fluid on the outside of the blanket and could be used to drive turbines to extract energy [2].

7 Conclusion

There are several configurations for magnetic plasma confinement that are widely studied, among them the tokamak and the stellarator. These toroidal devices garner the largest financial contributions and have shown the most promise towards the goal of steady-state, controlled fusion for a powerplant. Billions of dollars have been invested into the ITER, a large international fusion test-reactor which will use the tokamak configuration. ITER will validate many of the concepts of magnetic confinement and is expected to deliver a net power production in steady state operation. Although there are still many hurdles to cross, in the future, fusion reactors could provide a way to replace existing powerplants, and produce cleaner, more abundant energy.

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