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COLLISIONAL CASCADING: THE LIMITS OF POPULATION GROWTH IN LOW EARTH ORBIT

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ABSTRACT

Predictions have been made by several authors that random collisions between made-made objects in Earth orbit will lead to a significant source of new orbital debris, possibly within the next century. The authors have also concluded that there are a number of uncertainties in these models, and additional analysis and data are required to fully characterize the future environment. However, the nature of these uncertainties are such that while the future environment is uncertain, the fact that collisions will control the future environment is less uncertain. The data that already exist is sufficient to show that cascading collisions will control the future debris environment with no, or very minor increases in the current low Earth orbit population. Two populations control this process: Explosion fragments and expended rocket bodies and payloads. Practices are already changing to limit explosions in low Earth orbit; it is now necessary to begin limiting the number of expended rocket bodies and payloads in orbit.

INTRODUCTION

In 1978, collisional cascading was predicted to be an important source of new orbital debris, possibly before the year 2000, if debris control techniques were not adopted /1/. These predictions were reenforced in 1986 and 1990 /2,3,4,5/. In 1981, NASA adopted procedures which would minimize the possibility of explosions in orbit. By 1989, most other nations and organizations appear to have adopted similar procedures. These procedures will result in a near-term reduction in the projected debris environment; however, they do not address the long term problem. In the long term, once a critical population density of objects is reached, the rate of fragment production from random collisions exceeds the rate of removal by atmospheric drag, and the procedures to prevent explosions becomes less important. Once this critical density is reached, the debris population will increase without placing any more objects into orbit. This increase will stop only when the population of large objects is sufficiently reduced, either by active removal, or by fragmentation. However, by the time fragmentation reduces the population of large objects, the resulting debris environment is likely to be too hostile for future space use. Consequently, it is important to develop operational techniques which maintain a population below the critical density. The objective of this paper is to define the conditions producing this critical density.

KEY OBSERVATIONS AND ASSUMPTIONS

By definition, a critical population density is reached when that population will produce fragments from random collisions at an increasing rate and at a rate that is greater than the rate of removal by natural processes. The rate of fragment production will be controlled in the size regions where mass and area are concentrated. More than 99% of the orbiting population mass and area is contained in orbital debris larger than 10 cm /6/. The average payload or rocket body mass is about 1000 kg /7/, and the average 10 cm to 20 cm mass is estimated to be about 1 kg /1/. At an average velocity of 10 km/sec, an average collision between any two orbiting objects would result in the catastrophic breakup of both objects. Consequently, the rate that collision fragments are generated will be controlled by the collision rate in this size range, and the conditions describing the critical density can be defined in terms of the density of objects 10 cm and larger. Following a collision, the amount of the mass that go into smaller than 10 cm sizes depends on a large number of factors, and is significantly uncertain. However, most fragments larger than 10 cm to 20 cm are observed by US Space Command following a breakup, and the number is not as uncertain. Therefore, the current data is capable of describing the conditions of critical density with more certainty than the time dependant consequence of the smaller than 10 cm population. A fundamental assumption of this analysis is that the current population of large, intact objects, is maintained by replacement when they decay, reducing the analysis to determining the source and sink rates for fragments.

Breakup models have been developed to describe the number and velocity of fragments relative to the center of mass. These models are based on observations of satellite breakups in space and in the lab, and attempt to cover a large range of sizes /8,9/. The velocity of 10 cm fragments ranges from 0 to 200 m/sec. The velocity distribution given in /9/ was used and new orbits calculated for 10 cm fragments. An atmospheric model, assuming a solar activity of 110 F10.7 units, was used to propagate these orbits as a function of time. Spatial density was calculated at various time intervals using the techniques in /10/. It was assumed that the breakup of a 1000 kg spacecraft would produce 300 fragments larger than 10 cm, and that the fragment area to mass ratios are given by the expression given in /1/. The results for a break-up at 1000 km altitude is show in figure 1. As

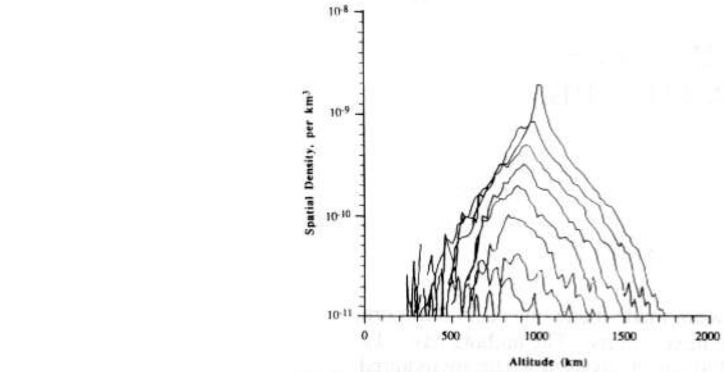

Fig.1. Satellite breakup at 1000 km. Top curve is initial spatial density distribution with altitude. Time interval between lower curves is 400 years.

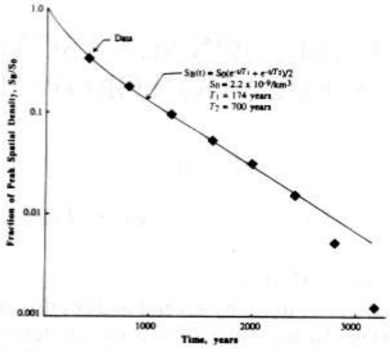

Fig. 2. Decay in Spatial Density following a breakup. Spatial density decays exponentially until it reaches .01 its initial value.

shown in figure 2, the decay in spatial density at the break-up altitude can be accurately expressed as the sum of two exponentials containing a "mean life", just as in natural radioactivity. That is, Spatial density at the breakup altitude as a function of time following a breakup can be expressed as

$$S_B\ (t) = S_0\ (e^{-t/\tau_1} + e^{-t/\tau_2})\ /2 \tag{1}$$

where S_0 is the initial spatial density following the breakup, t is time and τ_1 and τ_2 are constants and can be thought of as the mean life of two components of the debris cloud. These two components are the half of the cloud that go into higher orbits, and the half that go into lower orbits. The mean life of the half that go into higher orbits is controlled by the atmospheric density at the breakup altitude, while the mean life of the half that go into lower orbits is controlled by the atmospheric density at some lower altitude. For 300 fragments, $S_0=2.2X10^{-9}/km^3$, and at 1000 km τ_1 =174 years and τ_2 =700 years. At 500 km, τ_1 =.72 years and τ_2 =7.2 years. Each mean life is approximately inversely proportional to the atmospheric density at the breakup altitude.

If the satellite breakup rate is given by dB/dt, and breakups are equally likely to occur within some small altitude band (small enough not to significantly change the mean life), then by integrating over time, the equilibrium spatial density of the break-up fragments is given by

$$\bar{S}_B = \frac{dB}{dt}\bar{S}_0\tau = \frac{dB}{dt}\frac{\bar{N}_0}{\Delta U}\tau \tag{2}$$

where $\bar{S}_0$ is the initial spatial density averaged over breakups within the altitude band. The value of τ is simply the average of τ_1 and τ_2. It is convenient to write $\bar{S}_0$ in terms of the average number of fragments, $\bar{N}_0$, which are initially in the volume element ΔU defined by the altitude band. The value of $\bar{N}_0$ asymptotically approaches the number of fragments generated per average breakup as ΔU increases. The value of ΔU should be large, to account for all of the fragments; however, if too large, the value of τ will significantly change within the volume element. A value corresponding to an altitude band of about 100 km accounts for about one third of the fragments and is sufficiently small that mean life fairly accurately represents an average for the altitude band.

SATELLITE BREAKUP RATE

The satellite breakup rate due to random collisions is a function of spatial density, and can be calculated the following two ways:

$$\frac{dB}{dt} = \frac{1}{2}S^2V\sigma\Delta U = \sum_{i=1}^{n}\ \sum_{j=i+1}^{n-1}\ \int_{volume} S_iS_jV_{ij}\ \sigma_{ij}\,dU \tag{3}$$

The first expression assumes spherical symmetry about the Earth, and some average size and velocity. The value of S is the average spatial density at some altitude, and is calculated using the objects in the US Space Command catalogue by the techniques given in /10/. Values for the relative velocity v and collision cross-section σ, can either be assumed, or calculated by setting the first expression equal to the more exact, 2nd expression. In this expression, S_i is the spatial density of object i at some altitude and latitude, n is the number of catalogued objects in orbit, and V_{ij} and σ_{ij} are the calculated relative velocities and collision cross-sections, respective, between objects i and j in volume element dU.

Satellite physical dimensions and mass were obtained from /7/. When no physical dimensions were available, the radar cross-section was assumed to be a measure of the physical dimensions as described in /1/ and /9/. A numerical program which used a series of 10% random samples to perform the integrations and summations in

(3). A collision rate for catalogued objects of 0.05 collisions per year between 300 km and 1800 km altitude was obtained. By requiring the first expression to equal the same collision rate, the properly weighted average velocity and cross-section are v = 7.5 km/sec and σ = 10 sq m. In addition, the amount of mass involved with each collision was used to determine the amount of mass that would become fragments. It was assumed that if the ratio of the mass of the larger satellite times the relative velocity squared to the mass of the smaller satellite times 10 km/sec squared was less than 1000, than all of the mass of both satellites went into fragments. If this ratio was greater than 1000, then none of the mass went into fragments. This calculation determined that an average collision between 300 km and 1800 km produced 1600 kg of fragments. This means that a significant fraction of the collisions are between larger objects, so that an average collision would produce about 480 fragments larger than 10 cm, with 1/3 of them initially within a 100 km altitude band, or $\overline{N}_0 = 160$.

Combining equations (2) and (3), and assuming that one-half of the catalogue is intact satellites and the other half is breakup fragments, the average critical density of catalogued objects is given by

$$S = \frac{1}{V \tau \sigma \overline{N}_0} \qquad (4)$$

Using the values previously discussed, figure 3 compares the resulting average critical density with the spatial density of the current catalogue. The results indicate that much of the current population above 900 km is in the unstable region. This is an "average" critical density because it assumes that the satellite size and orbital inclination distributions are independent of altitude. We know this is not exactly true. For example, larger spacecraft are generally launched to lower altitudes, and orbital inclinations do vary with altitude, producing varying collision velocities. Assuming that these local size and inclination preferences will always exist, then an adjusted critical density can be calculated. Effective values for v, σ and $\overline{N}_0$ were obtained as a function of altitude by requiring the two expression in (3) to be equal to one another. It was concluded that the inclination effects do increase the collision rates over the average around 900 km and 1500 km by about 30%, and decreased it slightly at other altitudes. However, the effects of size were more dramatic...slightly increasing the average fragment generation rates at low altitudes, and significantly decreasing the rate at higher altitudes. These adjustments are shown in figure 4.

The results leave the 900 km to 1000 km altitude region firmly in the unstable region, while reducing the region of instability between 1000 km and 1400 km. This latter region of space contains mostly breakup fragments and few large satellites. Because of the low number density in these regions, it has been suggested that they might be used as the region to discard nuclear payloads. Such an action would change the current size distribution, decreasing the critical density line from that shown in figure 4, as well as increase the spatial density; consequently a more limited amount of material could be placed into the region than suggested by figure 4...figure 3 would be more representative, if we could be sure that there were no other factors to consider, such as a significant uncatalogued population.

Recent telescopic measurements, discussed in /11/ indicate that the US Space Command catalogue is only about 50% complete down to a limiting size of 10 cm. Although this means that the current population density should be double over that shown in figures 3 and 4, the corresponding critical density lines would also increase. By placing smaller objects into the population, the effective values for both σ and $\overline{N}_0$ would decrease, increasing the allowable critical density. Consequently, although this uncatalogued population will have an effect, it will be less than a factor of two. In addition, it is difficult to make meaningful statements about the number and size of objects when these objects are near the threshold of the measurements capabilities, especially when the quantity of interest is mass, and the limits of the sensors are expressed in terms of radar cross-section or limiting magnitude. A full analysis must include all of the distributions which effect the returned signal strength, such as object shape, albedo, and a number of other parameters /5/. Similar difficulties occur in /12/ which show the average fragment mass decreasing more rapidly than in /1/ (which is based on ground explosion data) as the 10 cm threshold is approached.

CONCLUDING REMARKS

This analysis indicates that certain regions of low Earth orbit are already unstable. Most of the larger debris generated in these regions will be confined to the unstable regions...this is one of the reasons they are unstable. However, smaller debris will be ejected to greater distances than large debris, and these unstable regions will act as an increasing source of small debris in all of low Earth orbit for centuries. If nothing more is added to the unstable regions, the rate that the debris environment will grow is slow...about one collisional breakup every 10 to 20 years, depending on the nature of the uncatalogued population, with about half of these breakups occurring within the unstable regions. Therefore, if objects are removed from these altitudes, it may still be possible to reverse the instability. However, if objects are continued to be placed into the unstable and surrounding regions,

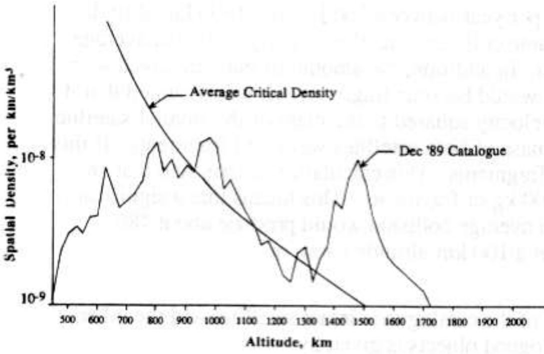

Fig. 3. Average Critical Density. Assumes size and orbital inclinations are independent of altitude.

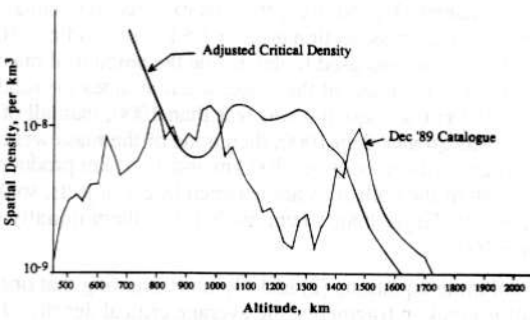

Fig. 4. Adjusted Critical Density. Assumes local size and orbital inclinations.

the breakup rate would be significantly higher and the region of instability would expand. In the last 9 years, the population density between 900 km and 1000 km has doubled. If the current population above 800 km were to double, the region of instability could expand, to include 800 km, and the breakup rate would increase to one every 2.5 years to 5 years. Under these conditions, it would be more difficult to reverse the instability.

There is some uncertainty in the exact threshold of instability; however, this uncertainty is small compared to the certainty that actions must begin now in order to be effective. Some actions could almost begin immediately. Future upper stages that have restart capability, such as the Delta 2nd stage, could be reentered after delivering their payload. Other actions, such as the controlled reentry at the end-of-life of payloads, or the retrieval of payloads will require at least 10 years of engineering development. That engineering development should begin now, since there is little uncertainty that it will be required.

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