Rip Currents

1. Theoretical Investigations

ANTHONY J. BOWEN

Tidal Institute, University of Liverpool
Birkenhead, Cheshire, England

The nearshore circulation of water on a plane beach produced by a wave train, normally incident on the beach, which has a longshore variation in wave height is investigated theoretically. The radiation stress arising from the excess flux of momentum due to the presence of the waves (M. S. Longuet-Higgins and R. W. Stewart, 1964) is found to provide driving terms for a steady flow pattern only inside the surf zone. A circulation pattern is thus produced by a longshore variation in the radiation stress in the surf zone. In shallow water, the radiation stress is proportional to the square of the wave height. The nearshore circulation is therefore directly related to longshore variation in breaker height, currents flowing seaward where the breaker height is low. When the inertial terms are included in the vorticity equation, an increase in the effective Reynolds number produces a narrowing, and consequently a strengthening, of the seaward flow, which suggests an explanation for the existence of the strong, narrow currents known as rip currents.

INTRODUCTION

Rip currents are strong, narrow currents that flow seaward from the surf zone. Although they are known to occur in models, in lakes and in bays rip currents are particularly well developed on gently sloping beaches exposed to large, regular, oceanic swell. In southern California the existence of these strong currents was further emphasized by the number of bathing fatalities directly attributable to them, and it was there that the first scientific observations of rip currents were made [*Shepard et al.* 1941]. The current intensity and the distance the rips extended seaward were both found to be related to the height of the incoming waves. The positions of the currents in relation to the underwater topography were also described; rip currents were observed to occur at the center of beach cusps and on either side of regions in which the breaker height is large.

A comprehensive series of field measurements at La Jolla, California, was made by *Shepard and Inman* [1950, 1951]. These observations showed that the nearshore movement of water could be described in terms of a circulation cell consisting of (1) a shoreward mass transport due to the wave motion carrying water through the breaker zone in the direction of wave propagation, (2) a movement of this water parallel to the coast as a longshore current, (3) a seaward flow along a concentrated lane, known as a rip current, and (4) longshore movement of the expanding rip head (Figure 1). This study also stressed the importance of the wave refraction due to the variations in the offshore bottom topography. The refraction patterns produce regions of wave convergence and divergence and the corresponding high and low breakers. It was recognized, however, that regular circulation patterns exist on long straight beaches with rather even bottom relief. More recent field studies in Australia [*McKenzie*, 1958] and South Africa [*Harris*, 1961, 1964] have shown that each incident wave system appears to form a characteristic pattern of longshore and rip currents. In particular, they noticed that in heavy seas only a few strong rips were produced and that, when the waves were smaller, the rips were weaker and more numerous.

The first suggestions as to the cause of rip currents were based on the concept of an onshore mass transport of water due to the incoming waves. This water, piled up on a beach or behind a sand bar, provided a head for the outflowing currents. In a purely two-dimensional

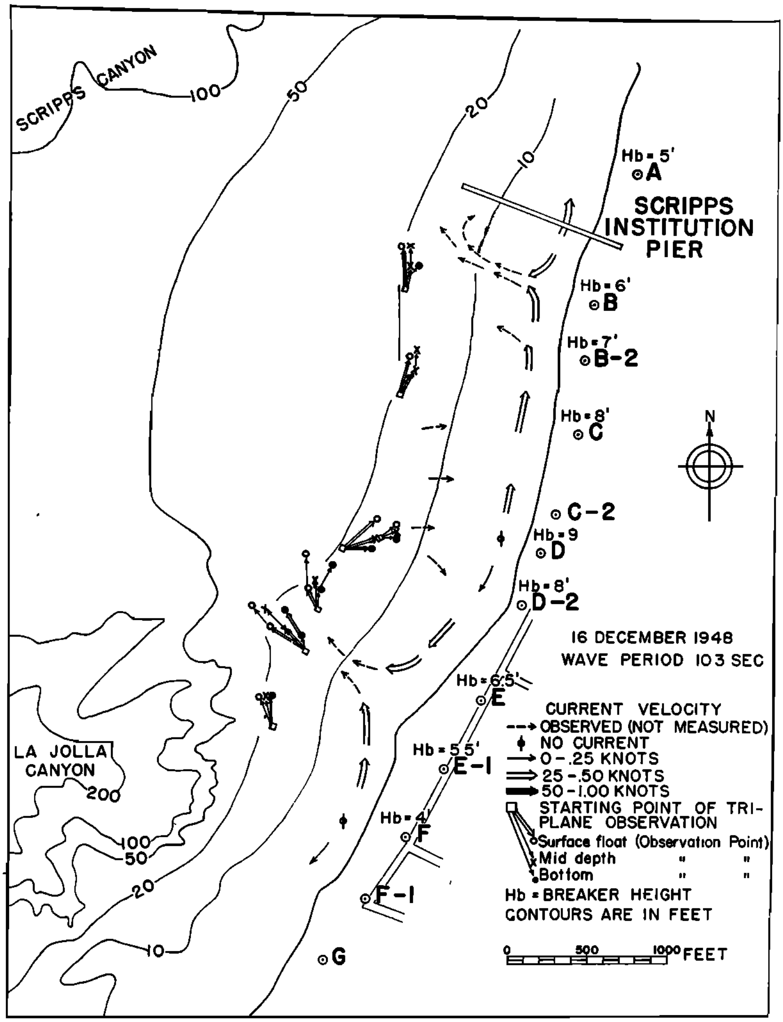

Fig. 1. Typical nearshore circulation pattern at La Jolla, California [from *Shepard and Inman,* 1950].

case, such as that of a wave flume, it was assumed that the pressure head on the beach provided a seaward flow equal to the onshore mass transport due to the waves, thus satisfying the continuity condition [*Munk,* 1949]. *Longuet-Higgins* [1953] showed theoretically and *Russell and Osorio* [1958] showed experimentally that, in this case, there would be an onshore drift at the surface and bottom, balanced by an offshore flow at intermediate depths. However, measurements in the ocean off La Jolla [*Inman and Quinn,* 1952; *Inman and Nasu,* 1956] showed no net offshore transport of water through the breakers at intermediate depths. The mass transport was found to be onshore at all depths except in the rip currents, where it was offshore at all depths. Thus, although a two-dimensional vertical circulation pattern in the ocean is theoretically possible, all the observations suggest that a horizontal circulation pattern is usually dominant.

Few quantitative estimates of this horizontal circulation were made because of the difficulty of evaluating the onshore mass transport near the breakers as a function of the wave parameters. This difficulty was aggravated by the extensive use of solitary wave theory, with its very artificial transport velocities, to describe the wave conditions near the breakers [e.g., *Putnam et al.*, 1949].

The changes in mean sea level due to the incoming waves have recently been the object of considerable theoretical interest. By considering the continuity of momentum flux in a normally incident wave train, rather than the continuity of mass, *Longuet-Higgins and Stewart* [1960, 1962, 1963, 1964], *Whitham* [1962], and *Lundgren* [1963] derived theoretical expressions for these changes in sea level. They predicted a lowering of the water level (set-down) as the waves approach the break point and a steady rise in sea level (set-up) shoreward of the breakers.

Longuet-Higgins and Stewart introduced the concept of a radiation stress to describe some of the nonlinear properties of surface gravity waves, the radiation stress being defined as the excess flow of momentum due to the presence of waves [*Longuet-Higgins and Stewart,* 1964]. The present study uses this concept to investigate how the wave field interacts with longshore variations in the nearshore region to produce circulation patterns. These longshore variations may be produced by longshore changes in either the bottom topography or the wave field, or they may be caused by the presence of edge waves, which are normal modes of longshore oscillation of the nearshore region. In each case, the longshore perturbations close to the shore should lead to the existence of rip currents.

Set-Up and Set-Down

Longuet-Higgins and Stewart [1964] described several of the theoretical second-order effects of surface gravity waves in terms of a radiation stress. One of the phenomena discussed was the change in mean sea level $\bar{\eta}$ that occurs when water waves encounter a sloping beach.

If the velocity potential of the incoming wave is given locally by

$$\phi = \frac{H\sigma}{2k} \cdot \frac{\cosh [k(z + h)]}{\sinh kh} \cos (kx + \sigma t) \qquad (1)$$

where H is the wave height, h is the still water depth, $k = 2\pi/L$ is the wave number, L is the wavelength, $\sigma = 2\pi/T$ is the radian frequency, and x and z are the horizontal and vertical coordinates, then the x component of the radiation stress is given by

$$S_{xx} = E\left(\frac{2kh}{\sinh 2kh} + \frac{1}{2}\right) \qquad E = \frac{1}{8}\rho g H^2 \quad (2)$$

where E is the wave energy per unit surface area. In shallow water, $kh \rightarrow 0$, and this case

$$S_{xx} = \tfrac{3}{2}E = \tfrac{3}{16}\,\rho g H^2 \qquad (3)$$

In the steady state, the shoreward flux of momentum must be independent of x, the coordinate perpendicular to the shore. Momentum balance then gives

$$\frac{dS_{xx}}{dx} + \rho g(\bar{\eta} + h)\frac{d\bar{\eta}}{dx} = 0 \qquad (4)$$

If the beach slope is sufficiently small and the wave reflection is negligible, two distinct regions can be considered, one seaward and one shoreward of the break point.

Seaward of the breakers, wave energy is approximately conserved,

$$ECn = \text{constant} \qquad (5)$$

where Cn is the group velocity of the wave train. *Longuet-Higgins and Stewart* [1962] showed that by using (5) equation 4 could be integrated to give

$$\bar{\eta} = -\frac{1}{8}\frac{H^2k}{\sinh 2kh} \qquad (6)$$

where $\bar{\eta}$, the difference between the still water level and the mean sea level in the presence of waves, is always negative seaward of the break point. By means of (6), $\bar{\eta}$ can be expressed as a function of the local depth, h, and the deep water height H_0 and wave number k_0:

$$\bar{\eta} = -\frac{1}{8}H_0^{\,2}k_0\frac{\coth^2 kh}{2kh + \sinh 2kh} \qquad (7)$$

and since the dispersion relation for the waves is given by

$$kh \tanh kh = (\sigma^2 h/g) = k_0 h \qquad (8)$$

then

$$\bar{\eta} = -\tfrac{1}{4}H_0^{\,2}\,k_0\,f(k_0h) \qquad (9)$$

where f, a function of only the nondimensional depth k_0h, is shown in Figure 2. As the depth decreases, the mean water level is lowered by the presence of unbroken waves; there is a set-down because the radiation stress increases steadily when no energy is dissipated.

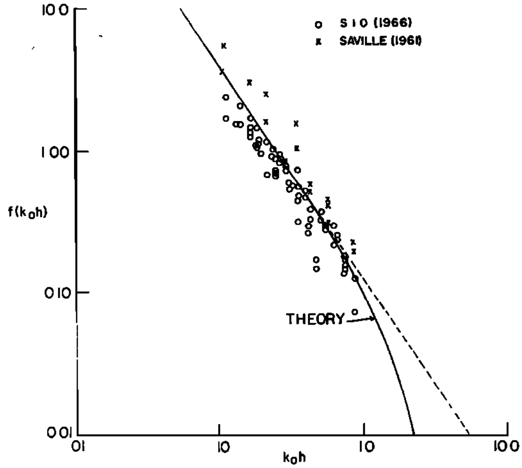

Fig. 2. Set-down outside the break point, expressed as a function of the nondimensional depth, k_0h.

Inside the break point the wave energy decreases shoreward, which leads to a decrease in the radiation stress. Using similarity arguments, we can assume that the height of the broken wave, or bore, remains an approximately constant proportion of the mean water depth

$$H = \gamma(\bar{\eta} + h) \tag{10}$$

Although the waves are now far too steep for the second-order theory to remain valid, it is not unreasonable to assume that $S_{xx} = \frac{3}{2}E$ [*Longuet-Higgins and Stewart,* 1964]. This gives

$$S_{xx} = \tfrac{3}{16}\,\rho g \gamma^2(\bar{\eta} + h)^2 \tag{11}$$

then, from (4) and (11), the gradient of the set-up is given by

$$\frac{d\bar{\eta}}{dx} = -K\frac{dh}{dx} \qquad K = \left[1 + \frac{8}{3\gamma^2}\right]^{-1} \tag{12}$$

Thus, for a plane beach where $h = x \tan \beta$, the slope of the mean sea level should be constant and proportional to the beach slope, $\tan \beta$.

The laboratory measurements made by *Saville* [1961] were found to be in reasonable agreement with the theoretical predictions, and more detailed laboratory measurements [*Bowen et al.,* 1968] show that the theory predicts both the set-down outside the surf zone and the set-up inside the surf zone remarkably well. The agreement between experiment and theory for the set-down outside the surf zone is shown by Figure 2, where the experimental values of $f(k_0h)$, given from (9) by $-4\bar{\eta}/H_0^2k_0$ are plotted against k_0h, the nondimensional depth. The measurements that lie below the theoretical curve tend to be those made near the break point, where the set-down was found to be consistently less than the theory predicts. The profile of mean sea level for a typical experiment is shown in Figure 3.

Equation 12 seems to provide a good qualitative description of the set-up. The quantitative nature of the agreement was examined by plotting K, the ratio of the set-up slope to the beach slope, against $\bar{\gamma}$, the mean of the observed values of γ across the surf zone.

$$K = -\frac{d\bar{\eta}}{dx}\left[\frac{dh}{dx}\right]^{-1} = -\frac{1}{\tan\beta}\frac{d\bar{\eta}}{dx} \tag{13}$$

From (13), the experimental value of K was taken as

$$(1/\tan\beta)\ [(\bar{\eta}_{max} - \bar{\eta}_b)/\bar{x}_b]$$

$\bar{\eta}_b$ is the set-down at the breakers, $\bar{\eta}_{max}$ is the maximum set-up on the beach, and $\bar{x}_b$ is the width of the surf zone from break point to mean run-up (Figure 4). The theoretical curve is given from equation 12 as

$$K = \left[1 + \frac{8}{3\gamma^2}\right]^{-1}$$

In view of the fact that this analysis includes the regions of plunge and rebound, where conditions are generally complex, the agreement between the measured and the theoretical values is quite good.

The basis for this analysis of set-up is the similarity argument used to obtain equation 10; the measurements of *Bowen et al.* [1968] show that, once the bore is well established, this assumption is reasonable.

The radiation stress has been defined as the excess flow of momentum due to the presence of waves, and it can be regarded as the stress exerted on the water by the wave field. In the two-dimensional case this stress induces changes in the mean water level, creating steady pressure gradients that balance the gradient of the radiation stress, (4). The general applicability of this idea is shown by the good agreement between the values of the set-down and set-up predicted by the theory and the values measured in laboratory experiments.

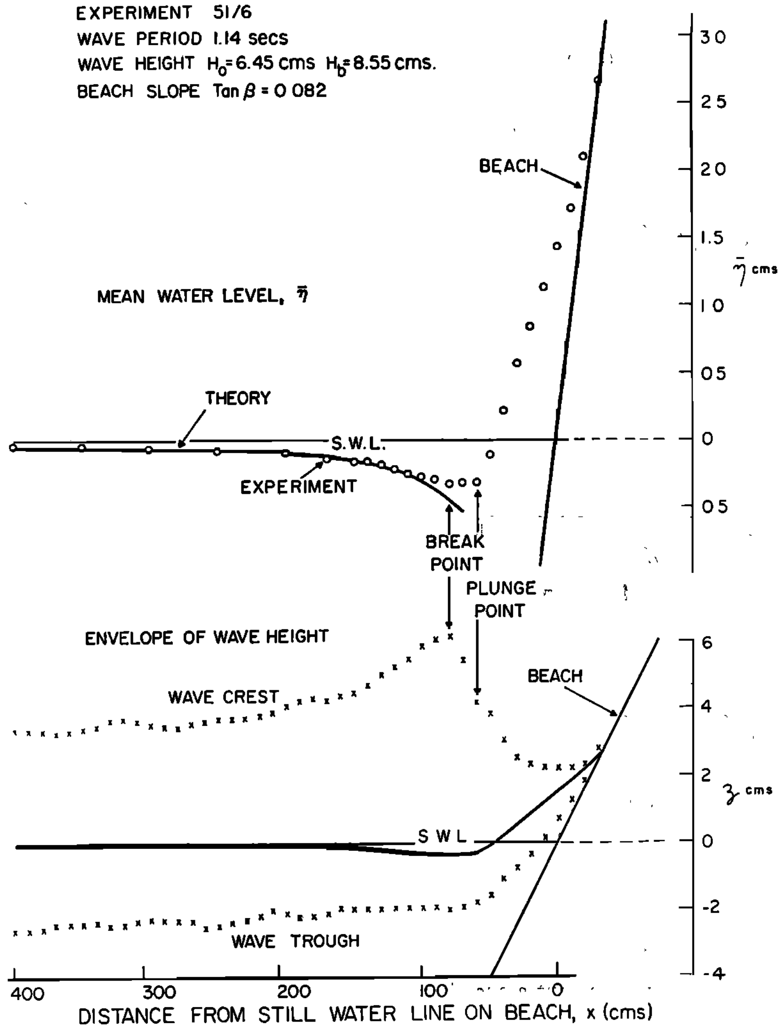

Fig. 3. The profile of the mean water level and the wave height for a typical experiment.

When these ideas are extended to the three-dimensional situation, a question arises as to whether a steady distribution of radiation stress can always be balanced by a pressure field or whether an imbalance can occur that might lead to flow patterns. *Longuet-Higgins and Stewart* [1962] showed that a second-order interaction between two sets of waves in a conservative field produces an effect that can be expressed by a boundary condition at the free surface of the water, the result of the interaction being expressed mathematically as a fluctuating pressure field at the free surface. Now the radiation stress is the component of the second-order self-interaction of a wave train that is steady in time. It should therefore lead to changes in the pressure field at the sea surface that are also stationary with time. Then, outside the surf zone, the only effect produced by the radiation stress should be an alternation in the configuration of the sea surface.

To discuss the application of this idea to a realistic situation, it is convenient to consider the case in which the wave height close to the break point varies along the beach. If the waves approach normal to a plane beach, then two profiles normal to the beach, some distance apart, have been illustrated in Figure 5 by the

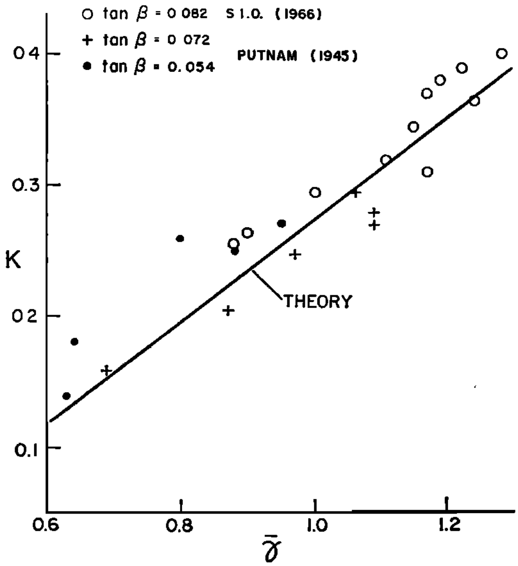

Fig. 4. K as a function of $\bar{\gamma}$, showing the good agreement between the theory and the experiments.

data from experiments 51/4 and 51/6 of *Bowen et al.* [1968]; the wave height was greater in experiment 51/6, otherwise the experimental conditions were the same.

Outside the surf zone the set-down due to the x component of the radiation stress, S_{xx}, is given from (6) as

$$\bar{\eta} = -\frac{1}{8}\frac{kH^2}{\sinh 2kh}$$

then, as $k(x)$ and $h(x)$ are constant along shore

$$\frac{\partial\bar{\eta}}{\partial y} = -\frac{1}{4}\frac{kH}{\sinh 2kh}\cdot\frac{\partial H}{\partial y} \tag{14}$$

where y is the longshore coordinate. There is a pressure field tending to accelerate water from the region of low waves toward the region of higher waves resulting from the greater set-down under the large waves. However, there is a y component of the radiation stress S_{yy} [*Longuet-Higgins and Stewart,* 1964], where

$$S_{yy} = \frac{1}{8}\rho g H^2\left[\frac{kh}{\sinh 2kh}\right] \tag{15}$$

Now the expression for the y component of momentum flux, analogous to (4), is

$$\frac{\partial F_y}{\partial y} = \frac{\partial\bar{\eta}}{\partial y} + \frac{1}{\rho g(\bar{\eta}+h)}\frac{\partial S_{yy}}{\partial y} \tag{16}$$

outside the surf zone $\bar{\eta} \ll h$; then from (15)

$$\frac{\partial F_y}{\partial y} = \frac{\partial\bar{\eta}}{\partial y} + \frac{1}{4}\frac{kH}{\sinh 2kh}\frac{\partial H}{\partial y} \tag{17}$$

Then using (14), we have $\partial F_y/\partial_y = 0$. To the present order of the calculations, the gradient of the radiation stress is balanced by the induced pressure field and there are no net forces outside the surf zone that might produce circulation patterns.

As the waves move into shallow water, they break when the ratio of wave height to water depth reaches some critical value, γ. Consequently, the larger wave breaks in deeper water, and the set-up therefore begins further seaward than it does for the smaller wave. Then, although the gradients of the set-up are approximately equal, the actual set-up associated with the high waves is everywhere considerably greater than the set-up due to the lower waves. In addition, the wave height of 51/6 continues to be generally larger than that of 51/4 after breaking (Figure 5). In the shallow water of the surf zone, we have from (15)

$$S_{yy} = (1/16)\rho g H^2$$

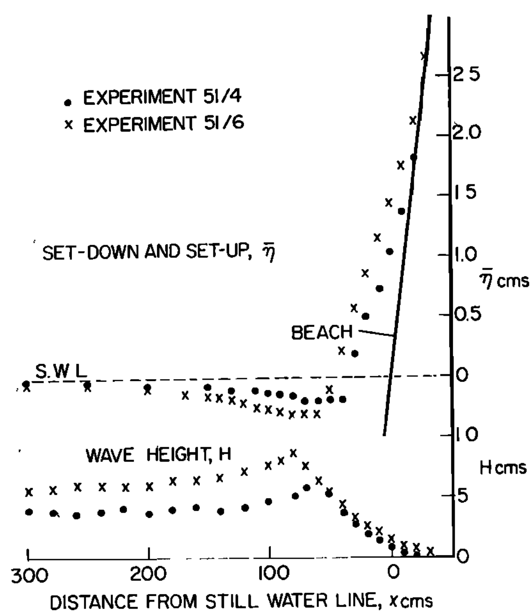

Fig. 5. Profiles of mean water level and wave height for experiments 51/4 and 51/6 [*Bowen et al.,* 1968], showing the effect of a difference in wave height.

and from (16)

$$\frac{\partial F_y}{\partial y} = \frac{\partial \bar{\eta}}{\partial y} + \frac{1}{\rho g(\bar{\eta} + h)} \frac{\partial S_{yy}}{\partial y}$$

In this region from (10) $H = \gamma(\bar{\eta} + h)$

$$\therefore \frac{\partial F_y}{\partial y} = \frac{\partial \bar{\eta}}{\partial y} + \frac{1}{8}\gamma^2 \frac{\partial \bar{\eta}}{\partial y} \tag{18}$$

The pressure field induced by the x component of the radiation stress is not in longshore equilibrium with the y component of the radiation stress as it was outside the surf zone. The gradients of both stresses act in the same direction and must therefore produce a flow of water in the surf zone away from the region of high waves toward the region of low waves. Field observations clearly illustrate this movement in the surf zone away from regions of high waves.

In the surf zone, where the dissipation of wave energy takes place, the forces are not conservative and there is no requirement for the conservation of vorticity. It therefore appears that the surf zone is the region in which interactions can occur that may provide forcing functions in the equations for the horizontal motion. The wave induced flow patterns in the nearshore region are driven from inside the surf zone; the flow outside the surf zone is not driven, its configuration being determined by the boundary conditions imposed at the line of the breakers. One might therefore expect the most intense flows to occur in the surf zone, and observations show that longshore currents and the longshore current components of nearshore circulation cells are essentially confined to the surf zone.

Equations of Motion

To formulate a tractable flow problem, it is necessary to make some rather general assumptions about the flow conditions in and near the surf zone. Following *Arthur* [1962] it is assumed that:

1. The currents are steady; there are no time dependent terms when the forcing function is itself steady.
2. The velocity is independent of depth, $u(x, y)$ and $v(x, y)$.
3. The water is homogeneous and incompressible; density ρ is constant.
4. The pressure is hydrostatic, $p = \rho g(\bar{\eta} - z)$.
5. The Coriolis force can be neglected.
6. The currents are sufficiently small, so that their interactions with the waves are negligible.

Observations in the laboratory suggest that statements 1 and 5 are good assumptions; statements 3 and 4 are certainly reasonable assumptions in the surf zone. Assumption 2 is necessary for a simple mathematical formulation of the nonlinear terms in the equation; assumption 6 is a necessary first step in the solution of the equations.

The equations of motion and continuity are now similar to the shallow-water equations of *Stoker* [1957]:

$$\frac{u\partial u}{\partial x} + \frac{v\partial u}{\partial y} = -g\frac{\partial \bar{\eta}}{\partial x} + R_x + \tau_x \tag{19}$$

$$\frac{u\partial v}{\partial x} + \frac{v\partial v}{\partial y} = -g\frac{\partial \bar{\eta}}{\partial y} + R_y + \tau_y \tag{20}$$

$$\frac{\partial}{\partial x}\left[u(\bar{\eta} + h)\right] + \frac{\partial}{\partial y}\left[v(\bar{\eta} + h)\right] = 0 \tag{21}$$

where R_x and R_y are frictional terms arising from either a horizontal eddy viscosity, A_H, in which case

$$\begin{aligned} R_x &= A_H\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ R_y &= A_H\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \end{aligned} \tag{22}$$

or from a linearized bottom friction, c, where

$$R_x = -\frac{cu}{\bar{\eta} + h} \qquad R_y = -\frac{cv}{\bar{\eta} + h} \tag{23}$$

τ_x and τ_y are derived from the radiation stress tensor [*Longuet-Higgins and Stewart,* 1964] where

$$\begin{aligned} \tau_x &= -\frac{1}{\rho(\bar{\eta} + h)}\left(\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}\right) \\ \tau_y &= -\frac{1}{\rho(\bar{\eta} + h)}\left(\frac{\partial S_{yy}}{\partial y} + \frac{\partial S_{yx}}{\partial x}\right) \end{aligned} \tag{24}$$

and

$$S_{xy} = S_{yx} = \int_{-h}^{\eta} \overline{uv}\, dz = 0$$

In the absence of flow, $R_x = R_y = 0$, and (19) and (20) reduce to (4) and (16) as expected.

Introduce a vorticity ξ, defined by

$$\xi = (\partial v/\partial x) - (\partial u/\partial y) \tag{25}$$

and a transport stream function [*Arthur*, 1962], where

$$\begin{aligned} u(\bar\eta + h) &= -\partial\psi/\partial y \\ v(\bar\eta + h) &= \partial\psi/\partial x \end{aligned} \tag{26}$$

Let $h + \bar\eta = d$, then, from (25) and (26)

$$\xi = \frac{1}{d}\left[\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right] - \frac{1}{d^2}\left[\frac{\partial\psi}{\partial y}\frac{\partial d}{\partial y} + \frac{\partial\psi}{\partial x}\frac{\partial d}{\partial x}\right] \tag{27}$$

Cross-differentiating equations 19 and 20 and subtracting, we have

$$\underbrace{\frac{\partial\psi}{\partial y}\cdot\frac{\partial}{\partial x}\left[\frac{\xi}{d}\right] - \frac{\partial\psi}{\partial x}\cdot\frac{\partial}{\partial y}\left[\frac{\xi}{d}\right]}_{\text{(I)}} = \underbrace{\frac{\partial R_x}{\partial y} - \frac{\partial R_y}{\partial x}}_{\text{(II)}} + \underbrace{\frac{\partial \tau_x}{\partial y} - \frac{\partial \tau_y}{\partial x}}_{\text{(III)}} \tag{28}$$

Equation 28 has three components arising from (I) the nonlinear terms, (II) the frictional terms, and (III) the forcing terms. *Arthur* [1962], considering only the nonlinear terms, showed that in the absence of friction the quantity ξ/d, which is similar to the potential vorticity [*Rossby*, 1940], should be conserved along a streamline. If a current moves from shallow into deeper water, the streamlines tend to move closer together producing a fast, narrow current. The nonlinear terms in the full equation will act in a similar way tending to broaden an onshore flow and to concentrate an offshore flow, producing idealized rip currents. The nonlinear problem is not analytically tractable, however, and must be solved numerically. It is therefore convenient first to consider a linear problem that can be treated analytically.

Forcing terms. Outside the surf zone $\partial\tau_x/\partial_y - \partial\tau_y/\partial_x = 0$, the motion is not driven. In general $\bar\eta \ll h$, so that $d \simeq h = x \tan\beta$. Inside the breakers the forcing term is given by equation 24.

$$\begin{aligned} \frac{\partial \tau_x}{\partial y} - \frac{\partial \tau_y}{\partial x} &= -\frac{\partial}{\partial y}\left[\frac{1}{\rho d}\frac{\partial S_{xx}}{\partial x}\right] + \frac{\partial}{\partial x}\left[\frac{1}{\rho d}\frac{\partial S_{yy}}{\partial y}\right] \\ &= -\frac{\partial}{\partial y}\left[\frac{3}{8}\frac{qH}{d}\frac{\partial H}{\partial x}\right] + \frac{\partial}{\partial x}\left[\frac{1}{8}\frac{qH}{d}\frac{\partial H}{\partial y}\right] \end{aligned}$$

because the water is shallow. If the flow is sufficiently slow that changes in $\bar\eta$ due to the flow can be neglected in comparison with $\bar\eta + h$, then equation 10 may be retained, $H = \gamma(\bar\eta + h) = \gamma d$; the forcing function then becomes

$$\frac{\partial \tau_x}{\partial y} - \frac{\partial \tau_y}{\partial x} = -\frac{1}{4} g\gamma \frac{\partial^2 H}{\partial x\,\partial y} \tag{29}$$

Now $d = \bar\eta + h = 0$ at the point of maximum set-up, $x = -x_s$, if a new coordinate $\bar x$ is defined with its origin at the mean point of maximum set-up, then

$$\bar x = x + x_s$$

and $\bar\eta + h = m\bar x$, where $m = (1 - K)\tan\beta$. Then in the two-dimensional case

$$H = \gamma m \bar x \tag{30}$$

If there is a steady longshore perturbation in the wave height, H will be given by

$$H = \gamma m\bar x(1 + \epsilon\cos\lambda y) \qquad \epsilon \ll 1 \tag{31}$$

where $\lambda = 2\pi/(\text{longshore wavelength})$. Equation 29 then leads to a forcing function of the form

$$\frac{\partial \tau_x}{\partial y} - \frac{\partial \tau_y}{\partial x} = +\frac{1}{4} g\gamma^2\, m\epsilon\lambda \sin\lambda y \tag{32}$$

and the depth is given by $d = m\bar x(1 + \epsilon\cos\lambda y)$. If $\epsilon \ll 1$, the y variation of d contributes only a second-order term, so that a reasonable approximation is

$$d = m\bar x \tag{33}$$

Solutions using bottom friction. If the nonlinear terms are omitted from (28), the friction terms calculated from (23) and the forcing terms from (32), then

$$c\left[\frac{\xi}{d} - \frac{mv}{d^2}\right] = B\sin\lambda y$$

where $B = -\,\frac{1}{4}\, g\gamma^2 m\epsilon\lambda$ inside the surf zone and $B = 0$ outside the surf zone.

In terms of the mass transport stream function ψ, this becomes

$$\frac{1}{d^2}\left[\frac{\partial^2\psi}{\partial\bar{x}^2}+\frac{\partial^2\psi}{\partial y^2}\right]-\frac{2}{d^3}\frac{m\partial\psi}{\partial\bar{x}}=\frac{B}{c}\sin\lambda y \quad (34)$$

The solution inside the surf zone, subject to the boundary condition $\psi = 0$ at $\bar{x} = 0$, is

$$\psi(\bar{x}, y) = \sin\lambda y\left\{P(\lambda\bar{x}\cosh\lambda\bar{x}-\sinh\lambda\bar{x}) + \frac{Bm^2}{c\lambda^4}[2-(\lambda\bar{x})^2+2\lambda\bar{x}\cdot\sinh\lambda\bar{x}-2\cosh\lambda\bar{x}]\right\} \quad (35)$$

then

$$\frac{\partial\psi}{\partial x} = \sin\lambda y\,[P\lambda^2\bar{x}\sinh\lambda\bar{x} + \frac{Bm^2}{c\lambda^4}2\lambda^2\bar{x}(\cosh\lambda\bar{x}-1)]$$

therefore, at $\bar{x} = 0$, the longshore component of the velocity v is given by

$$v(0, y) = \left[\frac{1}{m\bar{x}}\frac{\partial\psi}{\partial\bar{x}}\right]_{\bar{x}=0} = 0$$

and both components of the velocity are therefore zero at the shoreline, the line at which the water depth d goes to zero.

The free solution outside the surf zone must be patched to the forced solution at the breakers and must therefore have the same longshore variation. The solution subject to the boundary condition that ψ must remain bounded as $x \to \infty$ is given by

$$\psi(x, y) = Q(\lambda x + 1)e^{-\lambda x}\sin\lambda y \quad (36)$$

then

$$\frac{\partial\psi}{\partial x} = -Q\lambda^2 x e^{-\lambda x}\sin\lambda y$$

The longshore component of velocity is given by

$$v(x, y) = \frac{1}{x\tan\beta}\frac{\partial\psi}{\partial x} = -\frac{Q\lambda^2}{\tan\beta}e^{-\lambda x}\sin\lambda y$$

so that, at any given value of y, the longshore velocity outside the surf zone is unidirectional and decays monotonically seaward. The continuity condition requires the net offshore flow to be away from the rip current, the flow toward the rip current is therefore entirely confined to the surf zone.

The constants P and Q are determined by the condition that ψ and $\partial\psi/\partial x$ must be continuous at the breaker line, the position of the break being given by

$$\bar{x} = \bar{x}_b \qquad x = x_b = \bar{x}_b - x_s$$

A typical solution is shown in Figure 6, where $Bm^2/c\lambda^4 = -1.6$, $\lambda\bar{x}_b = \pi/2$, and $\lambda x_b = 2\pi/5$. The position of maximum set-up is then $\lambda x = -\lambda x_s = -\pi/10$. The flow in the surf zone is from the region of high waves, $\lambda y = 0$, toward the region of low waves, the rip current occurring at $\lambda y = \pi$ where the waves are lowest.

Solutions using eddy viscosity. If the frictional terms are assumed to arise from eddy viscosity (equation 22), a linear equation may be obtained equivalent to (34). In this case, however, the expression is fourth order instead of second and consequently there are more constants to be evaluated. As numerical methods must be used to examine the effects of the nonlinear terms, it is convenient to treat the linear problem at the same time.

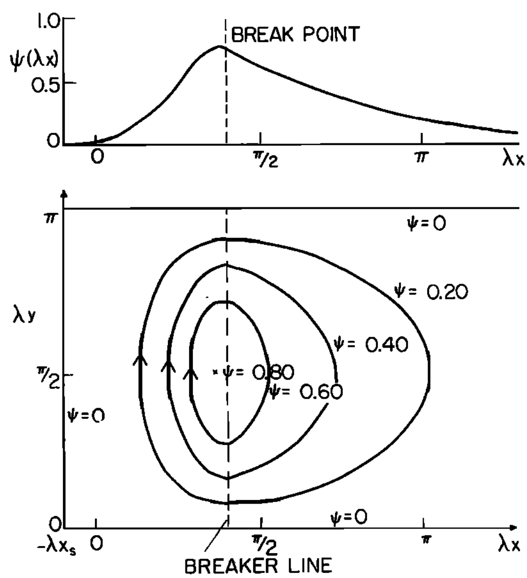

Fig. 6. A linear solution using bottom friction. The offshore dependence of the transport stream function ψ (λx), where $\psi = 0$ at the position of maximum set-up, is shown above the full solution, ψ $(\lambda x) \cdot \sin\lambda y$.

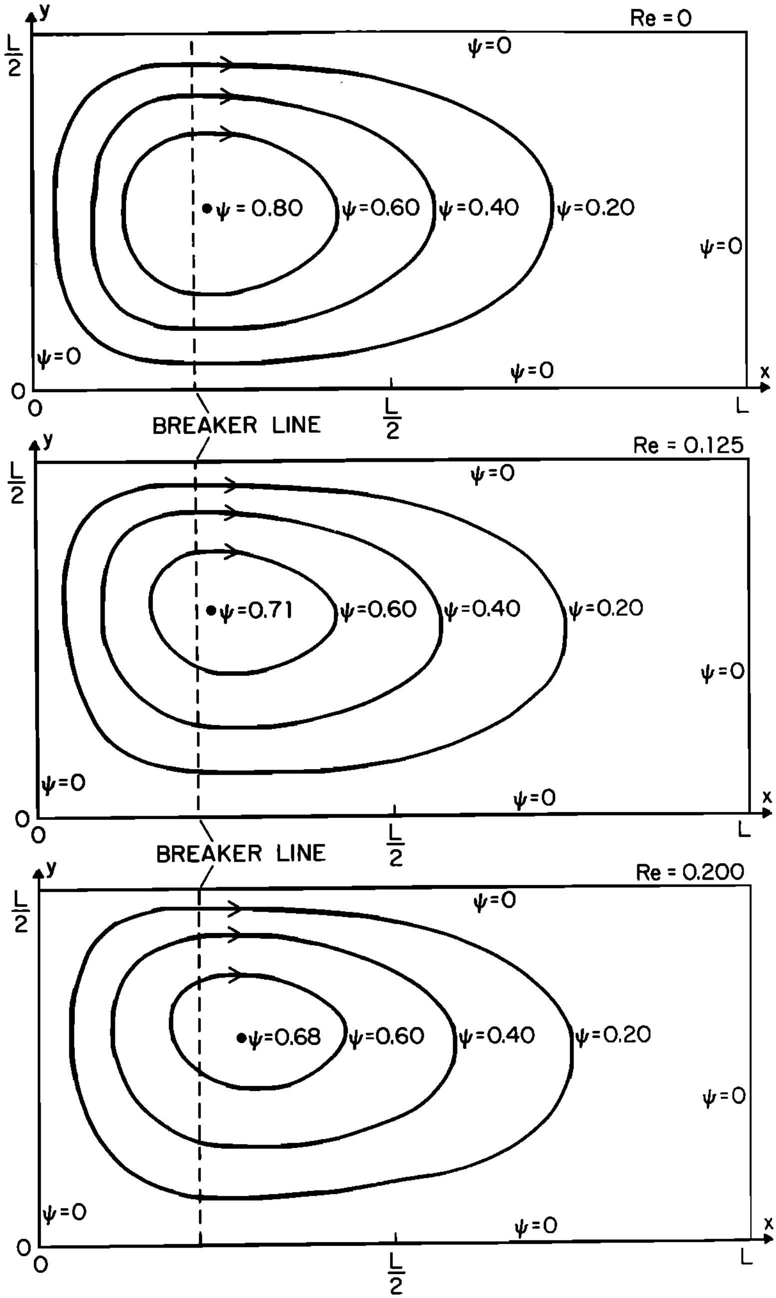

Fig. 7. Solutions using eddy viscosity.

The complete nonlinear equation can be derived from equations 22 and 28, where

$$\frac{\partial \psi}{\partial y} \cdot \frac{\partial}{\partial x}\left[\frac{\xi}{d}\right] - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\left[\frac{\xi}{d}\right] + A_H\left(\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2}\right) = B \sin \lambda y$$

where $B = -\frac{1}{4}g\gamma^2 m\, \epsilon\, \lambda$ inside the surf zone and $B = 0$ outside the surf zone.

In conjunction with (27), this gives the fourth-order equation in the transport stream function, ψ. If the variables are made nondimensional by introducing a scale velocity U and a scale length L, where $x' = x/L$, $y' = y/L$, $d' = d/L$, $\xi' = \xi L/U$, and $\psi' = \psi/UL^2$, $\lambda = 2\pi/L$, then

$$\frac{U^2}{L^2 B}\left\{\frac{\partial \psi'}{\partial y'}\frac{\partial}{\partial x'}\left[\frac{\xi'}{d'}\right] - \frac{\partial \psi'}{\partial x'}\frac{\partial}{\partial y'}\left[\frac{\xi'}{d'}\right]\right\} + \frac{A_H U}{BL^3}\left[\frac{\partial^2 \xi'}{\partial x'^2} + \frac{\partial^2 \xi'}{\partial y'^2}\right] = \sin 2\pi y'$$

defining the two important parameters, the frictional coefficient $W_f = A_H U/BL^3$ and the effective Reynolds number, the ratio between the inertial and viscous forces, $Re = UL/A_H$. A change in the frictional force at constant Reynolds number changes only the magnitude of the solution. The interesting case is the change in Re at constant W_f.

The method of solution was the successive over relaxation procedure used by *Holland* [1966]. A 21 × 21 square matrix was used for values of x' and y' from 0 to 1. A typical set of solutions is shown in Figure 7. Only half of each solution is shown as they are symmetrical about the line $y' = 0.5$. This expected symmetry proved a useful check on the validity of the solutions. In the example shown, the depth was taken as $d = 0.01 + x' \tan \beta$, where $\tan \beta = 0.10$, and the effect of the set-up was neglected. It was convenient to cut off the solution at some small value of d, because the nonlinear terms cause problems if $d \rightarrow 0$. In consequence, a slippery boundary condition was used at $x' = 0$. The break point was taken to be $x' = 0.23$.

The solutions shown in Figure 7 illustrate the effect of increasing Reynolds number for the case of $W_f = -8 \times 10^{-6}$; B is negative and the offshore current occurs at $\lambda y = 2\pi y/L = \pi$, as before. The effect of increasing the Reynolds number is to skew the solutions in several ways. As expected from *Arthur* [1962], the outflowing current tends to narrow and the onshore current to broaden. The general values of the transport stream function decrease with increasing Re due to an increase in the dissipation caused by the narrowing of the outflowing current. Unfortunately, a method has not been found to make the equations converge satisfactorily for Reynolds number greater than 0.20; a finer grid may be necessary to describe the conditions in the outflowing currents more exactly. As expected, the longshore flow is essentially confined to the surf zone, although the maximum value of $\psi(\lambda x)$ seems to occur just outside the surf zone.

In general, the results obtained from the analysis are qualitatively quite satisfactory. The longshore currents are largely confined to the surf zone and the velocities in the surf are much greater than the velocities outside, as the mass transport streamlines are closer together and the water is shallower. The introduction of nonlinear terms in the equations of motion shows the tendency for the outward flowing current to become narrower with increasing Reynolds number. The theory also showed that the rip currents occur in regions in which the wave height is low in agreement with the field observations.

Quantitatively, the results are less satisfactory. The nature of the frictional effects required to achieve steady conditions have to be assumed rather arbitrarily and the results are not altogether independent of the choice. It is doubtful if assumption 6, that the interaction between the currents and the wave field are negligible, remains valid as the rip current narrows and its velocity increases. In fact, it appears that an interaction of this type may be important in the generation of edge waves in this region.

Acknowledgments. This study was supported by the Coastal Engineering Research Center and the Office of Naval Research under contract with the Scripps Institution of Oceanography, University of California, and by the Institute of Marine Resources, University of California.

References

Arthur, R. S., A note on the dynamics of rip currents, *J. Geophys. Res.*, *67*(7), 2777–2779, 1962.

Bowen, A. J., D. L. Inman, and V. P. Simmons, Wave 'set-down' and set-up, *J. Geophys. Res., 73*(8), 2569–2577, 1968.

Harris, T. F. W., The nearshore circulation of water, *CSIR Symp. S2, South Africa,* 18–30, 1961.

Harris, T. F. W., A qualitative study of the nearshore circulation off a Natal beach with a submerged longshore sand bar, M.Sc. thesis, University of Natal, Durban, South Africa, 1964.

Holland, W. R., Wind driven circulation in an ocean with bottom topography, Ph.D. thesis, University of California, San Diego., 1966.

Inman, D. L., and W. H. Quinn, Currents in the surf zone, *Proc. 2nd Conf. Coastal Eng.,* pp. 24–36, Council on Wave Research, 1952.

Inman, D. L., and N. Nasu, Orbital velocity associated with wave action near the breaker zone, *Beach Erosion Board, U.S. Army Corps Engrs., Tech. Memo 79,* 72 pp., 1956.

Longuet-Higgins, M. S., Mass transport in water waves, *Phil. Trans Roy. Soc., London, A, 245,* 535–581, 1953.

Longuet-Higgins, M. S., and R. W. Stewart, Changes in the form of short gravity waves on long waves and tidal currents, *J. Fluid Mech., 8,* 565–583, 1960.

Longuet-Higgins, M. S., and R. W. Stewart, Radiation stress and mass transport in gravity waves with application to surf beats, *J. Fluid Mech., 13,* 481–504, 1962.

Longuet-Higgins, M. S., and R. W. Stewart, A note on wave set-up, *J. Mar. Res., 21,* 4–10, 1963.

Longuet-Higgins, M. S., and R. W. Stewart, Radiation stress in water waves, a physical discussion with applications, *Deep-Sea Res., 11*(4), 529–563, 1964.

Lundgren, H., Wave thrust and energy level, *Proc. Int. Ass. Hydraul. Res. Congr., London,* 147–151, 1963.

McKenzie, R., Rip current systems, *J. Geol. 66*(2), 103–113, 1958.

Munk, W. H., The solitary wave theory and its application to surf problems, *Ann. N. Y. Acad. Sci., 51*(3), 376–424, 1949.

Putnam, J. A., Preliminary report of model studies on the transition of waves in shallow water, *Univ. Calif., Berkeley, College Eng. Rep. HE-116-106,* 1945.

Putnam, J. A., W. H. Munk, and M. A. Traylor, The prediction of long-shore currents, *Trans. Amer. Geophys. Union, 30*(3), 337–345, 1949.

Rossby, C. G., Planetary flow patterns in the atmosphere, *Quart. J. Roy. Meterol. Soc., 66,* 72 pp., 1940.

Russell, R. C. H., and J. D. C. Osorio, An experimental investigation of drift profiles in a closed channel, *Proc. 6th Conf. Coastal Eng.,* pp. 171–183, Council on Wave Research, 1958.

Saville, T., Experimental determination of wave set-up, *Proc. 2nd Tech. Conf. Hurricanes,* 242–252, 1961.

Shepard, F. P., K. O. Emery, and E. C. LaFond, Rip currents: A process of geological importance, *J. Geol., 49*(4), 337–369, 1941.

Shepard, F. P., and D. L. Inman, Nearshore circulation related to bottom topography and wave refraction, *Trans. Amer. Geophys. Union, 31*(4), 555–565, 1950.

Shepard, F. P., and D. L. Inman, Nearshore circulation, *Proc. 1st Conf. Coastal Eng.,* pp. 50–59, University of California, Council on Wave Research, 1951.

Stoker, J. J., *Water Waves,* Interscience, New York, 1957.

Whitham, G. B., Mass, momentum, and energy flux in water waves, *J. Fluid Mech., 12,* 135–147, 1962.

(Received January 27, 1969;
revised July 3, 1969.)